Distributed state estimation and model predictive control of linear interconnected system: Application to fault tolerant control

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Abstract: In this paper, a distributed and networked control system architecture based on independent Model Predictive Control/Kalman-Filter (MPC/KF) architectures, is proposed. Interconnected subsystems, possibly located at different sites, exchange information through the digital communication network. For the partial local state measurement, the key component for realistic Distributed Predictive Model Control (DMPC) formulation is the state estimations. These state estimations are generated by Kalman filters. In this distributed framework, MPC and KF algorithms may require information from other sub-controllers to achieve their task in a cooperative way. The given distributed and cooperative control system architecture may be suitable for Fault Tolerant Control (FTC) in a network of distributed subsystems. The proposed approach is used to implement a Fault Tolerant Control system under actuator faults within the distributed architecture.

1. INTRODUCTION

Production processes of modern industries are generally composed by different subsystems, which are interconnected and characterized by significant interactions. At the same time, due to the high performance requirements, modern control systems are becoming more and more complex. For these processes, different control solutions can be developed. A centralized control solution, where all the interactions are considered, and provide better performance, which in turn suffer from potential problems associated with computations and maintenance due to their size, and their higher risk of failure due to their centralized nature. A decentralized control structure still remains the most widely used control structure in the process industries, the reasons for this control choice solution are many, the prominent being its ability to effectively solve problem of dimensionality, uncertainty and information structure constraints (Siljak et al. [2005]). The vast body of existing literature on the decentralized control of large and interconnected systems has been reviewed in a number of survey papers and books (Ikeda et al. [1986], Zecevic et al. [2005]). Most decentralized controller design approaches approximate or ignore the interactions between the various subsystems (Lunze [1992]). However, the effectiveness of the decentralized controllers depends on the magnitude of inherent process interaction (Bristol [1966]). In the presence of strong process interactions, decentralized controllers can lead to performance deterioration or even instability (Cui et al. [2002]). An alternative to both decentralized and full multivariable (centralized) controllers, is the distributed controllers. This class has a structure that lies in between the preceding two extreme control structures. Consequently, it is an attractive option for the situations where the global objective, such as closed-loop stability and performance requirements, cannot be met by decentralized controllers while the complexity in the design and high cost in the installation of centralized controllers are to be avoided. In order to fulfil the global objective for the global system, cooperation between the controllers through a digital communication network might be necessary. Thanks to the digital network, the required cooperation can be achieved by means of a proper information exchange between the controllers. From the control algorithms standpoint, it is well known that the Model Predictive Control algorithm allows to deal with complex, multivariable, nonlinear and constrained systems (Jia et al. [2001]). The MPC strategy is based on an on-line optimization problem and uses a process model to predict the effect of potential control action on the evolving state of the plant. Typically, MPC is implemented in a centralized fashion. The complete system is modeled and all the control inputs are computed in one optimization problem. However, for large and interconnected systems, it may be necessary to have a distributed control scheme as mentionned above, where local control inputs are computed using local measurements and small order models of the local dynamics. But with information exchange between the controllers, the objective is to achieve some degree of cooperation between sub-controllers that are solving MPC problem with locally relevant variables, costs and constraints.

Previous works on distributed MPC are reported in (Ventak et al. [2006a], Mercangoz et al. [2007], Venkat et al. [2005], Patton et al. [2005], Vaccarini et al. [2006]). A preliminary analysis of control performance of distributed MPC has been performed in Vaccarini et al. [2006]. In Ventak et al. [2006a] a distributed state estimation strategy is developed for supporting distributed output feedback MPC of large-scale and interconnected systems. Mercangoz et al. [2007] propose a Distributed Model Predictive Control (DMPC) architecture, based on the fully decentralized estimation and control structure, where at each node linear model and local measurements are used to estimate the plant states. In Venkat et al. [2005], two approaches to realize a coordination between sub-controllers are proposed: the so-called communication and cooperation based MPC. In the cooperation based MPC, each sub-controller knows the global objective in order to improve optimality and stability and makes the decentralized strategy very close to the centralized one. When only the local objectives are known, a hierarchical decentralized control architecture uses a supervisor to compute the global optimum and to coordinate the sub-controllers, this is called communication based MPC (Patton et al. [2005]).

In this paper, we deal with the unconstrained distributed model predictive control of complex and interconnected systems and provide an extension of the work of Vaccarini et al. [2009] to achieve global performance based on the use of a cooperative strategy between sub-controllers. Here, a local state feedback is designed based on the distributed MPC scheme. However, not all the states are measured, and the control input is computed based on the state estimations provided by Distributed Kalman Filters (DKF). Thanks to the flexibility and the online optimization process inherent to MPC algorithms, we apply DMPC/DKF to a Fault Tolerant Control (FTC) problem in a distributed framework. It is worth stressing that the FTC problem is becoming an important subject in modern control theory and practice (Aubrun et al. [2003], Patton [1997], Sun et al. [2008], Maciejowski [1999]). That is to say, an FTC structure has the ability to continue operating to fulfill specified objectives despite of the occurrence faults in systems.

2. REVIEW OF MODELING FOR INTEGRATING MPC

Let the centralized model for the overall system be represented as a discrete, linear time-invariant (LTI) model has the form

$$S \equiv \begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$ and $y(k) \in \mathbb{R}^{n_y}$ are state, control input and output, respectively with corresponding dimensions, n_x, n_u, n_y.

In the decentralized modeling framework, it is assumed that the subsystem-subsystem interactions have a negligible effect on system variables, *i.e*, we suppose the previous overall model S is composed of N subsystems S_{ii}, each of the subsystem is represented by the following state space equations

$$S_{ii} \equiv \begin{cases} x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) \\ y_i(k) = C_{ii}x_i(k) \qquad i = 1,2,\ldots,N \end{cases} \tag{2}$$

where $x_i(k) \in \mathbb{R}^{n_{x_i}}$, $u(k) \in \mathbb{R}^{n_{u_i}}$ and $y(k) \in \mathbb{R}^{n_{y_i}}$ are the local state, control input and output, respectively and $n_x = \sum_i n_{x_i}$, $n_u = \sum_i n_{u_i}$ and $n_y = \sum_i n_{y_i}$. Frequently, components of the interconnected system are tightly coupled due the material/energy and/or information flow between them. In such cases, the decentralized assumption leads to a loss in achievable control performance. It is natural to view the previous overall model S composed of N subsystems S_i which are interacting with each other through linear interconnections. Each of the subsystem is represented by the following state space equations

$$S_i \equiv \begin{cases} x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + w_i(k) \\ y_i(k) = C_{ii}x_i(k) + v_i(k) \qquad i = 1,2,\ldots,N \end{cases} \tag{3}$$

where the state and output interaction vectors w_i and v_i are given by

$$\begin{aligned} w_i(k) &\triangleq \sum_{j=1;j\neq i}^{N} A_{ij}x_j(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} B_{ij}u_j(k) \\ v_i(k) &\triangleq \sum_{j=1;j\neq i}^{N} C_{ij}x_j(k) \end{aligned} \tag{4}$$

These vectors represent the interaction of subsystem $j \neq i$ on subsystem i. The proposed distributed control architecture fosters implementation of cooperation-based strategy for several interacting processes (3), (4) in order to get closer on the benefits achievable with centralized control.

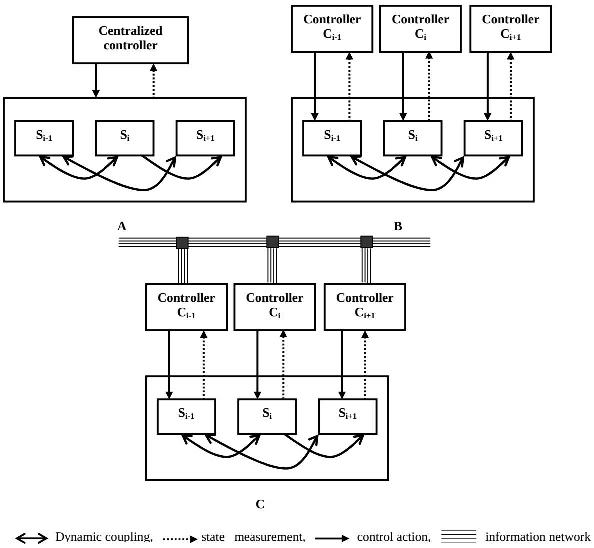

Fig. 1. A schematic representation of: A) Centralized control, B) Decentralized control C) Distributed control.

3. PROBLEM FORMULATION

By means of (3) and (4), the future state and output prediction over horizon p are given by

$$\hat{x}_i(k+l|k) = A_{ii}^l \hat{x}_i(k|k) + \sum_{s=1}^{l} A_{ii}^{s-1} \Big[B_{ii}u_i(k+l-s|k) + \hat{w}_i(k+l-s|k-1) \Big] \tag{5}$$

$\hat{y}_i(k+l|k) = C_{ii}\hat{x}_i(k+l|k) + \hat{v}_i(k+l|k-1)$ for $l = 1,2,\ldots.,p$

Given overall system S composed by N interactive subsystems S_i, $i = 1,2,\ldots,N$, the unconstrained Distributed Model Predictive Control problem with prediction horizon $p_i > 0$ and control horizon $m_i > 0$ two integer values, with $m_i < p_i$ consists of finding, at time k, a set of independent sub-controllers $\mathscr{C}_i$ such that each $\mathscr{C}_i$ minimizes its local cost function J_i

$$J_i = \sum_{l=1}^{p} \| \hat{y}_i(k+l|k) - y_i^d(k+l|k) \|_{Q_i}^2 + \sum_{l=1}^{m} \| \Delta u_i(k+l-1|k) \|_{R_i}^2 \tag{6}$$

$$\text{subject to} \begin{cases} \text{predictive model constraints given by(5)} \\ \text{initial condition} : \hat{x}_i(k|k) = \hat{x}_i(k) \end{cases}$$

where

$\| \chi \|_\Lambda^2 \triangleq \chi^T \Lambda \chi$.

$y_i^d(k+l|k)$: the desired output.

$\Delta u_i(k+l-1|k)$: future control increment at time k, with $u_i(k) = u_i(k-1) + \Delta u_i(k|k)$.

Q_i : symmetric and positive semi definite (SPSD) matrix of appropriate dimension ($Q_i \succeq 0$).

R_i : symmetric and positive definite (SPD) matrix of appropriate dimension ($R_i \succ 0$).

4. COOPERATION BASED DISTRIBUTED MPC PROBLEM

In order to find an explicit solution to the DMPC problem, each sub-controller $\mathscr{C}_i$ is decomposed in three connected function blocks: an optimizer, a state predictor and an interaction predictor. In the sequel of this paper, the following assumptions are made

- the prediction and control horizons are the same for each sub-controller, i.e, $m_i = m_j = m,\ p_i = p_j = p,\ \forall i, j = 1,2,...,N, j \neq i$;
- the sub-controllers are synchronous;
- the sub-controllers communicate only once within a sampling interval;
- the communication channel introduces a delay of one sampling period.

To simplify the mathematical expressions, some notations are adopted in following. Given the scalar numbers a, $b \in \mathbb{N}$.

- $0_{a\times b}$ is the $a\times b$ null matrix;
- 0_a is the $a\times a$ null matrix;
- $I_{a\times b}$ is the $a\times b$ identity matrix;
- I_a is the $a\times a$ identity matrix;
- $diag_a\{\mathscr{A}\}$ is a diagonal block matrix made by a blocks equal to $\mathscr{A}$;

Interaction prediction : Under the assumptions, at step k, the predictions of the interaction vectors are given by

$$\begin{aligned}\hat{W}_i(k,p|k-1) &= \tilde{A}_i\hat{X}(k,p|k-1) + \tilde{B}_i\tilde{\Gamma}_iU(k-1,m|k-1)\\ \hat{V}_i(k,p|k-1) &= \tilde{C}_i\hat{X}(k,p|k-1)\end{aligned} \tag{7}$$

$$\tilde{A}_i \triangleq [diag_p\{A_{i,1}\}....diag_p\{A_{i,i-1}\}\ 0_{n_{x_i}}\ diag_p\{A_{i,i+1}\} ...diag_p\{A_{i,N}\}]$$

$$\tilde{B}_i \triangleq [diag_p\{B_{i,1}\}....diag_p\{B_{i,i-1}\}\ 0_{n_{x_i}}\ diag_p\{B_{i,i+1}\} ...diag_p\{B_{i,N}\}]$$

$$\tilde{C}_i \triangleq [diag_p\{C_{i,1}\}....diag_p\{C_{i,i-1}\}\ 0_{n_{x_i}}\ diag_p\{C_{i,i+1}\} ...diag_p\{C_{i,N}\}]$$

Where

$$\hat{X}_i(k,p|k) \triangleq \begin{bmatrix}\hat{x}_i(k|k)\\ \hat{x}_i(k+1|k)\\ \vdots\\ \hat{x}_i(k+p-1|k)\end{bmatrix},\quad \hat{X}(k,p|k) \triangleq \begin{bmatrix}\hat{X}_1(k,p|k)\\ \hat{X}_2(k,p|k)\\ \vdots\\ \hat{X}_N(k,p|k)\end{bmatrix}$$

$$U_i(k,m|k) \triangleq \begin{bmatrix}u_i(k|k)\\ u_i(k+1|k)\\ \vdots\\ u_i(k+m-1|k)\end{bmatrix},\quad U(k,m|k) \triangleq \begin{bmatrix}U_1(k,m|k)\\ U_2(k,m|k)\\ \vdots\\ U_N(k,m|k)\end{bmatrix}$$

$$\tilde{\Gamma}_i \triangleq \begin{bmatrix}0_{(m-1)n_{u_i}\times n_{u_i}} & I_{(m-1)n_{u_i}}\\ 0_{n_{u_i}\times(m-1)n_{u_i}} & I_{n_{u_i}}\\ 0_{n_{u_i}\times(m-1)n_{u_i}} & I_{n_{u_i}}\\ \vdots & \vdots\\ 0_{n_{u_i}\times(m-1)n_{u_i}} & I_{n_{u_i}}\end{bmatrix}$$

State predictor : Under the assumptions, at step k, the local state prediction for the sub-controller $\mathscr{C}_i$ is expressed by

$$\hat{X}_i(k+1,p|k) = \bar{L}_i\hat{x}_i(k|k) + \bar{M}_iU_i(k,m|k) + \bar{S}_i\hat{W}_i(k,p|k-1) \tag{8}$$

$$\bar{S}_i \triangleq \begin{bmatrix}A_{ii}^0 & \dots & 0_{n_{x_i}}\\ \vdots & \ddots & \vdots\\ A_{ii}^{p-1} & \dots & A_{ii}^0\end{bmatrix}, \bar{L}_i \triangleq \bar{S}_i\begin{bmatrix}A_{ii}\\ 0_{pn_{x_i}\times n_{x_i}}\end{bmatrix}$$

$$\bar{M}_i \triangleq \bar{S}_i\begin{bmatrix} & diag_m\{B_{ii}\} & & \\ \hline 0_{n_{u_i}} & ...... & 0_{n_{u_i}} & B_{ii}\\ \vdots & \ddots & \vdots & \vdots\\ 0_{n_{u_i}} & ...... & 0_{n_{u_i}} & B_{ii}\end{bmatrix}$$

Optimal Control Sequence : Under the assumptions, at step k, the optimal control sequence $U_i(k,m|k)$ is

$$\begin{aligned}U_i(k,m|k) = \Gamma'_iu_i(k-1) + \bar{\Gamma}_i\bar{K}_i[(Y_i^d(k+1,p|k) - M_iu_i(k-1)\\ -\bar{C}_i\bar{L}_i\hat{x}_i(k|k) - \bar{C}_i\bar{S}_i\hat{W}_i(k,p|k-1)] - T_i\hat{V}_i(k,p|k-1)\end{aligned} \tag{9}$$

$$\bar{C}_i \triangleq diag_p\{C_{ii}\}, \mathscr{H}_i \triangleq \sum_{s=1}^{l} C_{ii}A_{ii}^{s-1}B_{ii}$$

$$M_i \triangleq \begin{bmatrix}\mathscr{H}_i^1\\ \vdots\\ \mathscr{H}_i^p\end{bmatrix},\ \underset{(m\ blocs)}{\Gamma'_i} \triangleq \begin{bmatrix}I_{n_{u_i}}\\ \vdots\\ I_{n_{u_i}}\end{bmatrix},\ \underset{(m\times m\ blocs)}{\bar{\Gamma}_i} \triangleq \begin{bmatrix}I_{n_{u_i}} & \dots & 0\\ \vdots & \ddots & \vdots\\ I_{n_{u_i}} & \dots & I_{n_{u_i}}\end{bmatrix}$$

$$T_i \triangleq \begin{bmatrix}0_{(p-1)n_{y_i}\times n_{y_i}} & I_{(p-1)n_{y_i}}\\ 0_{n_{y_i}\times(p-1)n_{y_i}} & I_{n_{y_i}}\end{bmatrix}$$

the explicit form of the control action applied by the $\mathscr{C}_i$ of the subsystem S_i is given by

$$\begin{aligned}u_i(k) = u_i(k-1) + K_i\Big[(Y_i^d(k+1,p|k) - M_iu_i(k-1)\\ -\bar{C}_i\bar{L}_i\hat{x}_i(k|k) - \bar{C}_i\bar{S}_i\hat{W}_i(k,p|k-1)\Big] - T_i\hat{V}_i(k,p|k-1)\end{aligned} \tag{10}$$

$$\Gamma_i \triangleq [I_{n_{u_i}}0_{n_{u_i}\times(m-1)n_{u_i}}], \bar{K}_i = [N_i^T\bar{Q}_iN_i + \bar{R}_i]^{-1}N_i^T\bar{Q}_i, K_i \triangleq \Gamma_i\bar{K}_i$$

$$\bar{Q}_i \triangleq diag_p\{\bar{Q}_i\}, \bar{R}_i \triangleq diag_p\{\bar{R}_i\}, N_i = \bar{C}_i\bar{S}_i\bar{B}_i\bar{\Gamma}_i$$

We refer to Vaccarini et al. [2009] for more details. In the next section, we review the distributed Kalman-filtering algorithm to generate the optimal states estimation, in order to use it in state feedback distributed MPC law.

5. DISTRIBUTED STATE ESTIMATION WITH MEASUREMENTS EXCHANGE

For large, networked systems, organizational and geographic constraints may preclude the use of centralized estimation strategies. The Kalman filter addresses the problem of estimating the state of a linear discrete-time for each subsystem (3) augmented with gaussian white noises, given by

$$\begin{aligned}x_i(k+1) &= A_{ii}x_i(k) + B_{ii}u_i(k) + w_i(k) + G_i\omega b_{x_i}\\ y_i(k) &= C_{ii}x_i(k) + v_i(k) + \nu b_i \qquad i = 1,2,...,N\end{aligned} \tag{11}$$

The conditional density of the subsystem state x_i, given the set of measurements y_i, $i = 1,2,...,N$, is assumed to be normally distributed. For each subsystem i, the vectors $\omega b_{x_i} \in \mathbb{R}^{n_{w_i}}$ and $\nu b_i \in \mathbb{R}^{n_{y_i}}$ denote the disturbances on the subsystem model state equation and output equation respectively, and are modeled as uncorrelated, zero mean, white sequences with corresponding covariance matrices Q_{x_i} and R_{v_i} respectively. The matrices, Q_{x_i} and R_{v_i} are considered to be block diagonal and $G_i \in \mathbb{R}^{n_i\times n_{w_i}}$ denotes the shaping matrix for the state disturbance ωb_{x_i}. Based on (3) and (4), the observer predictor equation for subsystem i is written as

$$\hat{x}_i(k+1|k) = A_{ii}\hat{x}_i(k|k-1) + B_{ii}u_i(k) + \sum_{j=1,j\neq i}^{N}\Big[A_{ij}\hat{x}_j(k|k-1) + B_{ij}u_j(k)\Big]$$

$$+ L_{ii}\Big[y_i(k) - C_{ii}\hat{x}_i(k|k-1)\Big] + \sum_{j=1,j\neq i}^{N} L_{ij}\Big[y_j(k) - C_{ij}\hat{x}_j(k|k-1)\Big] \quad (12)$$

The objective is to design distributed observers consisting of N separate communicating observers which minimize the local state estimation error covariance matrix. Let (A_{ii}, C_{ii}) be detectable for each $i = 1, 2, ..., N$, then, it is possible to construct a local observer of the form (12) for subsystem S_i. The Kalman gain $L_{ii}(k)$ can be computed at every step k so that it minimizes the estimation error covariance matrix $P_{ii}(k)$. Then, assuming that at step $k-1$, the prediction error covariance matrix is $P_{ii}(k-1)$, the estimation error covariance matrix and the Kalman gain at step k are expressed by

$$P_{ii}(k) = \Big(I - L_{ii}(k)C_{ii}\Big)\Big(A_{ii}P_{ii}(k-1)A_{ii}^T + \sum_{j=1,j\neq i}^{N} A_{ij}P_{jj}(k-1)A_{ij}^T + G_iQ_{x_i}G_i^T\Big) \times \Big(I - L_{ii}(k)C_{ii}\Big)^T + L_{ii}(k)R_{v_i}L_{ii}^T(k) + \sum_{j=1,j\neq i}^{N} L_{ii}(k)C_{ij}P_{jj}(k-1)\Big(L_{ii}(k)C_{ij}\Big)^T \quad (13)$$

$$L_{ii}(k) = \Big(C_{ii}(A_{ii}P_{ii}(k-1)A_{ii}^T + \sum_{j=1,j\neq i}^{N} A_{ij}P_{jj}(k-1)A_{ij}^T + G_iQ_{x_i}G_i^T\Big)^T \times \left[\left(C_{ii}\Big(A_{ii}P_{ii}(k-1)A_{ii}^T + \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}P_{jj}(k-1)A_{ij}^T + G_iQ_{x_i}G_i^T\Big)C_{ii}^T + R_{v_i} + \sum_{j=1;j\neq i}^{N} C_{ij}P_{jj}(k-1)C_{ij}^T\right)^{-1}\right]^T \quad (14)$$

The following lemma establishes a design procedure for distributed estimation.

Lemma(Ventak et al. [2006a]): Let the couple (A, C) of the overall system (1) be detectable and let (A_{ii}, C_{ii}) also be detectable for each $i = 1, 2, ..., N$. The set of subsystem-based distributed observers is given by (12) with

- L_{ii} from (14)
- $L_{ij} = A_{ij}C_{jj}^T(C_{jj}C_{jj}^T)^{-1}$

In order to implement the distributed Kalman filter, we proceed by the following two steps

- Prediction

$$\hat{x}_i(k|k-1) = A_{ii}\hat{x}_i(k-1|k-1) + B_{ii}u_i(k-1) + \sum_{j=1;j\neq i}^{N}\Big[A_{ij}\hat{x}_j(k-1|k-1) + B_{ij}u_j(k-1)\Big] \quad (15)$$

$$P_{ii}(k|k-1) = A_{ii}P_{ii}(k-1|k-1)A_{ii}^T + \sum_{j=1;j\neq i}^{N}\Big[A_{ij}P_{jj}(k-1|k-1)A_{ij}^T\Big] + G_iQ_{x_i}G_i^T \quad (16)$$

  At each site, this prediction steps are performed locally prior to information exchange between the different sites.

- Estimation (update or correction):

$$\hat{x}_i(k+1) = \hat{x}_i(k|k-1) + L_{ii}\Big[y_i(k) - C_{ii}\hat{x}_i(k|k-1)\Big] + \sum_{j=1;j\neq i}^{N} L_{ij}\Big[y_j(k) - C_{ij}\hat{x}_j(k|k-1)\Big] \quad (17)$$

$$P_{ii}(k) = \Big(I - L_{ii}(k)C_{ii}\Big)\Big(P_{ii}(k|k-1)\Big)\Big(I - L_{ii}(k)C_{ii}\Big)^T + L_{ii}(k)R_{v_i}L_{ii}^T(k) + \sum_{j=1;j\neq i}^{N} L_{ii}(k)C_{ij}P_{jj}(k|k-1)\Big(L_{ii}(k)C_{ij}\Big)^T \quad (18)$$

For the ith sub-controller $\mathscr{C}_i$, the algorithm is described as follows. The desired output $Y_i^d(k+l|k)$ for sub-controller $\mathscr{C}_i$ is provided by a proper reference generator. Each sub-controller $\mathscr{C}_i$ implements the following steps

(1) Set $k = 1$
(2) Acquire by network the predicted future state trajectories $\hat{X}_j(k, p|k-1)$ and control inputs $U_j(k-1, m|k-1)$ from sub-controllers $\mathscr{C}_j$.
(3) Build $\hat{X}(k, p|k-1)$ and $U(k-1, m|k-1)$ by combining the local state trajectory $\hat{X}_i(k, p|k-1)$ and control input $U_i(k-1, m|k-1)$ with the acquired information, and compute the corresponding predictions of the interactions see (7).
(4) Get state estimations $\hat{x}_i(k)$ from the local Kalman filter and the desired trajectory $Y_i^d(k+l|k)$ over the horizon p.
(5) Compute the optimal control sequence and broadcast it by network to sub-controllers $\mathscr{C}_j$, see (9).
(6) Apply the first element $u_i(k) = u_i(k|k) = \Gamma_i U_i(k, m|k)$ of the optimal sequence $U_i(k, m|k)$ as control input to S_i.
(7) Compute the future state trajectory of subsystem S_i over the horizon p and broadcast it by network to sub-controllers $\mathscr{C}_j$ see (8).
(8) Increment the sample time index $k \leftarrow k+1$ and go to step 2.

A diagram which represents the structure of the DMPC controller is presented in Fig. 1

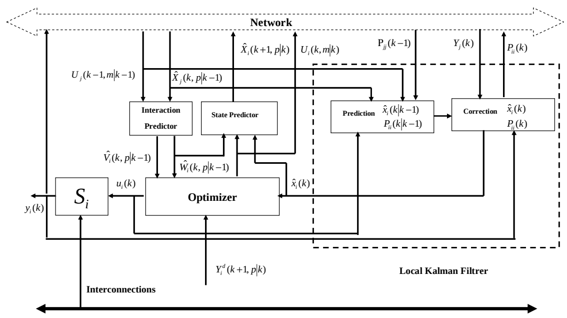

Fig. 2. Internal structure of the ith MPC sub-controller $\mathscr{C}_i$.

6. NUMERICAL EXAMPLE AND APPLICATION TO AN FTC ISSUES

In this section we illustrate the DMPC/DKF on a nominal model and apply it also to an FTC problem through an example. Component failures such as actuator, sensor and even network failure are inevitable during process runtime. Such faults may change the dynamics of system, lead to performance degradation, and even result in instability. In this section, we illustrate the fault tolerance capability of the DMPC/DKF to handle failures on controlled system trough simulation. Consider the following randomly generated discrete-time system

6.1 Nominal case

$$x(k+1) = \begin{bmatrix} 0.52 & 0.028 & 0 & 0.1 & 0 & 0 \\ 0 & 0.772 & 0.002 & 0 & 0 & 0.2 \\ 0 & 0 & 0.0407 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0.107 & 0 & 0 \\ 0.02 & 0 & 0.02 & 0 & 0.21 & 0.034 \\ 0 & 0 & 0.02 & 0.02 & 0 & 0.99 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 0.001 & 0 \\ 0.787 & 0 \\ 0 & 0.787 \\ 0 & 0 \\ 0 & 0.001 \end{bmatrix} u(k) + \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0\,0 \\ 0 & 0.1 & 0 & 0 & 0\,0 \\ 0 & 0 & 0.1 & 0 & 0\,0 \\ 0 & 0 & 0 & 0.1 & 0\,0 \\ 0 & 0 & 0 & 0 & 0.10 \\ 0 & 0 & 0 & 0 & 00.1 \end{bmatrix} \omega b(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \qquad (19)$$

The nominal control objective is to keep the output at specified desired values in the face of states and output disturbance. The examination of the process model leads to decomposition into two interconnected subsystems S_1 and S_2 of the form (3) with state-space realization.

$$A_{11} = \begin{bmatrix} 0.52 & 0.028 & 0 \\ 0 & 0.772 & 0.02 \\ 0 & 0 & 0.0407 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0.2 \\ 0.05 & 0 & 0 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} 0.107 & 0 & 0 \\ 0 & 0.21 & 0.034 \\ 0.02 & 0 & 0.99 \end{bmatrix}, A_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0.02 & 0 & 0.02 \\ 0 & 0 & 0.02 \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 0 \\ 0.001 \\ 0.787 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0.787 \\ 0 \\ 0.001 \end{bmatrix}$$

$$C_{11} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

Based on the decomposition, the DMPC/DFK sub-controllers

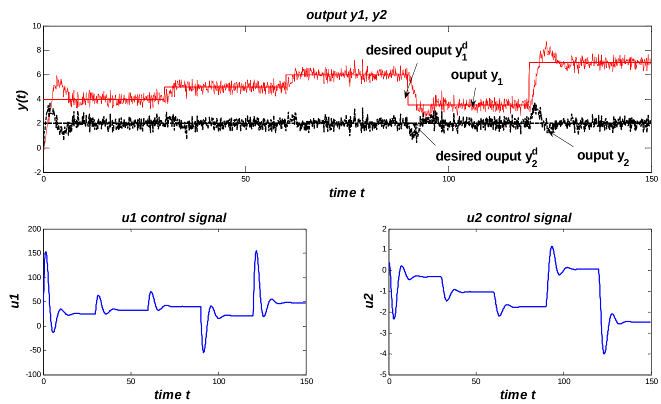

Fig. 3. Output responses with controllers implementing the proposed distributed MPC/KF coordination strategy.

$\mathscr{C}_1$ and $\mathscr{C}_2$ of Fig.2 are computed and implemented on the subsystems. The simulation result illustrated in Fig.3 shows clearly that the proposed DMPC/DFK algorithm performs quite very well. Here each MPC needs his local KF to estimate the relevant state from noisy measurement and input disturbance model to eliminate steady-state offset.

6.2 *Fault case*

In the context of the set-point tracking problem, we consider partial actuator failure as evidenced by a reduction of control effectiveness. When the ith actuator fails, such fault can be expressed by

$$u_i^f = \left(I_{n_{u_i}} + \gamma_i(k)\right) u_i \qquad (20)$$

where $-1 \leq \gamma_i(k) \leq 0$, $i = 1, 2, \ldots, N$ are control effectiveness factors. The two extreme cases $\gamma_i(k) = 0$ and $\gamma_i(k) = -1$ relates to the faulty-free case and to the complete actuator failure case respectively. However, the system should be remain controllable under actuator fault and thus excludes (*de facto*) the case $\gamma_i(k) = -1$ for all $i = 1, 2, \ldots, N$. The state equation with partial actuator failures reads as

$$\begin{aligned} x_i(k+1) &= A_{ii}x_i(k) + B_{ii}\left(I_{u_{y_i}} + \gamma_i(k)\right)u_i(k) + w_i(k) + G_i\omega b_{x_i} \\ y_i(k) &= C_{ii}x_i(k) + v_i(k) + \nu b_i \qquad i = 1, \ldots., N \end{aligned} \qquad (21)$$

We assume knowledge of the evolution and the estimation of the control effectiveness factors $\gamma_i(k)$ which might be provided by local FDI modules. This fault information is passed on-line to the sub-controller to yield the internal faulty model (21) in the place of the fault free model given by (11). The internal faulty model matches therefore the actual plant dynamics for the MPC formulation.
For the plant (19), we consider a partial actuator failure with γ_2=30% occurring at time $t = 80s$, and we assume that at time $t = 100s$ the fault is detected and isolated by a FDI module. Using the updated on-line local faulty model, the DMPC/DKF strategy is illustrated in Fig. 4 where it is shown that the proposed cooperative-based algorithm has the ability to cope with actuator failure in interconnected system.

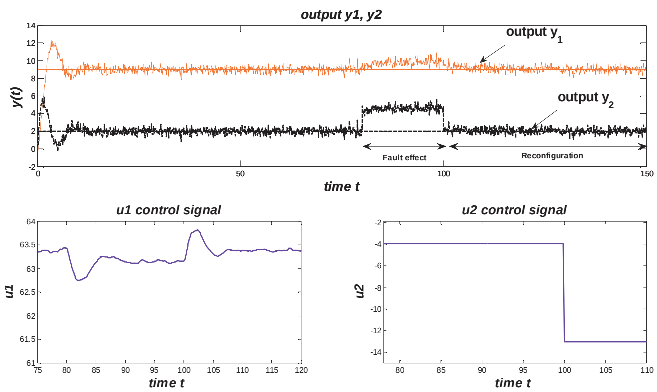

Fig. 4. Fault accommodation effect.

7. CONCLUSIONS AND FUTURE WORKS

7.1 *Conclusions*

In this present study, a formulation of distributed state prediction and model predictive control for linear interconnected process is presented. The solution is based on local Kalman filters that provides state estimations, which are used in MPC formulation when the states are unmeasurable. The methodology is demonstrated on an example and track the set-point without error. Furthermore, a fault tolerant DMPC/DFK scheme is shown to be easily implementable under actuator failure in some part of the overall system.

7.2 Future Works

The challenge posed by distributed control systems is autonomy in the presence of faults. This implies that the system should be renconfigurable and fail-safe. Thus it is necessary to define a suitable robust observers scheme for diagnosis issues to generate the fault information. This information provided by a local FDI module is used to update the on-line model of the MPC formulation. The future work will focus on the distributed FDI/FTC design based on DMPC formulation.

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