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Abstract: This paper proposes to investigate the use of a coordinate free approach for the mapping of geometrical requirements along the product life cycle. The geometry of the studied assembly is represented using a Gram matrix that is derived from a parametric model constituted of points and vectors. This parametric model is updated for all the relevant phases of the product life cycle. More precisely, this paper demonstrates how to combine two Gram matrices used as coordinate free representations for two states of an assembly into a global one in order to map the geometrical requirement evolution. This association is carried out using the Cholesky factorization technique. The application presented in this paper is constituted of a simple 2D case.
Keywords : Geometrical requirement; Life Cycle ; Metric tensor ; Gram matrices ; Non-Cartesian geometry.

1 - Introduction

In most cases, a mechanical product is subjected to dimensional variations along its life cycle due to mechanical strains. These variations can affect the value of the functional or geometrical requirements for the product functionalities. Generally there exist several useful values of a requirement. For instance, the value of a requirement taken under operating condition and at the assembly stage of its life-cycle is useful for the designer to fulfill the expected product functionalities. As a matter of fact, considering the designer point of view, several problems appear: "Does the chosen dimension allow meeting the requirements under operating conditions?" or "Which dimensions must be specified on the drawing to ensure a given value of the functional requirement in operation?"

In order to compare the geometric configuration of a product at two distinct stages of its life-cycle, this paper proposes a technique to combine the coordinate free representation of the two states in a unique representation using Gram matrices [W1]. It is assumed that a Gram matrix representation of the two configurations to be compared already exists.

This paper will introduce the coordinate free model used in this research. The advantages and limitations will be lightly exposed and then an application case will be detailed.

2 - Mathematical Tools

The great majority of the concepts presented here have been introduced in a previous research by the authors [MS1]. They will be more detailed in the following sections.

2.1 - Notations

This paper will use the following conventions:

- Matrices will be noted with capital bold letters: **M** ;
- Vectors will be noted with bold letters: **u** ;
- Scalar number will be noted with italic letters: x.

2.2 - Coordinate free representation

This paper investigates the use of a coordinate free approach for the calculations. With the proposed approach, a given geometry described in two different coordinate systems would have a common, unique description if a coordinate free system were used. From there it becomes possible for the designer to describe intrinsic geometric properties of a part (or a component) using a compact mathematical model without paying attention to the coordinate system in which the object is described.

This choice has been motivated by the existence of generic constraints specification techniques (as in [S1]) and solvers (cf. in [M1]) to resolve these constraints. Authors decided to use a Gram matrix as the mathematical representation of choice for the mechanism.

As this approach is vector space based, it then becomes necessary to obtain vectors to represent the product model. There exists several possible ways to obtain such vectors.

Among these possibilities, it is conceivable to use the TTRS/MGRE model [D1] to obtain the relative position of the technological surfaces. From there it becomes possible to extract one or several vectors to represent the relative positions of two surfaces, parts or components. Moreover, during the early stages of the design of a product, there often exists a simple geometrical representation of the product such as a skeleton from which positioning vector parameters should be extracted

2.3 - Gram matrices

The following definition is found in [W1] : «the Gram matrix **G** (or Gramian matrix or Gramian) of a set of vectors $\{\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_k}\}$ in an inner product space is the Hermitian matrix of inner products, whose entries are given by equation (1)». G_{lm} represents the term of the row *l* and column *m* of the **G** matrix. In this paper the names of all Gram matrices start with a capital "G".

$$G_{lm} = <\mathbf{x_l},\mathbf{x_m}> \qquad (1)$$

A metric tensor is a particular case of a Gram matrix which has its rank equal to its dimension. For example, in the 3D Euclidian space, a 3 by 3 Gram matrix which has a nonzero determinant (which is equivalent to being constituted of 3 independent vectors) is called a metric tensor. In this paper, the names of all the metric tensors start with a capital "M".

2.4 - Application to assemblies

In any case, this paper supposes that a vectorial representation of the assembly already exists. The vectors included in this representation will be noted $\mathbf{un_l}$ where n represents the current life-cycle stage and l is an index used to count the vectors. For example, at the assembly stage noted n of the product life cycle, the set of k vectors is noted: $\mathbf{Sun} = \{\mathbf{un_1}, \mathbf{un_2}, \ldots, \mathbf{un_l}, \ldots, \mathbf{un_k}\}$.

From this set of vector **Sun**, and for each configuration of the product, the assembly is represented by its associated Gram matrix which will be noted **Gn** where n represents the current life-cycle stage.

2.5 - Geometrical requirement calculation

Functional requirements are expressed as an algebraic relation between the vectors (or combination of vectors) used in the Gram matrix. Basically, from the Gram matrices it is immediately possible to obtain the scalar product between two vectors. From there, it is easy to deduce the norm of a vector ($\|\mathbf{x}\| = \sqrt{<x,x>}$) and the angle between two vectors ($\cos(\mathbf{x},\mathbf{y}) = <\mathbf{x},\mathbf{y}> / (\|\mathbf{x}\| \bullet \|\mathbf{y}\|)$).

More generally, the geometrical requirement (noted *grn* at stage n) might be expressed thanks to linear combinations of scalar products between vectors included in the set used for the Gram matrix definition. Based on the set **Sun** presented above, equation (2) introduces β_l and γ_m which represent the weights of $<\mathbf{un_l},\mathbf{un_m}>$ in *grn*.

$$grn = \sum_{l,m=1}^{k} <\beta_l \mathbf{un_l}, \gamma_m \mathbf{un_m}>$$
$$grn = \sum_{l,m=1}^{k} \beta_l \, Gn_{lm} \, \gamma_m \qquad (2)$$

Equation (2) yields to the definition of two vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ that represent the weights of **Gn** in *grn*. From there, the relation (3) is obtained.

$$grn = \boldsymbol{\beta} \bullet \mathbf{Gn} \bullet \boldsymbol{\gamma}^{T} \qquad (3)$$

2.6 - Comparison between configurations

Two Gram matrices **Gi** and **Gf** representing the initial and final geometrical configurations are first computed using a coordinate free solver [M1]. **Gi** stand for Gram matrix corresponding to the initial stage and $\mathbf{Sui} = \{\mathbf{ui_1}, \ldots, \mathbf{ui_k}\}$ is its associated set of k vectors. In the same way, **Gf** stand for Gram matrix corresponding to the final stage and $\mathbf{Suf} = \{\mathbf{uf_1}, \ldots, \mathbf{uf_k}\}$ is its associated set of vectors.

In order to map the evolution of the geometrical requirement along the life cycle from an initial stage i to a final stage f, it becomes necessary to calculate the scalar products between vectors from both configurations such as $<\mathbf{ui_1},\mathbf{uf_4}>$. To perform such calculations it is necessary to propose a way to superpose (or to link) the two relevant configurations and to calculate a relative orientation matrix **Oif** as defined by equation (4).

$$Oif_{lm} = <\mathbf{ui_l},\mathbf{uf_m}> \qquad (4)$$

From there it become possible to build a global Gram matrix G, expressed in equation (6), including all the vectors from the two configurations that uses vectors of the set **Suif** which is the concatenation of the sets **Sui** and **Suf** as expressed in equation (5).

$$\mathbf{Suif} = \{\mathbf{ui_1}, \ldots \mathbf{ui_k}, \mathbf{uf_1}, \ldots, \mathbf{uf_k}\}$$
$$\mathbf{Suif} = \{\mathbf{uif_1}, \mathbf{uif_2}, \ldots, \mathbf{uif_m}\} \qquad (5)$$

$$G_{lm} = <\mathbf{uif_l},\mathbf{uif_m}> \qquad (6)$$

Moreover, in order to calculate a point deviation it is necessary to associate the configuration in an affine way. This association basically consists in assuming that a point in the two configurations has a known deviation (that could be null) during the shift from stage i to stage f.

2.7 - Vectors association of two configurations

The previous section pointed to the necessity to carry out the calculation of "relative orientation matrix" described in equation (4). As the Gram matrix approach is coordinate free, the coordinates of the vectors included in the sets **Sui** and **Suf** are not directly available. Consequently, the direct application of equation (4) is impossible.

Therefore, the relative orientation has to be calculated using another technique. Authors propose to use matrix factorization techniques to express any **Gi** and **Gf** as the

product specified in equation (7) where **Id** stands for the identity matrix.

$$\mathbf{Gi} = \mathbf{Fi}^T \bullet \mathbf{Fi} = \mathbf{Fi}^T \bullet \mathbf{Id} \bullet \mathbf{Fi} \quad \text{and}$$
$$\mathbf{Gf} = \mathbf{Ff}^T \bullet \mathbf{Ff} = \mathbf{Ff}^T \bullet \mathbf{Id} \bullet \mathbf{Ff} \quad (7)$$

Relation (7) is equivalent to (8) using Einstein's convention.

$$Gi_{lm} = Fi_{pl} \bullet Fi_{pm} = Fi_{pl} \bullet \delta_{pq} \bullet Fi_{qm} \quad \text{and}$$
$$Gf_{lm} = Ff_{pl} \bullet Ff_{pm} = Ff_{pl} \bullet \delta_{pq} \bullet Ff_{qm} \quad (8)$$

In equation (7), **Id** can be viewed as the metric tensor of an ortho-normal reference frame $\{\mathbf{e_1},\mathbf{e_2},\mathbf{e_3}\}$ (3 vectors in 3D Euclidian space). From there $\mathbf{Fi}^T$ and $\mathbf{Ff}^T$ can be viewed as the transformation matrix defined in (9). As **Gi** and **Gf** are Gram matrices that are positive, semidefinite, the terms of **Fi** and **Ff** remain real numbers.

$$\mathbf{un_l} = Fn_{lq} \bullet \mathbf{e_q} \text{ with n=i or n=f} \quad (9)$$

The application of relation (9) on the terms of relation (4) **Sui** and **Suf** gives the expression (10) that allows the calculation of **Oif**.

$$Oif_{lm} = \langle Fi_{lp} \bullet \mathbf{e_p}, Ff_{qm} \bullet \mathbf{e_q} \rangle = Fi_{lp} \bullet Ff_{mq} \langle \mathbf{e_p}, \mathbf{e_q} \rangle$$
$$Oif_{lm} = Fi_{lp} \bullet Ff_{mq} \bullet \delta_{pq} = Fi_{lp} \bullet Ff_{mp} \quad (10)$$

Finally, equation (10) is simplified as equation (11):

$$\mathbf{Oif} = \mathbf{Fi}^T \bullet \mathbf{Ff} \quad (11)$$

Currently, authors have looked into the use of two factorization techniques to determine the terms of equation (11) :

- A singular value decomposition
- A Cholesky based factorization (see in [W2])

The affine association consists simply in the appropriate choice of a point to connect a path from the initial to the final configuration. The application to the case presented in an upcoming section will clearly explicit this.

3 - Use of the Cholesky factorization

For the association of two configurations of the same model, the Cholesky factorization appears to be appropriate because the user can choose the vectors for building the identification orthonormal reference frame. Consequently, by choosing the appropriate vectors, the designer is able to set the desired association conditions.

However, the Cholesky factorization has a limitation. This method is only available for square matrices that have a non-zero determinant. This means that the matrix to be factorized has to be a metric tensor and so its size has to be equal to the dimension of the Euclidian space. In the general case, Gram matrices sizes are bigger than the dimension of the Euclidian space. Consequently, for the application of the Cholesky factorization, it first becomes necessary to be able to obtain the metric tensors from the Gram matrices and secondly to rebuilt the original Gram matrix from the metric tensor.

3.1 - From Gram matrix to metric tensor.

The first conversion is obvious: it is simply necessary to choose a subset of independent vectors in the set **Sun** (**Sui** and **Suf**) and extract the corresponding columns and rows of the Gram matrix to obtain the resulting metric tensor. For the initial configuration, **Mi** is extracted from **Gi**. The same rows and columns are used in the final configuration to extract **Mf** from **Gf**. Afterwards, the metric tensors **Mi** and **Mf** are factorized with the Cholesky technique and the relative orientation matrix **Oif** is calculated using relation (11). From there the relative orientations of the initial and final independent vectors are deduced using the **GOif** Gram matrix given by (12).

$$\mathbf{GOif} = \begin{array}{|c|c|} \hline \mathbf{Mi} & \mathbf{Oif} \\ \hline \mathbf{Oif}^T & \mathbf{Mf} \\ \hline \end{array} \quad (12)$$

3.2 - From metric tensor to Gram matrix

3.2.1 - Mathematical tools

This sub-section will present the two concepts of covariant and contravariant coordinates used in the conversion for metric tensor to Gram matrix [W3]. The covariant coordinates noted u_k of a vector **u** on the reference frame $\{\mathbf{e_1}, \dots, \mathbf{e_l}\}$ are defined by the scalar products of equation (13). These coordinates are those that are commonly used in engineering and science. One can notice that Gram matrices are constituted of covariant coordinates.

$$u_k = \langle \mathbf{u}, \mathbf{e_k} \rangle \quad (13)$$

Moreover the covariant coordinates lead to the definition of the metric tensor **Me** for the reference frame $\{\mathbf{e_1}, \dots, \mathbf{e_l}\}$. The term Me_{lm} of the row l and column m of **Me** is defined by (14). **Me** is invertible and its inverse matrix is noted **M-e**

$$Me_{lm} = \langle \mathbf{e_l}, \mathbf{e_m} \rangle \quad (14)$$

The contravariant coordinates define the u^k coefficients used to obtain a vector **u** as a unique linear combination of the vectors $\{\mathbf{e_1}, \dots, \mathbf{e_l}\}$ defining the reference frame (15).

$$\mathbf{u} = \sum_{p=1}^{l} u^p \, \mathbf{e_p} \quad (15)$$

The conversion from covariant to contravariant coordinates is achieved through a combination of relations (13), (14) and (15) and the application of Einstein's convention. From there, it is easy to deduce equations (16) and (17) .

$$u_m = u^p \, Me_{mp} \quad (16)$$

$$u^p = u_m \, M\text{-}e_{mp} \quad (17)$$

3.2.2 - Principle

The metric tensor **Mn** of the reference frame $\{\mathbf{e_1}, \dots, \mathbf{e_l}\}$ is given with two vectors **u** and **v** defined by their contravariant

coordinates (15) . The calculation of **<u,v>**, that leads to the calculation of any term of the Gram matrix, is given by equation (18) that uses Einstein's convention.

$$\begin{aligned} &<\mathbf{u},\mathbf{v}>=<u^m\mathbf{e_m},v^p\mathbf{e_p}> \\ &<\mathbf{u},\mathbf{v}>=u^m\ <\mathbf{e_m},\mathbf{e_p}>\ v^p \\ &<\mathbf{u},\mathbf{v}>=u^m\ Mn_{mp}\ v^p \end{aligned} \qquad (18)$$

Using equation (18), it becomes easy to calculate the terms of the **Gif** gram matrix as defined by equation (4).

3.2.3 - Contravariant coordinates of vectors

The contravariant coordinates of the vectors for the initial configuration will be calculated using relation (17). The inverse matrix of the **Mi** metric tensor is to be computed and the covariant coordinates of the vectors in the set **Sui** are to be extracted from the appropriate elements in the **Gi** Gram matrix. Contravariant coordinates of the final vectors included in the set **Suf** are computed using the same technique using the **Mf** and **Gf** matrices.

From there, the global Gram matrix including all vectors from the set **Suif** is computed and the evaluation of the geometrical requirement evolution becomes possible using relation (3). The procedure for this calculation is detailed in [MS1].

4 - Application case.

4.1 - Geometry and vectorization.

This section will illustrate the comparison of two configurations of the same mechanism at two different stages of its life-cyle on a simple 2D representation of a piston - connecting rod – crank shaft assembly presented in figure 1.

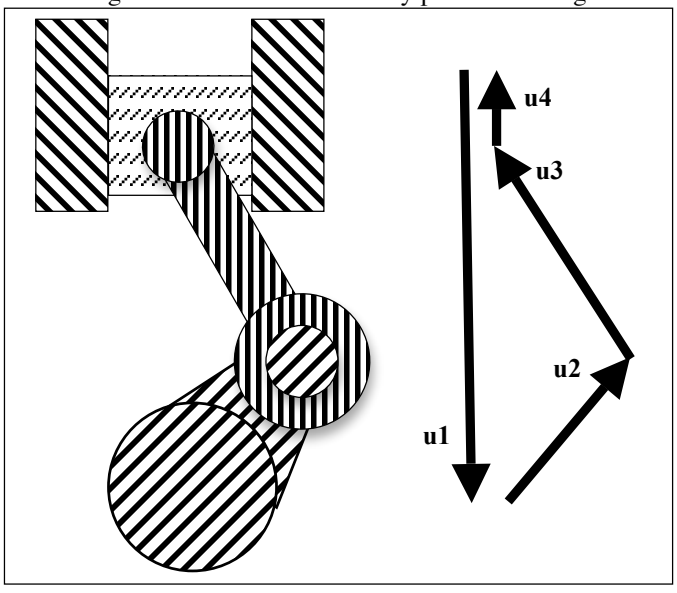

Figure 1 : piston - connecting rod – crank shaft assembly and its vectorization.

The following conditions are used to obtain the initial geometrical coordinate free representation with the solver [M1]:

- ||**ui1**|| = 11
- ||**ui2**|| = 4
- ||**ui3**|| = 7
- ||**ui4**|| = 3
- cos(**ui1**,**ui4**) = -1
- **ui1** + **ui2** + **ui3** + **ui4** = **0**

From there, the coordinate free solver gives the **Gi** Gram matrix for the initial configuration presented in table 1.

Gi	ui1	ui2	ui3	ui4
ui1	121	-21,3125	66,6875	-33
ui2	-21,3125	16	0,5	5,8125
ui3	66,6875	0,5	49	-18,1875
ui4	-33	5,8125	-18,1875	9

Table 1 : Initial Gram matrix Gi

4.2 - Dimensional evolution along the life cycle.

It has been arbitrarily decided by the authors that the geometrical conditions at the final stage are :

- ||**uf1**|| = 11
- ||**uf2**|| = 4,02
- ||**uf3**|| = 7,2
- ||**uf4**|| = 3,01
- cos(**uf1**,**uf4**) = -1
- **uf1** + **uf2** + **uf3** + **uf4** = **0**

For this final configuration, the coordinate free solver gives the **Gf** Gram matrix presented in table 2.

Gf	uf1	uf2	uf3	uf4
uf1	121	-19,3846	68,5054	-33,11
uf2	-19,3846	16,1604	2,0801	5,3043
uf3	68,5054	2,0801	51,84	-18,7456
uf4	-33,11	5,3043	-18,7456	9,0601

Table 2 : Final Gram matrix Gf

Moreover, it is also assumed that the vector **u1** is not varying along the life-cycle, meaning that **ui1** = **uf1**.

4.3 - Cholesky Association.

As explained in section 3, the user must choose a set of 2 independent vectors (the problem is in 2D) in the set {**u1**,**u2**,**u3**,**u4**} in order to perform the Cholesky factorization. In order to satisfy the condition **ui1**=**uf1**, the sets {**ui1**,**ui2**} and {**uf1**,**uf2**} are arbitrarily chosen (other combinations would also qualify such as {**ui1**,**ui3**} and {**uf1**,**uf3**}).

In accordance with section 3.2, the **Mi** and **Mf** metric tensors (see table 3 and table 4) are extracted from the two first rows and columns of **Gi** and **Gf**. These two tensors are factorized with the Cholesky technique to obtain the **Fi** and **Ff** matrices (see table 5 and table 6).

Mi	ui1	ui2
ui1	121	-21,3125
ui2	-21,3125	16

Table 3 : Mi metric tensor

Mf	uf1	uf2
uf1	121	-19,3846
uf2	-19,3846	16,1604

Table 4 : Mf metric Tensor

Fi		
	11,0000	-1,9375
	0,0000	3,4994

Table 5 : Fi matrix

Ff		
	11,0000	-1,7622
	0,0000	3,6132

Table 6 : Ff Matrix

Finally the **Oif** matrix (table 7) is calculated using equation (11)

Oif	uf1	uf2
ui1	121,0000	-19,3846
ui2	-21,3125	16,0584

Table 7 :Oif orientation matrix

Then with (17) the **GOif** matrix (table 8) is assembled.

GOif	ui1	ui2	uf1	uf2
ui1	121,0000	-21,3125	121,0000	-19,3846
ui2	-21,3125	16,0000	-21,3125	16,0584
uf1	121,0000	-21,3125	121,0000	-19,3846
uf2	-19,3846	16,0584	-19,3846	16,1604

Table 8 : GOif Gram matrix of relative positions.

4.4 - Contravariant coordinates of vectors

The computation of the contravariant coordinates is performed on the {**ui1**,**ui2**} reference frame for the initial configuration and on the {**uf1**,**uf2**} reference frame for the final one. The metric tensors of these reference frame **Mi** and **Mf** are already available and the covariant coordinates (see table 9) of the others vectors are to be read from the two firsts columns of the **Gi** and **Gf** Gram matrices. According to section 3.2.2 and relation (17), the contravariant coordinates presented in table 10 are finally obtained.

	ui1	ui2			uf1	uf2
ui1	121,0000	-21,3125		**uf1**	121,0000	-19,3846
ui2	-21,3125	16,0000		**uf2**	-19,3846	16,1604
ui3	66,6875	0,5000		**uf3**	68,5054	2,0801
ui4	-33,0000	5,8125		**uf4**	-33,1100	5,3043

Table 9 : covariant coordinates of the vectors in their Cholesky reference frame

	ui1	ui2			uf1	uf2
ui1	1,0000	0		**uf1**	1,0000	0
ui2	0,0000	1		**uf2**	0,0000	1
ui3	0,7273	1		**uf3**	0,7263	1
ui4	-0,2727	0		**uf4**	-0,2736	0

Table 10 : contravariant coordinates of the vectors in their Cholesky reference frame

From table 10, the contravariant coordinates of the vectors {**ui1**,**ui2**,**ui3**,**ui4**,**uf1**,**uf2**,**uf3**,**uf4**} are expressed in the reference frame {**ui1**,**ui2**,**uf1**,**uf2**}. As contravariant coordinates expressed are the coefficient for expressing any vector as a linear combination of a given reference frame, the initial vectors have null contravariant coordinates on the final reference frame. In the same way final vectors have null contravariant coordinates on the initial reference frame. From there, the matrix of contravariant coordinates **C** is deducted (see table 11). The application of relation (18) on each line of the **C** matrix lead to the calculation of the scalar products between all the vectors of the set {**ui1**,**ui2**,**ui3**,**ui4**,**uf1**,**uf2**,**uf3**,**uf4**}. It then becomes possible to compute the global Gram matrix **G** (see table 12) for the whole set {**ui1**,**ui2**,**ui3**,**ui4**,**uf1**,**uf2**,**uf3**,**uf4**} using relation (19) and table 11.

$$\mathbf{G} = \mathbf{C} \otimes \mathbf{GOif} \otimes \mathbf{C}^{T} \qquad (19)$$

C	ui1	ui2	uf1	uf2
ui1	1,0000	0,0000	0,0000	0,0000
ui2	0,0000	1,0000	0,0000	0,0000
ui3	0,7273	1,0000	0,0000	0,0000
ui4	-0,2727	0,0000	0,0000	0,0000
uf1	0,0000	0,0000	1,0000	0,0000
uf2	0,0000	0,0000	0,0000	1,0000
uf3	0,0000	0,0000	0,7263	1,0000
uf4	0,0000	0,0000	-0,2736	0,0000

Table 11 : contravariant coordinate of {ui1,ui2,ui3,ui4,uf1,uf2,uf3,uf4} in the {ui1,ui2,uf1,uf2} frame.

4.5 - Geometrical requirement evolution

Once the global **G** Gram matrix of the two configurations is obtained, the techniques presented in [MS1] are applied in order to calculate the geometrical requirement evolution along the corresponding stages of the assembly life-cycle.

G	ui1	ui2	ui3	ui4	uf1	uf2	uf3	uf4
ui1	121,0000	-21,3125	66,6875	-33,0000	121,0000	-19,3846	68,5054	-33,1100
ui2	-21,3125	16,0000	0,5000	5,8125	-21,3125	16,0584	0,5777	5,8319
ui3	66,6875	0,5000	49,0000	-18,1875	66,6875	1,9605	50,3999	-18,2481
ui4	-33,0000	5,8125	-18,1875	9,0000	-33,0000	5,2867	-18,6833	9,0300
uf1	121,0000	-21,3125	66,6875	-33,0000	121,0000	-19,3846	68,5054	-33,11
uf2	-19,3846	16,0586	1,9605	5,2867	-19,3846	16,1604	2,0806	5,3043
uf3	68,5054	0,5777	50,3999	-18,6833	68,5054	2,0806	51,8400	-18,7456
uf4	-33,1100	5,8319	-18,2481	9,0300	-33,1100	5,3043	-18,7456	9,0601

Table 12 : Global G Gram matrix

5 - Conclusion and perspectives

This paper has first presented some mathematical models and tools for a coordinate free approach applied to mechanisms. This model describes how to represent the mechanism in the proposed coordinate free model, and how to calculate the value of a geometrical requirement at a given stage of the product life-cyle. This research also shows how to link two configurations in order to map the evolution of a geometrical requirement using the Cholesky factorization technique. An application of this coordinate free approach on a simple 2D example has been carried out. However this generic methodology is also applicable on 3D cases.

Globally, this paper has demonstrated that a generic coordinate free approach is suitable for the association of two configurations of a mechanism. From there the coordinate free approach has been shown to be applicable to the analysis of the evolution of a geometrical requirement along the product life cycle.

As the approach is generic, the authors propose to go beyond this first validation with the application of the method on more complicated cases such as mobile mechanisms and hyperstatic mechanisms or structures.

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