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Optimization of the transmission of disruptions in assembly systems

*A. Resano Lázaro and C. J. Luis Pérez

Mechanical, Energetics and Materials Engineering Department. Public University of Navarre. Campus de Arrosadia s/n, 31006 Pamplona. Spain.

*Corresponding author. Email: arturo.resano@unavarra.es

Abstract

The dynamic analysis of the disruptions transmission in networks of closed loops formed by machines and intermediate buffers are of vital importance in most production systems. Nevertheless, little research has been done on optimization in this field up to the present. This study analyzes the disruption time transmission in a generic assembly system, which has been modelled as a network of closed loops of machines and intermediate buffers. In addition, this modelling has been used to analyze a real automobile assembly line, taking into account variables which have not been previously considered by the literature such as: working regimes of machines, their cycle times, capacities of the intermediate buffers and their minimum contents. The optimal configuration of the intermediate buffers is analyzed. Dynamic outlines of these kind of assembly systems are proposed in order to maximize the transmission of disruption times and hence, their availability. For this purpose, an algorithm for analyzing and optimizing availability in this kind of manufacturing systems has also been developed.

Keywords: Assembly lines; Closed loops; Conveyors; Transmission of disruptions.

1. Introduction

Serial transfer lines and closed loops of machines decoupled by intermediate buffers have been widely investigated. However, little research work has been done on networks of several closed loops up to the present. Gershwin (1987) establishes the first decomposition method of transfer lines with machines that have different processing rates. Dallery and Gershwin (1992) review the most relevant models of transfer lines decoupled by intermediate buffers. Levantesi *et al.* (2003) present an approximate analytical method for the performance evaluation of asynchronous serial transfer lines with deterministic processing times, multiple failure modes and finite buffer capacity. Onvural and Perros (1990) prove that the production rate of a closed loop depends on the population of pallets that exists in the system due to the transmission of blocking and starvation in the loop. Frein *et al.* (1996) propose the first analytical method for evaluating the performance of closed loop systems with unreliable machines and finite buffers. However, this method does not consider the correlation existing between the number of parts in the intermediate buffers and the propagation of blocking and starvation on each machine. Gershwin *et al.* (2001) and Werner (2001) decompose one closed loop in a similar way to those used for the evaluation of serial transfer lines. Paik *et al.* (2002) propose some decomposition and aggregation methods for finite-buffered one-closed-loop production systems. Nevertheless, they do not consider machine-processing pallets that are not univocally related to each other. Maggio *et al.* (2003) present an analytical method to predict the average production rate and the work in progress in each intermediate buffer of a closed loop of three machines and Gershwin and Werner (2003) extend it to simple closed loops with more than three machines. However, they do not analyze the performance of networks with more than one closed loop. Levantesi (2001) establishes one of the first analytical methods for the performance evaluation of networks of several closed-loops although he does not take into account buffers that can be formed by conveyors, which is one of the most common configurations in the automobile assembly lines.

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Even though closed-loop conveyors have been analyzed by different authors, few analytical models have been extended to networks with more than one closed-loop conveyor. Bastani (1998) studied a closed-loop conveyor with multiple unloading stations and provided an analytical solution to measure the effect of recirculation times, but this is not the real case in most automobile assembly lines. Bozer and Hiseh (2004, 2005) model a conveyor as a unidirectional closed-loop consisting of discrete spaces or windows of equal size, which hold at least only one load or unit. Nevertheless, the windows of most conveyors in the automobile industry can vary their size in different branches according to the cycle times of their downstream machines and the transportation speed of the conveyor in each branch. Zhuang *et al.* (1997) model and re-design one assembly system using a queueing network model. Nonetheless, they do not take into account that the processing times can vary depending on the number of pallets that exist in their upstream buffers. Kouikoglou (2002) extends the continuous flow simulators for assembly networks developed by Kouikoglou and Phillis (1995, 1997), which are based on the model applied by D'Angelo *et al.* (1998), developing a discrete event simulation model of assembly and disassembly production networks in which the traffic of discrete parts is approximated by a continuous flow. However, these models do not consider the phenomena of blocking and starvation transmission in networks formed by several closed loops.

Zhang (2006) considers new aspects in the analysis of networks of several closed loops such as buffer level matrices to represent the blocking and starvation relationships between machine failures and the limit state buffer levels. Nevertheless, the transmission times of the blocking and starvation phenomena from one machine to the others and the optimal capacities of the different buffers in order to maximize the transmission times and the availability of the entire manufacturing system have not been considered.

Resano Lázaro and Luis Pérez (2007a) establish one of the first analytical models of an automobile assembly line as a network of closed loops of machines decoupled by intermediate buffers formed by conveyors. They demonstrate that machines can work in both, stationary or transitory regimes, depending on the number of pallets that exist in their upstream buffers. The stationary regime is characterized by a continuous feeding of the machines from their upstream buffers with assembly pallets, processing them at a constant cycle time, according to the production rate of the complete manufacturing system. Meanwhile, the transient regime is characterized by a discontinuous feeding of the machines with assembly pallets, working at a cycle time, which depends on the number of pallets in their upstream buffers. In a later work, Resano Lázaro and Luis Pérez (2007b) analyze the influence of new factors on the blocking and starvation transmission in the automobile line considered, such as the working regimes of the machines involved or the four-door cars proportion between the doors' disassembly and assembly stations, represented by the variable x .

However, these previous research works of Resano Lázaro and Luis Pérez (2007a,b) do not consider the factors that influence on the blocking and starvation transmission times in generic assembly systems. This analysis has a great deal of importance because it enables production engineers to maximize the production times when any branch of the assembly system disrupts. Therefore, with this analysis it is possible to increase the number of parts that are being manufactured when a disruption appears.

The main aim of the present paper is to establish generic equations that describe the transmission of disruption times, analyzing the influence of different variables, which have not been previously considered in existing literature. Afterwards, an algorithm to maximize the transmission of disruption times in generic networks of closed loops is developed. This algorithm has been applied to the model previously proposed by Resano Lázaro and Luis Pérez (2007a,b) by means of implementing a system of conveyors with variable capacity. This analysis could be applied to optimize the behaviour of this kind of manufacturing systems, because with the algorithm developed, it is possible to minimize the effect of a disruption on the system.

2. Transmission of disruption times

2.1. Proposed model of a generic assembly system

Figure 1 represents a generic assembly system as a network of closed loops of machines decoupled by intermediate buffers, formed by conveyors. Each machine stands for a branch of the main assembly line or a preassembly line. The dotted arrows, circles and boxes represent an indefinite series of machines decoupled by intermediate buffers, which is the case of most automobile assembly systems.

Figure 1. *Generic assembly system*

The stationary working regime is characterized by a continuous feeding of the machines with transport and assembly pallets. Hence, the minimum contents of the intermediate buffer $B_{i,j}$ required for a stationary working regime of its downstream machine M_j is defined by the minimum number of transport and assembly pallets in this buffer ($m_{i,j}$) required to guarantee a continuous feeding of the downstream machine M_j . As Equation (1) shows, this value depends on: the cycle time of the downstream machine in stationary regime (c_{js}), the length ($l_{i,j}$) of the intermediate buffers and the transportation speed ($v_{i,j}$) of the assembly pallets.

$$m_{i,j} = l_{i,j} / (c_{js} \cdot v_{i,j}) \quad (1)$$

However, the transitory working regime is characterized by a discontinuous feeding of the machines with transport and assembly pallets, therefore the number of transport and assembly pallets ($n_{i,j}$), which exists in each intermediate buffer $B_{i,j}$ for a transitory working regime of their downstream machines (M_j), is defined by

$$0 < n_{i,j} < m_{i,j} = l_{i,j} / (c_{js} \cdot v_{i,j}) \quad (2)$$

The cycle times of machine M_j fed by one intermediate buffer, which work in transitory (c_{jt}) or stationary (c_{js}) regimes, if they suffer neither a starvation from their upstream buffers ($B_{i,j}$) nor a blockage from their downstream buffers ($B_{j,k}$), are defined by

$$\text{If } 0 < n_{i,j} < m_{i,j} \quad c_{jt} = l_{i,j} / (n_{i,j} \cdot v_{i,j}) \quad (3)$$

$$\text{If } n_{i,j} \geq m_{i,j} \quad c_{js} = l_{i,j} / (m_{i,j} \cdot v_{i,j}) \quad (4)$$

In case of machines fed by two intermediate buffers, their cycle times in transitory regime are determined by the intermediate buffer, which feeds them at a lower frequency.

The maximum capacity ($M_{i,j}$) of each intermediate buffer $B_{i,j}$ depends on the ratio between the length of the conveyors ($l_{i,j}$) and the average storing length per transport and assembly pallet ($l_{aspi,j}$), as is determined by

$$M_{i,j} = l_{i,j} / l_{aspi,j} \quad (5)$$

The governing equations of each closed loop relate the sum of the number of pallets stored at any time in the corresponding buffers ($B_{i,j}$) to functions $g(x_m, n_{a,b}, n_{b,c}, \dots, n_{j,a})$, as is shown in matrix Equation (6). These functions relate the number of pallets in the buffers of the corresponding closed loop to external variables x_m , which depend on the number of pallets required in each buffer for assembling one final unit, in our case study, one automobile. That is, buffers, which contain front axles or car body pallets, are univocally related to each other and they do not depend on external variables. However, buffers, which store car body pallets, require $1+x$ door pallets per final unit assembled; as a consequence of they are not univocally

related to each other and depend on the four-door cars proportion between the doors' disassembly and assembly stations.

$$\bar{M} = \bar{g} \quad (6)$$

Where

$$\bar{M} = \begin{bmatrix} n_{i,j} \pm n_{j,k} \pm \dots \pm n_{o,i} \\ n_{k,m} \pm n_{m,n} \pm \dots \pm n_{r,k} \\ \dots \\ n_{d,e} \pm n_{e,f} \pm \dots \pm n_{t,d} \end{bmatrix} \quad \bar{g} = \begin{bmatrix} g(x_1, n_{i,j}, n_{j,k}, \dots, n_{o,i}) \\ g(x_2, n_{k,m}, n_{m,n}, \dots, n_{r,k}) \\ \dots \\ g(x_s, n_{d,e}, n_{e,f}, \dots, n_{t,d}) \end{bmatrix}$$

In case of machine M_i disrupts for a period of time $k_i \cdot T_i$, once it has worked under normal conditions for T_i , its cycle time is governed by

$$\begin{cases} \text{If } t_0 \leq t \leq T_i & c_i = c_i \\ \text{If } T_i \leq t \leq T_i + k_i \cdot T_i & c_i = 0 \\ \text{If } t > T_i + k_i \cdot T_i & c_i = c_i \end{cases} \quad (7)$$

If a disruption in machine M_i appears for a long enough period of time ($k \cdot T_i \gg T_i$), the number of pallets in the first downstream buffer ($n_{i,j1}$) of one machine (M_i) is minimized. The number of pallets to be minimized is subjected to the governing matrix equation (6) of the closed loops and the lower and upper boundaries in both working regimes and is given by

Min $n_{i,j1}$

s.t. $\bar{M} = \bar{g}$

Subject to: $m_{i,j1} \leq n_{i,j1} \leq M_{i,j1}$ for a stationary working regime and $0 \leq n_{i,j1} \leq M_{i,j1}$ for a transitory working regime.

As a consequence of the upper boundaries ($M_{i,j}$) represent the maximum capacities of buffers, their values are the same in both working regimes. However, the lower boundaries in stationary regime ($m_{i,j1}$) differ from those established in transitory regime, which can be zero.

Once the value of $n_{i,j1}$ has been obtained, the number of pallets in the following buffers of the same closed loop must be progressively minimized. After blockage of any machine of the closed loop through one of its downstream buffers or starvation through one of its upstream buffers, the number of pallets in one of the other downstream buffers can be minimized, broadening the minimisation through the closed loops.

Thus, the results of the propagation of disruptions from one machine to the others depend on their working regimes and the external variables (x_m) of the governing equations of the closed loops, as Resano Lázaro and Luis Pérez (2007b) demonstrate.

2.2. Application of the generic model to a real automobile assembly line

Figure 2 represents the main automobile assembly line and the front axle and doors preassembly lines analyzed by Resano Lázaro and Luis Pérez (2007a), as a network of four closed loops of machines and intermediate buffers.

Figure 2. *Main assembly line and preassembly lines.*

Machine M_1 represents the car bodies sequencing store, machine M_2 represents the first branch of the main assembly line, machine M_3 represents the chassis assembly line, machine M_4 , which determines the production rate of the entire assembly system, represents the second and final branch of the main assembly line, machines M_5 and M_6 model the doors preassembly and inspection lines and machine M_7 represents the chassis assembly line.

The different branches of the main assembly line and the preassembly lines associated are decoupled by intermediate buffers ($B_{i,j}$). There are three kinds of pallets involved in the assembly and preassembly lines analyzed. The car body assembly pallets are processed by machines M_1 to M_4 and stored in the corresponding intermediate buffers ($B_{1,2}$, $B_{2,3}$, $B_{3,4}$ and $B_{4,1}$). Meanwhile, the front axle assembly pallets are processed by machines M_3 and M_7 and stored in buffers $B_{3,7}$ and $B_{7,3}$. Finally, the doors assembly pallets are processed by machines M_2 , M_5 , M_6 and M_4 and stored in the corresponding intermediate buffers.

The capacities of the car body and front axle assembly pallets lie on one workpiece. Meanwhile, each assembly door pallet carries a pair of doors, left and right. Thus, one and two door pallets have to be processed respectively every two and four doors car assembled in the main assembly line.

The governing matrix Equation of the assembly line analyzed represents an application case of generic Equation (6) and is given by

$$\begin{bmatrix} n_{1,2} + n_{2,3} + n_{3,4} + n_{4,1} \\ n_{2,5} + n_{5,6} + n_{6,4} + n_{4,2} \\ n_{3,7} + n_{7,3} \\ n_{2,5} + n_{5,6} + n_{6,4} - n_{3,4} - n_{2,3} \end{bmatrix} = \begin{bmatrix} 237 \\ 450 \\ 138 \\ 150 + (216 + n_{2,3} + n_{3,4}) \cdot x \end{bmatrix} \quad (8)$$

The sum of the number of pallets that exist in each intermediate buffer ($n_{i,j}$) of the three first closed loops remain constant in accordance with the three first rows of the matrix Equation (8). The first closed loop is defined by the car body transport and assembly conveyor, i.e. by machines M_1 , M_2 , M_3 and M_4 and the corresponding intermediate buffers $B_{1,2}$, $B_{2,3}$, $B_{3,4}$ and $B_{4,1}$, which decouple them. The sum of the number of car body assembly pallets remains invariant. Since machines M_1 , M_2 , M_3 and M_4 have a fixed number of stations, the sum of the number of pallets in the corresponding intermediate buffers ($n_{1,2}+n_{2,3}+n_{3,4}+n_{4,1}$) remains constant and equals the value of 237 units in our case study.

The doors transport and assembly conveyor represents the second closed loop, which relates machines M_2 , M_5 , M_6 and M_4 through the intermediate buffers $B_{2,5}$, $B_{5,6}$, $B_{6,4}$ and $B_{4,2}$. Since the numbers of stations with doors assembly pallets in machines M_1 , M_2 , M_3 and M_4 are constant, the sum of the number of pallets in buffers, which decoupled them ($n_{2,5}+n_{5,6}+n_{6,4}+n_{4,2}$), remains also invariable and reaches the value of 450 units, as is shown in the second row of matrix Equation (8).

The third closed loop is defined by the front axle transport and assembly conveyor, i.e. by machines M_3 and M_7 and by the corresponding buffers $B_{3,7}$ and $B_{7,3}$. As was previously explained for the two first loops, the sum of the number of pallets in the buffers of this closed loop ($n_{3,7}+n_{7,3}$) remains constant and equals 138 units.

Nevertheless, the number of pallets in buffers $B_{2,5}$, $B_{5,6}$ and $B_{6,4}$ ($n_{2,5}$, $n_{5,6}$ and $n_{6,4}$) of the second closed loop are related to the number of pallets in buffers $B_{2,3}$ and $B_{3,4}$ ($n_{2,3}$ and $n_{3,4}$) of the first closed loop via an external variable (x). This variable defines the ratio of four-door cars placed between the doors disassembly stations from the car body, located in the first branch of the assembly line, and the doors assembly stations in the car body, located in the second branch of the line. The value of x can vary between 0 and 1. This relationship defines the fourth closed loop, as is shown in the last row of the matrix Equation (8).

Table 1 represents the maximum capacities ($M_{i,j}$) and the minimum number of pallets ($m_{i,j}$) of each intermediate buffer ($B_{i,j}$) required for a stationary working regime of the downstream machine (M_j) for the most usual cycle times of the machines involved, according to Equations (1) and (5).

Table 1. Real values of $m_{i,j}$ and $M_{i,j}$

The minimum contents of buffers $B_{6,4}$ and $B_{4,2}$ in stationary regime depend on the values of x as a consequence of they contain doors' pallets and their downstream machines M_4 and M_6 process car bodies. As Resano Lázaro and Luis Pérez (2007b) demonstrated, the assembly line analyzed cannot operate for $x > 0.97$ in stationary regime and $x < 0.37$ in both regimes, as a result of the boundaries shown in Table 1 are not compatible with the matrix Equation (8).

2.3. Analysis of blockage transmission

Consider a segment of a production system, as shown in Figure 3 in which machine M_j feeds machine M_i . Between these machines there is a buffer $B_{j,i}$, whose capacity is $M_{j,i}$. Suppose that M_i breaks down at time t_i . Let the number of items in $B_{j,i}$ at that time be $n_{j,i(t_i)}$. Since the buffer is finite, its level will start to increase from time t_i onwards, until the buffer eventually becomes full. When this happens machine M_j will become blocked and will stop producing. The elapsed time from t_i to the time when M_j becomes blocked will be referred to as the blocking time of M_j , ($t_{j,i}$) and is given by

$$t_{j,i} = (M_{j,i} - n_{j,i(t_i)}) \cdot c_j \quad (9)$$

Where c_j is the cycle time of machine M_j in stationary or transitory regime, which depends on the number of pallets in its upstream buffers.

Figure 3. Blockage transmission from machine M_i to machine M_j .

In the case of machines M_k and M_j block progressively after disruption of machine M_i , as is shown in Figure 4, the blocking time of machine M_j from the disruption of machine M_i ($t_{j,i}$) is given by

$$t_{j,i} = t_{k,i} + t_{j,k} \quad (10)$$

Where $t_{k,i}$ and $t_{j,k}$ are the blocking times of machines M_k from the disruption of M_i and M_j from the blockage of M_k , respectively.

Figure 4. Blockage transmission from machine M_i to machine M_j through M_k .

The values of $t_{k,i}$ and $t_{j,k}$ depend on: the capacities of buffers $B_{k,i}$ and $B_{j,k}$ ($M_{k,i}$ and $M_{j,k}$), the cycle times of machines M_k and M_j (c_k and c_j) and the number of pallets in buffers $B_{k,i}$ and $B_{j,k}$ when machines M_i and M_k respectively stop ($n_{k,i(t_i)}$ and $n_{j,k(t_k)}$), as is given by

$$t_{k,i} = (M_{k,i} - n_{k,i(t_i)}) \cdot c_k \quad (11)$$

$$t_{j,k} = (M_{j,k} - n_{j,k(t_k)}) \cdot c_j \quad (12)$$

The number of pallets in buffer $B_{j,k}$ when machine M_k has been blocked ($n_{j,k(t_k)}$) is related to the number of pallets in this buffer when machine M_i disrupts ($n_{j,k(t_i)}$) and is determined by

$$n_{j,k(tk)} = n_{j,k(ti)} + (\eta_j - \eta_k) \cdot t_{k,i} \quad (13)$$

Where η_j and η_k are production rates of machines M_j and M_k , which correspond to the inverse of their cycle times in both regimes.

By means of: replacing the value of $t_{k,i}$ of Equation (11) in Equation (13), the value of $n_{j,k(tk)}$ of Equation (13) in Equation (12) and the results of Equations (11) and (12) in Equation (10), the value of $t_{j,i}$ as a function of the values of $M_{k,i}$, $M_{j,k}$, $n_{k,i(ti)}$, $n_{j,k(ti)}$ and c_j is given by

$$t_{j,i} = M_{k,i} \cdot c_j + M_{j,k} \cdot c_j - n_{k,i(ti)} \cdot c_j - n_{j,k(ti)} \cdot c_j \quad (14)$$

Following the same procedure, as is shown in Figure 5, the generic expression that describes the blocking time of machine M_{in} from the disruption of machine M_{i1} ($t_{in,i1}$) through machines M_{i2} to M_{in-1} , is given by

$$t_{in,i1} = \left(\sum_{j=2}^{i=n} M_{ij,ij-1} - \sum_{j=2}^{i=n} n_{ij,ij-1(ti1)} \right) \cdot c_j \quad (15)$$

Figure 5. Blockage transmission from machine M_{i1} to machine M_{in} .

Where $M_{i2,i1}$ to $M_{in,n-1}$ are the capacities of the corresponding intermediate buffers $B_{i2,i1}$ to $B_{in,in-1}$, $n_{i2,i1(ti1)}$ to $n_{in,in-1(ti1)}$ are the number of pallets in buffers $B_{i2,i1}$ to $B_{in,in-1}$ when machine M_{i1} stops and c_j is the cycle time of machine M_j in stationary or transient regime.

2.4. Analysis of starvation transmission

In the case of machine M_j starves through its upstream buffer $B_{i,j}$ after disruption of machine M_i , as shown in Figure 6, its starvation time from the disruption of M_i in stationary and transitory regimes ($t_{j,is}$ and $t_{j,it}$) is determined by Equations (16) and (17), respectively.

Figure 6. Starvation transmission from machine M_i to machine M_j .

$$t_{j,is} = (n_{i,j(ti)} - m_{i,j}) \cdot c_{js} \quad (16)$$

$$t_{j,it} = n_{i,j(ti)} \cdot c_{jt} \quad (17)$$

Where $m_{i,j}$ is the minimum number of pallets in buffer $B_{i,j}$ required for a stationary working regime of the downstream machine M_j and c_{js} and c_{jt} are the cycle times of machine M_j in stationary and transitory regimes, respectively.

In the case of machines M_j and M_j starve progressively after disruption of machine M_i , as shown in Figure 7, the starvation time of machine M_j from the disruption of machine M_i ($t_{j,i}$) is determined by

$$t_{j,i} = t_{k,i} + t_{j,k} \quad (18)$$

Figure 7. Starvation of machine M_j after disruption of M_i through buffers $B_{i,k}$ and $B_{k,j}$.

Where $t_{k,i}$ and $t_{j,k}$ are the starvation times of machines M_k from the disruption of M_i and M_j from the starvation of M_k , respectively.

As a consequence of machine M_k starves through its upstream buffer $B_{i,k}$, once machine M_i stops, its starvation time from the disruption of machine M_i is determined in stationary ($t_{k,is}$) and transitory ($t_{k,it}$) regimes by

$$t_{k,is} = (n_{i,k(t_i)} - m_{i,k}) \cdot c_{ks} \quad (19)$$

$$t_{k,it} = n_{i,k(t_i)} \cdot c_{kt} \quad (20)$$

Where $n_{i,k(t_i)}$ is the number of pallets in buffer $B_{i,k}$ when machine M_i disrupts, $m_{i,k}$ is the minimum number of pallets in this buffer to guarantee a stationary working regime of machine M_k and c_{ks} and c_{kt} are the cycle times of machine M_k in stationary and transitory regimes, respectively.

The starvation time of machine M_j from the disruption of machine M_k is determined in stationary ($t_{j,ks}$) and transitory ($t_{j,kt}$) regimes by

$$t_{j,ks} = (n_{k,j(t_k)} - m_{k,j}) \cdot c_{js} \quad (21)$$

$$t_{j,kt} = n_{k,j(t_k)} \cdot c_{jt} \quad (22)$$

Where $n_{k,j(t_k)}$ is the number of pallets in buffer $B_{k,j}$ when machine M_k starves, $m_{k,j}$ is the minimum number of pallets in this buffer to guarantee a stationary working regime of machine M_j and c_{js} and c_{jt} are the cycle times of machine M_j in both working regimes.

The number of pallets in buffer $B_{k,j}$ when machine M_k starves ($n_{k,j(t_k)}$) is related to the number of pallets in the above-mentioned buffer when machine M_i disrupts ($n_{i,k(t_i)}$) through the production rates of machines M_k and M_j in stationary and transitory regimes (η_k and η_j), according to

$$n_{k,j(t_k)} = n_{i,k(t_i)} + (\eta_k - \eta_j) \cdot t_{k,j} \quad (23)$$

Replacing the values of $n_{k,j(t_k)}$ from Equation (23) in Equations (21) and (22), the values of $t_{k,is}$, $t_{k,it}$, $t_{j,ks}$ and $t_{j,kt}$ from Equations (19) to (22) in Equation (18) and the production rates of machines M_k and M_j (η_k and η_j) by the inverse of their cycle times in both regimes, the values of $t_{j,is}$ and $t_{j,it}$ are obtained as a function of $n_{i,kmin}$, $n_{k,jmin}$, c_{js} , c_{jt} , $n_{i,k(t_i)}$ and $n_{k,j(t_i)}$, according to

$$t_{j,is} = n_{i,k(t_i)} \cdot c_{js} + n_{k,j(t_i)} \cdot c_{js} - m_{i,k} \cdot c_{js} - m_{k,j} \cdot c_{js} \quad (24)$$

$$t_{j,it} = n_{i,k(t_i)} \cdot c_{jt} + n_{k,j(t_i)} \cdot c_{jt} \quad (25)$$

Following the same procedure, the generic expressions, that describe the starvation time of machine M_{in} from the disruption of machine M_{i1} , through machines M_{i2} to M_{in-1} , as is shown in Figure 8, are determined in stationary ($t_{in,i1s}$) and transitory ($t_{in,i1t}$) regimes by

$$t_{in,i1s} = \left(\sum_{j=1}^{j=in-1} n_{ij,ij+1(t_{i1})} - \sum_{j=1}^{j=in-1} m_{i,j,ij+1} \right) \cdot c_{js} \quad (26)$$

$$t_{in,i1t} = \sum_{j=1}^{j=in-1} n_{ij,ij+1(t_{i1})} \cdot c_{jt} \quad (27)$$

Figure 8. Starvation transmission from machine M_{i1} to machine M_{in} .

2.5. Concurrent blockage and starvation transmission

Figure 9 shows the transmission of disruptions from machine M_{i1} to machine M_{in} combining blocking and starvation phenomena through the corresponding intermediate buffers. The generic expressions, that describe the blocking or starvation times of machine M_{in} from the disruption of machine M_{i1} , through machines M_{i2} to M_{in-1} , are determined by Equations (28) and (29) in stationary ($t_{in,i1s}$) and transitory ($t_{in,i1t}$) regimes in line with the generic blocking and starvation Equations (15), (26) and (27).

Figure 9. Concurrent blockage and starvation transmission from machine M_{i1} to machine M_{in} .

$$t_{in,i1s} = \left(\sum_{j=2}^{j=t} M_{ij,ij-1} - \sum_{j=2}^{j=t} n_{ij,ij-1(t_{i1})} + \sum_{j=t}^{j=n-1} n_{ij,ij+1(t_{i1})} - \sum_{j=t}^{j=n-1} m_{ij,ij+1} \right) \cdot c_{js} \quad (28)$$

$$t_{in,i1t} = \left(\sum_{j=2}^{j=t} M_{ij,ij-1} - \sum_{j=2}^{j=t} n_{ij,ij-1(t_{i1})} + \sum_{j=t}^{j=n-1} n_{ij,ij+1(t_{i1})} \right) \cdot c_{jt} \quad (29)$$

3. Proposed algorithm for optimizing the availability of a generic assembly system

In the case of any machine M_i of the network of closed loops stops for a long enough period of time, the disruption is transmitted to the remaining machines by means of blocking or starvation phenomena from their downstream or upstream buffers, respectively.

Once machine M_{in} , which determines the production rate of the entire manufacturing system, starves or blocks, the assembly system becomes unavailable. Thus, by increasing the maximum capacities of the affected buffers up to their saturation values, the blocking times can be maximized and the availability of the manufacturing system optimized, as is demonstrated by Equation (15). If starvation phenomena occur, by decreasing the minimum contents of the corresponding buffers in stationary regime, the saturation values approach those existing in transitory regime, increasing in this way the availability of the assembly system, as is proved in Equation (26).

In Figure 10 the flowchart of the algorithm for maximizing the transmission of blocking times after disruption of machine M_i is shown, which starts with the procedure described by Resano Lázaro and Luis Pérez (2007b) for analyzing the blocking and starvation paths from M_i to machine M_{in} , which determines the production rate of the manufacturing system.

Figure 10. Flowchart of the algorithm for maximizing the transmission of blocking times

After having determined the blocking and starvation paths from each machine to machine M_{in} , which sets the production rate of the manufacturing system, the capacities of the buffers $B_{i,j}$ to $B_{m,in}$ involved have to be increased for each value of x_i up to their saturation values, i.e. up to the involved capacities reach their maximum values. Once the saturation values have been calculated, the capacities of the remaining buffers of the networks of closed loops have to be changed in order to find the maximum saturation capacities of buffers $B_{i,j}$ to $B_{m,in}$.

The configuration which maximizes the transmission of disruption times and hence, the availability of the manufacturing system, corresponds to the highest value of $M_{i,j} + \dots + M_{m,in}$.

4. Application of the algorithm to a real automobile assembly line

4.1. Optimization of the transmission of disruption times from machine M_1

After disruption of machine M_1 , machine M_4 , which determines the production rate of the manufacturing system, blocks through its downstream buffer $B_{4,1}$, as Figure 11 shows.

Figure 11. Transmission of disruption from machine M_1 to machine M_4 .

By means of varying the capacity of buffer $B_{4,1}$ both, the value of $t_{4,1}$ and the availability of the manufacturing system can be maximized, according to

$$M_{4,1} = 123 + K_1 \quad (30)$$

The values of K_1 represent the increases and decreases of the capacities of the corresponding buffers, depending on whether they are positive or negative.

Figure 12. Values of $n_{4,1s}$ and $n_{4,1t}$ as a function of $M_{4,1}$ and x after disruption of machine M_1 .

As Figure 12 shows, after disruption of machine M_1 , the number of pallets in buffer $B_{4,1}$ in both working regimes ($n_{4,1s}$ and $n_{4,1t}$) increase as $M_{4,1}$ grows until they saturate. Thus, the values of $M_{4,1}$ should not be higher than the saturation values of $n_{4,1s}$ and $n_{4,1t}$ in order not to waste the capacity of buffer $B_{4,1}$ ($M_{4,1}$). Since the number of pallets in the intermediate buffers might be zero in transitory regime, the saturation values of $n_{4,1t}$ are higher than the saturation values of $n_{4,1s}$ for the same values of x .

Figure 13 represents the values of $n_{1,2}$, $n_{2,3}$, $n_{3,4}$, $n_{4,1}$ and $n_{4,2}$ as a function of x after disruption of machine M_1 , when the capacity of buffer $B_{4,1}$ ($M_{4,1}$) corresponds to its saturation values.

Figure 13. Values of $n_{1,2}$, $n_{2,3}$, $n_{3,4}$ and $n_{4,2}$ for $M_{4,1}$ in stationary and transitory regimes as a function of x after disruption of machine M_1 .

Since the number of pallets in buffer $B_{1,2}$ ($n_{1,2}$) reaches its lower limit and $n_{4,2}$ achieves its upper limit for $0.37 \leq x \leq 0.60$ and $0.37 \leq x \leq 0.70$ in both regimes, the saturation values of $n_{4,1}$ cannot reach their upper limits for the values of x mentioned above.

Thus, by means of varying the capacity of buffer $B_{4,2}$ ($M_{4,2}$), the saturation values of $n_{4,1}$ can reach their upper limits for any value of x in the operational range of the manufacturing system, according to

$$M_{4,2} = 124 + K_1 \quad (31)$$

Table 2 summarizes the maximum capacities of buffer $B_{4,2}$ ($M_{4,2}$) in order to get the saturation capacity of buffer $B_{4,1}$ ($M_{4,1}$) reaches its upper limit and hence the maximum value of $t_{4,1}$.

Table 2. Values of $M_{4,1}$ and $M_{4,2}$ to maximize $t_{4,1}$

In order to maximize the value of $t_{4,1}$, the addition of two parallel buffers $B'_{4,1}$ and $B'_{4,2}$ are proposed in both working regimes, as is shown in Figure 14, whose capacities correspond to the positive values of K_1 . Notice that the capacity of buffer $B_{4,2}$ must be reduced according to the negative values of K_1 in Table 2, by means of establishing a by-pass $B''_{4,2}$ of capacity ($M_{4,2} + K_1$).

Buffers $B'_{4,1}$, $B'_{4,2}$ and $B''_{4,2}$ would be enabled in the case of a disruption of machine M_1 in both working regimes, turning pallets away from the main buffers. Under normal conditions, buffers $B'_{4,1}$, $B'_{4,2}$ and $B''_{4,2}$ would remain disabled in order not to increase the minimal contents of these buffers so that their downstream machines (M_1 and M_2) work in stationary regime.

In the case of establishing by-passes $B''_{1,2}$, $B''_{2,3}$ and $B''_{3,4}$ in the buffers $B_{1,2}$, $B_{2,3}$ and $B_{3,4}$ in stationary regime, the saturation capacity of buffer $B_{4,1}$ increases and approximates to the value in transitory regime, which allows higher values of $t_{4,1}$ in stationary regime.

Figure 14. *Optimal configuration of buffers after disruption of machine M_1 , according to the values of Table 2.*

4.2. Optimization of the transmission of disruption times from machine M_2

After disruption of the first branch of the main automobile assembly line (M_2), machine M_4 blocks through its downstream buffer $B_{4,2}$ for $0.37 \leq x \leq 0.50$ in both working regimes, as shown in Figure 15.

Figure 15. *Transmission of disruption from machine M_2 to machine M_4 for $0.37 \leq x < 0.60$ in both working regimes*

However, for $0.60 \leq x \leq 0.97$ and $0.60 \leq x \leq 1.00$ in stationary and transitory regimes, machines M_1 and M_4 block progressively through their downstream buffers $B_{1,2}$ and $B_{4,1}$, as shown in Figure 16.

Figure 16. *Transmission of disruption from machine M_2 to machine M_4 for $0.60 \leq x \leq 0.97$ and $0.60 \leq x \leq 1.00$ in stationary and transitory regimes.*

Tables 3 and 4 present the results of the algorithm proposed in section 3 for maximizing $t_{4,2}$ for different values of x .

Table 3. Options to maximize $t_{4,2}$ for $0.37 \leq x \leq 0.50$

Table 4. Values of $M_{4,1}$ and $M_{4,2}$ to maximize $t_{4,2}$ for $0.60 \leq x \leq 0.97$ and $0.60 \leq x \leq 1.00$

In order to maximize the blocking time of machine M_4 from the disruption of machine M_2 ($t_{4,2}$), the addition of a parallel buffer $B'_{4,2}$ is proposed with a capacity represented by the positive values of K_1 in Tables 3 and 4. Meanwhile, a by-pass $B''_{4,2}$ of capacity $(M_{4,2} + K_1)$ is proposed according to the negative values of K_1 , as Figure 17 shows.

The addition of parallel buffers $B'_{1,2}$ or $B'_{4,1}$ or a combination of both, with a capacity described by the values of K_1 in Tables 3 and 4, maximizes the value of $t_{4,2}$.

By means of establishing in stationary regime the by-passes $B''_{2,3}$ and $B''_{3,4}$ in the buffers $B_{2,3}$ and $B_{3,4}$, as shown in Figure 17, their capacity is reduced and the saturation values of buffers $B_{1,2}$, $B_{4,1}$ and $B_{4,2}$ increase in this regime, approximating to those existing in transitory regime.

Figure 17. *Optimal configuration of buffers after disruption of machine M_2 , according to the values of Tables 3 and 4.*

4.3. Optimization of the transmission of disruption times from machine M_3

As shown in Figure 18, after disruption of the chassis assembly line (M_3), machine M_4 blocks through its downstream buffers $B_{4,2}$ and $B_{2,3}$, for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in stationary and transitory regimes.

Figure 18. *Transmission of disruption from machine M_3 to machine M_4 for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in stationary and transitory regimes.*

However, as shown in Figure 19, machine M_4 starves through its upstream buffer $B_{3,4}$, for $0.40 < x \leq 0.97$ and $0.50 < x \leq 1.00$ in stationary and transitory regimes.

Figure 19. *Transmission of disruption from machine M_3 to machine M_4 for $0.40 < x \leq 0.97$ and $0.50 < x \leq 1.00$ in stationary and transitory regimes.*

Table 5 presents the results of the algorithm proposed in section 3 for maximizing $t_{4,3}$ for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in stationary and transitory regimes.

Table 5. Values of $M_{4,2}$ and $M_{2,3}$ to maximize $t_{4,3}$ for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$

Thus, in order to maximize the value of $t_{4,3}$, the addition of a parallel buffer $B'_{4,2}$ is proposed with a capacity which corresponds to the positive values of K_1 , shown in Table 5. Meanwhile, a by-pass $B''_{2,3}$ of capacity $(M_{2,3} + K_1)$ should be enabled in accordance to the negative values of K_1 , as is shown in Figure 20. By means of establishing in stationary regime the by-pass $B''_{3,4}$ in the buffer $B_{3,4}$, the value of $t_{4,3}$ is optimized for the remaining values of x in the operational range in stationary regime.

Figure 20. *Optimal configuration of buffers after disruption of machine M_3 , according to the values of Table 5.*

4.4. Optimization of the transmission of disruption times from machine M_5

As shown in Figure 21, after disruption of machine M_5 , machines M_2 , M_1 and M_4 block progressively through their downstream buffers $B_{2,5}$, $B_{1,2}$ and $B_{4,1}$ for $0.60 < x \leq 0.97$ in stationary regime and $0.60 \leq x \leq 1.00$ in transitory regime.

Figure 21. *Transmission of disruption from machine M_5 to machine M_4 for $0.60 < x \leq 0.97$ and $0.60 \leq x \leq 1.00$ in stationary and transitory regimes.*

Meanwhile, as Figure 22 shows, machine M_4 starves through its upstream buffers $B_{6,4}$ and $B_{5,6}$ once machine M_5 disrupts, for $0.37 \leq x \leq 0.60$ and $0.37 \leq x < 0.60$ in stationary and transitory regimes.

Figure 22. *Transmission of disruption from machine M_5 to machine M_4 for $0.37 \leq x \leq 0.60$ and $0.37 \leq x < 0.60$ in stationary and transitory regimes.*

Table 6 presents the results of the algorithm proposed in section 3 for maximizing the value of $t_{4,5}$.

Table 6. Values of $M_{2,3} + M_{3,4}$, $M_{4,1}$ and $M_{2,5}$ to maximize $t_{4,5}$ for $0.60 < x \leq 0.97$ and $0.60 \leq x \leq 1.00$

Hence, in order to maximize the blocking time of machine M_4 from the disruption of machine M_5 ($t_{4,5}$) the addition of parallel buffers $B'_{2,5}$, $B'_{2,3}$ and $B'_{3,4}$ is proposed with capacities which correspond to the positive values of K_1 , shown in Table 6. Meanwhile, by-passes $B''_{2,3}$, $B''_{3,4}$ and $B''_{4,1}$ with capacities $(M_{2,3} + K_1)$, $(M_{3,4} + K_1)$ and $(M_{4,1} + K_1)$ have to be added in accordance with the negative values of K_1 , as shown in Figure 23. By means of establishing the by-passes $B''_{5,6}$ and $B''_{6,4}$ the value of $t_{4,5}$ is optimized for the remaining values of x in the operational range in stationary regime.

Figure 23. *Optimal configuration of buffers after disruption of machine M_5 , according to the values of Table 6.*

4.5. Optimization of the transmission of disruption times from machine M_6

As is shown in Figure 24, after disruption of the doors inspection line (M_6), machines M_5 , M_2 , M_1 and M_4 block progressively through their downstream buffers $B_{5,6}$, $B_{2,5}$, $B_{1,2}$ and $B_{4,1}$ for $0.80 \leq x \leq 0.97$ and $0.80 \leq x \leq 1.00$ in stationary and transitory regimes.

Figure 24. *Transmission of disruption from machine M_6 to machine M_4 , for $0.80 \leq x \leq 0.97$ and $0.80 \leq x \leq 1.00$ in stationary and transitory regimes.*

Meanwhile, machine M_4 starves through its upstream buffer $B_{6,4}$, once machine M_6 disrupts, for $0.37 \leq x < 0.80$ in both working regimes, as is shown in Figure 25.

Figure 25. *Transmission of disruption from machine M_6 to machine M_4 for $0.37 \leq x < 0.80$ in both regimes.*

Table 7 presents the results of the algorithm proposed in section 3 for maximizing $t_{4,6}$, for $0.80 \leq x \leq 0.97$ and $0.80 \leq x \leq 1.00$ in both regimes.

Table 7. Options to maximize $t_{4,6}$ for $0.80 \leq x \leq 0.97$ and $0.80 \leq x \leq 1.00$

Since $t_{4,6s}$ depends on $m_{6,4}$ for $0.37 \leq x < 0.80$ in stationary regime, a reduction of the minimum content of buffer $B_{6,4}$ in stationary regime ($m_{6,4}$) increases the values of $t_{4,6s}$ and approximates them to those in transitory regime (values of $t_{4,6t}$).

Thus, in order to maximize the blocking time of machine M_4 from the disruption of machine M_6 ($t_{4,6}$) the addition of parallel buffers $B'_{2,5}$ and $B'_{5,6}$ is proposed with capacities which correspond to the positive values of K_1 in Table 7. Meanwhile, a by-pass $B''_{4,1}$ with capacity $(M_{4,1} + K_1)$, according to the negative values of K_1 has to be added, as is shown in Figure 26.

By means of establishing the by-pass $B''_{6,4}$ in stationary regime, shown in Figure 26, the value of $t_{4,6}$ is optimized for the remaining values of x in the operational range in stationary regime.

Figure 26. *Optimal configuration of buffers after disruption of machine M_6 , according to the values of Table 7.*

4.6. Optimization of the transmission of disruption times from machine M_7

As shown in Figure 27, after disruption of the front axle preassembly line (M_7), machine M_3 starves through its upstream buffer $B_{7,3}$. Once machine M_3 starves, machines M_2 and M_4 block progressively through their downstream buffers $B_{2,3}$ and $B_{4,2}$ for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in stationary and transitory regimes.

Figure 27. *Transmission of disruption from machine M_7 to machine M_4 for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in stationary and transitory regimes.*

However, machine M_4 starves through its upstream buffers $B_{3,4}$ and $B_{7,3}$ once machine M_7 disrupts, for $0.40 < x \leq 0.97$ and $0.50 < x \leq 1.00$ in stationary and transitory regimes, as shown in Figure 28.

Figure 28. *Transmission of disruption from machine M_7 to machine M_4 for $0.40 < x \leq 0.97$ and $0.50 < x \leq 1.00$ in stationary and transitory regimes.*

The results of the algorithm proposed in section 3 for maximizing $t_{4,7}$ are equal to those discussed in Table 5 for maximizing the values of $t_{4,3}$.

Hence, in order to maximize the blocking time of machine M_4 from the disruption of machine M_7 ($t_{4,7}$) the addition of a parallel buffer $B'_{4,2}$ is proposed with a capacity which corresponds to the positive values of K_1 , shown in Table 5. Meanwhile, a by-pass $B''_{2,3}$ of capacity $(M_{2,3} + K_1)$ is proposed in line with the negative values of K_1 , as is shown in Figure 29.

By means of establishing the by-passes $B''_{7,3}$ and $B''_{3,4}$ in the buffers $B_{7,3}$ and $B_{3,4}$, the value of $t_{4,7s}$ is optimized for the remaining values of x in the operational range in stationary regime.

Figure 29. *Optimal configuration of buffers after disruption of machine M_7 .*

5. Conclusions

In this study the generic equations that describe the transmission of disruption times in assembly systems, modelled as networks of closed loops of machines and intermediate buffers have been developed. The transmission of disruption times in generic assembly systems depend on different factors, which have not yet been analyzed by existing literature, such as: the external variables, which govern the populations of the closed loops, the different working regimes of the machines involved, their cycle times, the maximum capacities of the affected buffers and their minimum contents.

In this study an algorithm has been put forward in order to maximize the transmission of disruption times from each machine to the machine which determines the production rate of the entire manufacturing system by means of varying the capacities of the intermediate buffers affected. This methodology allows the manufacturing engineers of any assembly line to improve its availability whenever one branch disrupts.

The algorithm proposed has been used in a real automobile assembly line to maximize the transmission of disruptions between the different branches and preassembly lines and hence, the availability of the manufacturing system.

The application of the algorithm to a real assembly system has demonstrated that the transmission of disruption times depends on: the four-door-car proportion, the working regimes of the affected machines and the disrupted machines.

The fluctuation in the optimal capacities of the affected buffers has been shown by the variable K_1 which allows the intermediate buffers to vary their capacities by means of adding branches with capacities K_1 , if they are positive, or $M_{i,j} + K_1$ if they are negative.

In accordance with the capacities optimized, an implementation of flexible layouts has been proposed, which can be carried out by enabling and disabling branches of the conveyors in order to vary the capacities of the affected buffers in a flexible way.

With this study a better knowledge of the transmission of disruptions from different preassembly lines and the optimal capacities of the buffers involved is attained, thus improving the availability of this kind of manufacturing systems in the case of a disruption of any preassembly line.

In future studies, several expert systems to control the speed of the pallets in the intermediate buffers of a generic assembly system could be developed in order to compare the results with the methodology proposed in this paper.

Appendix: List of symbols

| | |
|-------------|---|
| $B_{i,j}$ | Intermediate buffer between machines M_i and M_j |
| $B'_{i,j}$ | By-pass in buffer $B_{i,j}$ with capacity K_1 |
| $B''_{i,j}$ | By-pass in buffer $B_{i,j}$ with capacity $M_{i,j} + K_1$ |
| c_j | Cycle time of machine M_j in stationary or transitory regimes |
| c_{js} | Cycle time of machine M_j in stationary regime |
| c_{jt} | Cycle time of machine M_j in transitory regime |

| | | |
|----|-----------------------------------|--|
| 1 | | |
| 2 | | |
| 3 | $g(x_m, n_{a,b}, \dots, n_{j,a})$ | Functions that relate the number of pallets in the buffers of the corresponding closed loop |
| 4 | | to external variables x_m |
| 5 | \bar{g} | |
| 6 | | Matrix that defines the relationship between numbers of pallets in buffers and external |
| 7 | | variables x_m |
| 8 | K_l | Variation of capacity of buffer $B_{i,j}$ |
| 9 | $k_i \cdot T_i$ | Disruption time of machine M_i |
| 10 | $l_{aspi,j}$ | Average storing length per pallet in buffer $B_{i,j}$ |
| 11 | $l_{i,j}$ | Length of buffer $B_{i,j}$ |
| 12 | $m_{i,j}$ | Minimum content of buffer $B_{i,j}$ |
| 13 | $M_{i,j}$ | Maximum capacity of buffer $B_{i,j}$ |
| 14 | $n_{i,js}$ | Number of pallets in buffer $B_{i,j}$ in stationary regime |
| 15 | $n_{i,jt}$ | Number of pallets in buffer $B_{i,j}$ in transitory regime |
| 16 | $n_{j,i}(t_i)$ | Number of pallets in buffer $B_{j,i}$ at time t_i |
| 17 | \bar{M} | |
| 18 | | Matrix that defines the sum of the number of pallets in buffers of each loop |
| 19 | $v_{i,j}$ | Transportation speed of pallets in buffer $B_{i,j}$ |
| 20 | T_i | Working time of machine M_i before any disruption appears |
| 21 | $t_{i,js}$ | Elapsed time from the disruption of M_i to the time when M_j becomes starved in stationary |
| 22 | | regime |
| 23 | $t_{i,jt}$ | Elapsed time from the disruption of M_i to the time when M_j becomes starved in transient |
| 24 | | regime |
| 25 | $t_{j,i}$ | Elapsed time from the disruption of M_i to the time when M_k becomes blocked |
| 26 | x | Four-door cars proportion between the doors' disassembly and assembly stations |
| 27 | x_m | Required number of pallets in buffer $B_{i,j}$ for assembling one final unit in the assembly |
| 28 | | system |
| 29 | η_i | Production rate of machine M_i |
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| 60 | | |

References

- Bastani A.S., 1988, Analytic Solution of closed loop conveyor systems with discrete and deterministic material flow. *European Journal of Operational Research*, **35**, 187-192.
- Bozer, Y.A. and Hiseh Y.J., 2004, Expected waiting times at loading stations in discrete-space closed-loop conveyors. *European Journal of Operational Research*, **155 (2)**, 516-532.
- Bozer, Y.A. and Hiseh Y.J., 2005, Troughput performance analysis and machine layout for discrete-space closed-loop conveyors. *IIE Transactions*, **37 (1)**, 77-89.
- Dallery, Y. and Gershwin, S.B., 1992, Manufacturing flow line systems: a review of models and analytical results. *Queueing Systems*, **12**, 3-94.
- D'Angelo, H., Caramanis, M., Finger, S., Mavretic, A., Phillis, Y.A. and Ramsden, E., 1988, Event-driven model of unreliable production lines with storage. *International Journal of Production Research*, **26 (7)**, 1173-1182.
- Frein, Y., Commault, C. and Dallery, Y., 1996, Modelling and analysis of closed loop production lines with unreliable machines and finite buffers. *IIE Transactions*, **28**, 545-554.

- 1
2
3 Gershwin, S.B., 1987, Representation analysis of transfer lines with machines that have different
4 processing rates. *Annals of Operations Research*, **9**, 511-530.
5
6
7 Gershwin, S.B., Maggio, N., Matta, A., Tolio, T. and Werner, L.M., 2001, Analysis of loop networks by
8 decomposition. *Third Aegean International Conference on Design and Analysis of Manufacturing*
9 *Systems*. Tinos Island. Greece.
10
11 Gershwin, S.B. and Werner, L.M., 2003, An approximate analytical method for evaluating the
12 performance of closed loop flow systems with unreliable machines and finite buffers- Part II: Large
13 Loops. *Papers of the MIT Manufacturing Systems Engineering Laboratory*.
14
15
16 Kouikoglou, V.S. and Phillis, Y.A., 1995, An efficient discrete-event model for production networks of
17 general geometry. *IIE Transactions*. **27(1)**, 32-42.
18
19
20 Kouikoglou, V.S. and Phillis, Y.A., 1997, A continuous-flow model for production networks with finite
21 buffers, unreliable machines, and multiple products. *International Journal of Production Research*, **35(2)**,
22 381-397.
23
24
25 Kouikoglou, V.S., 2002, An efficient discrete event model of assembly/disassembly production networks.
26 *International Journal of Production Research*. **40 (17)**, 4485-4503
27
28
29 Levantesi, R., 2001, Analysis of multiple loop assembly and disassembly networks. *PhD Thesis*.
30 *Politecnico di Milano*.
31
32
33 Levantesi, R., Matta, A. and Tolio, T., 2003, Performance evaluation of continuous production lines
34 having different processing times and multiple failure modes. *Performace Evaluation*, **51**, 247-268.
35
36
37 Maggio, N., Matta, A., Gershwin, S.B. and Tolio, T., 2003, An approximate analytical method for
38 evaluating the performance of closed loop flow systems with unreliable machines and finite buffers- Part I:
39 Small Loops. *Papers of the MIT Manufacturing Systems Engineering Laboratory*.
40
41
42 Paik C.H., Kim H.G. and Cho H.S., 2002, Performance analysis for closed-loop production systems with
43 unreliable machines and random processing times. *Computers & Industrial Engineering*, **42 (2-4)**, 207-
44 220.
45
46
47 Resano Lázaro, A. and Luis Pérez, C.J., 2007a, Analysis of an automobile assembly line as a network of
48 closed loops working in both, stationary and transitory regimes. *International Journal of Production*
49 *Research*. Paper in Press. DOI: 10.1080/00207540601182294.
50
51
52 Resano Lázaro, A. and Luis Pérez, C.J., 2007b, Dynamic analysis of an automobile assembly line
53 considering starving and blocking. *Robotics and Computer Integrated Manufacturing*. Paper in Press.
54 DOI:10.1016/j.rcim.2007.11.002
55
56
57 Onvural R. and Perros, H.G., 1990, Throughput analysis of cyclic queuing network with blocking.
58 Technical report. *C.S. Dept, North Carolina State University*
59
60
61 Werner, L.M., 2001, Analysis and design of closed loop manufacturing systems. *Master's Thesis*.
62 *Massachusetts Institute of Technology*.

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Zhang, Z., 2006, Analysis and design of manufacturing systems with multiple-loop structures. *PhD Thesis. Massachusetts Institute of Technology.*

Zhuang, L., Wong, Y.S., Fuh, J.Y.H. and Yee, C.Y., 1997, On the role of a queueing network model in the design of a complex assembly system. *Robotics and Computer-Integrated Manufacturing*, 14, 153-161.

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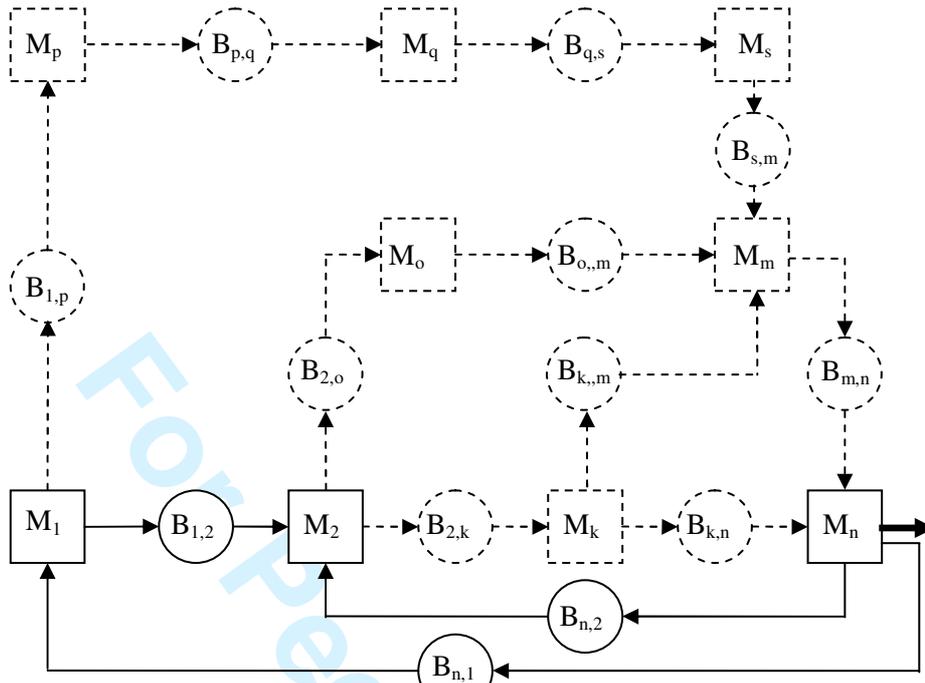


Figure 1. Generic assembly system

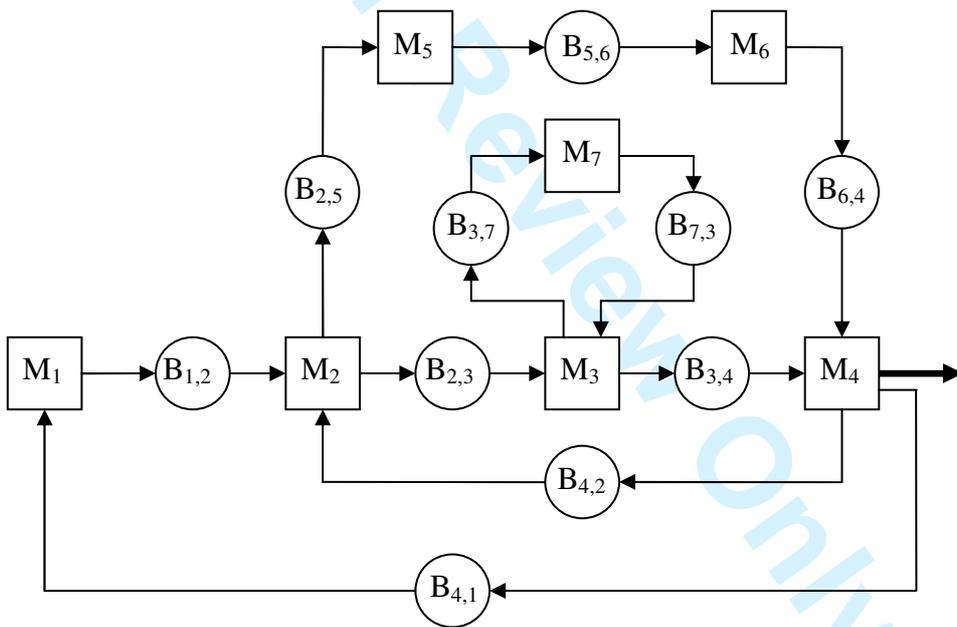


Figure 2. Main assembly line and preassembly lines.

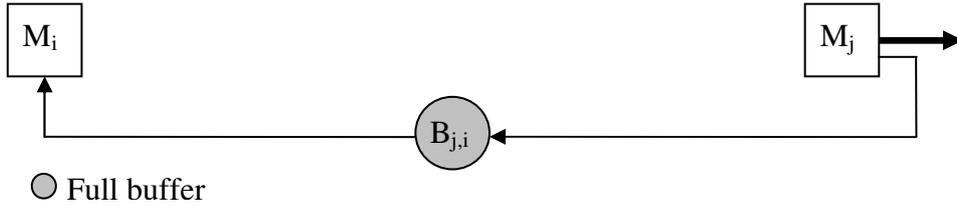


Figure 3. Blockage transmission from machine M_i to machine M_j .

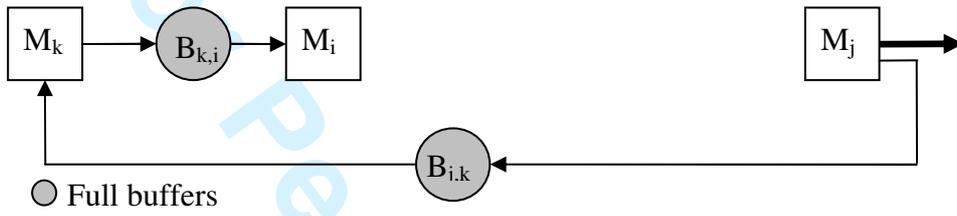


Figure 4. Blockage transmission from machine M_i to machine M_j through M_k .

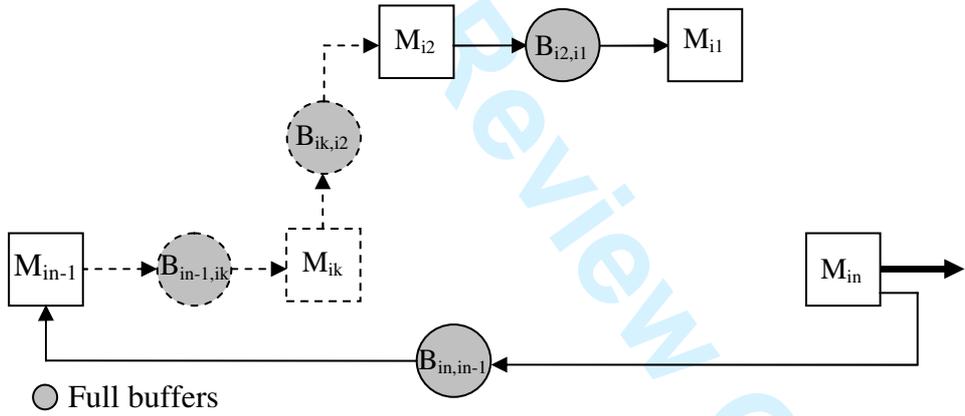


Figure 5. Blockage transmission from machine M_{i1} to machine M_{in} .



Figure 6. Starvation transmission from machine M_i to machine M_j .

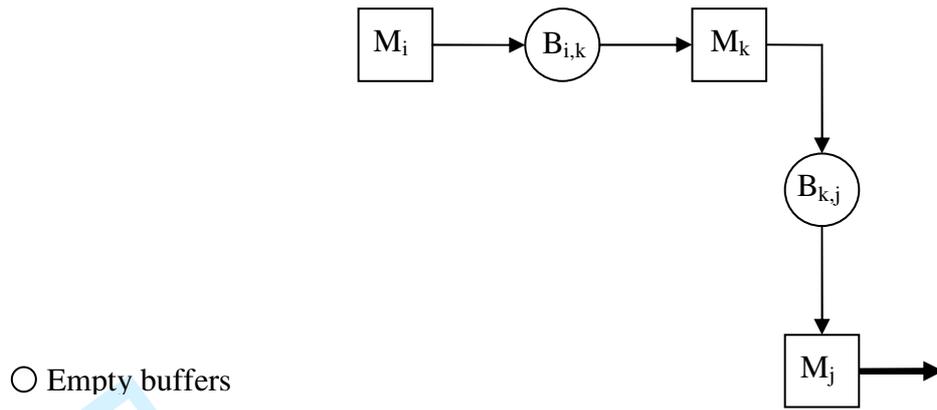


Figure 7. Starvation of machine M_j after disruption of M_i through buffers $B_{i,k}$ and $B_{k,j}$.

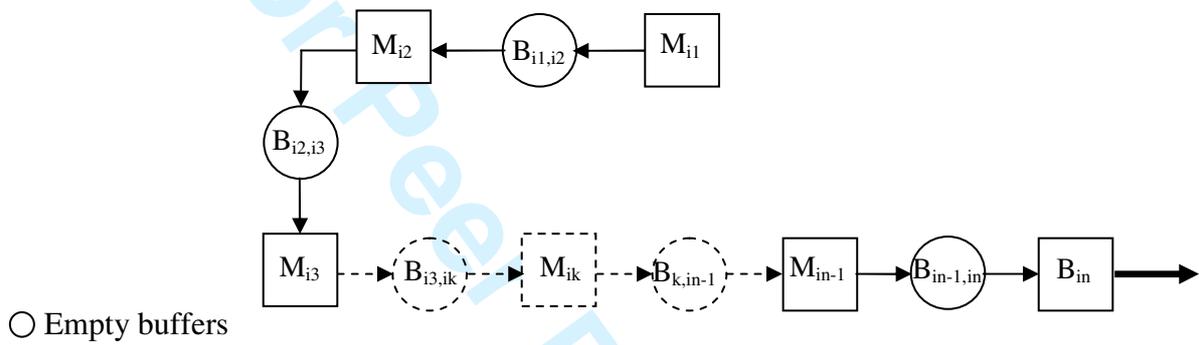


Figure 8. Starvation transmission from machine M_{i1} to machine M_{in} .

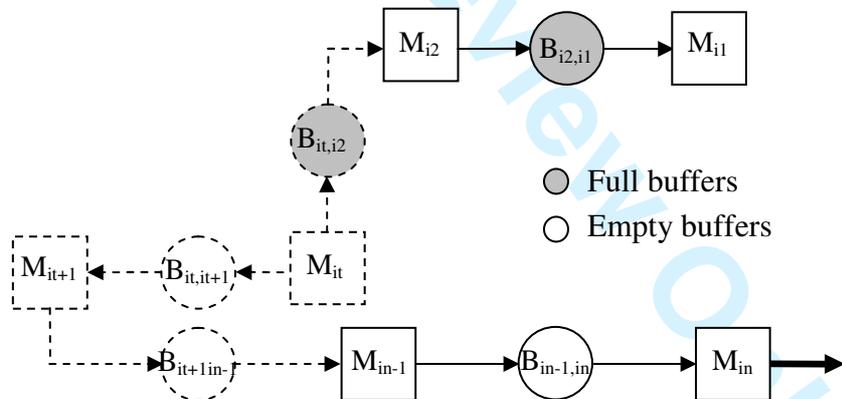


Figure 9. Concurrent blockage and starvation transmission from machine M_{i1} to machine M_{in} .

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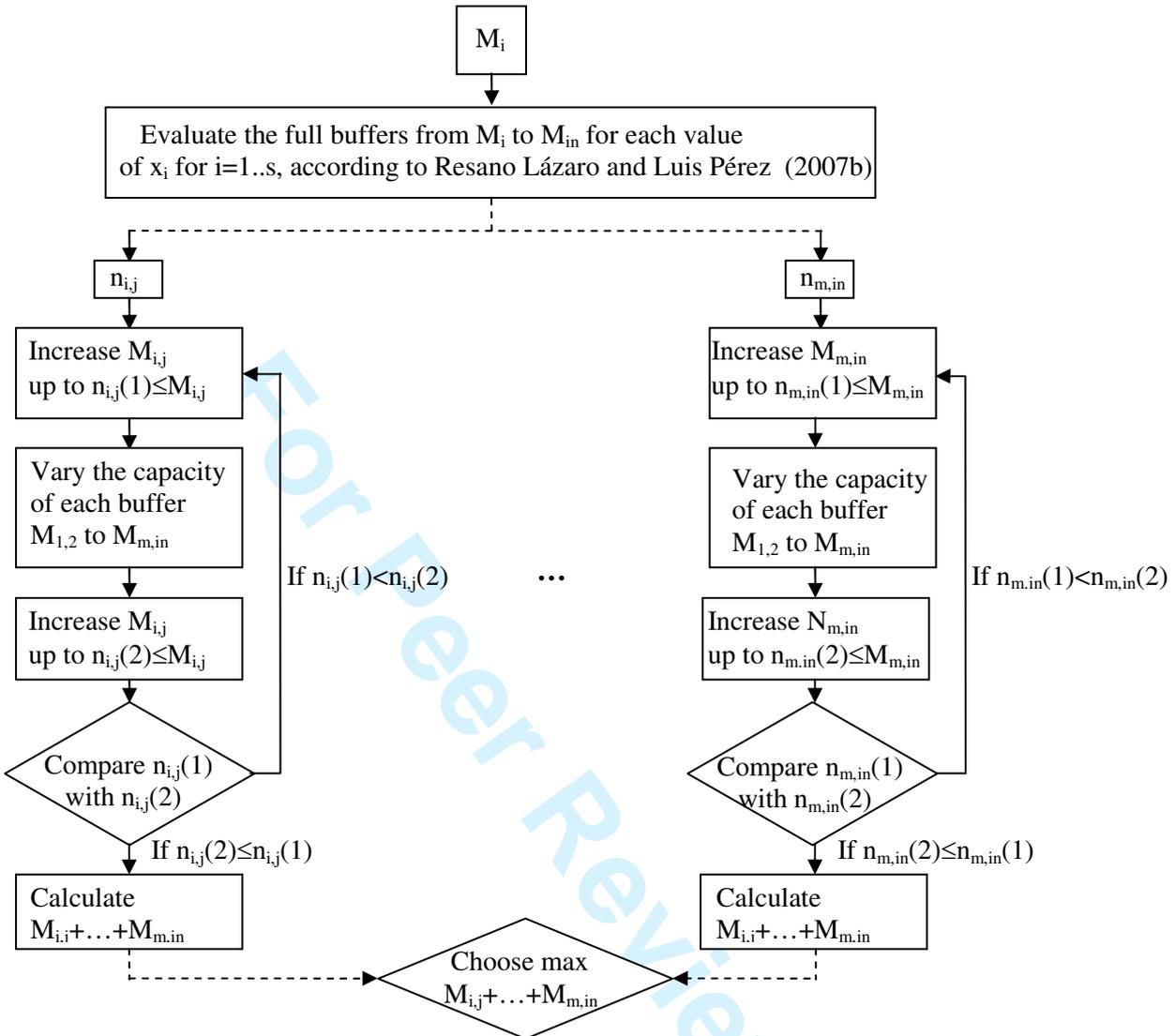


Figure 10. Flowchart of the algorithm for maximizing the transmission of blocking times

● Full buffer for $0.37 \leq x \leq 0.97$ y $0.37 \leq x \leq 1.00$ in stationary and transitory regimes

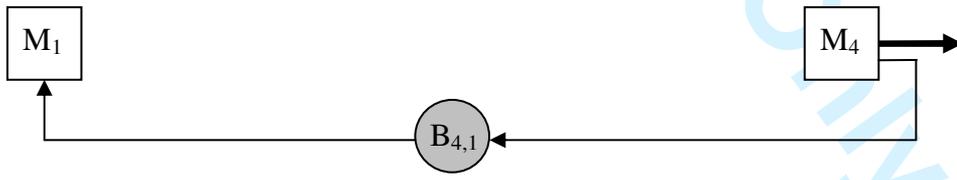


Figure 11. Transmission of disruption from machine M1 to machine M4.

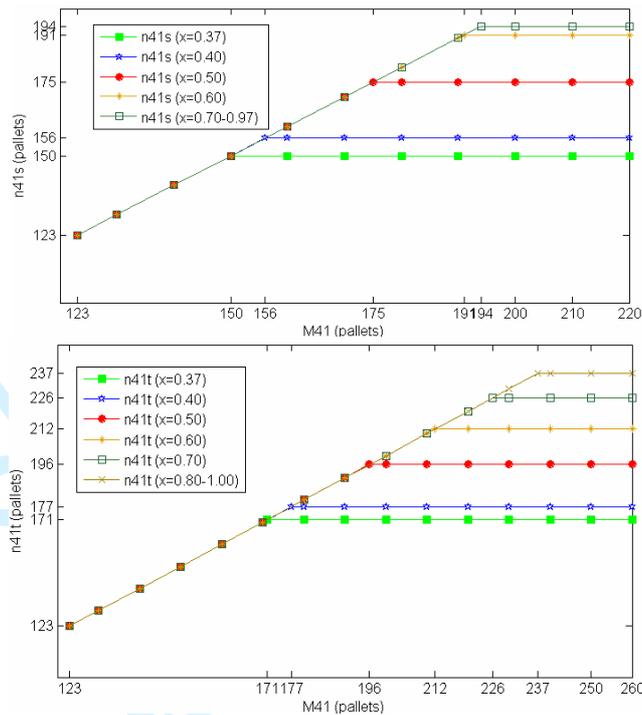


Figure 12. Values of $n_{4,1s}$ and $n_{4,1t}$ as a function of $M_{4,1}$ and x after disruption of machine M_1 .

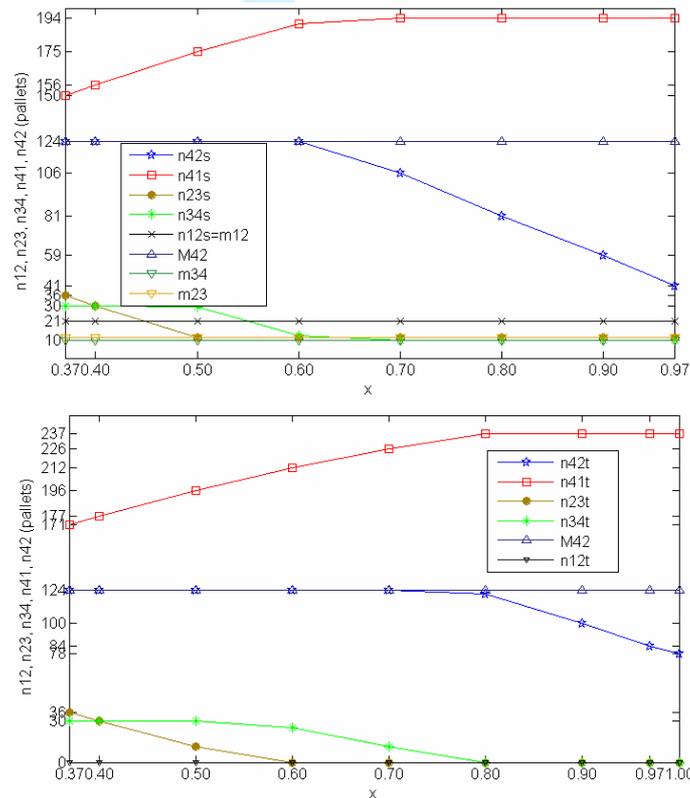


Figure 13. Values of $n_{1,2}$, $n_{2,3}$, $n_{3,4}$ and $n_{4,2}$ for $M_{4,1}$ in stationary and transitory regimes as a function of x after disruption of machine M_1 .

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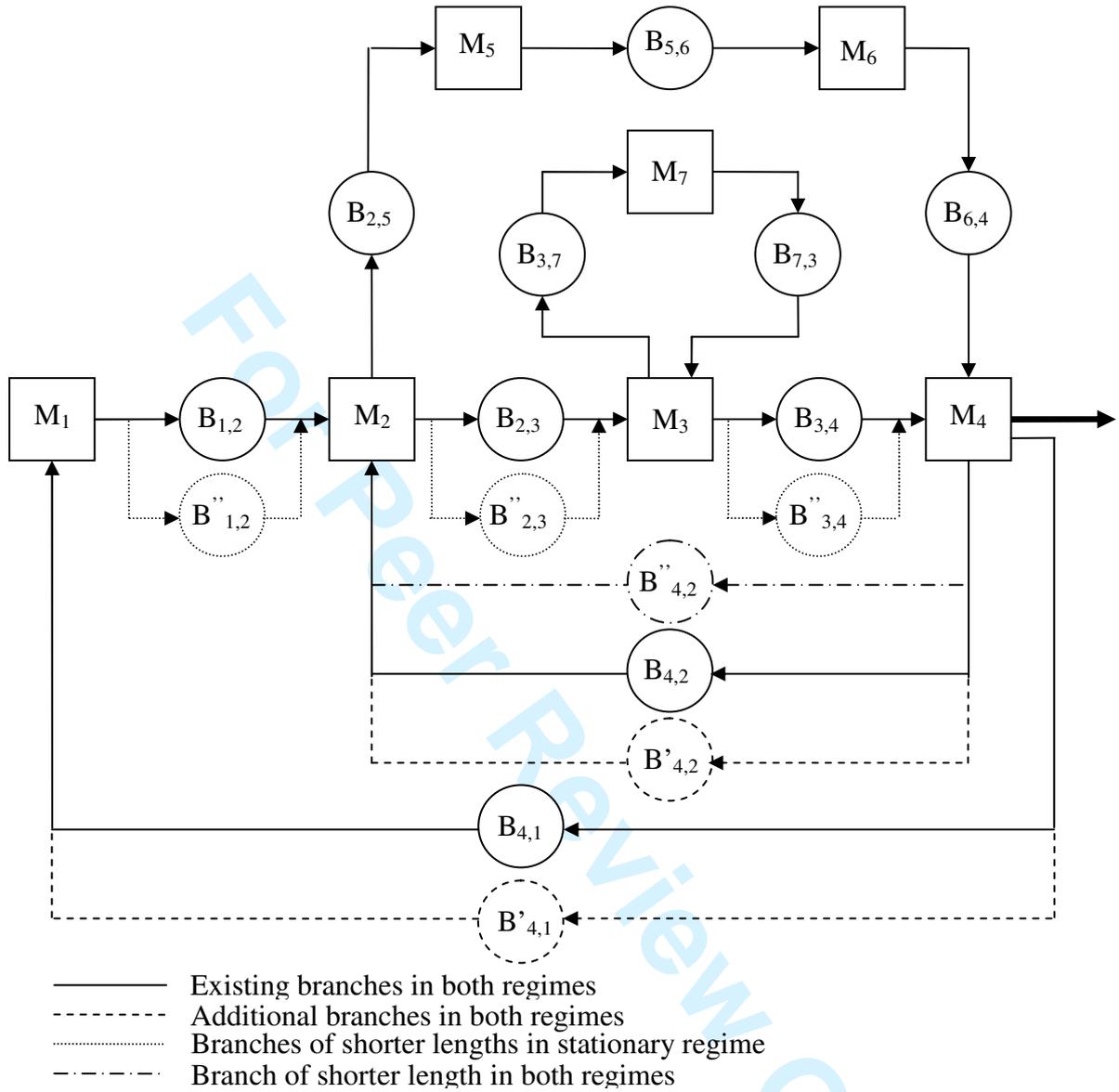


Figure 14. Optimal configuration of buffers after disruption of machine M_1 , according to the values of Table 2.

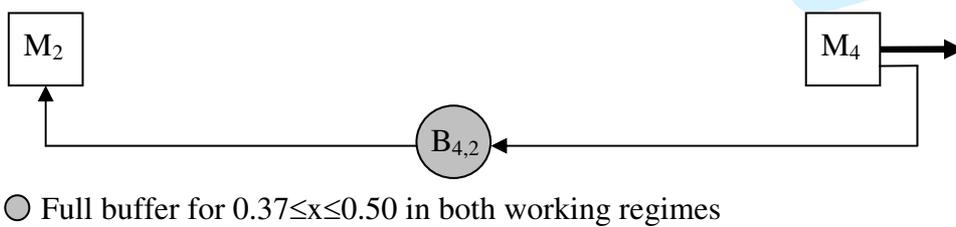


Figure 15. Transmission of disruption from machine M_2 to machine M_4 for $0.37 \leq x < 0.60$ in both working regimes

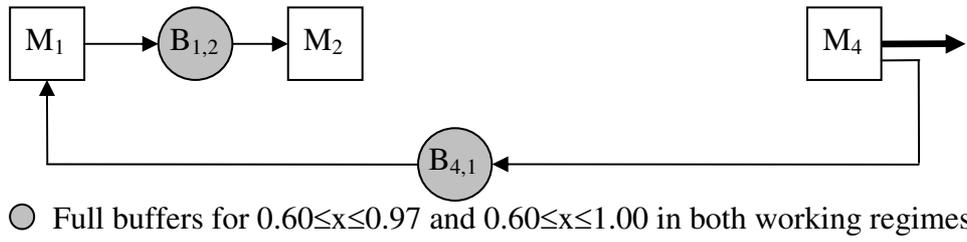


Figure 16. Transmission of disruption from machine M_2 to machine M_4 for $0.60 \leq x \leq 0.97$ and $0.60 \leq x \leq 1.00$ in stationary and transitory regimes.

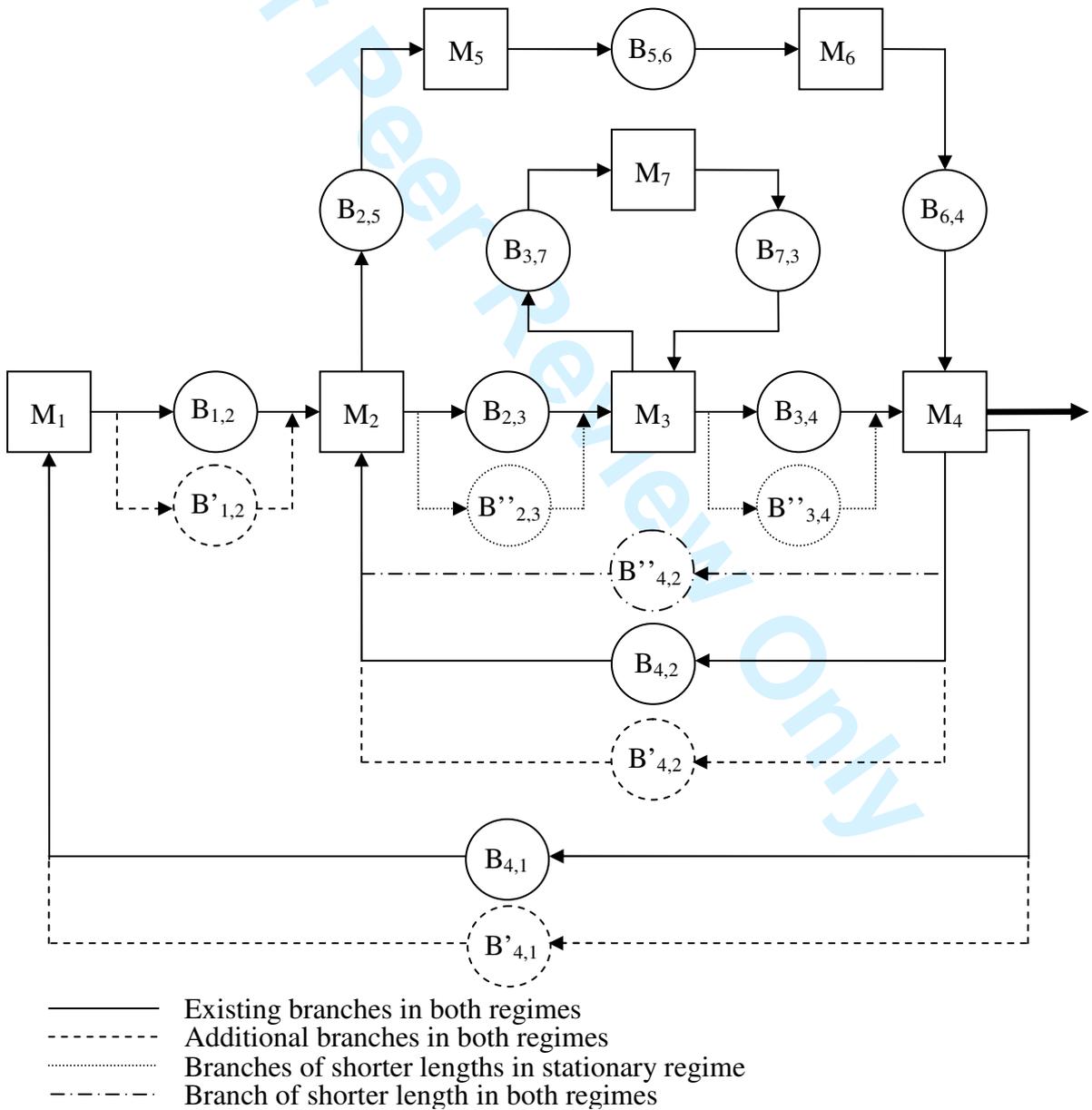
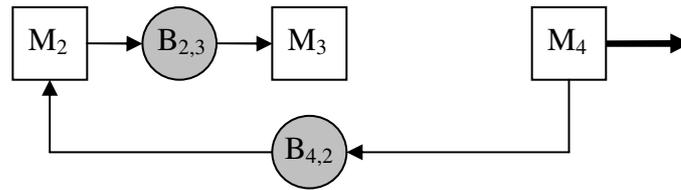
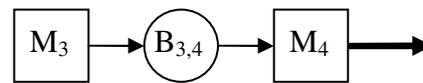


Figure 17. Optimal configuration of buffers after disruption of machine M_2 , according to the values of Tables 3 and 4.



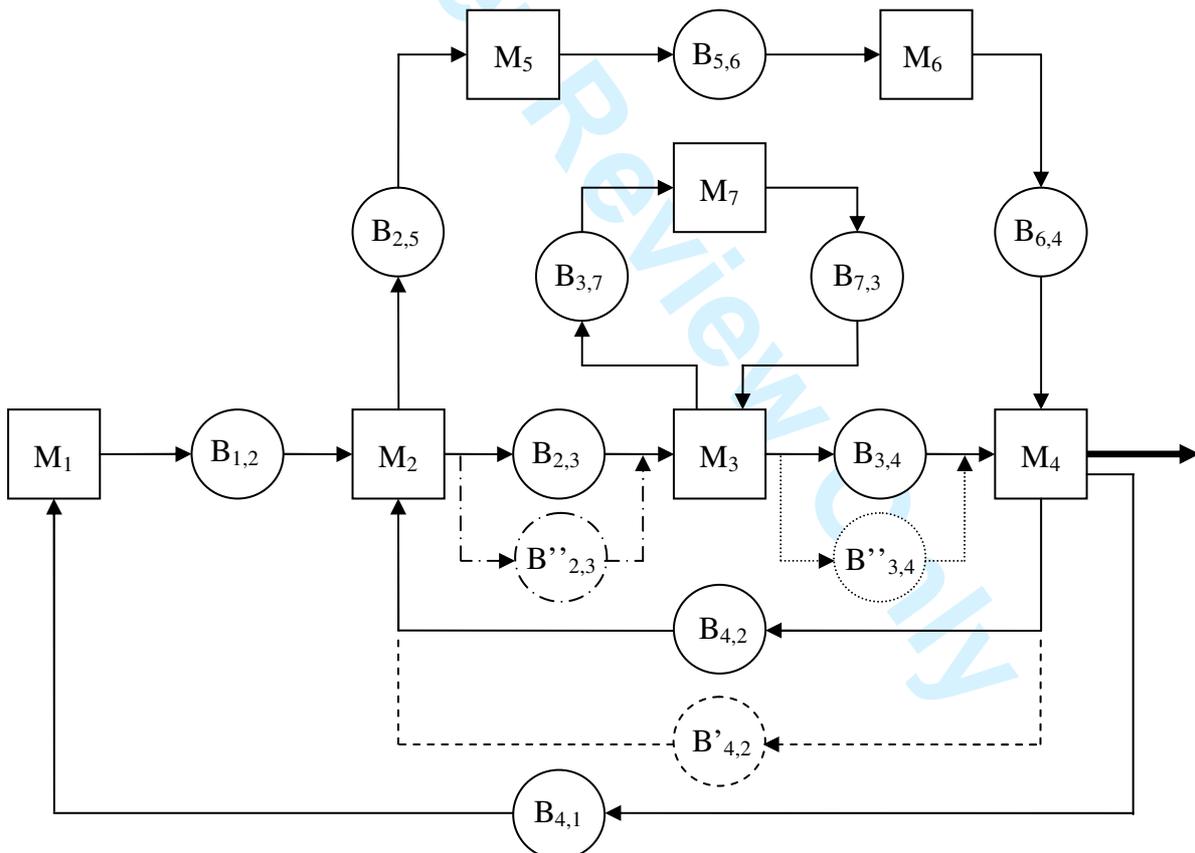
● Full buffers for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in both working regimes

Figure 18. Transmission of disruption from machine M_3 to machine M_4 for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in stationary and transitory regimes.



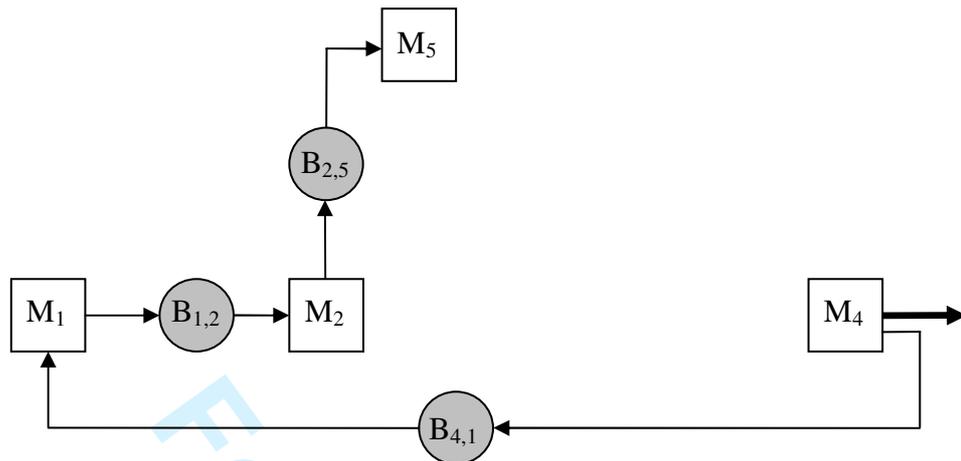
○ Empty buffer for $0.40 < x \leq 0.97$ and $0.50 < x \leq 1.00$ in both working regimes

Figure 19. Transmission of disruption from machine M_3 to machine M_4 for $0.40 < x \leq 0.97$ and $0.50 < x \leq 1.00$ in stationary and transitory regimes.



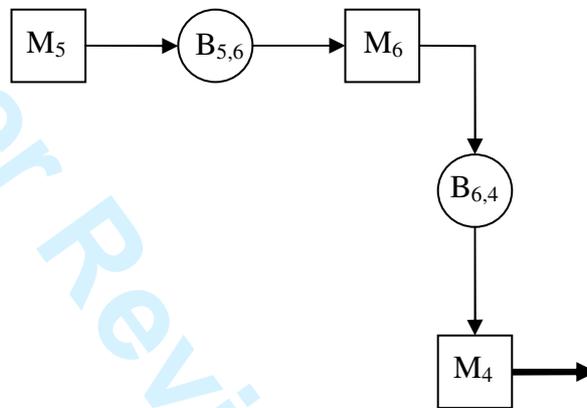
- Existing branches in both regimes
- - - - - Additional branches in both regimes
- Branches of shorter lengths in stationary regime
- . - . - Branch of shorter length in both regimes

Figure 20. Optimal configuration of buffers after disruption of machine M_3 , according to the values of Table 5.



● Full buffers for $0.60 < x \leq 0.97$ and $0.60 \leq x \leq 1.00$ in stationary and transitory regimes

Figure 21. Transmission of disruption from machine M_5 to machine M_4 for $0.60 < x \leq 0.97$ and $0.60 \leq x \leq 1.00$ in stationary and transitory regimes.



○ Empty buffers for $0.37 \leq x \leq 0.60$ and $0.37 \leq x < 0.60$ in both working regimes

Figure 22. Transmission of disruption from machine M_5 to machine M_4 for $0.37 \leq x \leq 0.60$ and $0.37 \leq x < 0.60$ in stationary and transitory regimes.

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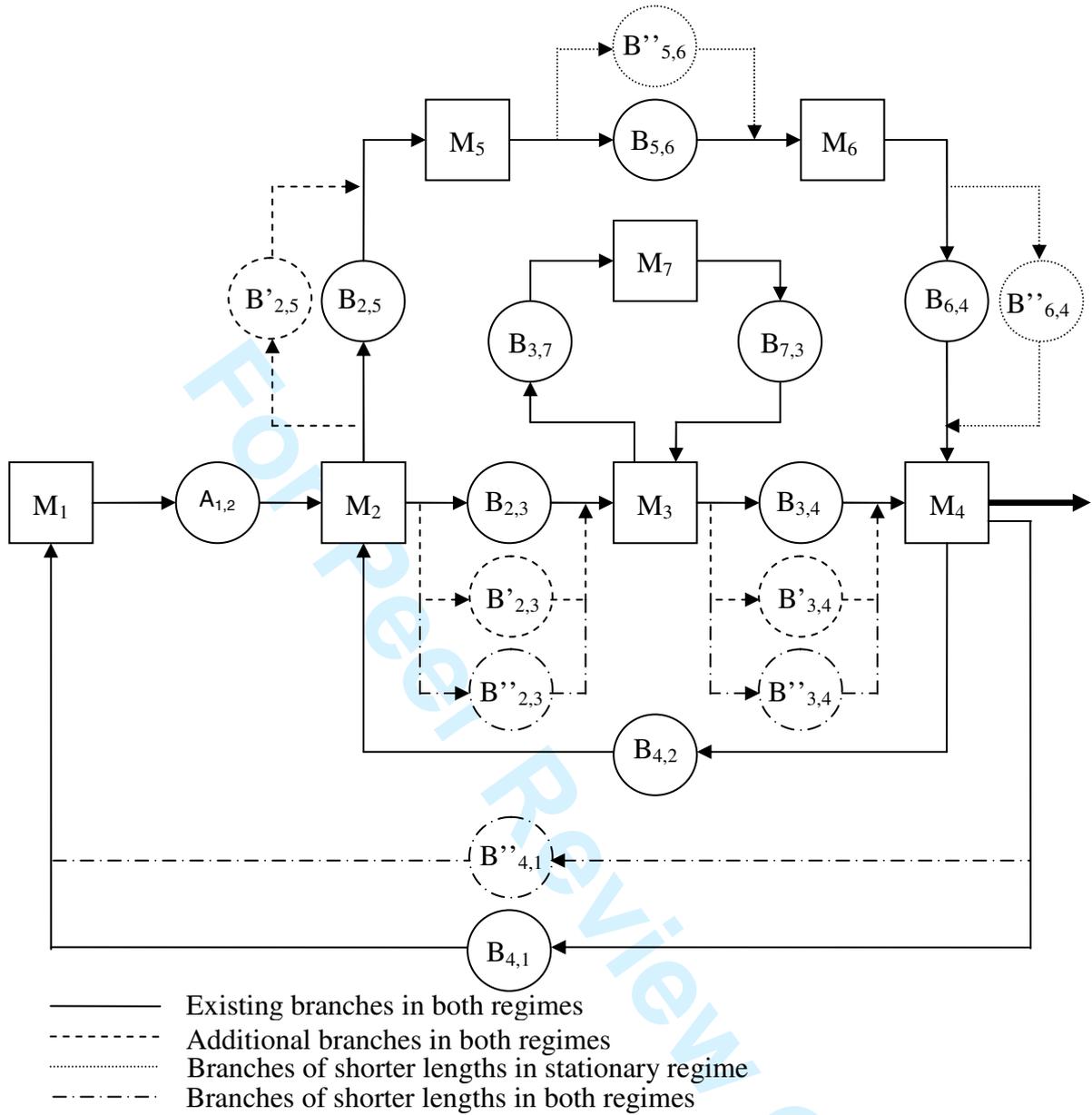
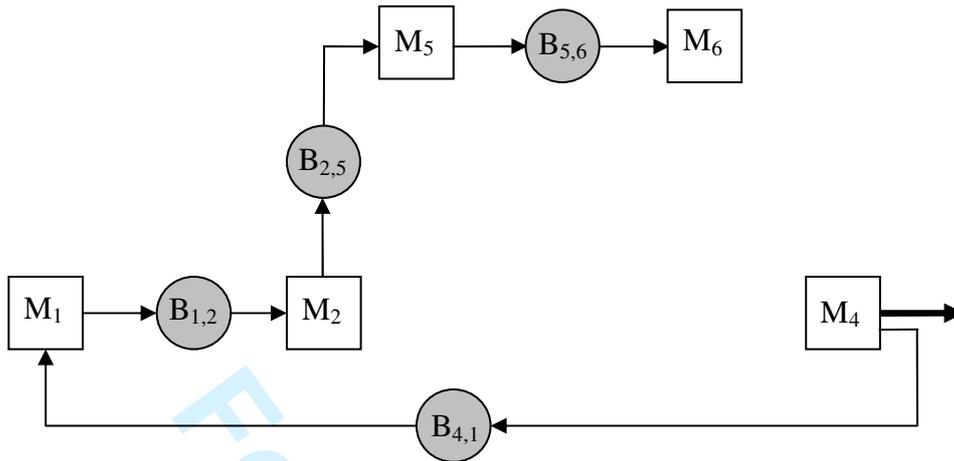
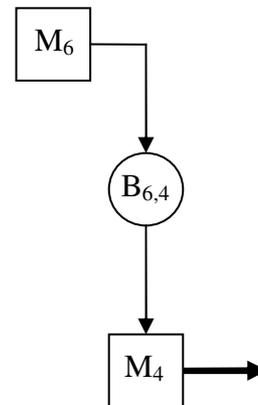


Figure 23. Optimal configuration of buffers after disruption of machine M5, according to the values of Table 6.



● Full buffers for $0.80 \leq x \leq 0.97$ and $0.80 \leq x \leq 1.00$ in stationary and transitory regimes

Figure 24. Transmission of disruption from machine M_6 to machine M_4 , for $0.80 \leq x \leq 0.97$ and $0.80 \leq x \leq 1.00$ in stationary and transitory regimes.



○ Empty buffer for $0.37 \leq x < 0.80$ in both working regimes

Figure 25. Transmission of disruption from machine M_6 to machine M_4 for $0.37 \leq x < 0.80$ in both regimes.

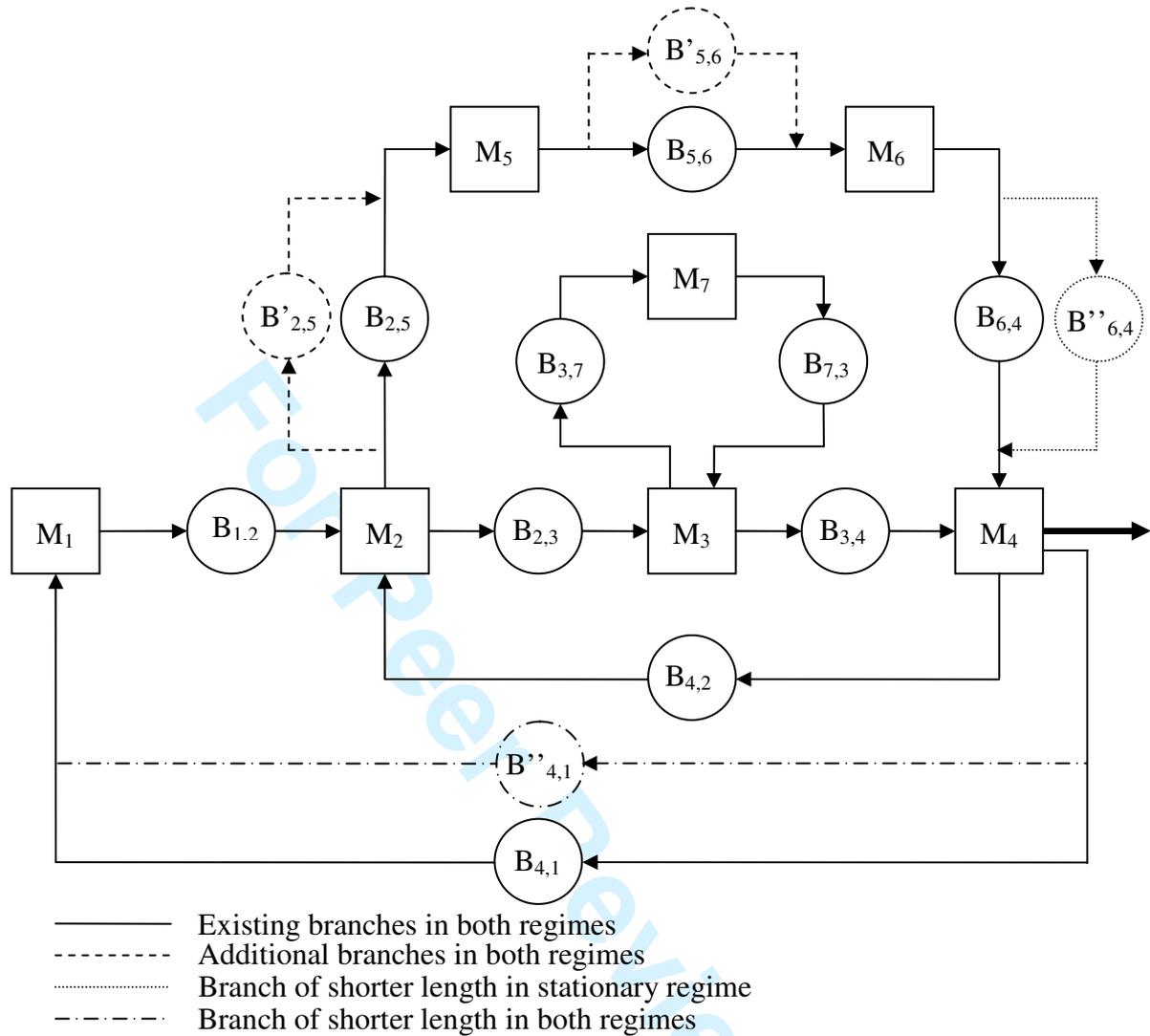
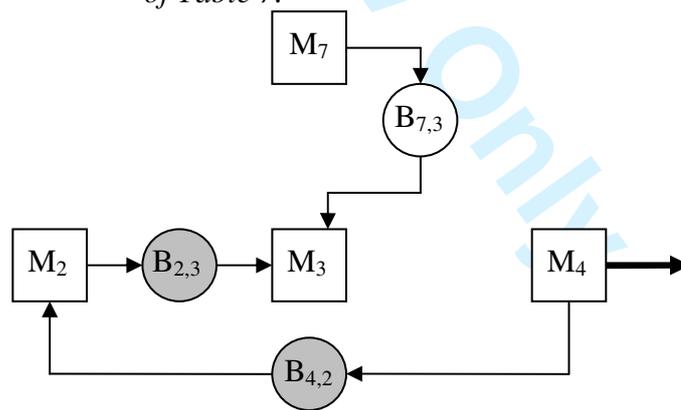
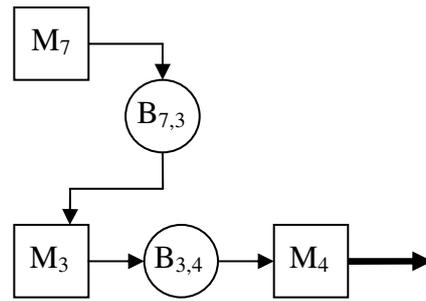


Figure 26. Optimal configuration of buffers after disruption of machine M_6 , according to the values of Table 7.



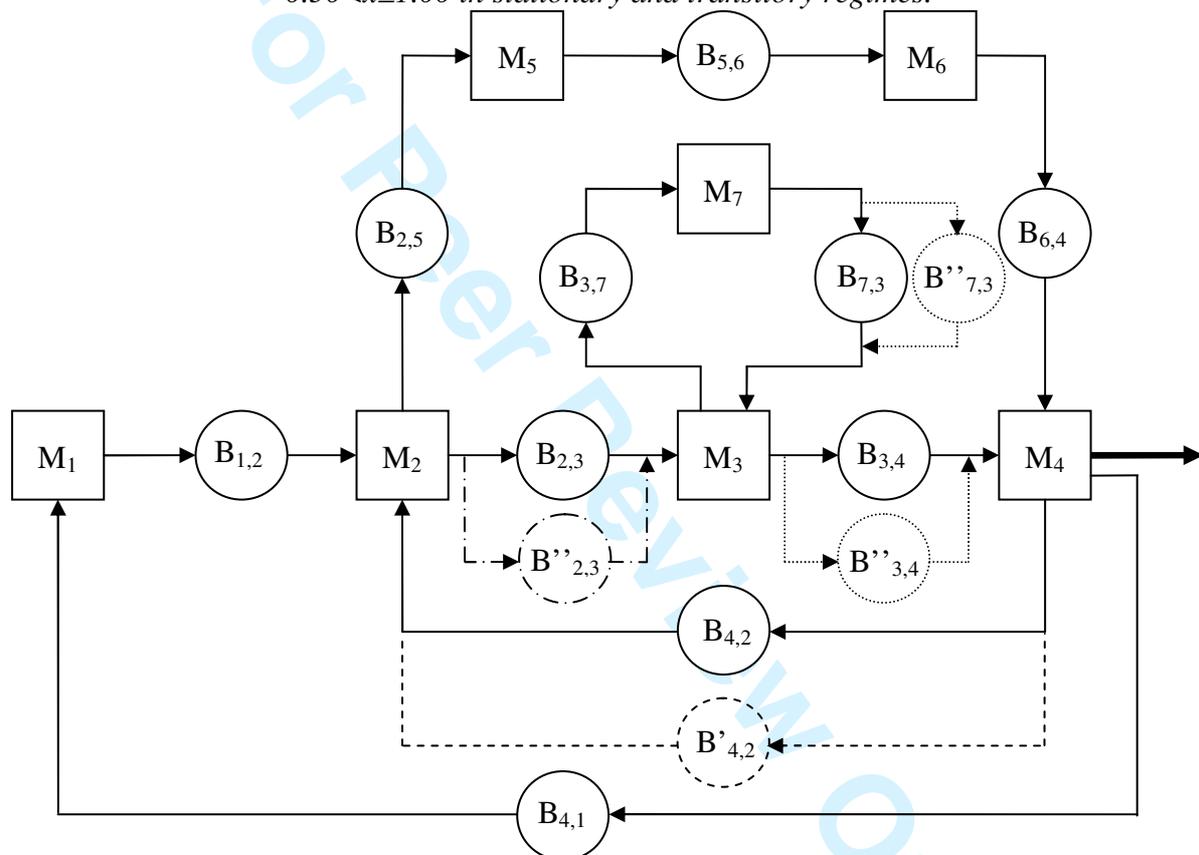
- Empty buffer for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in stationary and transitory regimes
- Full buffers for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in stationary and transitory regimes

Figure 27. Transmission of disruption from machine M_7 to machine M_4 for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$ in stationary and transitory regimes.



○ Empty buffers for $0.40 < x \leq 0.97$ and $0.50 < x \leq 1.00$ in both working regimes

Figure 28. Transmission of disruption from machine M_7 to machine M_4 for $0.40 < x \leq 0.97$ and $0.50 < x \leq 1.00$ in stationary and transitory regimes.



- Existing branches in both regimes
- - - Additional branches in both regimes
- Branches of shorter lengths in stationary regime
- · - · Branch of shorter length in both regimes

Figure 29. Optimal configuration of buffers after disruption of machine M_7 .

Table 1. Real values of $m_{i,j}$ and $M_{i,j}$

| $B_{i,j}$ | $m_{i,j}$ (pallets) | $M_{i,j}$ (pallets) |
|-----------|------------------------------|---------------------|
| $B_{1,2}$ | $m_{1,2} = 21$ | $M_{1,2} = 80$ |
| $B_{2,3}$ | $m_{2,3} = 12$ | $M_{2,3} = 37$ |
| $B_{3,4}$ | $m_{3,4} = 10$ | $M_{3,4} = 30$ |
| $B_{4,1}$ | $m_{4,1} = 31$ | $M_{4,1} = 123$ |
| $B_{2,5}$ | $m_{2,5} = 40$ | $M_{2,5} = 330$ |
| $B_{5,6}$ | $m_{5,6} = 5$ | $M_{5,6} = 39$ |
| $B_{6,4}$ | $m_{6,4} = 11.1 \cdot (1+x)$ | $M_{6,4} = 161$ |
| $B_{4,2}$ | $m_{4,2} = 8.6 \cdot (1+x)$ | $M_{4,2} = 124$ |
| $B_{3,7}$ | $m_{3,7} = 10$ | $M_{3,7} = 157$ |
| $B_{7,3}$ | $m_{7,3} = 10$ | $M_{7,3} = 135$ |

Table 2. Values of $M_{4,1}$ and $M_{4,2}$ to maximize $t_{4,1}$

| x | Stationary Regime (pallets) | Transitory Regime (pallets) |
|------|---|--|
| 0.37 | $M_{4,1} = 194$ ($K_1 = 71$) and $M_{4,2} = 184$ ($K_1 = 60$) | $M_{4,1} = 237$ ($K_1 = 114$) and $M_{4,2} = 214$ ($K_1 = 90$) |
| 0.40 | $M_{4,1} = 194$ ($K_1 = 71$) and $M_{4,2} = 177$ ($K_1 = 53$) | $M_{4,1} = 237$ ($K_1 = 114$) and $M_{4,2} = 208$ ($K_1 = 84$) |
| 0.50 | $M_{4,1} = 194$ ($K_1 = 71$) and $M_{4,2} = 153$ ($K_1 = 29$) | $M_{4,1} = 237$ ($K_1 = 114$) and $M_{4,2} = 186$ ($K_1 = 62$) |
| 0.60 | $M_{4,1} = 194$ ($K_1 = 71$) and $M_{4,2} = 129$ ($K_1 = 5$) | $M_{4,1} = 237$ ($K_1 = 114$) and $M_{4,2} = 164$ ($K_1 = 40$) |
| 0.70 | $M_{4,1} = 194$ ($K_1 = 71$) and $M_{4,2} = 106$ ($K_1 = -18$) | $M_{4,1} = 237$ ($K_1 = 114$) and $M_{4,2} = 143$ ($K_1 = 19$) |
| 0.80 | $M_{4,1} = 194$ ($K_1 = 71$) and $M_{4,2} = 81$ ($K_1 = -43$) | $M_{4,1} = 237$ ($K_1 = 114$) and $M_{4,2} = 121$ ($K_1 = -3$) |
| 0.90 | $M_{4,1} = 194$ ($K_1 = 71$) and $M_{4,2} = 59$ ($K_1 = -65$) | $M_{4,1} = 237$ ($K_1 = 114$) and $M_{4,2} = 100$ ($K_1 = -24$) |
| 0.97 | $M_{4,1} = 194$ ($K_1 = 71$) and $M_{4,2} = 41$ ($K_1 = -83$) | $M_{4,1} = 237$ ($K_1 = 114$) and $M_{4,2} = 84$ ($K_1 = -40$) |
| 1.00 | ----- | $M_{4,1} = 237$ ($K_1 = 114$) and $M_{4,2} = 78$ ($K_1 = -46$) |

Table 3. Options to maximize $t_{4,2}$ for $0.37 \leq x \leq 0.50$

| x | Stationary Regime (pallets) | Transitory Regime (pallets) |
|------|--------------------------------|--------------------------------|
| 0.37 | $M_{4,2}=184$ ($K_1=60$) and | $M_{4,2}=214$ ($K_1=90$) and |
| | $M_{1,2}=92$ ($K_1=12$) or | $M_{1,2}=114$ ($K_1=34$) or |
| | $M_{4,1}=135$ ($K_1=12$) or | $M_{4,1}=157$ ($K_1=34$) or |
| | $M_{1,2}+M_{4,1}=215$ | $M_{1,2}+M_{4,1}=237$ |
| 0.40 | $M_{4,2}=177$ ($K_1=53$) and | $M_{4,2}=208$ ($K_1=84$) and |
| | $M_{1,2}=92$ ($K_1=12$) or | $M_{1,2}=114$ ($K_1=34$) or |
| | $M_{4,1}=135$ ($K_1=12$) or | $M_{4,1}=157$ ($K_1=34$) or |
| | $M_{1,2}+M_{4,1}=215$ | $M_{1,2}+M_{4,1}=237$ |
| 0.50 | $M_{4,2}=153$ ($K_1=29$) and | $M_{4,2}=186$ ($K_1=62$) and |
| | $M_{1,2}=92$ ($K_1=12$) or | $M_{1,2}=114$ ($K_1=34$) or |
| | $M_{4,1}=135$ ($K_1=12$) or | $M_{4,1}=157$ ($K_1=34$) or |
| | $M_{1,2}+M_{4,1}=215$ | $M_{1,2}+M_{4,1}=237$ |

Table 4. Values of $M_{4,1}$ and $M_{4,2}$ to maximize $t_{4,2}$ for $0.60 \leq x \leq 0.97$ and $0.60 \leq x \leq 1.00$

| x | Stationary Regime (pallets) | Transitory Regime (pallets) |
|------|--------------------------------|--------------------------------|
| 0.60 | $M_{4,1}=135$ ($K_1=12$) and | $M_{4,1}=157$ ($K_1=34$) and |
| | $M_{4,2}=129$ ($K_1=5$) | $M_{4,2}=164$ ($K_1=40$) |
| 0.70 | $M_{4,1}=135$ ($K_1=12$) and | $M_{4,1}=157$ ($K_1=34$) and |
| | $M_{4,2}=106$ ($K_1=-18$) | $M_{4,2}=143$ ($K_1=19$) |
| 0.80 | $M_{4,1}=135$ ($K_1=12$) and | $M_{4,1}=157$ ($K_1=34$) and |
| | $M_{4,2}=81$ ($K_1=-43$) | $M_{4,2}=121$ ($K_1=-3$) |
| 0.90 | $M_{4,1}=135$ ($K_1=12$) and | $M_{4,1}=157$ ($K_1=34$) and |
| | $M_{4,2}=58$ ($K_1=-66$) | $M_{4,2}=100$ ($K_1=-24$) |
| 0.97 | $M_{4,1}=135$ ($K_1=12$) and | $M_{4,1}=157$ ($K_1=34$) and |
| | $M_{4,2}=41$ ($K_1=-83$) | $M_{4,2}=84$ ($K_1=-40$) |
| 1.00 | ----- | $M_{4,1}=157$ ($K_1=34$) and |
| | | $M_{4,2}=78$ ($K_1=-46$) |

Table 5. Values of $M_{4,2}$ and $M_{2,3}$ to maximize $t_{4,3}$ for $0.37 \leq x \leq 0.40$ and $0.37 \leq x \leq 0.50$

| x | Stationary Regime (pallets) | Transitory Regime (pallets) |
|------|--|---|
| 0.37 | $M_{4,2}=168$ ($K_1=44$) and $M_{2,3}=24$ ($K_1=-13$) | $M_{4,2}=168$ ($K_1=44$) and $M_{2,3}=34$ ($K_1=-3$) |
| 0.40 | $M_{4,2}=160$ ($K_1=36$) and $M_{2,3}=24$ ($K_1=-13$) | $M_{4,2}=160$ ($K_1=36$) and $M_{2,3}=34$ ($K_1=-3$) |
| 0.50 | ----- | $M_{4,2}=135$ ($K_1=11$) and $M_{2,3}=34$ ($K_1=-3$) |

Table 6. Values of $M_{2,3}+M_{3,4}$, $M_{4,1}$ and $M_{2,5}$ to maximize $t_{4,5}$ for $0.60 < x \leq 0.97$ and $0.60 \leq x \leq 1.00$

| x | Stationary Regime (pallets) | Transitory Regime (pallets) |
|------|---|--|
| 0.60 | ----- | $M_{2,5}=450$ ($K_1=120$) and $M_{2,3}+M_{3,4}=103$ ($K_1=36$) and $M_{4,1}=54$ ($K_1=-69$) |
| 0.70 | $M_{2,5}=411$ ($K_1=81$) and $M_{2,3}+M_{3,4}=58$ ($K_1=-9$) and $M_{4,1}=82$ ($K_1=-41$) | $M_{2,5}=450$ ($K_1=120$) and $M_{2,3}+M_{3,4}=84$ ($K_1=17$) and $M_{4,1}=73$ ($K_1=-50$) |
| 0.80 | $M_{2,5}=409$ ($K_1=79$) and $M_{2,3}+M_{3,4}=58$ ($K_1=-9$) and $M_{4,1}=99$ ($K_1=-24$) | $M_{2,5}=450$ ($K_1=120$) and $M_{2,3}+M_{3,4}=67$ ($K_1=0$) and $M_{4,1}=90$ ($K_1=-33$) |
| 0.90 | $M_{2,5}=406$ ($K_1=76$) and $M_{2,3}+M_{3,4}=44$ ($K_1=-23$) and $M_{4,1}=112$ ($K_1=-11$) | $M_{2,5}=450$ ($K_1=120$) and $M_{2,3}+M_{3,4}=53$ ($K_1=-14$) and $M_{4,1}=104$ ($K_1=-19$) |
| 0.97 | $M_{2,5}=406$ ($K_1=76$) and $M_{2,3}+M_{3,4}=42$ ($K_1=-25$) and $M_{4,1}=115$ ($K_1=-8$) | $M_{2,5}=450$ ($K_1=120$) and $M_{2,3}+M_{3,4}=43$ ($K_1=-24$) and $M_{4,1}=114$ ($K_1=-9$) |
| 1.00 | ----- | $M_{2,5}=450$ ($K_1=120$) and $M_{2,3}+M_{3,4}=39$ ($K_1=-28$) and $M_{4,1}=118$ ($K_1=-5$) |

Table 7. Options to maximize $t_{4,6}$ for $0.80 \leq x \leq 0.97$ and $0.80 \leq x \leq 1.00$

| x | Stationary Regime (pallets) | Transitory Regime (pallets) |
|------|-------------------------------------|-------------------------------------|
| 0.80 | $M_{2,5} = 375$ ($K_1 = 45$) or | $M_{2,5} = 411$ ($K_1 = 81$) or |
| | $M_{5,6} = 374$ ($K_1 = 335$) and | $M_{5,6} = 450$ ($K_1 = 411$) and |
| | $M_{4,1} = 99$ ($K_1 = -24$) | $M_{4,1} = 90$ ($K_1 = -33$) |
| 0.90 | $M_{2,5} = 373$ ($K_1 = 43$) or | $M_{2,5} = 411$ ($K_1 = 81$) or |
| | $M_{5,6} = 372$ ($K_1 = 333$) and | $M_{5,6} = 450$ ($K_1 = 411$) and |
| | $M_{4,1} = 113$ ($K_1 = -10$) | $M_{4,1} = 104$ ($K_1 = -19$) |
| 0.97 | $M_{2,5} = 372$ ($K_1 = 42$) or | $M_{2,5} = 411$ ($K_1 = 81$) or |
| | $M_{5,6} = 371$ ($K_1 = 332$) and | $M_{5,6} = 450$ ($K_1 = 411$) and |
| | $M_{4,1} = 123$ ($K_1 = 0$) | $M_{4,1} = 114$ ($K_1 = -9$) |
| 1.00 | ----- | $M_{2,5} = 411$ ($K_1 = 81$) or |
| | | $M_{5,6} = 450$ ($K_1 = 411$) and |
| | | $M_{4,1} = 118$ ($K_1 = -5$) |