Construction of full diversity space-time codes based on half-rate invertible linear binary codes

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***Abstract*—In this paper, a novel approach to construct full diversity space-time codes from half-rate invertible linear binary codes is proposed. The proposed construction benefits from the bijective relation between information and redundancy bits to exploit the whole spatial diversity. It also uses the principle of separation between threads to guarantee the full spatial diversity. Thanks to such construction, we are able to design full diversity space-time codes from half-rate invertible linear binary codes whatever the modulation type.**

I. Introduction

Space-time coding for Multi-Input Multi-Output (MIMO) systems has attracted much attention in the last decade, mainly as a coding technique to combat fading in a MIMO environment. It enables to exploit the full spatial diversity in order to improve the reliability of a data transmission in wireless MIMO systems. Alamouti [1] has presented a simple transmission scheme for two transmit antennas ($n_t = 2$), which provides the whole spatial diversity. Based on the pairwise error probability, derived under the assumption of a Maximum Likelihood (ML) decoding at the receiver, Tarokh *et al.* have introduced some criteria [2] to exploit the whole spatial diversity and to maximize the space-time coding gain. Follow-on work by Tarokh *et al.* [3] has generalized for a large number of antennas ($n_t > 2$) the transmit diversity technique proposed by Alamouti. Due to their Orthogonal Designs (OD), these Orthogonal Space-Time Block Codes (OSTBCs) allow a ML decoder complexity linear with the number of transmit antennas. These designs and others that have same properties, referred to as Coordinate Interleaved Orthogonal designs (CIOD), are said to be single-symbol ML decodable linear Space-Time Block (STB) codes [4]. But unfortunately, the rate of such STB codes decay with the number of transmit antennas. Full rate and full diversity linear dispersion STB codes were developed as a solution to not only guarantee a maximum diversity advantage but also to maximize the mutual information between the transmitter and the receiver [5]-[7]. These codes suffer from a mandatory complexity, beyond ML decoding, which increases exponentially with the number of transmit antennas and the modulation efficiency.

However, all the above approaches optimize the space-time encoder and not the whole transmitter structure that inevitably includes an error-correcting code. A joint design of error correction coding, modulation and space-time scheme has been considered in [3] in order to construct the space-time trellis codes (STTCs) that provide a substantial coding gain and a high diversity advantage. These codes have a better performance than STB codes, but since STTCs are based on trellis codes they have a high decoding complexity. An interesting method to systematically construct a family of full diversity STB codes from linear error correcting codes has been proposed in [8], [9]. These codes referred to, in this paper, as Space-Time Error Correcting Codes (STECCs) have an error correction feature at the center of their designs. An explicit construction of such STECCs was proposed in [10] and it was improved from a space-time design point of view in [11]. It has been proved in [12] that when forward error correcting (FEC) code is serially concatenated with such STB codes, the concatenated code can be formatted as a turbo product code, which enables the receiver to better exploit the channel diversity. This feature enhances that such STB codes are more suitable than others for a serial concatenation with FEC codes.

For 2 transmit antennas, we present in this paper a systematic construction of full diversity STECCs built from a class of half-rate invertible linear binary codes. We recall that a half-rate code is said to be invertible if, knowing only the parity-check bits of a codeword, the corresponding information bits can be uniquely determined by an inversion process. Combining the binary rank criterion introduced by Hammons and the thread separation approach used in Threaded Algebraic Space-Time (TAST) construction [6], we prove that a full diversity with a maximum rate, given according to the rate/diversity trade-off [8], can be ensured using half-rate invertible linear binary codes.

The remainder of this paper is organized as follows. In section II, we recall the space-time design criteria for coherent block fading channels where the Channel State Information (CSI) is available only at the receiver. Moreover, we prove that the unified construction of ST codes, proposed by Lu & Kumar, is unable to ensure the full transmit diversity for a class of half-rate invertible linear error correcting codes. In section III, we propose a technique to construct ST codes from such linear error correcting codes, for 2 transmit antennas. In section IV, for the sake of self completeness we exhibit an iterative receiver, which can exploit the available channel diversity, based on the cooperation of an MMSE detector

(low complexity at the detection stage) and a turbo product decoder (good error rate performance). In section V, we present simulation results and finally we provide a conclusion in section VI.

II. System Model

A. Space-time design criteria

We consider a coherent system over an $n_t \times n_r$ non frequency-selective block fading channel. Let T be the number of time slots for transmitting the space-time codeword $\mathbf{X}_{n_t \times T}$, thus the $n_r \times n_t$ receiver signal may be written as

$$\mathbf{Y}_{n_r \times T} = \mathbf{H}_{n_r \times n_t} \mathbf{X}_{n_t \times T} + \mathbf{N}_{n_r \times T} \quad (1)$$

where $\mathbf{H}$ is the $n_r \times n_t$ channel matrix with independent and identically distributed (i.i.d.) zero mean unit variance circularly symmetric complex Gaussian entries and $\mathbf{N}$ is assumed to be the $n_r \times T$ i.i.d. zero mean circularly symmetric complex Gaussian noise.

Under a ML decoding assumption, the pairwise error probability of estimating a codeword $\hat{\mathbf{X}} \neq \mathbf{X}$, at the receiver while $\mathbf{X}$ has been sent, is upper bounded by

$$P(\mathbf{X} \to \hat{\mathbf{X}}) \leq \left(G \frac{E_b}{4N_0}\right)^{-rn_r} \quad (2)$$

where $\frac{E_b}{N_0}$ represents the average signal-to-noise ratio per transmitted bit, $r = \text{rank}(\mathbf{X} - \hat{\mathbf{X}})$ and $G = (\prod_{j=1}^{r} \lambda_j)^{1/r}$. Therefore, the famous space-time design criteria can be summarized as follows: over all pairs of distinct codewords $\mathbf{X}$ and $\hat{\mathbf{X}}$ ($\mathbf{X} \neq \hat{\mathbf{X}}$),

- *Rank criterion*: Maximize the transmit diversity gain r.
- *Product distance criterion*: Maximize the coding gain G.

These design criteria lead to a good performance in terms of Bit Error Rate (BER). From another viewpoint, it was shown in [2], [9] that the transmit diversity gain r and the modulation symbol rate is governed by the following tradeoff

$$R \leq \nu(n_t - r + 1) \quad (3)$$

where ν is the extension degree of the transmission symbol set drawn from the modulation symbol set. Thus the maximum achievable rate obtained by a full transmit diversity ST code is equal to ν modulation symbols per channel use (p.c.u.). For STECCs , we have $\nu = 1$, i.e., each transmit symbol is composed only from one modulation symbol.

B. STECCs [8]

A STECC is a ST code constructed from a linear error correcting code by arranging, according to a judicious mapping, the components of each error correcting codeword in the form of an $n_t \times T$ matrix. Interesting from the implicit relation between components of error correcting codewords, a full transmit diversity can be ensured.

As the unified construction proposed by Lu & Kumar encompasses many STECC constructions, we recall briefly such a basic approach. For the sake of simplicity, we present this construction for Quadrature Amplitude Modulation (QAM), although it can be applied to other types of modulation. Let $\mathcal{C}_I$ be a linear binary $(n_t \times T)$ matrix code, where the elements of each codeword are spatially formatted into an $n_t \times T$ matrix. Moreover, we consider a linear error correcting code such that the binary rank criterion [8] is satisfied, i.e., every binary matrix $\mathbf{C} \in \mathcal{C}_I$ has full rank over the binary field $\mathbb{F} = \{0, 1\}$. Each entry of the space-time error correcting codeword $\mathbf{X}$ can be written as

$$x_{p,q} = \alpha \sum_{u=0}^{U-1} 2^u \theta^{[\mathbf{C}_1^u + 2\mathbf{C}_2^u]_{p,q}} \quad 1 \leq p \leq n_t, \ 1 \leq q \leq T \quad (4)$$

where $\mathbf{C}_1^u$, $\mathbf{C}_2^u$, $i = \sqrt{-1}$, $\theta = i$, $\alpha = 1 + i$ and $m_b = 2U$ is the modulation efficiency. $x_{p,q}$ is the (p, q)th component of the matrix $\mathbf{X}$. Therefore, the resulting STECC achieves a full transmit diversity.

C. Can Lu & Kumar construction ensures a full diversity from half-rate invertible linear binary codes or not ?

According to the rate-diversity tradeoff (3), one can see that the maximum achievable rate for a full spatial diversity STECC is equal to one modulation symbol p.c.u. ($1/n_t$ modulation symbol per antenna and time slot). Thus the optimal number of transmit antennas (optimal rate-diversity tradeoff) to design a full diversity STECC, based on half-rate invertible linear binary codes, is equal to $n_t = 2$. Firstly, we investigate the Lu & Kumar construction on such linear error correcting codes. Let $\mathcal{C}_I^b(2k, k)$ be a half-rate invertible linear binary code, with k its dimension. The binary matrix associated to $\mathcal{C}_I^b$ can be written as

$$\mathbf{C}_{2\times k} = \begin{bmatrix} \mathbf{mP}_1 \\ \mathbf{mP}_2 \end{bmatrix} \quad (5)$$

where $\mathbf{m}$ is a $1 \times k$ vector consisting of information bits, $\mathbf{P}_1$ and $\mathbf{P}_2$ are $k \times k$ binary matrices that represent 2 linear transformations. Then $\mathbf{C}_{2\times k}$ satisfy the binary rank criterion if and only if

$$\begin{cases} \mathbf{mP}_1 = 0 \Leftrightarrow \mathbf{m} = 0, & \mathbf{P}_1 \text{ has full rank over } \mathbb{F} \\ \mathbf{mP}_2 = 0 \Leftrightarrow \mathbf{m} = 0, & \mathbf{P}_2 \text{ has full rank over } \mathbb{F} \\ \mathbf{m}(\mathbf{P}_1 \oplus \mathbf{P}_2) = 0 \Leftrightarrow \mathbf{m} = 0, & (\mathbf{P}_1 \oplus \mathbf{P}_2) \text{ has full rank over } \mathbb{F} \end{cases} \quad (6)$$

where $\oplus$ stands for modulo 2 addition. Multipling by $(\mathbf{P}_1)^{-1}$ (the inverse exists as $\mathbf{P}_1$ has full rank), we obtain

$$\mathbf{C}_{2\times k}\mathbf{P}_1^{-1} = \begin{bmatrix} \mathbf{m} \\ \mathbf{mP}_2\mathbf{P}_1^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{m} \\ \mathbf{mP} \end{bmatrix} \quad (7)$$

where $\mathbf{P}$ can be defined such that $\mathbf{G}_{k\times 2k} = [\mathbf{I}_{k\times k} \ \mathbf{P}_{k\times k}]$ is a systematic generator matrix of $\mathcal{C}_I^b(2k, k)$. Therefore, taking into account a half-rate invertible linear binary code, whatever the spatial arrangement used to form a binary matrix $\mathbf{C}_{2\times k}$ associated to such a code the binary rank criterion is satisfied if and only if $(\mathbf{I} \oplus \mathbf{P})$ is of full rank over $\mathbb{F}$. Such constraint is a hard restriction on the class of half-rate invertible linear binary codes. In other words, this means that if $\mathbf{c}_{1\times 2k} = [\mathbf{m} \ \mathbf{mP}] \in \mathcal{C}_I^b$, thus $\mathbf{m}$ must be different from $\mathbf{mP}$

for all $\mathbf{m} \in \mathbb{F}^k$, $\mathbf{m} \neq \mathbf{0}_{1\times k}$. Thus, as an explicit example, for half-rate invertible linear binary codes that contain all-1 codeword (e.g. Extended Hamming, Extended Golay code, ...), no spatial format can achieve the whole spatial diversity beyond the unified construction proposed by Lu & Kumar.

Motivated to construct full transmit diversity STECC based on half-rate invertible linear binary codes and without any constraints or restriction on such codes, we propose in the next section an approach that allows to ensure the whole spatial diversity. Moreover, the proposed approach has high flexibility as it does not need to determine an appropriate space-time arrangement related to the employed error correcting code.

III. $2 \times T$ STECCs Based on Half-Rate Invertible Linear Binary Codes

In this section, we firstly describe the construction approach, then we prove that full transmit diversity can be achieved by such a construction. We consider codes such that their dimension is greater than 2 ($k \geq 3$) and their minimal Hamming distances are equal or more than 3 ($d^{\min}_{\text{Hamming}} \geq 3$). Let $\mathbf{c}_u \in \mathcal{C}_I^b$, $1 \leq u \leq m_b$ and let $\mathcal{S}$ be the ring of complex numbers that contains the constellation set whatever the modulation efficiency (for example for QAM, $\mathcal{S}$ is the Gaussian integer set $\mathbb{Z}[i]$). In a systematic form a codeword $\mathbf{c}_u \in \mathcal{C}_I^b$ can be written as

$$\mathbf{c}_u = [\underbrace{m_1^u \dots m_k^u}_{\text{information}} \ \underbrace{r_1^u \dots r_k^u}_{\text{redundancy}}] = [\mathbf{m}_u \ \mathbf{r}_u] \tag{8}$$

Thus the STECC can be defined by the following map

$$\mu : (\mathbf{c}_1, \dots, \mathbf{c}_{m_b}) \in (\mathcal{C}_I^b)^{m_b} \mapsto \mathbf{X}_{2\times k} \in \mathcal{S}^{2\times k}, \ T = k \tag{9}$$

such that for $q = 1, \dots, N = \lfloor \frac{k+1}{2} \rfloor$, where $\lfloor z \rfloor$ stands for the largest integer not greater than z, the entries of $\mathbf{X}_{2\times k}$ can be defined as

$$\begin{cases} x_{1,2q} = \phi_q x_{2q} & x_{1,2q-1} = x_{2q-1} \\ x_{2,2q} = y_{2q} & x_{2,2q-1} = \phi_q y_{2q-1} \end{cases} \tag{10}$$

where

$$\begin{aligned} (m_p^1, \dots, m_p^b) &\xrightarrow{\text{Modulation}} x_p \in \mathcal{S} \\ (r_p^1, \dots, r_p^b) &\xrightarrow{\text{Modulation}} y_p \in \mathcal{S} \end{aligned}$$

Moreover elements of $\mathcal{L} = \{\phi_1, \dots, \phi_N\} \in \mathbb{C}^N$ satisfy the following constraints

$$\begin{cases} |\ \phi_p\ | = 1, \ p = 1, \dots, N, \text{to ensure an energy efficiency.} \\ \phi_l \phi_p \notin \mathcal{S}, \ \forall\ p, l = 1, \dots, N \text{ in particular } \phi_p^2 \notin \mathcal{S}. \\ \phi_l \phi_p^* \notin \mathcal{S} \ \forall\ p \neq l, p, l = 1, \dots, N. \end{cases} \tag{11}$$

To demonstrate that such a STECC can realize the full transmit diversity, we must verify that the rank of two different space-time error correcting codewords is equal to $n_t = 2$.

Let $\hat{\mathbf{X}} = \mu(\acute{\mathbf{c}}_1, \dots, \acute{\mathbf{c}}_{m_b})$ be a space-time error correcting codeword such that $\hat{\mathbf{X}} \neq \mathbf{X} \Rightarrow (\mathbf{c}_1, \dots, \mathbf{c}_{m_b}) \neq (\acute{\mathbf{c}}_1, \dots, \acute{\mathbf{c}}_{m_b})$. Thus, there exists at least u, $1 \leq u \leq m_b$, such that $\mathbf{c}_u \neq \acute{\mathbf{c}}_u$ or thanks to the bijection property of $\mathbf{C}_I^b$, we have

$$\mathbf{c}_u \neq \acute{\mathbf{c}}_u \Leftrightarrow \mathbf{m}_u \neq \acute{\mathbf{m}}_u \text{ and } \mathbf{r}_u \neq \acute{\mathbf{r}}_u \tag{12}$$

In addition, since $d^{\min}_{\text{Hamming}} \geq 3$ then either $\mathbf{m}_u$ (resp. $\mathbf{r}_u$) differs from $\acute{\mathbf{m}}_u$ (resp. $\acute{\mathbf{r}}_u$) by two bits (resp. one bit) at least or $\mathbf{r}_u$ (resp. $\mathbf{m}_u$) differs from $\acute{\mathbf{r}}_u$ (resp. $\acute{\mathbf{m}}_u$) by two bits (resp. one bit) at least. Thus there exists at least two different integers l and p, $l, p \in \{1, \dots, k\}$ such that either $(x_l, y_l, x_p) \neq (\acute{x}_l, \acute{y}_l, \acute{x}_p)$ or $(x_l, y_l, y_p) \neq (\acute{x}_l, \acute{y}_l, \acute{y}_p)$, i.e., there exists at least one 2×2 sub-matrix $\mathbf{M}_{2\times 2}$ of the difference matrix $\hat{\mathbf{X}} - \mathbf{X}$, composed from two columns (or their permutations) of the difference matrix, that can be defined as one of the following equations

- if $\lfloor p - l \rfloor_2 = 1$ thus let l is the odd number, p is the even number and $p \neq l + 1$

$$\mathbf{M}_{2\times 2} = \begin{bmatrix} x_l - \acute{x}_l & \phi_{\frac{p}{2}}(x_p - \acute{x}_p) \\ \phi_{\frac{l+1}{2}}(y_l - \acute{y}_l) & y_p - \acute{y}_p \end{bmatrix} \tag{13}$$

- if $\lfloor p - l \rfloor_2 = 1$ thus let l is the odd number, p is the even number and $p = l + 1$

$$\mathbf{M}_{2\times 2} = \begin{bmatrix} x_l - \acute{x}_l & \phi_{\frac{p}{2}}(x_p - \acute{x}_p) \\ \phi_{\frac{p}{2}}(y_l - \acute{y}_l) & y_p - \acute{y}_p \end{bmatrix} \tag{14}$$

- if l, p are odd

$$\mathbf{M}_{2\times 2} = \begin{bmatrix} x_l - \acute{x}_l & x_p - \acute{x}_p \\ \phi_{\frac{l+1}{2}}(y_l - \acute{y}_l) & \phi_{\frac{p+1}{2}}(y_p - \acute{y}_p) \end{bmatrix} \tag{15}$$

- if l, p are even

$$\mathbf{M}_{2\times 2} = \begin{bmatrix} \phi_{\frac{l}{2}}(x_l - \acute{x}_l) & \phi_{\frac{p}{2}}(x_p - \acute{x}_p) \\ y_l - \acute{y}_l & y_p - \acute{y}_p \end{bmatrix} \tag{16}$$

where $\phi_0 = 1$, $\lfloor . \rfloor_2$ denotes the modulo 2 operation and either $(x_l - \acute{x}_l)(y_l - \acute{y}_l)(x_p - \acute{x}_p) \neq 0$ or $(x_l - \acute{x}_l)(y_l - \acute{y}_l)(y_p - \acute{y}_p) \neq 0$. One can see that the sub-matrix $\mathbf{M}_{2\times 2}$ has a nonzero determinant whatever the modulation type if the elements of $\mathcal{L}$ satisfy the second and the third conditions of (11). Therefore, the minimum rank of $\hat{\mathbf{X}} - \mathbf{X}$ is equal to $n_t = 2$.

For QAM constellations, the modulation symbols belong to $\mathbb{Z}[i]$ thus a sufficient condition to ensure the full transmit diversity of these STECCs can be rewritten as follows, $\forall\ p = 1, \dots, N$, $\phi_p = e^{i\theta_p}$ ($|\ \phi_p\ | = 1$) where

$$\theta_p \in \left]0 : \frac{\pi}{4}\right[\cup \left]\frac{\pi}{4} : \frac{\pi}{2}\right[\tag{17}$$

In general, determined values of θ_p, $p = 1, \dots, N$ can be chosen subject to the maximization of the space-time coding gain. But as the STECC is constructed in the aim to be serially concatenaded with a FEC, the space-time coding gain is negligible with respect to the gain offered by the FEC code. Therefore, an optimization of such parameters does not provide a remarkable improvement in the performance, moreover optimum values can depend on the inner code used in the construction and the modulation order.

In fact, these parameters can be viewed as rotation factors. It always guarantees that the two threads, which constitute a space-time codeword, belong to different algebraic sub-spaces. Such a technique, inspired from TAST construction, is very interesting as it does not induce any penalty on the complexity at the receiver. It extends the construction of full diversity

STECC to the class of half-rate invertible linear binary error correcting codes. In addition, it is clear that no constraint are imposed on the spatial arrangement of parity-check bits with respect to information bits. Indeed, this flexibility makes the construction not only extensible to those known in the litterature but also it is easier as no spatial arrangement must be found for redundancy bits with respect to information ones.

IV. Iterative Receiver Structure

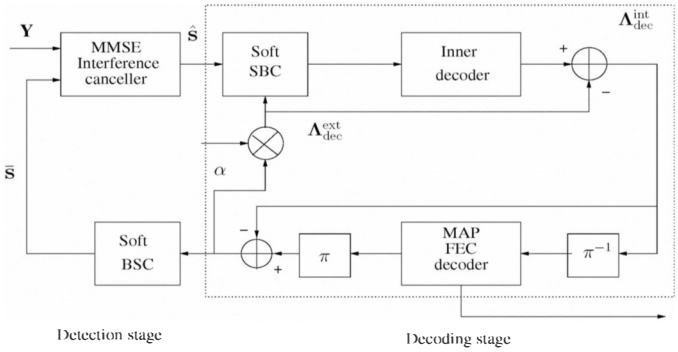

Fig. 1. MMSE interference canceller with SISO turbo product code decoding structure.

This section deals with iterative decoding that can exploit performance of concatenated STECCs with a reasonable complexity. We describe a low complexity modified iterative receiver structure of the one presented in [12]. In our previous work, a *maximum a posteriori* (MAP) symbol detection on the observed signal $\mathbf{Y}$ at each time slot is performed taking into account *a priori* information provided by a Soft-Input Soft-Output (SISO) turbo product code decoder. Despite the BER improvement achieved by such an iterative receiver, unfortunately a MAP symbol detector involves a significant amount of computational complexity for high order modulations and large numbers of transmit antennas. Motivated to reduce the complexity at the detection stage, we replace the MAP symbol detector by a linear detector optimized according to the minimum mean square error (MMSE) criterion (Figure 1). No change is applied to the SISO decoder, which always works according to the turbo principle developed in [13]. It is based on the reliability information exchange between two elementary SISO decoders: the inner one, which exploits the error correcting code used in the construction of the space-time code, and the MAP outer FEC decoder. The modification brought to the detection stage of the iterative receiver presented in [12] reduces its complexity without any degradation on the BER performance thanks to the reliability of the *a priori* information provided by the turbo product code decoder.

V. Simulation Results

In this section, we present simulation results for uncoded and coded (i.e. concatenated) STECCs based on either the extended Hamming code or the extended Golay code. We assume a block Rayleigh flat fading channel constant over τ time slots. Information symbols are carved from a $4-$QAM constellation with Gray mapping. For the FEC code, we use the half-rate convolutional code $CC(7,5)_{oct}$.

At the receiver side, we fix $n_r = 2$ receive antennas. For uncoded STECCs based on the extended Hamming code, we apply a ML decoder. But as the complexity of such receiver becomes prohibitive for uncoded STECCs based on the extended Golay code, we resort to an iterative receiver structure to decode these space-time codes . To be more precise, we apply a MAP symbol detector on the signal observed at each time slot, then the soft *extrinsic* information of this detector is delivered to a SISO inner decoder of the Golay code. The soft *extrinsic* information of the last decoder, based on a sliding encoding window algorithm [14] is injected as *a priori* information to the MAP symbol detector. This iterative procedure continues in such a way until the convergence of the receiver.

For coded STECCs, a channel decoder based on the BCJR algorithm [15] is adopted to decode the convolutional code and for the extended Hamming code, a MAP inner decoder is employed .

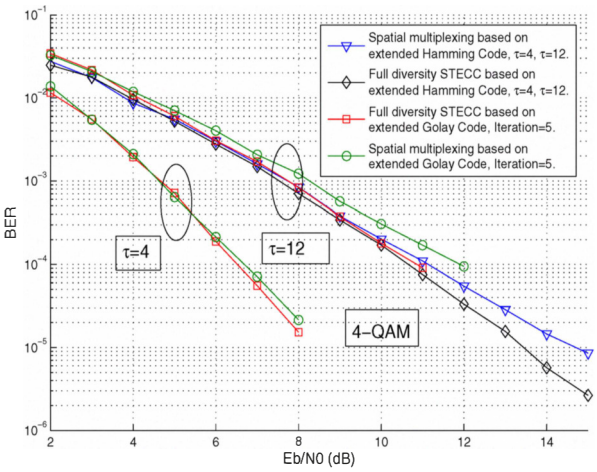

Fig. 2. Performance comparison of the proposed full diversity STECCs and the spatial multiplexing scheme, both are based on either the extended Hamming code or the extended Golay code. $\phi_1^{\text{Hamming}} = e^{i\pi/5}$, $\phi_2^{\text{Hamming}} = e^{i2\pi/5}$. $\phi_q^{\text{Golay}} = e^{qi\pi/25}$, $q = 1, \ldots, 6$

In Figure 2, we compare the BER performance of the full diversity STECCs and the spatial multiplexing scheme, both are based on either the extended Hamming code or the extended Golay code. By spatial multiplexing scheme based on an inner code we refer to a transmission system where data streams are encoded by this inner code and are sent using the multiple transmit antennas without caring to provide a full transmit diversity.

For STECCs based on the extended Hamming code, one can see that the diversity advantage, i.e. the asymptotic slope of BER versus signal-to-noise ratio, for the proposed code (diamond marked curve) is greater than the diversity advantage

realized by the spatial multiplexing scheme (triangle marked curve). This means that the transmit diversity is exploited by the proposed code as it was theoritically proved in section III. We note that, as space-time error correcting codewords are independent of each others, the BER performance remains the same for every τ multiple of T. For $\tau = T = 4$ and $\tau = 3T = 12$, a gain of 1 dB is achieved at a BER of 10^{-5}.

For STECCs based on the extended Golay code, the sub-optimal iterative receiver can not fully exploit all the spatial diversity. Simulations show a gain offered by the proposed code (square marked curve) with respect to the spatial multiplexing scheme (circle marked curve). For $\tau = 12$ (resp. $\tau = 4$) a gain of 1 dB (resp. 0.2 dB) is achieved at a BER of 10^{-4} (resp. 2.10^{-5}). For $\tau = 4 < T$, we note that STECCs based on the Golay code also benefit from the time diversity, which is not available for $\tau = T = 12$

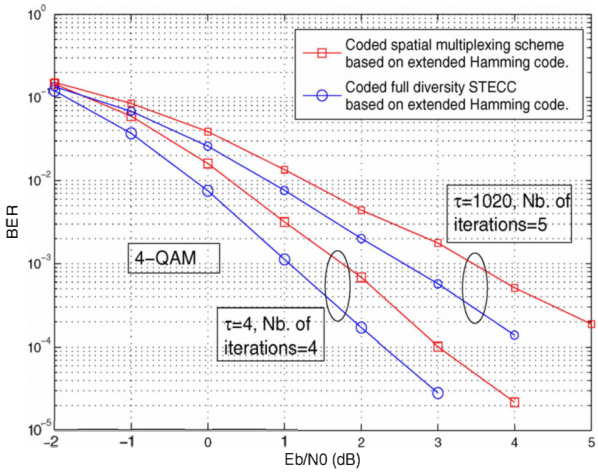

Fig. 3. Performance comparison of full diversity STECCs and the spatial multiplexing scheme, both are based on the extended Hamming code, and are serially concatenated with a convolutional code $CC(7,5)_{\text{oct}}$. 4×1018 information bits. $n_r = 2$. 4−QAM. $\phi_1^{\text{Hamming}} = e^{i\pi/5}$, $\phi_2^{\text{Hamming}} = e^{i2\pi/5}$.

In Figure 3, we show the BER performance of previous schemes based on the extended Hamming code in the case of a serial concatenation with a FEC code. The receiver presented in section IV, is adopted to decode these coded schemes. For $\tau = 4$ (roughly quasi fast fading channel), the coded full diversity STECC outperforms the coded multiplexing scheme by 0.9 dB at a BER=2.10^{-5}. For $\tau = 1020$ (slow fading channel), at a BER=10^{-4} a gain of 1.2 dB is offered by the concatenated full diversity STECC with respect to the coded spatial multiplexing scheme.

It is important to note that, based on a half-rate invertible linear binary code, the receiver complexity for the full diversity STECC is the same as for the spatial multiplexing scheme in the case of uncoded systems as well as for coded systems.

VI. Conclusion

In this paper, we highlight the construction of full diversity space-time codes based on half-rate invertible linear binary error correcting codes. We have developed an approach that benefits from the invertible property of the error correcting code to construct full diversity space-time codes for two transmit antennas whatever the modulation efficiency. We have shown also an iterative receiver structure with low complexity, which can exploit the performance of a system composed of such space-time codes when serially concatenated with a FEC code. Simulations have validated the gains offered by the proposed code with respect to spatial multiplexing scheme for a same complexity receiver structure.

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