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RESEARCH NOTE

A note on: An optimal batch size for an imperfect production system with quality assurance and rework

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In Ojha *et al.* 2007 three models are introduced in the context of imperfect production systems. By analysing the proposed models we identify some problems. In particular we focus on six issues of the models developed in Ojha *et al.* and propose some modifications. All corrections are presented in detail and the numerical example of Ojha *et al.* is used to show the influence of the issues addressed in this paper.

Keywords: Imperfect production, Lot sizing, Remanufacturing

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Analysis

In Ojha *et al.*, a static deterministic model with imperfect production is considered. Raw materials are received from a supplier, processed and delivered to the customer. Three scenarios are distinguished. Within the most complex one, multiple lots using a batch of raw material and each production lot is delivered to the customer in multiple instalments. Furthermore, two special cases of this model are considered. In the second and third model all produced items of a lot are delivered in a single instalment. The third model additionally applies a lot-for-lot policy, i.e. the raw material is ordered in quantity Q_r that is required for one production run. In spite of the contribution of Ojha *et al.* understanding imperfect production in the context of inventory-production-delivery systems, some issues addressed in their paper remain unclear. In the following we discuss six issues in more detail.

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2
3
4 Firstly, Ojha *et al.* assume merely that production rate P must be higher than demand rate D_f . However,
5 when dealing with imperfect production, this does not necessarily avoid shortages in the system. Using the
6 same equipment, rework is done directly after production. Since sequential movement is assumed
7 additionally, the sum of production and rework time $(t_1 + t_2)$ must not exceed the cycle time $T = Q_f/D_f$.
8 Recall that production uptime t_1 is defined as Q_f/P and the rework time t_2 is given by $(\beta Q_f)/P$, the
9 aforementioned condition only holds if $P \geq (1 + \beta)D_f$ is valid. Since this is a general condition, it affects all
10 three models developed by Ojha *et al.*

11 Secondly, the price for an item of raw material in Ojha *et al.* is C . In addition, the conversion factor f_0
12 specifies how many finished goods can be made out of one raw material item. It should be noted that C is
13 used as raw material price for a raw material part as well as for the raw material price of a finished product.
14 This is only true for the special case of $f_0 = 1$. Therefore, we have to differentiate between C_r , the price for
15 one unit raw material, and C_f , the effective value of raw material for a finished product. The relation of C_r
16 and C_f is obviously $C_f = C_r/f_0$. In this context, the inventory carrying cost for reworkable items I_{RE} and for
17 finished products I_F are affected.

18 Thirdly, we address the finished goods inventory carrying cost where a production lot is distributed to the
19 customer in multiple instalments. In this case the finished goods inventory carrying cost consists of those
20 costs which are incurred during production and those which are incurred during delivery: $I_F = I_{F,prod} + I_{F,del}$.
21 While $I_{F,prod}$ is calculated correctly, we examine $I_{F,del}$. Following Ojha *et al.* the values of non-defective
22 items and reworked items are $C_f + V$ and $C_f + 2V$, respectively. Unfortunately, this fact is ignored while
23 calculating $I_{F,del}$. The average cost per unit of the finished product is

$$\frac{(C_f + V)(1 - \beta)Q_f + (C_f + 2V)\beta Q_f}{Q_f} = C_f + V(1 + \beta). \quad (1)$$

24 Thus, the correct finished goods inventory carrying costs are

$$I_F = i(C_f + V) \frac{D_f Q_f}{2P} [1 + \beta(1 - \beta)] + i[C_f + V(1 + \beta)] \frac{x}{2} (m - 1); \quad (2)$$

25 or

$$I_F = i(C_f + V) \left(\frac{D_f Q_f}{2P} [1 + \beta(1 - \beta)] + \frac{x}{2} (m - 1) \right) + \frac{\beta i x V}{2} (m - 1).$$

26 Fourthly, usually in static deterministic inventory-models the cost function is formulated in \$ per year.
27 However, Ojha *et al.* specify inspection, late delivery and lost production costs as cycle cost. The
28 corresponding expressions have to be divided by the cycle length T . The correct expressions are
29 $C_I = C_i D_f$; $C_L = (\beta C_i D_f)/P$; $C_{LP} = \beta D_f V$. In consequence, these cost components have no impact on the
30 optimal values of the decision variables.

In the following the issues described above are incorporated into the total cost functions. As a consequence the functions of Q^* , n^* and m^* need to be adjusted slightly.

$$\begin{aligned}
 TC^I(m, n) = & \frac{imxC_f}{2} \left[\frac{D_f}{P} + n - 1 \right] + \frac{D_f}{mx} \left(\frac{A_r}{n} + A_f \right) \\
 & + i(C_f + V) \frac{mxD_f}{2P} [1 + \beta(1 - \beta)] + i(C_f + V(1 + \beta)) \frac{x}{2} (m - 1) \\
 & + VD_f(1 + \beta) + \frac{imx\beta D_f}{2P} (1 + \beta)(C_f + V) + \frac{imx\beta^2 D_f V}{2P}
 \end{aligned} \tag{3}$$

$$m^{I*} = \sqrt{\frac{A_f D_f}{\Delta_1 x}} \tag{4}$$

$$n^{I*} = \sqrt{\frac{2\Delta_1 A_r}{ixA_f C_f}} \tag{5}$$

$$\begin{aligned}
 \Delta_1 = & \frac{ixC_f}{2} \left(\frac{D_f}{P} - 1 \right) + \frac{ixD_f}{2P} (C_f + V) [1 + \beta(1 - \beta)] \\
 & + \frac{ix}{2} (C_f + V(1 + \beta)) + \frac{\beta ix D_f}{2P} (1 + \beta)(C_f + V) + \frac{\beta^2 ix D_f V}{2P}
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 TC^{II}(Q_f, n) = & \frac{iC_f Q_f}{2} \left[\frac{D_f}{P} + n - 1 \right] + \frac{D_f}{Q_f} \left(\frac{A_r}{n} + A_f \right) \\
 & + \frac{iQ_f D_f}{2P} [1 + \beta(1 - \beta)] (C_f + V) + VD_f(1 + \beta) \\
 & + \frac{i\beta Q_f D_f}{2P} (1 + \beta)(C_f + V) + \frac{i\beta^2 Q_f D_f V}{2P} \\
 & + C_i D_f + \frac{\beta C_i D_f}{P} + \beta D_f V
 \end{aligned} \tag{7}$$

$$Q_f^{II*}(n) = \sqrt{\frac{D_f \left(\frac{A_r}{n} + A_f \right)}{\frac{inC_f}{2} + \Delta_2}} \tag{8}$$

$$Q_f^{II*} = \sqrt{\frac{A_f D_f}{\Delta_2}} \quad (9)$$

$$n^{II*} = \sqrt{\frac{2\Delta_2 A_r}{iA_f C_f}} \quad (10)$$

$$\Delta_2 = \frac{iC_f}{2} \left(\frac{D_f}{P} - 1 \right) + i(C_f + V) \frac{D_f}{2P} [1 + \beta(1 - \beta)] + \frac{\beta^2 i D_f V}{2P} + \frac{\beta i D_f}{2P} (1 + \beta)(C_f + V) \quad (11)$$

$$\begin{aligned} TC^III(Q_f) &= \frac{iQ_f D_f C_f}{2P} + \frac{D_f}{Q_f} (A_r + A_f) \\ &+ \frac{iQ_f D_f}{2P} [1 + \beta(1 - \beta)] (C_f + V) + V D_f (1 + \beta) \\ &+ \frac{i\beta Q_f D_f}{2P} (1 + \beta)(C_f + V) + \frac{i\beta^2 Q_f D_f V}{2P} \\ &+ C_i D_f + \frac{\beta C_i D_f}{P} + \beta D_f V \end{aligned} \quad (12)$$

$$Q_f^{III*} = \sqrt{\frac{2D_f}{\Delta_3} (A_r + A_f)} \quad (13)$$

$$\Delta_3 = \frac{iC_f D_f}{P} + i(C_f + V) \frac{D_f}{P} [1 + \beta(1 - \beta)] + \frac{\beta i D_f}{P} (1 + \beta)(C_f + V) + \frac{\beta^2 i D_f V}{P} \quad (14)$$

Fifthly, taking the first derivative of $TC^I(m, n)$ with respect to m we see that the optimal value of m depends on the given n , thus $m^*(n)$. Conversely, the optimal value of n is also a function of m . Therefore, it is not sufficient to verify only the four combinations $TC^I(\lfloor m^* \rfloor, \lfloor n^* \rfloor)$, $TC^I(\lfloor m^* \rfloor, \lceil n^* \rceil)$, $TC^I(\lceil m^* \rceil, \lfloor n^* \rfloor)$, $TC^I(\lceil m^* \rceil, \lceil n^* \rceil)$ to derive the optimal solution. For example, a Branch&Bound-procedure could be applied to obtain the optimal integer values m_{opt} and n_{opt} . The first step is to round m^{I*} and n^{I*} to the nearest integer value and to insert them in (3). The resulting costs C_b represent the first upper bound. Now assume that, based on (3), merely m is pre-specified and we are interested in determining the lowest possible costs for given m -values. Obviously, this requires us to solve the first order condition $\partial(TC^I(m, n))/\partial(n) = 0$ for n :

$$n(m) = \frac{1}{m} \sqrt{\frac{2A_r D_f}{ix^2 C_f}}$$

Substituting of this formula into (3) provides

$$TC(m) = \Delta_1 m + \frac{A_f D_f}{xm} + \Delta_4 \quad (15)$$

with

$$\Delta_4 = -i \left(C_f + V(1 + \beta) \right) \frac{x}{2} + D_f V + \beta D_f V + C_i D_f + \frac{\beta C_l D_f}{P} + \beta D_f V + \sqrt{2i C_f A_r D_f}.$$

Since (3) is convex in m , we are able to establish a lower bound m_l and an upper bound m_u within which the optimal integer value of m is located. To compute both boundary values, we equate the costs C_b with the right-hand side of (15) and solve the resulting expression for m . In doing so, we obtain a quadratic equation which provides the following two integer values:

$$m_l = \max(1; \lceil \alpha_1 - \alpha_2 \rceil) \quad \text{and} \quad m_u = \max(1; \lfloor \alpha_1 + \alpha_2 \rfloor)$$

with

$$\alpha_1 = -\frac{\Delta_4 - C_b}{2\Delta_1} \quad \text{and} \quad \alpha_2 = \sqrt{\left(\frac{\Delta_4 - C_b}{2\Delta_1} \right)^2 - \frac{A_f D_f}{\Delta_1}}.$$

Due to the convexity of TC in n , it is possible to calculate the optimal integer n -value for a given m by $\lfloor n(m) \rfloor$ or $\lceil n(m) \rceil$. For all m -values between the lower and upper bound the optimal n -value is determined by comparing the resulting costs. Each time a new temporary optimal solution is found, we update the bounds m_l and m_u by using cost TC^l as the new C_b -value. This successive tightening of these bounds offers - in contrast to using static bounds - the potential to considerably reduce the solution effort needed to ascertain the cost minimizing solution. After examining all relevant pairs (m, n) , the best solution so far represents the optimal solution for the underlying planning problem.

Sixthly, in the second model of Ojha *et al.* the lot size Q_f and the number n of production lots have to be determined. Obviously, n_{opt} has to be an integer value. Therefore, Ojha *et al.* compare $TC''(Q^*, \lfloor n^* \rfloor)$ and $TC''(Q^*, \lceil n^* \rceil)$ to find n_{opt} . However, it would be favorable to calculate the optimal Q_f for a given value of n and compare the resulting cost afterwards, i.e. $TC''(Q(\lfloor n^* \rfloor), \lfloor n^* \rfloor)$ and $TC''(Q(\lceil n^* \rceil), \lceil n^* \rceil)$.

Numerical example

For better understanding the impact of the issues described above, we use the numerical example of Ojha *et al.* The problem parameters are: $\beta_1 = 0.05$; $E_1 = 0.1$; $E_2 = 0.1$; $A_f = 50$ [\$ per setup]; $A_r = 150$ [\$ per order]; $C_r = 2$ [\$ per unit raw material]; $C_i = 0.2$ [\$ per unit]; $C_l = 0.1$ [\$ per year]; $D_f = 2000$ [units per year]; $f_0 = 0.95$; $i = 0.18$ [per year]; $P = 3000$ [units per year]; $V = 1$ [\$ per unit]; $x = 20$ [units]; $C_f = C_r/f_0 = 2.11$. The procedure to get the optimal solution for model I is shown in the following table.

**** Place table 1 here ****

The procedure proposed by Ojha *et al.* does not consider the combination $m = 22$ and $n = 3$, which is the optimal solution.

The results using the formulation presented in this paper and those of Ojha *et al.* are given in table 2. Whereas a plus sign is used to indicate the results obtained in this note, a hash marks the results of Ojha *et al.* It should be noted that $TC^\#$ is calculated by inserting the production policy derived in Ojha *et al.* into the cost functions derived in this note to get an impression of the induced cost deviations for the given example.

**** Place table 2 here ****

Due to the corrected total cost function, the optimal expressions of the decision variables m , n and Q_f show significant differences. Finally, it is important to point out that the main idea and contribution of the paper of Ojha *et al.* have a great value.

References

Ojha, D., Sarker, B.R., and Biswas, P., 2007. An optimal batch size for an imperfect production system with quality assurance and rework. *International Journal of Production Research*, 45 (14), 3191-3214.

Table 1. Iterations of Branch&Bound for model I.

iteration	m	n	$TC^l(m,n)$	C_b	range of m	current $m_{opt}; n_{opt}$
start	23	3	3865.97	3865.97	$21 \leq m \leq 25$	23; 3
1	21	2	3905.37	3865.97	$21 \leq m \leq 25$	23; 3
2	21	3	3865.90	3865.90	$21 \leq m \leq 25$	21; 3
3	22	2	3895.26	3865.90	$21 \leq m \leq 25$	21; 3
4	22	3	3864.99	3864.99	$22 \leq m \leq 24$	22; 3
5	23	2	3887.51	3864.99	$22 \leq m \leq 24$	22; 3
6	23	3	3865.97	3864.99	$22 \leq m \leq 24$	22; 3
7	24	2	3881.81	3864.99	$22 \leq m \leq 24$	22; 3
8	24	3	3868.59	3864.99	$22 \leq m \leq 24$	22; 3

Table 2 Comparison of results.

	Q_f^+	$Q_f^\#$	n^+	$n^\#$	m^+	$m^\#$	TC^+	$TC^\#$
I	440	340	3	4	22	17	3864.99	3885.87
II	671	443	2	3	-	-	3705.29	3741.48
III	1045	757	-	-	-	-	3725.23	3765.45