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Reconfigurable Control Design for Over-actuated Systems based on Reliability Indicators

Ahmed Khelassi, Philippe Weber and Didier Theilliol

***Abstract*— Control allocation is a solution to distribute the control efforts among a redundant set. A new approach to manage the actuators redundancy in the presence of faults is proposed based on reliability indicators. The aim is to preserve the health of the actuators and the availability of the system both in the nominal behavior and in the presence of actuator faults. In degraded functional, a reconfigured control allocation strategy is proposed based on the on-line re-estimation of actuators reliability. A benefit of incorporate the reliability indicators on the over-actuated control system design is to manage smartly the redundant actuators and improve the safety of the system. The proposed approach is illustrated with a flight control application.**

I. Introduction

In order to respect the growing of economic demand for high plant availability, and system safety, dependability is becoming an essential need in the industrial automation. In this context and to satisfy these requirements, fault-tolerant control (FTC) is introduced. The aim of FTC is to keep plant available by the ability to achieve the objectives that have been assigned in the faulty behavior and accept reduced performance when critical faults occur [2], [16]. In most safety critical systems, the actuators redundancy is often used such as the three major control effectors in aircraft flight control (aileron, elevator and rudder). They are usually designed utilizing one control effector or actuator for each rational degree of freedom. However, due to the increased requirements on the reliability maneuverability and survivability of modern and future aircraft, control effectors are no longer limited to these three conventional control effectors and many more control actuators have been introduced.

Several tools and approaches have been proposed and used to manage the redundancy and to distribute the desired control efforts among a set of actuators. A common approach is to use the optimal control theory to shape the closed-loop dynamics and to distribute desired control efforts in one step. Optimized methods like linear quadratic control [14] and robust control [24] are readily available.

An alternative strategy is to separate the regulation task from the control distributing. Indeed, the control law specify only the desired control efforts to be produced, and a separate *control allocation module* is introduced in the control loop to distribute the control among the actuators [17]. This strategy is used in practical applications in aerospace control. In [8], the control allocation is applied for several airplane flight. [4] illustrate this technique for an F-18 fighter with seven independent moments. A review of existing methods can be found in [9]. In degraded functional, the reconfigurable control allocation is utilized. The advantage of this strategy is the ability to accommodate the control surface damages without modify the controller parameters. Different approaches of control re-allocation have been proposed for the flight control systems. In [1], an on-line control allocation with a sliding modes controller is proposed. [5] reformulates the control allocation problem based on a quadratic programming problem where the solution can be found. A reconfigurable control allocation based on Pseudo-Inverse Methods is illustrated in [23].

The main goal of these methods is to improve the safety and the reliability of the system, which is rarely associated with an objective criterion that guides a design [20]. Some works have introduced the reliability analysis for fault-tolerant control systems in order to take into account the health of the actuators in the reconfiguration strategy [12]. The reconfigurability analysis has been investigated for a reliable fault-tolerant control design in [13]. In this context, a reconfigurable control allocation design is proposed in this paper based on the actuators reliability. The aim is to manage smartly the redundant actuators in order to satisfy the performance requirements and improve the probability of the mission success.

The paper is organized as follows: the reconfigurable control allocation issue for actuator faults is presented in Section II. The reformulation of the reconfigurable control allocation problem with integrating the reliability requirements is proposed in section III. In section IV, solutions for a reliable reconfigurable control of over-actuated systems is presented based on the actuators reliability indicators. The proposed approach is illustrated and applied to a linearized aircraft model from ADMIR simulator in section V. Finally, concluding remarks are given in the last part of this work.

II. Reconfigurable control allocation problem

Let us consider the LTI system be given by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_u u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B_u \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are respectively, the state, the control and the output matrices. $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the system output, and (A, B_u) is stabilizable. Control allocation is generally used for over-actuated systems, where the number of operable control is greater than the controlled

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variables. Let us assume that $rank(B_u) = k < m$. This implies that B_u can be factorized as:

$$B_u = B_v B$$

where $B_v \in \mathbb{R}^{n\times k}$ and $B \in \mathbb{R}^{k\times m}$. An alternative description of (1) can be given as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_v v(t) \\ v(t) &= Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

where $v \in \mathbb{R}^k$ is the virtual control input, called as the total control efforts produced by the actuators and defined by the controller. For simplicity and for this study, the case $k = p$, i.e., when the number of virtual controls equals the number of variables to be controlled is considered.

The control allocation problem can be expressed as a constrained linear mapping problem based on the relation ship,

$$v(t) = Bu(t) \tag{3}$$

$$u_{min} \leq u \leq u_{max} \tag{4}$$

where (4) is the physical actuators saturation. Optimized based control allocation methods aim to find an optimal solution. If there is no exact solution, the optimal control is the feasible one such that $Bu(t)$ approximates $v(t)$ well as possible. The optimal control input can be obtained by a two-step optimization, namely sequential quadratic programming:

$$\psi = \underset{u_{\min}\leq u\leq u_{\max}}{\arg\min} \|Bu - v\|_2 \tag{5}$$

$$u = \arg\min_{u\in\psi} \|W_u u\|_2 \tag{6}$$

where ψ is the set of feasible solutions subject to the objective criterion (6). The weighting matrix $W_u \in \mathbb{R}^{m\times m} \succ 0$ is used to give a specific priority level to the actuators.

Due to abnormal operation or material aging, actuator faults can occur and increase the complexity to solve the control allocation problem on-line. In this work, loss of effectiveness control is considered where the system (2) can be written in degraded functional as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_v v(t) \\ v(t) &= B_f u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{7}$$

The matrix B_f can be written according to the nominal control input matrix B and the control effectiveness factors $\gamma_i \in [0 \;\; 1]$, $i = 1, \ldots, m$, as follows:

$$B_f = B(I - \Gamma), \quad \Gamma = \begin{pmatrix} \gamma_1 & & & 0 \\ & \gamma_2 & & \\ & & \ddots & \\ 0 & & & \gamma_m \end{pmatrix}$$

Indeed, if $\gamma_i = 0$, then the ith actuator is considered in fault-free case. Nevertheless, when $0 < \gamma_i < 1$, a fault which present a partial loss of effectiveness control is considered. Moreover, when $\gamma_i = 1$ failure is considered and the ith actuator is out of order.

In order to improve the safety of the system and preserve the actuators, a specific choice of the weighting matrix W_u is proposed based on the actuators reliability indicators. The weighing matrix W_u is considered as a key to manage the redundant actuators and contribute to a reliable controller which improve the system reliability. This technique can increase the life time of the system and prevent additional faults from occurring.

III. Reliable control allocation design

Reliability $R(t)$ is defined as the probability that units, components, equipments and systems will accomplish its intended function for a specified period of time under some stated conditions and specific environments [11].

In many situations and especially in the considered study, failure rates are obtained from components under different levels of loads. Several mathematical models have been developed to define the failure level in order to estimate the failure rate λ [15]. Proportional hazard model introduced by [6] is used in this paper.

***Definition** 1:* The failure rate is modeled as follows:

$$\lambda_i = \lambda_i^{bl} \times g_i(\ell, \vartheta) \tag{8}$$

where λ_i^{bl} represents the baseline failure rate (nominal failure rate) for the ith subsystem or component and $g_i(\ell, \vartheta)$ is a function (independent of time) taking into account the effects of applied loads with ℓ presenting an image of the load and ϑ defining some parameters of the subsystem or component.

***Definition** 2:* Different definitions of the load function $g_i(\ell, \vartheta)$ exist in the literature [15]. However, the exponential form is commonly used [7]. Moreover, the failure rate functions for the exponential distribution change according to the load level, which are assumed to be directly associated to the control input.

$$g_i(\ell, \vartheta) = g(\|u_i\|) = e^{f(u_i)} \tag{9}$$

where $f(.)$ is an increased monotonic function.

***Definition** 3:* For the exponential distribution of reliability, the main time before the first failure (MTTF) can be adopted as a reliability indicator defined such as:

$$MTTF = \int_0^\infty R(t)dt = \frac{1}{\lambda} \tag{10}$$

Moreover, for m redundant components, the overall system reliability can be computed at the end of the mission defined by $t = t_M$ as follows:

$$R_g(t_M) = 1 - \prod_{i=1}^{m} (1 - R_i(t_M)) \tag{11}$$

In order to prevent the actuators and the safety of the system, the desired efforts in the control allocation problem will be distributed to the different actuators taken into account their reliability characteristics. As given in (8), the failure rate of the actuator can be defined according to the load level which is proportional to the applied control input,

$$\lambda_i^{bl} \geq \lambda_i, \quad i = 1, \ldots m \tag{12}$$

Moreover, the optimal control input $u^* = (u_1^*, u_2^*, \ldots, u_m^*)$, solution of the control allocation problem (5) and (6) is defined according to the values of the weighing matrix $W_u = diag\{w_1, w_2, \ldots, w_m\}$. To perform the solution of the control allocation problem, and keep the set of the actuators available as long as possible, the desired efforts $v(t)$ defined by the controller can be distributed proportionally to the actuators reliability as follows:

$$W_u = \begin{pmatrix} \frac{\lambda_1^{bl}}{\lambda_{max}^{bl}} & & & 0 \\ & \frac{\lambda_2^{bl}}{\lambda_{max}^{bl}} & & \\ & & \ddots & \\ 0 & & & \frac{\lambda_m^{bl}}{\lambda_{max}^{bl}} \end{pmatrix} \succ 0 \tag{13}$$

where $\lambda_{max}^{bl} = \max(\lambda_i^{bl})$ is the upper failure rate corresponding to the least reliable actuator.

As a direct consequence, for $\lambda_i^{bl} << \lambda_{max}^{bl}$, $\gamma_i \to 0$ and so the associated control component u_i^* (solution of the optimization problem (6)) becomes very large. In addition, if $\lambda_i^{bl} \to \lambda_{max}^{bl}$, $\gamma_i \to 1$ and the associated control input is weighted heavily. The actuators are utilized in the control allocation proportionally to their healths. This off-line synthesis of the control allocation strategy less damage the sensible actuators which, improve the system safety and minimize the actuators aging. Indeed, the following relation can be achieved:

$$\lambda_i^{bl} \to \lambda_{max}^{bl} : u_i^* \to 0 \ \} \tag{14}$$

where λ_{min}^{bl} is the failure rate of the most reliable actuator.

IV. On-line control re-allocation synthesis based on reliability indicators

In the degraded behavior and after fault occurrence, the desired efforts are distributed to the actuators based on the re-estimation of their reliability indicators. The control inputs $u_i,\ i = 1, \ldots m$ are re-allocated taken into account the actuators aging.

A. On-line reliability estimation

The control redistributing in the faulty case involve the update of the weighting matrix and the estimation of the actuators reliabilities according to the time of fault occurrence. In the next of this work, fault is assumed detected and isolated at time $t = t_f$. As presented previously, the MTTF is defined like the expected value of the failure distribution $V(t)$:

$$MTTF = \int_0^\infty tV(t)dt = \int_0^\infty -t\frac{dR(t)}{dt}dt \tag{15}$$

Indeed, for a reconfigurable system at $\tau = t_f + \Delta t$, the mean time before the first failure MTTF can be written according to $t = \tau$ as follows:

$$MTTF_\tau = \int_\tau^\infty tV(t)dt \tag{16}$$

where $MTTF_\tau$ can be seen as an estimation of the mean time before failures for a new reconfiguration. In fact, for the exponential distribution of reliability, (16) can be evaluated as follows:

$$MTTF_\tau = \frac{1}{\lambda^\tau} = \int_\tau^\infty \lambda t e^{-\lambda t} dt \tag{17}$$

from where the relation defining the evaluation of the failure rate according to t_f can be obtained in as follows:

$$\lambda^f = \left.\frac{\lambda}{\lambda t + 1} e^{\lambda t}\right|_{t=\tau} \tag{18}$$

where λ is calculated according to the load level defined for $t \in [0, t_f]$ as in (8). In order to integrate the actuators degradation in the reconfigured control allocation strategy, the control input can be obtained by solving the optimization problem (5) and (6) where, for $t \in [\tau, t_M]$, B is replaced by B_f and the weighting matrix W_u is re-estimated and changed on-line according to the new failure rates values λ_i^f.

$$W_u = \begin{pmatrix} \frac{\lambda_1^f}{\lambda_{max}^f} & & & 0 \\ & \frac{\lambda_2^f}{\lambda_{max}^f} & & \\ & & \ddots & \\ 0 & & & \frac{\lambda_m^f}{\lambda_{max}^f} \end{pmatrix} \succ 0$$

In fact, $\lambda_{max}^f = \max(\lambda_i^f), i = 1, \ldots, m$; is the upper value of λ_i^f corresponding to the most degraded actuator.

Indeed, if an actuator fault occurs, the weighing matrix will be changed on-line W_u and the control input $u(t)$ is re-allocated smartly in order to minimize the use of the sensible actuators.

B. Pseudo-inverse method

In the faulty-case, the control re-allocation problem consist in finding the control input $u(t)$ minimizing (6) and satisfy $B_f u(t) = v(t)$. If the above control constraint (4) is not considered and $u_d = 0$, an explicit solution can be obtained from minimization of the above quadratic problem (5),(6) as follows:

$$\begin{aligned} &\min_u J = \|W_u u\| \\ &s.q \quad B_f u(t) = v(t) \end{aligned} \tag{19}$$

and the solution is given based on a weighted pseudo-inverse as follows:

$$u(t) = W_u^{-1}(B_f W_u^{-1})^+ v(t) \tag{20}$$

where $(+)$ is the pseudo-inverse operator. Obviously, there is no guarantee that the solution will not exceed the constraints. Improved approaches have been proposed to accommodate to the limits. The Redistributed pseudoinverse method (RPI) proposed by [21] is an alternative solution, in which all control inputs that violate their bounds in the pseudoinverse solution are saturated and removed from the optimization. Then the control problem is resolved with only the remaining control inputs as free variables. The cascaded generalized inverse (CGI) method proposed by [4] is an iterative redistributed pseudoinverse. All control inputs that violate their bounds are set saturated values and removed at each step,

and the redistribution process is continued until either the pseudoinverse solution is feasible or all control inputs are saturated. [9] also suggests to compute the pseudoinverse solution iteratively, as in CGI but to only saturate one control input per iteration. For the proposed approach and in order to respect the reliability of the actuators, the most reliable actuator will be saturated first at each step and removed from the CGI optimization problem.

C. Active set method for constrained control allocation

While the actuators saturation are considered, a common technique is to approximately reformulate the sequential optimization problem (5) and (6) as a mixed optimization problem as follows:

$$u^* = \underset{u_{\min} \leq u \leq u_{\max}}{\arg\min} \|Bu - v\|_2 + \gamma \|W_u(u - u_d)\|_2 \quad (21)$$

where $\gamma >> 1$. u_d is the desired control input. In fact, the Weighted Least Squares algorithm based on the set active method [18] can be used to found a good approximation of the virtual control input.

The bound and equality constrained least squares problem may write as follows:

$$u = \arg\min_u \|\bar{A}u - \bar{b}\|_2 \quad (22)$$

$$\begin{aligned} Bu &= v \\ \bar{C}u &\geq U \end{aligned} \quad (23)$$

where $\bar{C} = \begin{bmatrix} I \\ -I \end{bmatrix}$ and $U = \begin{bmatrix} u_{min} \\ -u_{max} \end{bmatrix}$.

The active set method solves this problem by solving a series of equality constraints problem. Indeed, the control allocation problem (21) can be written as the following cost function:

$$\|Bu - v\|_2 + \gamma \|W_u(u - u_d)\|_2 = \left\| \begin{pmatrix} \gamma^{\frac{1}{2}} B \\ W_u \end{pmatrix} u - \begin{pmatrix} \gamma^{\frac{1}{2}} v \\ W_u u_d \end{pmatrix} \right\|^2 \quad (24)$$

This problem is equivalent to the constrained least squares problem (22) where,

$$\bar{A} = \begin{pmatrix} \gamma^{\frac{1}{2}} B \\ W_u \end{pmatrix}, \quad \bar{b} = \begin{pmatrix} \gamma^{\frac{1}{2}} v \\ W_u u_d \end{pmatrix},$$

The optimal control input can be found as following: Let u^0 be a feasible starting point, satisfying the constraints (23). Let the working set $\mathcal{W}$ contain the active inequality constraints at u^0. Given a sub-optimal iterate $u^i,\ i = 1, \ldots m$, find the optimal perturbation p considering the inequality constraints in the working set as equality constraints and disregarding the remaining inequality constraints. Solve

$$\min_p \|\bar{A}(u^i + p) - \bar{b}\| \quad (25)$$

$$\begin{aligned} Bp &= 0 \\ p_i &= 0, i \in \mathcal{W} \end{aligned} \quad (26)$$

For one situation, if $u^i + p$ is feasible, set $u^{i+1} = u^i + p$ and compute the Lagrange multipliers in the following form,

$$\bar{A}^T(\bar{A}u - \bar{b}) = (B^T \bar{C}_0^T)\begin{pmatrix} \mu \\ \psi \end{pmatrix} \quad (27)$$

where $\bar{C}_0$ contains the rows of $\bar{C}$ that correspond to constraints in the working set. μ and ψ are associated to with the active constraints in (23).

In fact, if all $\psi \geq 0$, u^{i+1} is the optimal solution. Iteration will stop with $u = u^{i+1}$, else, remove the constraints associated with the most negative ψ from the working set. However, for another situation, if $u^i + p$ is infeasible, the maximum step α length should be determined such that $u^{i+1} = u^i + \alpha p$ is feasible. Then the bounding constraints at u^{i+1} is added to the working set.

V. Flight control example

The ADMIRE model has been used by several researchers (e.g. [17]) and within the Group of Aeronautical Research ans Technology in Europe (GARTEUR). The linear model used here has been obtained at a low speed flight condition of Mach 0.22 at an altitude of $3000m$ and is similar to the one in [18]. The states are $x = [\alpha\ \beta\ p\ q\ r]^T$ with controlled outputs $y = [\alpha\ \beta\ p]$; where α is the angle of attack (rad), β is the sideslip angle (rad), and p is the roll rate (rad/s), q defines the pitch rate (rad/s) and r is the yaw rate (rad/s). The control surfaces are $\sigma = [\sigma_c\ \sigma_{re}\ \sigma_{le}\ \sigma_r]^T$, which represent the deflections of the canard, right eleven, left eleven and rudder respectively. A linearized model [18] is :

$$A = \begin{bmatrix} -0.5432 & 0.0137 & 0 & 0.9778 & 0 \\ 0 & -0.1179 & 0.2215 & 0 & -0.9661 \\ 0 & -10.5128 & -0.9967 & 0 & 0.6176 \\ 2.6221 & -0.0030 & 0 & -0.5057 & 0 \\ 0 & 0.7075 & -0.0939 & 0 & -0.2127 \end{bmatrix},$$

$$B_u = \begin{bmatrix} 0.0069 & -0.0866 & -0.0866 & 0.0004 \\ 0 & 0.0119 & -0.0119 & 0.0287 \\ 0 & -4.2423 & 4.2423 & 1.4871 \\ 1.6532 & -1.2735 & -1.2735 & 0.0024 \\ 0 & -0.2805 & 0.2805 & -0.8823 \end{bmatrix},$$

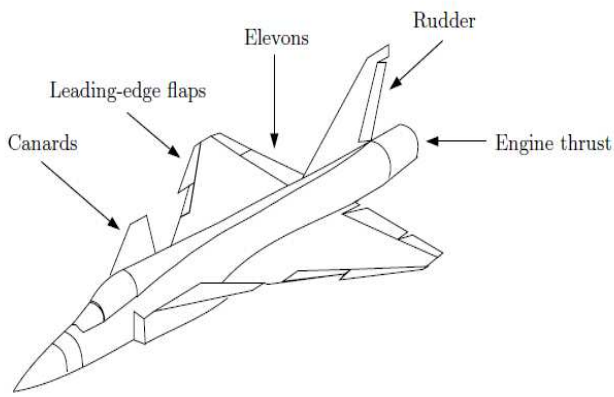

Fig. 1. Aircraft configuration

TABLE I

FAILURE RATES OF ELEMENTARY COMPONENTS

Failure rates	
λ_1	1e-2 min^{-1}
λ_2	6e-3 min^{-1}
λ_3	5e-3 min^{-1}
λ_4	1e-3 min^{-1}

In this example, the actuator dynamics are neglected, and the approximate model can be given where:

$$B_u = B_v B$$

and where

$$B_v = \begin{bmatrix} 0_{2\times 3} \\ I_{3\times 3} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & -4.2423 & 4.2423 & 1.4871 \\ 1.6532 & -1.2735 & -1.2735 & 0.0024 \\ 0 & -0.2805 & 0.2805 & -0.8823 \end{bmatrix},$$

The resulting virtual control input $v(t)$, contains the angular accelerations in roll, pitch, and yaw produced by the control surfaces.

In order to illustrate the proposed approach in the short time window, we adapts the values of the actuators failure rates with the time of the considered scenario. The failure rates are considered with a very huge value and given in the table.1. To model the effects of the applied loads 8 and evaluate the actuators degradation, the root-mean-square (RMS) of the control input applied to each actuator during the mission is considered. In fact the RMS is seen as an image on the average applied load of the different actuators.

In this example, an optimal solution of the reconfigurable control problem is calculated in order to manage smartly the set of the actuators and increase the overall system reliability. The desired efforts are distributed against an optimal choice of the weighting matrix W_u based on the reliability indicators (13). After fault occurrence $\tau = 10s$, an on-line estimation of actuators reliability values $\lambda_i^f, i = 1, \ldots m$ are considered, a new weighting matrix is calculated and a new control re-allocation is applied. The considered control inputs are compared to the an arbitrary choice $W_u = I$.

Figure(2) shows the simulation results when the actuator constraints are not included in the aircraft model. In the considered scenario, partials loss of effectiveness control correspond to the left eleven $\gamma_3 = 0.5$ and $\gamma_3 = 0.7$ are considered successively. Figure(3) shows the control inputs generated by the reconfigured allocation module. In the nominal behavior, the desired efforts for the proposed choice are distributed with respect to the reliability of each actuators. However, after fault occurrence, the distribution is considered taken into account the actuators aging and the rest of their life times. The overall system reliability is evaluated in figure(4) for the different choices of the weighing matrix. In the proposed approach, the distribution of the desired efforts is achieved with a high overall system reliability. This strategy improves the availability of the plant and can also prevent additional faults from occurring.

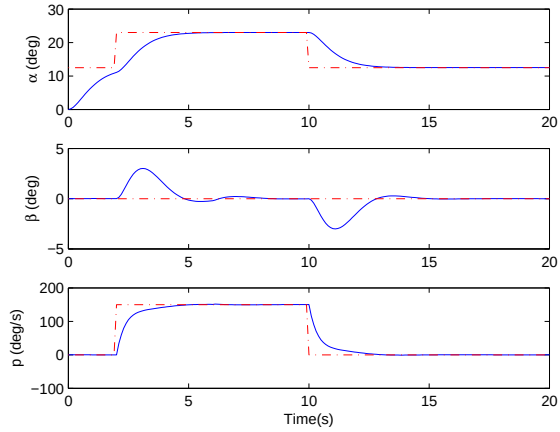

Fig. 2. Aircraft trajectory

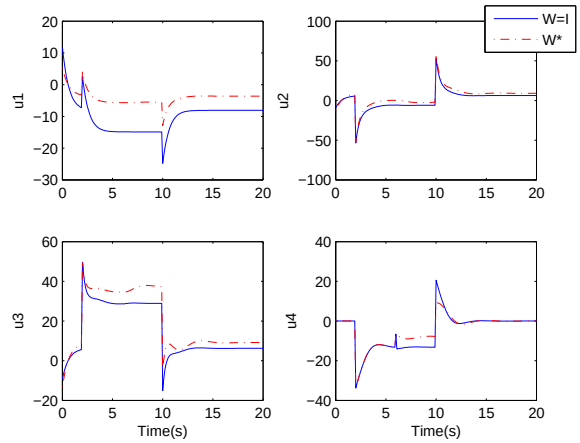

Fig. 3. Control inputs

TABLE II

APPLIED LOAD OF THE ACTUATORS

$\Gamma = diag\{1\ 1\ 1\ 0.5\}$				
	λ_1^{bl}	λ_2^{bl}	λ_3^{bl}	λ_4^{bl}
RMS	0.7948	0.8290	1.3898	0.7264
RMS^*	0.3274	0.8839	1.7000	0.6264
$\Gamma = diag\{1\ 1\ 1\ 0.7\}$				
RMS	0.7948	0.8292	1.3902	0.7263
RMS^*	0.3264	0.8851	1.7246	0.5877

The table.II shows the evolution of the considered applied load according to the energy consumption by each actuator for different scenarios. For the proposed approach, the actuators are requested in the control allocation due to their baseline failure rates where the less sensible actuator is less used in the efforts distribution. The applied loads change in the degraded behavior according to health of the actuators.

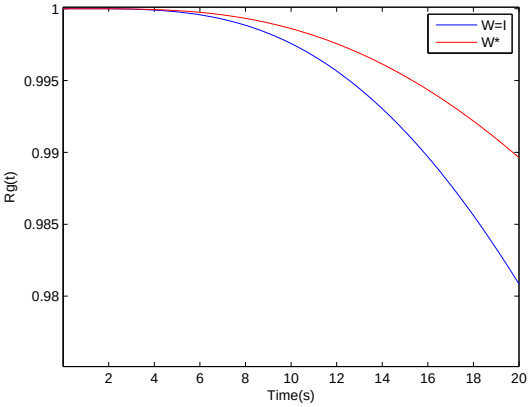

Fig. 4. Overall system reliability evolution

VI. Conclusion

A contribution for an optimal reconfigurable control allocation strategy against reliability is proposed. The distribution of the desired efforts computed by the control law is considered based on the actuators reliability. An optimal choice of the weighing matrix is proposed based on the characteristic of the implemented actuators. the actuators aging is considered in the control re-allocation problem where, an on-line estimation of the reliability is integrated and a new choice of the weighing matrix is obtained. This strategy can preserve and improve the availability of the actuators during the mission with a high overall system reliability.

References

[1] H. Alwi and C. Edwards. Fault tolerant control using sliding modes with on-line control allocation. *Automatica*, vol. 4, 2008, pp 1859–1866.

[2] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki. Diagnosis and fault tolerant control. *Control Systems Series, Springer-Verlag London*, 2006.

[3] M. Bodson. Evaluation of optimization methods for control allocation. *Journal of Guidance, Control, and Dynamics*, vol. 25, 2002, pp 703–711.

[4] K.A. Bordignon. Constrained control allocation for systems with redundant control effector. *PhDThesis*, Vergenia Polytechnic Institute and State University, 1996.

[5] J. Burken, P. Lu, Z. Wu and C. Bahm. Two reconfigurable flight control design methods: Robust servomechanism and control allocation. *Journal of Guidance, Control, and Dynamics*, vol. 24, 2001, pp 482–493.

[6] D.R. Cox. Regression models and life tables. *Journal of the Royal Statistical Society*, vol. 34, 1972, pp 187–220.

[7] J.C. Dale. Application of the proportional hazards model in the reliability field. *Reliability Engineering*, vol. 10, 1985, pp 1–14.

[8] W.C. Durham. Constrained control allocation. *Journal of Guidance, Control, and Dynamics*, vol. 16, 1993, pp 717–125.

[9] D. Enns. Control allocation approaches. *AIAA Guidance, Navigation, and Control Conference*, VA, 1998, pp 98–108.

[10] Z. Gao and P. Antsaklis. Stability of the pseudo-inverse method for reconfigurable control systems. *International Journal of Control*, vol. 53, 1991, pp 717-729.

[11] I. Gertsbakh. Reliability theory with applications to preventive maintenance. *Springer*, 2000.

[12] F. Guenab, D. Theilliol, P. Weber and Y.M. Zhang. Fault tolerant control design: A reconfiguration strategy based on reliability analysis under dynamic behaviour constraints. *In Proc. of 6th IFAC symposium Safeprocess*, Beijing, RP China, 2006.

[13] A. Khelassi, D. Theilliol, and P. Weber. Reconfigurability for reliable fault-tolerant control design. *I7th Workshop on Advanced Control and Diagnosis*, Zielon Gora, Poland, 2009.

[14] H. Kawakernaak and R. Sivan. Linear Optimal Control Systems. *New York, Wiley-Interscience*, 1972.

[15] S. Martorell, A. Sanchez, and V.Serradell. Age-dependent reliability model considering effects of maintenance and working conditions. *Reliability Engineering and System Safety*, vol. 64, 2009, pp 19–31.

[16] H. Noura, D. Theilliol, J.C. Ponsart, and A. Chamssedine. Fault tolerant control systems: Design and practical application. *Springer Dordrecht Heidelberg London*, 2009.

[17] O. Harkegard. Backstepping and Control Allocation with Applications to Flight Control. *PhDThesis*, Department of Electrical Engineering Linkoping University, Sweden, 2003.

[18] O. Harkegard and S.T. Glad. Resolving actuator redundancy-optimal control vs. control allocation. *Automatica*, vol. 41, 2004, pp 137-144.

[19] R.J. Patton. Fault-tolerant control: The 1997 situation. *In Proc. IFAC SafeProcess97*, 1997, pp 1033–1055.

[20] D. Theilliol, A. Chamsseddine, Y. Zhang, and P. Weber. Optimal reconfigurable control allocation design based on reliability analysis. 7th *IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, Barcelona, Spain, 2009.

[21] J.C. Virnig and D.S. Bodden. Multivariable control allocation and control law conditioning when control effectors limits. *AIAA Guidance, Navigation, and Control Conference*, Scottsdale, AZ, 1994, pp 572–582.

[22] N.E. Wu. Reliability and fault tolerant control system Part II. *IEEE Conference on decision and control*, Orlando, Florida, USA, 2001.

[23] Y. Zhang, V.S. Suresh, B. Jiang, and D. Theilliol. Reconfigurable control allocation against aircraft control effector failures. *IEEE International Conference on Control Applications*, 2007.

[24] K. Zhou, J.C. Doyle, and K. Glover. Robust and optimal control. *Upper Saddle River*, NJ, 1996.