



Assembling coordinate free representations for the calculation of geometric variations

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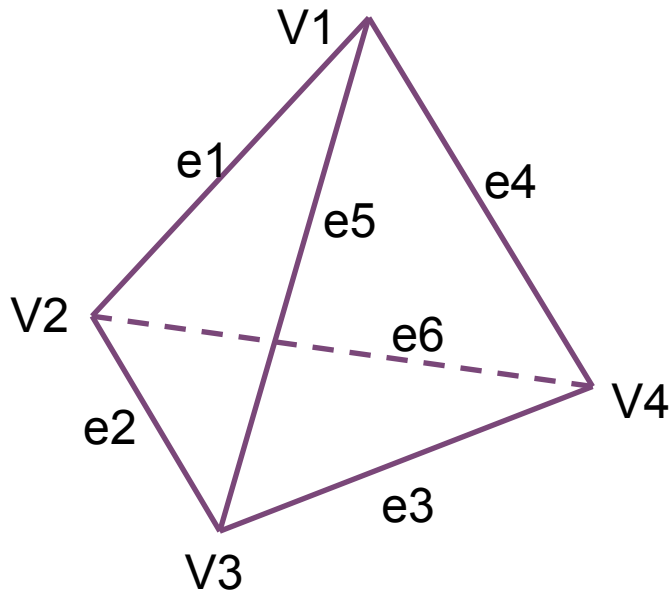
+ Introduction

- Parts within mechanisms are generally specified for the assembly stage of their life cycle;
- Functional Requirement are translated as Geometrical Requirements;
- Useful values of Geometrical Requirements are usually defined under operating conditions (at higher temperature and strains...).
- Challenge : How to study Geometrical Requirement evolution during the product life cycle ?

+ Context

- Computation of the geometrical requirements along the product life cycle at the early phases of product design.
- Use of simplified 3D representations of the product.
- Use of a coordinate free approach to represent geometries.

+ Coordinate Free : Geometrical Model



C	V1	V2	V3	V4
e1	-1	1	0	0
e2	0	-1	1	0
e3	0	0	-1	1
e4	1	0	0	-1
e5	-1	0	1	0
e6	0	-1	0	1

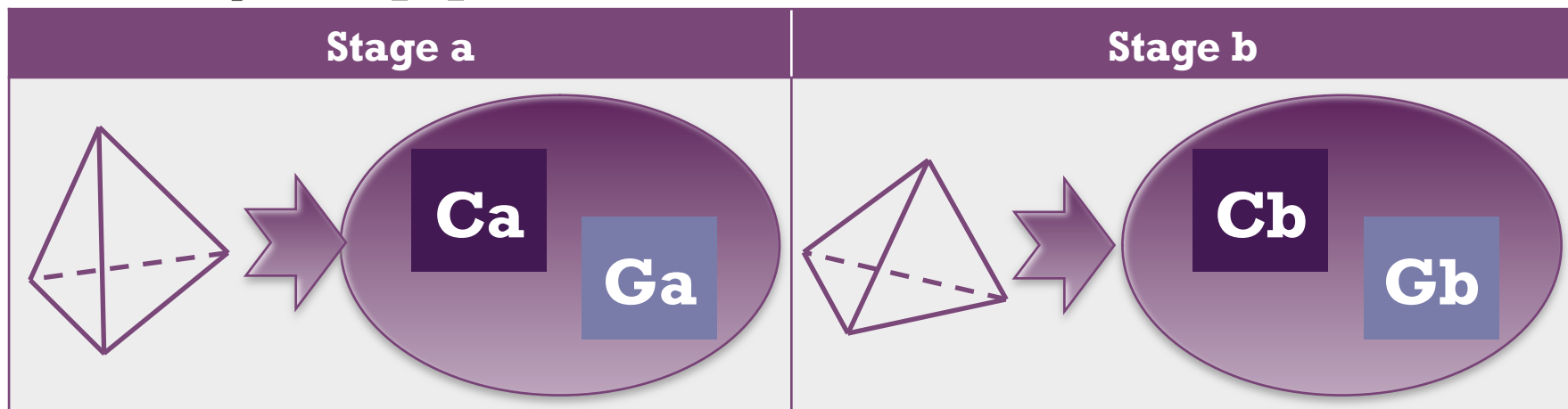
G	e1	e2	e3	e4	e5	e6
e1
e2	
e3		
e4		Sym	
e5				
e6						...

Each object is represented by 2 elements :

- Topology : Connection between vertices and edges (**C**)
- Geometry : Orientation and length of edges (**G**)

+ Hypothesis

- There exists a geometrical (coordinate free) representation of the product at two stages of its life-cycle. [1]



- The topology of the product is to remain unchanged : $\mathbf{Ca} = \mathbf{Cb}$.

+ Objectives

- Association of 2 objects in a single and common representation
- Calculation of the evolution of a geometrical requirement (or parameter) between the 2 objects that have been associated before.

+ Why a coordinate free approach ?

- Ability to specify intrinsic geometric properties of a part or component into a compact mathematical model;
- Availability of a coordinate free geometrical solver at LISMMA based on Gram matrices;
- Declaration of the association of two configurations of a single part that do not exist at the same time (hidden with a Cartesian approach).

+ Tools : Gram matrices

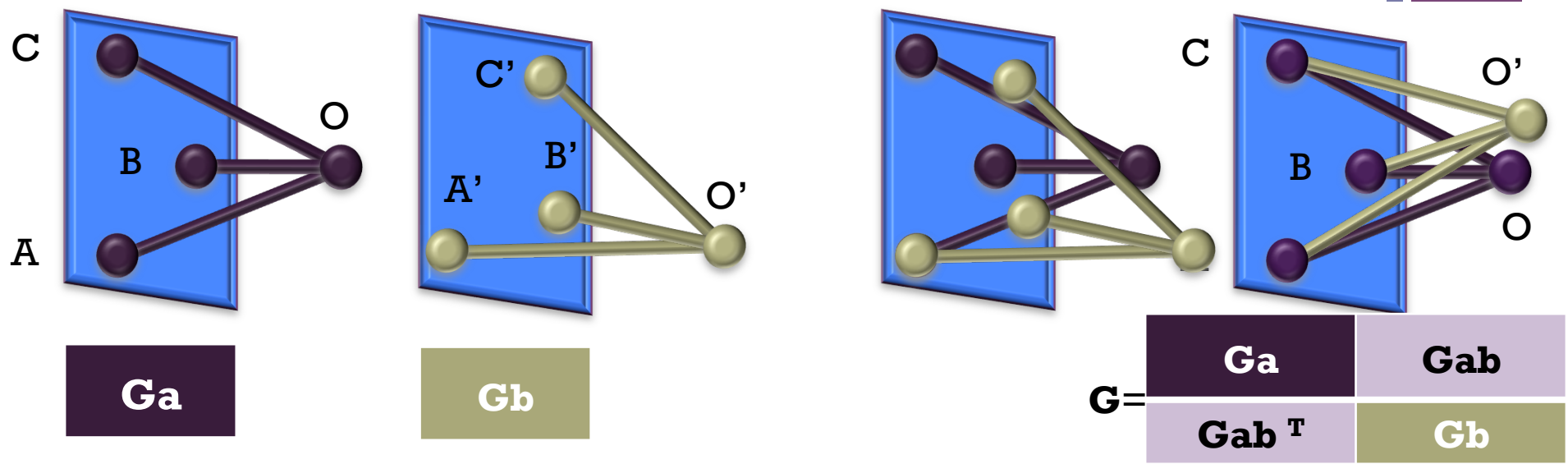
- Calculated from a set of vectors.

OA, OB, OC, AB, AC, BC

G	OA	OB	OC	AB	AC	BC
OA	$\langle \text{OA}, \text{OA} \rangle$	$\langle \text{OA}, \text{OB} \rangle$	$\langle \text{OA}, \text{OC} \rangle$	$\langle \text{OA}, \text{AB} \rangle$	$\langle \text{OA}, \text{AC} \rangle$	$\langle \text{OA}, \text{BC} \rangle$
OB	$\langle \text{OB}, \text{OA} \rangle$	$\langle \text{OB}, \text{OB} \rangle$	$\langle \text{OB}, \text{OC} \rangle$	$\langle \text{OB}, \text{AB} \rangle$	$\langle \text{OB}, \text{AC} \rangle$	$\langle \text{OB}, \text{BC} \rangle$
OC	$\langle \text{OC}, \text{OA} \rangle$	$\langle \text{OC}, \text{OB} \rangle$	$\langle \text{OC}, \text{OC} \rangle$	$\langle \text{OC}, \text{AB} \rangle$	$\langle \text{OC}, \text{AC} \rangle$	$\langle \text{OC}, \text{BC} \rangle$
AB	$\langle \text{AB}, \text{OA} \rangle$	$\langle \text{AB}, \text{OB} \rangle$	$\langle \text{AB}, \text{OC} \rangle$	$\langle \text{AB}, \text{AB} \rangle$	$\langle \text{AB}, \text{AC} \rangle$	$\langle \text{AB}, \text{BC} \rangle$
AC	$\langle \text{AC}, \text{OA} \rangle$	$\langle \text{AC}, \text{OB} \rangle$	$\langle \text{AC}, \text{OC} \rangle$	$\langle \text{AC}, \text{AB} \rangle$	$\langle \text{AC}, \text{AC} \rangle$	$\langle \text{AC}, \text{BC} \rangle$
BC	$\langle \text{BC}, \text{OA} \rangle$	$\langle \text{BC}, \text{OB} \rangle$	$\langle \text{BC}, \text{OC} \rangle$	$\langle \text{BC}, \text{AB} \rangle$	$\langle \text{BC}, \text{AC} \rangle$	$\langle \text{BC}, \text{BC} \rangle$

+ Strategy for the declaration of the association.

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- Definition of a common reference point (topology)

- Definition of a common 3D orientation (vectors)

➔ Calculation of the Global Gram Matrix.

+ Definition of the common orientation.

- Use of usual matrix factorization techniques for the definition of an orthogonal reference frame from the Gram matrix of each configuration. (dependent of the user declaration)
- Coincidence of the two reference frames.
- Calculation of the orientation matrix (**F_a** and **F_b**) of each configuration in the previously defined reference frame.

+ Calculation of relative orientation matrix

G_a	G_{ab}
G_{ab}^T	G_b

From matrix factorization (Cholesky, SVD)
we have :

- $G_a = F_a^T \otimes F_a$
- $G_b = F_b^T \otimes F_b$

From there it's deduced that :

- $G_{ab} = F_a^T \otimes F_b$

+ Calculation of geometrical requirements [1]

- Once the global \mathbf{G} Gram matrix of the 2 configuration is obtained, it become possible to calculate the evolution of geometrical requirements.
 - A geometrical requirement (noted grn) can be viewed as a combination of elements from \mathbf{G} .
 - $grn = \beta_l G_{lm} \gamma_m$
- In a matricial point of view :
- $grn = \beta \cdot \mathbf{G} \cdot \gamma^T$

+ Conclusion

- Calculation of the deviation possible with a generic coordinate free method
- Validation with a Cartesian method on an example.
- From there it become possible
 - To determinate the zone in which the vertex O' is.
 - To adjust the initial lengths of the bars to keep the vertex O' in the specified zone.

+ Feedback : advantages of coordinate free method

- The two configurations never exists at the same time.
- The user has to declare how to associate the references frames of the two configurations

+ On going work

- This technique is to be used on simplified representations of the product such as used for preliminary design.
- Definition of a 3D physical & geometrical model for preliminary design.
- Definition of acceptable geometrical variations for the detailed 3D model.



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