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Hassan Hammouri, Pousga Kabore, Sami Othman, Jean Biston

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Hassan Hammouri, Pousga Kabore, Sami Othman, Jean Biston. Failure diagnosis and nonlinear observer. Application to a hydraulic process. Journal of the Franklin Institute of the State of Pennsylvania, 2002, 339 (4-5), pp.455-478. 10.1016/S0016-0032(02)00027-3. hal-00524444

HAL Id: hal-00524444 https://hal.science/hal-00524444

Submitted on 11 Oct 2010 $\,$

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Université de Lyon, Lyon, F-69003, France; Université Lyon 1; CNRS UMR 5007 LAGEP (Laboratoire d'Automatique et de GEnie des Procédés), 43 bd du 11 novembre, 69100 Villeurbanne, France Tel +33 (0) 4 72 43 18 45 - Fax +33 (0) 4 72 43 16 99 <u>http://www-lagep.univ-lyon1.fr/ http://www.univ-lyon1.fr</u> <u>http://www.cnrs.fr</u>

Failure Diagnosis and Nonlinear Observer. Application to a Hydraulic Process. H. Hammouri, P. Kabore, S. Othman and J. Biston LAGEP, UPRES-A CNRS Q 5007/ CPE Bat G308, Universite Claude Bernard, 43 bd du 11 Nov. 1918, 69622 Villeurbanne Cedex e-mail: hammouri@lagep.cpe.fr

Abstract : This paper deals with the problem of fault detection and isolation for nonlinear systems. The proposed approach is based on nonlinear disturbance decoupling techniques and nonlinear observers. First, we give sufficient conditions and a design procedure for the synthesis of residual generators. Next, we characterize a class of nonlinear systems for which a high gain observer can be designed. Finally, the two above developments are illustrated through a real application dealing with the detection and isolation of three failures in a hydraulic process. The design of the residual generators is shown and their performances are discussed.

Key words: Nonlinear system, Diagnosis, Observer, Hydraulic process.

1 Introduction

Over the last two decades, fault diagnosis has become an issue of primary importance in modern process automation as it provides a basis for the reliable and safe fundamental design features of many complex engineering systems. Failure diagnosis consists of providing information on the time and on the location of faults that occur in the supervised process. The former task is called fault detection problem while the latter is referred to as fault isolation. Several methods for Fault Detection and Isolation (FDI) have been presented since the original works of Beard [3] and Jones [23]. The most important ones are based on the concept of analytical redundancy, which consists of observing certain variables that are identically equal to zero (or under a predetermined threshold) as long as no failure occurs. Such observed signals are called residuals, and their generating system is referred to as a residual generator. If a residual is not altered by a subset of failures, we say that this residual is unaffected by the subset. In fact, such a residual is aptly known as a structured residual. Making a diagnosis therefore consists of comparing various structured residuals with different decoupling properties. The analysis of the distinguishably nonzero residuals leads to fault isolation. Parity space techniques (see e.g. [13] and the references therein), parameter estimation techniques (see e.g. [21], [7], [9]), statistical techniques [2] and observer-based techniques (see e.g. [26]; [27]; [8]; [29]; [1]; [18]; [19] and the references therein) are amongst the most important contributions to the analytical redundancy-based failure diagnosis approach.

Many solutions for the FDI problem that use an observer-based approach have been proposed. These include an Eigenstructure Assignment approach, an Unknown Input Observer approach, and a geometric approach proposed in [26] and [27] (see e.g. [8]; [29] as surveys of existing techniques). In fact, observers are used to estimate a part of the state vector which allows the reconstruction of the output functions and the innovation is then taken as a residual. The choice of such output functions depends on their decoupling properties with respect to failures ([26]; [27]). For nonlinear systems, the FDI problem was initially introduced by P. M. Frank [7]. More recently, many technics were proposed in order to solve the FDI problem for nonlinear systems:

- L. R. Seliger and P. M. Frank [31] propose some solutions based on robust observer approach.
- Combining the geometric decoupling technics together with the nonlinear observer synthesis, the authors in ([30], [1]; [18]; [19]; [20]) gave sufficient conditions permetting to solve the FDI problem.

The outline of this paper is as follows. In section 2, we define a sufficient condition for the solution of the FDI problem for of nonlinear systems. In section 3, we show how one can extend the high gain observer (stated in [11]) to a more general class of nonlinear systems. In section 4, we apply the theoretical results stated in sections 2 and 3 to a hydraulic process.

2 Fault detection and isolation for nonlinear systems

We consider the following class of nonlinear systems:

$$\begin{cases} \dot{x} = f(x) + \sum_{i=1}^{m} u_i g_i(x) + \sum_{i=1}^{m_f} v_i e_i(x) \\ y = h(x) \end{cases}$$
(1)

where the state $x(t) \in V$ is an open set of \mathbb{R}^n , the input $u(t) = (u_1(t), \ldots, u_m(t)) \in U$ is an open set of \mathbb{R}^m and the output $y(t) = (y_1(t), \ldots, y_p(t)) \in \mathbb{R}^p$, the v_i are unknown scalar failure modes and the vector failure \underline{v}_i is given by $(v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_{m_f})$.

The following definition extends the definition of the FPRG (fundamental problem in residual generation) proposed by M. A. Massoumnia et al. in [27].

Definition 1 The FDI w.r.t. v_i admits a solution if we can find a dynamical system called residual filter (or residual generator):

$$\dot{z} = F(u, z, y)$$

$$r = H(y, z)$$
(2)

satisfying the following conditions:

C1) The residual r depends upon v_i in the following sense: There exists an initial state (x(0), z(0)) of the augmented system (1), (2) and time varying signals u, \underline{v}_i such that the input-output map $v_i \to r(x(0), z(0), u, \underline{v}_i, v_i)$ is not trivial one (It is not a constant map).

C2) If $v_i = 0$, then

i) For every initial state (x(0), z(0)) of (1), (2) and for every signals u, \underline{v}_i , $\lim_{t \to \infty} r(t) = 0$

ii) r doesn't depend on \underline{v}_i .

For linear systems, the residual generator proposed by Massoumnia et al. is a linear one, and the authors gave a geometric condition (necessary and sufficient condition) that allows them to solve the FDI problem of definition 1.

In what follows, we give a sufficient condition together with a design procedure to solve the FDI w.r.t. v_1 . Concerning the standard geometric notions of distributions, the reader is referred to the well-known books of Isidori [22], and Nijmeijer and Van Der Schaft [28].

For the sake of simplicity, we will consider the FDI w.r.t. v_1 (i = 1). Consider the following distributions $\Delta_1 \subset \Delta_2 \subset \ldots \subset \Delta_{k-1} \subset \Delta_k \subset \ldots$ where: Δ_1 is spanned by $\{e_2, \ldots, e_{m_f}\}$ and for $k \geq 2$, Δ_k is the distribution spanned by Δ_{k-1} and $[\mathcal{G}, \Delta_{k-1}]$ where $\mathcal{G} = \{f, g_1, \ldots, g_m\}$ and $[\mathcal{G}, \Delta_{k-1}]$ denotes the family of all vector fields $[X, Y], X \in \mathcal{G}$ and $Y \in \Delta_{k-1}, [,]$ is the Lie Bracket operator.

Assuming that the Δ_i 's are of constant dimension (i.e. dim $\Delta_i(x) = \dim \Delta_i(x')$, $\forall x, x' \in V$, i = 1, 2, ...). Then, there exists an integer k such that:

$$\forall i \ge k, \ \Delta_{i+1} = \Delta_i = \Delta^*$$

Now, consider the functional space Ω of smooth functions $\sigma : \mathbb{R}^p \longrightarrow \mathbb{R}$ such that $L_X(\sigma \circ h_j) = 0$, for j = 1, ..., p. Here $L_X(\psi) = \sum_{l=1}^m X_i \frac{\partial \psi}{\partial x_i}$ is the Lie derivative of ψ w.r.t. the vector field X. Noticing that Ω is not empty (since 0 is a solution). Denote by $d\Omega$ the co-distribution spanned by all one-forms $d(\sigma \circ h_j), \sigma \in \Omega$ and assume that $d\Omega$ is of constant dimension on V. Let $\varphi_1, \ldots, \varphi_{\bar{p}} \in \Omega$ such that $(d\varphi_1, \ldots, d\varphi_{\bar{p}})$ forms a basis $d\Omega$. In what follows, φ denotes the vector map $\begin{bmatrix} \varphi_1 \end{bmatrix}$

$$\vdots \\ \varphi_{ar{p}}$$

Now, consider the nominal system (1) in which $v_2 = \dots = v_{m_f} = 0$ with the output map $\varphi \circ h$:

$$\begin{cases} \dot{x} = f(x) + \sum_{i=1}^{m} u_i g_i(x) \\ \bar{y} = \varphi \circ h(x) = \bar{h}(x) = (\bar{h}_1(x), \dots, \bar{h}_{\bar{p}}(x))^\top \end{cases}$$
(3)

Denote by $O_{\varphi \circ h}$ the observation space of system (3): the smallest real vector space containing $\bar{h}_1, \ldots, \bar{h}_{\bar{p}}$ and such that for every $\sigma \in O_{\varphi \circ h}, L_X \sigma \in O_{\varphi \circ h}$, where $X = f + \sum_{i=1}^m u_i g_i, (u_1, \ldots, u_m)$ is any m-tuple of U. Assume that the codistribution defined by $dO_{\varphi \circ h}$ is of constant dimension on V and $(d\Psi_1, \ldots, d\Psi_d)$ is a basis of $dO_{\varphi \circ h}$. Consider a local system of coordinates $\xi = (\xi_1, \ldots, \xi_n)$ such that $\xi_i = \Psi_i(x)$ for $i = 1, \ldots, d$, and set $\xi^1 = (\xi_1, \ldots, \xi_d)^\top, \xi^2 = (\xi_{d+1}, \ldots, \xi_n)^\top$, we obtain:

Lemma 2 In this system of coordinates, system (3) takes the following form:

$$\dot{\xi}^{1} = F^{1}(\xi^{1}) + \sum_{i=1}^{m} u_{i} G^{1}_{i}(\xi^{1})$$
(4)

$$\dot{\xi}^2 = F^2(\xi) + \sum_{i=1}^m u_i G_i^2(\xi)$$
(5)

$$\bar{y} = H(\xi^1) \tag{6}$$

The decomposition (4), (5), (6) is well-known and can be found in the general literature on nonlinear systems. In what follows, we assume that the system of coordinates (ξ_1, \ldots, ξ_n) is a global one: i.e. the map $x \mapsto (\xi_1(x), \ldots, \xi_n(x))$ is a diffeomorphism from V into its range. If this is not the case, we restrict ourselves to an open subset of V on which (ξ_1, \ldots, ξ_n) is well defined.

Now, denote by ker $dO_{\varphi \circ h}$ the distribution spanned by vector fields X such that $L_X(\psi) = 0$ for every $\psi \in O_{\varphi \circ h}$.

We can state our main result :

Theorem 3 Assume that the following conditions hold:

 $i) \ e_1 \notin \ker dO_{\varphi \circ h}$

ii) system (4), (6) admits an asymptotic observer then the FDI w.r.t. v_1 possesses a solution.

Proof. In what follows, we will construct a residual filter that can be used to detect and isolate the failure mode v_1 (in other words, a residual generator satisfying conditions C1) and C2) of definition 1). To do so, using the construction of Δ^* and the definition of $\varphi \circ h$, we obtain $L_{e_j}(\tau) = 0$ for $2 \leq j \leq m_f$ and for every $\tau \in O_{\varphi \circ h}$. In particular, $L_{e_2}(\xi_i) = \ldots = L_{e_{m_f}}(\xi_i) = 0$, for $1 \leq i \leq d$.

Using lemma 2 and taking into account this last fact, system (1) takes the form:

$$\dot{\xi}^{1} = F^{1}(\xi^{1}) + \sum_{i=1}^{m} u_{i}G^{1}_{i}(\xi^{1}) + v_{1}E^{1}_{1}(\xi)$$
(7)

$$\dot{\xi}^2 = F^2(\xi) + \sum_{i=1}^m u_i G_i^2(\xi) + \sum_{i=1}^{m_f} v_i E_i^2(\xi)$$
(8)

$$\bar{y} = H(\xi^1) \tag{9}$$

From condition ii), system (4), (6) admits an observer which takes the general form:

$$\left\{ \begin{array}{l} \dot{\zeta} = \Lambda(\zeta, u, \bar{y} - H(\hat{\xi}^1)) \\ \hat{\xi}^1 = \Phi(\zeta) \end{array} \right.$$

where \bar{y} is the output of system (4), (6) and $\hat{\xi}^1$ is the variable which estimates ξ^1 .

Our candidate residual generator is then of the form:

$$\dot{\zeta} = \Lambda(\zeta, u, \bar{y} - H(\Phi(\zeta)))$$

$$r = H(\Phi(\zeta)) - \bar{y}$$
(10)

where \bar{y} is the output of (7), (8). In what follow, we will show that system (10) satisfies conditions C1), C2) of definition 1.

Condition C2):

Condition C(2) - i), comes from the fact that system (10) is an asymptotic observer for system (7), (9) in which $v_1 = 0$.

Condition C2)-ii). Using the structure of system (7), (9), it is easy to see that if $v_1 = 0$, then \bar{y} doesn't depend on $v_2, ..., v_{m_f}$ and in similar way, r doesn't depend on $v_2, ..., v_{m_f}$.

Condition C1):

There exists an initial state (x(0), z(0)) of the augmented system (1), (2) and time varying signals u, \underline{v}_i such that the Frechet derivative of the functional map $v_i \to r(x(0), z(0), u, \underline{v}_i, v_i)$ does not vanish.

Assume that C1 is not satisfied. Then for every initial state $(\xi(0), \varsigma(0))$ of the augmented system (7), (8) and (10) and every signals $u, (v_2, ..., v_{m_f}) = \underline{v}_i$, the Frechet derivative of the functional map $v_i \to r(x(0), z(0), u, \underline{v}_i, v_i)$ is equal to zero. This means that the residual r doesn't depend on v_1 . using the structure of system (10), it follows that \overline{y} doesn't depend on v_1 . This implies that the output \overline{y} of system (7), (8), (9) in which $v_2 = \ldots = v_{m_f} = 0$ doesn't depend on v_1 . Set $E_1(\xi) = [E_1^{1T}(\xi), E_1^{2T}(\xi)]^T$ and applying proposition 4.14 (cf pp 136-137 [28]), it follows that for every $\tau \in O_H$, $L_{E_1}(\tau) = 0$ (where O_H is the observability space of system (7), (8), (9)), or equivalently, $e_1 \in \ker dO_{\varphi \circ h}$. This contradicts condition i) of theorem 3. Consequently, C1 is true.

To end this section, we will extend this result to more general systems.

Consider the following control affine system:

$$\dot{x} = f(x) + \sum_{i=1}^{m} u_i g_i(x) + \tilde{f}(x) + \sum_{i=1}^{m} u_i \tilde{g}_i(x) + \sum_{i=1}^{m} v_i e_i(x)$$
(11)
$$y = h(x)$$
(12)

Using the above procedure and consider Δ^* and $O_{\varphi \circ h}$ obtained from $\{f, g_1, \ldots, g_m\}$ and $\{e_2, \ldots, e_{mf}\}$. Let $(d\Psi_1, \ldots, d\Psi_d)$ be a basis of $dO_{\varphi \circ h}$, (ξ_1, \ldots, ξ_n) be a system of coordinates s.t. $(\xi_1, \ldots, \xi_d) = (\Psi_1(x), \ldots, \Psi_d(x))$ and assume the following hypothesis:

H1) For $1 \leq i \leq d$, $L_{\tilde{f}}(\psi_i)$, $L_{\tilde{g}_j}(\psi_i)$ depend only on (ξ_1, \ldots, ξ_d) and y = h(x). It means that:

$$\begin{array}{lcl} L_{\tilde{f}}(\xi_i) & = & \tilde{F}_i(\xi^1, y), \ L_{\tilde{g}_j}(\xi_i) = \tilde{G}_{ij}(\xi^1, y) \\ \text{for } 1 & \leq & i \leq d \end{array}$$

As above (see lemma 2), system (11) with output $\bar{y} = \varphi \circ h$ takes the following

form:

$$\dot{\xi}^{1} = F^{1}(\xi^{1}) + \sum_{i=1}^{m} u_{i}G^{1}_{i}(\xi^{1}) + \tilde{F}^{1}(\xi^{1}, y) + \sum_{i=1}^{m} u_{i}\tilde{G}^{1}_{i}(\xi^{1}, y) + v_{1}E^{1}_{1}(\xi)$$
(13)

$$\dot{\xi}^2 = F^2(\xi) + \sum_{i=1}^m u_i G_i^2(\xi) + \tilde{F}^2(\xi) + \sum_{i=1}^m u_i \tilde{G}_i^2(\xi) + \sum_{i=1}^{m_f} v_i E_i^2(\xi)$$
(14)

$$\bar{y} = H(\xi^1) \tag{15}$$

Now, we can state the following extension of theorem 3

Theorem 4 Under hypothesis H1), the FDI w.r.t. v_1 has a solution if the following conditions hold:

ii) For $v_1 = 0$, system (13), (15) admits an asymptotic observer

The proof of this theorem is similar to that of theorem 3.

3 Observer Synthesis

In this section, we give some observer algorithms that can be used in the construction of residual filters. Several contributions to nonlinear observer synthesis can be found in the literature:

- A Luenberger observer can be extended to a linear system up to output injection. Nonlinear systems which can be steered by a change of coordinates to such systems are characterized in [24], [25] and [32].
- The Kalman observer approach has been extended to state affine systems up to output injection by the authors in [5] and [14]. The characterization of nonlinear systems which are equivalent (using a change of coordinates) to such systems has been stated in [15], [16] and [17].
- The High gain observer is a general nonlinear extension of a Luenberger observer. Systems which are concerned by this observer design, are those which are uniformly observable. A nonlinear system is called uniformly observable if for every two different initial states x, \bar{x} and every admissible control, the associated outputs $y(x, u, t), y(\bar{x}, u, t)$ are not identically equal on their time interval of definition. In [10] (see for a short proof [11]), the authors showed for the single output case that if a nonlinear system of the form (1) (in the absence of faults) is uniformly observable, then locally generically the following map:

$$(h, L_f(h), \dots, L_f^{n-1}(h))^T$$
 (16)

i) $e_1 \notin \ker dO_{\varphi \circ h}$

is a local diffeomorphism which transforms system (1) into the following form:

$$\begin{cases} \dot{x}(t) = Ax(t) + \gamma(u(t), x(t)) \\ y(t) = Cx(t) \end{cases}$$
(17)

where $\gamma_i(x, u) = \gamma_i(x_1, \ldots, x_i, u)$,

$$A = \begin{pmatrix} 0 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & 1 \\ 0 & 0 & \dots & 0 \end{pmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \dots & 0 \end{bmatrix}$$

Moreover, the authors in [11], showed that an observer for (17) takes the following form:

$$\dot{\hat{x}} = A\hat{x} + \gamma(u, \hat{x}) - S_{\theta}^{-1}C^{\top}(C\hat{x} - y)$$

where S_{θ} is the symmetric positive definite matrix satisfying:

$$\theta S_{\theta} + A^{\top} S_{\theta} + S_{\theta} A = C^{\top} C$$

Several extensions of this observer can be found in [4], [6], [12], ... In what follows, we will extend the above observer synthesis to a more general class of nonlinear system.

3.1 Single output case

Consider the class of nonlinear system of the form:

$$\begin{cases} \dot{x}(t) = a(s(t), u(t))Ax(t) + \gamma(s(t), u(t), x(t)) \\ y(t) = Cx(t) \end{cases}$$
(18)

where A, C are as above, s(t), u(t) are known signal and input, and γ possesses the triangular structure: $\gamma_i(s, u, x) = \gamma_i(s, u, x_1, \dots, x_i)$. To design an observer for system (18), we will need the following hypotheses:

H2) We assume that one of the following conditions hold:

$$\begin{array}{ll} \mathrm{i)} & 0<\alpha < a(s(t),u(t)) <\beta \\ \mathrm{ii)} & 0>-\alpha >a(s(t),u(t))>-\beta \end{array}$$

for every $t \ge 0$, where α and β are constants.

H3) $\gamma(s, u, x)$ is a global Lipschitz function w.r.t. x locally uniformly w.r.t. (s, u): $\exists M_1, M_2 > 0; \forall s, u$ with $||s|| \leq M_1, ||u|| \leq M_2; \exists \gamma_0 > 0; \forall x, \bar{x}$, we have $||\gamma(s, u, x) - \gamma(s, u, \bar{x})|| \leq \gamma_0 ||x - \bar{x}||.$

Remark 5 a) If condition H_2) – *ii*) holds, then after a simple change of coordinates, the obtained system takes the form (18) and satisfies condition H_2) – *i*) (the change of coordinates can take the form $z = \delta x$, where δ is the $n \times n$ diagonal matrix diag($\delta_1, ..., \delta_i$) and $\delta_i = (-1)^i$).

b) Hypothesis H3) can be omitted in the case when trajectories of system (17) are bounded. Indeed, in this case, we can prolong the nonlinear term of system (18) to a global Lipschitz function (see for instance [11]).

Theorem 6 Assume that hypotheses H2), H3) hold, then for every constant $\rho \geq \frac{\beta}{2}$, the following system:

$$\dot{\hat{x}} = a(s, u)A\hat{x} + \gamma(s, u, \hat{x}) - \rho S_{\theta}^{-1}C^{\top}(C\hat{x} - y)$$

is an exponential observer, for θ sufficiently large where S_{θ} is as above.

Proof. We proceed as in [11]:

Set $e(t) = \hat{x}(t) - x(t)$, and consider the change of coordinates: $\varepsilon = \Delta_{\theta} e$, where $\Delta_{\theta} = diag(\frac{1}{\theta}, ..., \frac{1}{\theta^n})$ we get:

$$\dot{\varepsilon} = \theta[a(s(t), u(t))A - \rho S_1^{-1} C^T C]\varepsilon + \Delta_\theta \tilde{\gamma}$$

where $\tilde{\gamma} = \gamma(s(t), u(t), \hat{x}(t)) - \gamma(s(t), u(t), x(t))$ and $S_1 = S_{\theta}$ for $\theta = 1$. Now set $V(t) = \varepsilon(t)^T S_1 \varepsilon(t)$, a simple calculation gives:

$$\begin{aligned} \dot{V} &= -\theta a(s(t), u(t))V - \theta (2\rho - a(s(t), u(t)))(C\varepsilon)^2 + 2\varepsilon^T S_1 \Delta_{\theta} \tilde{\gamma} \\ &\leq -\theta a(s(t), u(t))V - \theta (2\rho - a(s(t), u(t)))(C\varepsilon)^2 + 2\sqrt{V} \sqrt{(\Delta_{\theta} \tilde{\gamma})^T S_1(\Delta_{\theta} \tilde{\gamma})} \end{aligned}$$

Using the triangular structure of $\tilde{\gamma}$ and hypothesis H3), it follows that:

$$\sqrt{(\Delta_{\theta}\tilde{\gamma})^T S_1(\Delta_{\theta}\tilde{\gamma})} \le \varkappa \|\varepsilon\|$$
(19)

for θ sufficiently large, where \varkappa is a constant which does not depend on θ . Thus,

$$\dot{V} \le -\theta a(s(t), u(t))V - \theta(2\rho - a(s(t), u(t)))(C\varepsilon)^2 + 2\frac{\varkappa}{\lambda_{\min}}V$$

where λ_{\min} is the smallest eigenvalue of S_1 . Hence for $\rho \geq \frac{\beta}{2}$, we obtain:

$$\dot{V} \le -(\theta \alpha - 2\frac{\varkappa}{\lambda_{\min}})V \tag{20}$$

Finally choose $\theta > 2\frac{\varkappa}{\lambda_{\min}}$, $\|\varepsilon(t)\|$ converges exponentially to zero.

3.2 A multi-output case

In what follows, we will extend the above observer synthesis to the multi-output cascade system:

$$\begin{pmatrix} \dot{x}^{1}(t) = a_{1}(s(t), u(t))A_{1}x(t) + \gamma^{1}(s(t), u(t), x(t)) \\ \vdots \\ \dot{x}^{i+1}(t) = a_{i+1}(s(t), u(t), x^{1}(t), ..., x^{i}(t))A_{i+1}x^{i+1}(t) + \gamma^{i+1}(s(t), u(t), x(t)) \\ 1 \le i \le p - 1 \\ y(t) = \begin{bmatrix} C_{1}x^{1}(t) \\ \vdots \\ C_{p}x^{p}(t) \end{bmatrix} = \begin{bmatrix} x_{1}^{1}(t) \\ \vdots \\ x_{1}^{p}(t) \end{bmatrix}$$

$$(21)$$

Where,
$$x^{i} = \begin{bmatrix} x_{1}^{i} & \dots & x_{n_{i}}^{i} \end{bmatrix}^{T} \in \mathbb{R}^{n_{i}}, x = \begin{bmatrix} x^{1} \\ \vdots \\ x^{p} \end{bmatrix} \in \mathbb{R}^{n}, \gamma = \begin{bmatrix} \gamma^{1} \\ \vdots \\ \gamma^{p} \end{bmatrix}$$
 and

 A_i is the $n_i \times n_i$ matrix:

$$\left(\begin{array}{ccccc} 0 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & 1 \\ 0 & 0 & \dots & 0 \end{array}\right)$$

s(t), u(t) are respectively a known signal and input, and the $a_i(s, u, x^1, ..., x^i)$ are smooth w.r.t. $(x^1, ..., x^i)$.

As above, we assume that the system possesses the following triangular structure:

The j th components of γ^i satisfy: $\gamma^i_j(s, u, x) = \gamma^i_j(s, u, ..., x^{i-1}, x^i_1, ..., x^i_j)$, for $1 \le j \le n_i$.

To design a high gain observer, we need the following hypotheses:

 H_4) We only concerned by a set of bounded admissible controls \mathcal{U} and compact sets $\mathcal{K} \subset \mathcal{K}'$ such that for every input $u \in \mathcal{U}$ and every initial state $x \in \mathcal{K}$, the associated trajectory $x_u(t)$ belongs to \mathcal{K}' for every $t \geq 0$.

 H_5) We assume that one of the following conditions hold:

- i) $0 < \alpha_i < a_{i+1}(s(t), u(t), x^1(t), ..., x^i(t)) < \beta_i$
- ii) $0 > -\alpha_i > a_{i+1}(s(t), u(t), x^1(t), ..., x^i(t)) > -\beta_i$

for every $t \ge 0$; for very $u \in \mathcal{U}$ and $1 \le i \le q$, where α_i and β_i are constants. Using hypothesis H_4), we can always assume that the nonlinear term γ is global Lipschitz w.r.t. x. Indeed, if this is not the case, we can prolong γ outside \mathcal{K}' to a global Lipschitz function (see for instance the construction given in section 4 formula (41)).

As above consider the symmetric positive definite matrix S^i_{θ} satisfying $\theta S^i_{\theta} + A^{\top}_i S^i_{\theta} + S^i_{\theta} A_i = C^{\top}_i C_i$, and set

$$A = \begin{pmatrix} A_1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & A_p \end{pmatrix}$$
$$S_{\theta} = \begin{pmatrix} S_{\theta}^1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & S_{\theta}^p \end{pmatrix}$$

As stated in remark 5 -a), H_5) can be reduced to $H_5 - i$). Now we can state our main result:

Theorem 7 Under hypotheses H_4 , $H_5 - i$, the following system:

$$\dot{\hat{x}} = \Omega(s, u, \breve{x})A\hat{x} + \gamma(s, u, \breve{x}) - \Lambda S_{\theta}^{-1}C^{\top}(C\hat{x} - y)$$

is an exponential observer, for θ sufficiently large, where,

$$\breve{x} = \begin{bmatrix} \breve{x}^{1} \\ \cdot \\ \cdot \\ \cdot \\ \breve{x}^{p} \end{bmatrix} \text{ and } \breve{x}^{i} = \begin{bmatrix} y_{i} \\ \hat{x}_{2}^{i} \\ \cdot \\ \cdot \\ \hat{x}_{n_{i}}^{i} \end{bmatrix}$$

$$\begin{split} \Lambda &= \left[\begin{array}{cccc} \lambda_{1}I_{1} & 0 & \dots & 0 \\ 0 & \cdot & \ddots & \vdots \\ \vdots & \ddots & \cdot & 0 \\ 0 & \dots & 0 & \lambda_{p}I_{p} \end{array} \right], \\ \Omega(s, u, \breve{x}) &= \left[\begin{array}{cccc} a_{1}(s, u)I_{1} & 0 & \dots & 0 \\ 0 & a_{1}(s, u, \breve{x}^{1}) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & a_{p}(s, u, \breve{x}^{1}, \dots, \breve{x}^{p-1})I_{p} \end{array} \right] \end{split}$$

 I_i is the $n_i \times n_i$ identity matrix, and $\lambda_i \geq \frac{\beta_i}{2}$.

Proof. For the sake of simplicity, we only give the proof for p = 2. As above, set $e^i(t) = \hat{x}^i(t) - x^i(t)$, $\bar{x}^i = \Delta^i_\theta x^i$, $\varepsilon^i = \Delta^i_\theta e^i$, where Δ^i_θ is the $n_i \times n_i$ matrix $diag(\frac{1}{\theta}, .., \frac{1}{\theta^{n_i}})$ we get:

$$\begin{aligned} \dot{\varepsilon}^{1} &= \theta[a_{1}(s, u)A_{1} - \lambda_{1}(S_{1}^{1})^{-1}C_{1}^{T}C_{1}]\varepsilon^{1} + \Delta_{\theta}\tilde{\gamma}^{1} \\ \dot{\varepsilon}^{2} &= \theta[a_{2}(s, u, \breve{x}^{1}(t))A_{2} - \lambda_{2}(S_{2}^{2})^{-1}C_{2}^{T}C_{2}]\varepsilon^{2} + \Delta_{\theta}\tilde{\gamma}^{2} \\ &+ \theta[a_{2}(s, u, \breve{x}^{1}) - \theta a_{2}(s, u, x^{1}(t))]A_{2}\bar{x}^{2} \end{aligned}$$

where $\tilde{\gamma}^i = \gamma^i(s, u, \breve{x}) - \gamma^i(s, u, x)$ and $S_1^i = S_{\theta}^i$ for $\theta = 1$.

Now set $V_i = (\varepsilon^i)^T S_1^i \varepsilon^i$, and choose $\lambda_1 \ge \frac{\beta_1}{2}$, as in the single output case (see formula (20)), we obtain:

$$\dot{V}_1 \le -(\theta \alpha_1 - 2\frac{\varkappa_1}{\lambda_{\min}^1})V_1 \tag{22}$$

where \varkappa_1 is a constant which can be obtained in a similar way as in formula (19) by replacing $\tilde{\gamma}$ and S by $\tilde{\gamma}^1$ and S^1 and λ_{\min}^1 stands from the smallest eigenvalue of S^1 .

Now taking $\theta > 2\frac{\varkappa_1}{\lambda_{\min}^1}$, it follows that:

$$\left\|e^{1}(t)\right\| \leq c_{1}(\theta) \exp\left[-\frac{\left(\theta\alpha - 2\frac{\varkappa_{1}}{\lambda_{\min}^{1}}\right)t}{2}\right]$$
(23)

for some constant $c_1(\theta) > 0$. Similarly, we have:

$$\dot{V}_2 = -\theta a_2(s, u, \breve{x}^1) V_2 - \theta (2\lambda_2 - a(s, u, \breve{x}^1) (C_2 \varepsilon^2)^2 + 2(\varepsilon^2)^T S_1^2 \Delta_\theta \tilde{\gamma}^2 - 2\theta [a_2(s, u, \breve{x}^1) - a_2(s, u, x^1)] (\varepsilon^2)^T S_1^2 A_2 \bar{x}^2$$

Using hypothesis $H_5 - i$) and formula (23), it follows that there exists $t_0 > 0$, such that for every $t \ge t_0$,

$$\alpha_2 < a_2(s(t), u(t), \breve{x}^1(t)) < \beta_2$$

As for (20), take $\lambda_2 \geq \frac{\beta_2}{2}$, the following inequality hold for every $t \geq t_0$:

$$\dot{V}_2 \le -(\theta\alpha_2 - 2\frac{\varkappa_2}{\lambda_{\min}^2})V_2 - 2\theta[a_2(s, u, \breve{x}^1) - a_2(s, u, x^1)](\varepsilon^2)^T S_1^2 A_2 \bar{x}^2$$

where \varkappa_2 is obtained as in formula (19) in which $\tilde{\gamma}$ and S_1 are replaced by $\tilde{\gamma}^2$ and S_1^2 and λ_{\min}^2 stands from the smallest eigenvalue of S_1^2 .

Combining formula (23) and the fact that $x^1(t)$ is bounded (H_4)) and $a_2(s, u, x^1)$ is smooth w.r.t. x^1 , we obtain:

$$\left\|a_{2}(s(t), u(t), \breve{x}^{1}(t)) - a_{2}(s(t), u(t), x^{1}(t))\right\| \leq \sigma(\theta) \exp\left[-\frac{(\theta\alpha - 2\frac{\varkappa_{1}}{\lambda_{\min}^{1}})t}{2}\right]$$

for some positive constant $\sigma(\theta)$.

To end the proof, it suffices to take $\theta > 2 \max\{\frac{\varkappa_1}{\lambda_{\min}^1}, \frac{\varkappa_2}{\lambda_{\min}^2}\}$.

4 Failure diagnosis in a hydraulic process

4.1 Process description and modelling

The hydraulic process consists of three cylindrical water tanks T1, T2, T3 (see Figure 1). The tank T1 contains cold water, and the water in the tank T2 is heated by means of a resistance. The water from tanks T1 and T2 are mixed in T3. Pneumatic values V1, V2, V3, V4 and V5 are used as local regulators of the water flow rates through the system.

The model of the process can be obtained by combining the material and energy balances:

Material Balance:

This material balance is a classical one and uses only the relation between flow rates, levels of water and difference of pressure between the inlet and the outlet of valves.

We use the following notations:

 x_i : the level of water in tank T_i , $1 \le i \le 3$

- u_i : the aperture of value $i, 1 \le i \le 5$
- $\alpha_i :$ the flow rate coefficient of valve Vi

P: the measured relative pressure at the inlet of valve V1 and V2.

 ρ : the water density

g: the acceleration due to gravity

l: the length of water column between the outlet of valves V3 and V4 and the inlet of tank T3 (see Figure 1). The cross sectional area of cylindrical tanks T1, T2, T3 are s_1 , s_2 and s_3 respectively.

We obtain:

$$\dot{x}_1 = \frac{\alpha_1 \sqrt{P}}{s_1} u_1 - \alpha_3 \sqrt{\rho g} \frac{\sqrt{x_1 + l}}{s_1} u_3 \tag{24}$$

$$\dot{x}_2 = \frac{\alpha_2 \sqrt{P}}{s_2} u_2 - \alpha_4 \sqrt{\rho g} \frac{\sqrt{x_2 + l}}{s_2} u_4 \tag{25}$$

$$\dot{x}_3 = \alpha_3 \sqrt{\rho g} \frac{\sqrt{x_1 + l}}{s_3} u_3 + \alpha_4 \sqrt{\rho g} \frac{\sqrt{x_2 + l}}{s_3} u_4 - \alpha_5 \sqrt{\rho g} \frac{\sqrt{x_3}}{s_3} u_5$$
(26)

Energy balance:

For a given tank Ti, we consider two global heat exchange coefficients: the first one, h_i , corresponds to the heat exchange coefficient between the free surface of the liquid in a tank (of area $s_i = \pi r_i^2$) and the surrounding environment, r_i being the radius of the cylinder. The second one h'_i corresponds to heat exchange coefficient through the surface S_i ($S_i = \pi r_i^2 + 2\pi r_i x_i$) of the jacket of tank Ti.

The experimental results showed that the heat exchange between the free surface T1 and the environment can be neglected. Thus we only consider the energy balances of tank T2 and T3.

Denote by x_4 and x_5 the temperatures of water in T2 and T3 which are assumed to be stirred. τ and τ' denote the respective temperatures of water in T1 and the ambient which are assumed to be constant. Now set u_6 the power of the heating resistance in T2 and denote by C_p the water heat capacity, the energy balance can be expressed as follows:

$$\dot{x}_{4} = u_{2} \frac{\alpha_{2} \sqrt{P}}{x_{2} s_{2}} (\tau - x_{4}) + \frac{u_{6}}{\rho C_{p} s_{2} x_{2}} + \left[\frac{h_{2}}{\rho C_{P} x_{2}} + h_{2}^{\prime} \frac{2\pi r_{2} x_{2} + s_{2}}{\rho s_{2} C_{P} x_{2}} \right] (\tau^{\prime} - x_{4})$$
$$= u_{2} \frac{\alpha_{2} \sqrt{P}}{x_{2} s_{2}} (\tau - x_{4}) + \frac{u_{6}}{\rho C_{p} s_{2} x_{2}} + \left[\frac{h_{2} + h_{2}^{\prime}}{\rho C_{P} x_{2}} + \frac{2h_{2}^{\prime}}{\rho C_{P} r_{2}} \right] (\tau^{\prime} - x_{4})$$
(27)

$$\dot{x}_{5} = \frac{u_{3}\alpha_{3}}{x_{3}s_{3}}\sqrt{\rho g}\sqrt{x_{1}+l}(\tau-x_{5}) + \frac{u_{4}\alpha_{4}}{x_{3}s_{3}}\sqrt{\rho g}\sqrt{x_{2}+l}(x_{4}-x_{5}) \\ + \left[\frac{h3}{\rho C_{P}x_{3}} + h'3\frac{2\pi r_{3}x_{3}+s_{3}}{\rho s_{3}C_{P}x_{3}}\right](\tau'-x_{5}) \\ = \frac{u_{3}\alpha_{3}}{x_{3}s_{3}}\sqrt{\rho g}\sqrt{x_{1}+l}(\tau-x_{5}) + \frac{u_{4}\alpha_{4}}{x_{3}s_{3}}\sqrt{\rho g}\sqrt{x_{2}+l}(x_{4}-x_{5}) \\ + \left[\frac{h_{3}+h'_{3}}{\rho C_{P}x_{3}} + \frac{2h'_{3}}{\rho C_{P}r_{3}}\right](\tau'-x_{5})$$
(28)

To illustrate the methodology of section 2, we only consider the three following failure modes:

- The failure mode v_1 (resp. v_2) on the value V3 (resp. V4) can be modelled as an additional unknown signal on the well-known control u_3 (resp. u_4).
- v_3 is the failure mode on the heating resistance. This failure can be considered as an additional unknown signal on u_6 .

To achieve our objective, we take three output measurements at our disposal: $y_1 = x_2$, the level of water in T2, $y_2 = x_3$ that in T3, and $y_3 = x_5$ is the water temperature in T3.

The constraints on the inputs u_i of the process are:

$$0.33 \le u_i \le 1, \ i = 1, \dots, 5; \ 0 \le u_6 \le 4.28$$

The experimental physical domain V of state variables that we considered is given by the following constraints:

$$\begin{cases}
0 < x_1 < 0.5; \ 0.3 < x_2 < 0.5; \ 0.1 < x_3 < 0.5 \\
max(\tau, \tau') < x_4 < 80; \ max(\tau, \tau') < x_5 < 80
\end{cases}$$
(29)

4.2Detection and Isolation of v_1

Noticing that the failure mode v_1 and v_2 occur in the material balance:

$$\dot{x}_{1} = \frac{\alpha_{1}\sqrt{P}}{s_{1}}u_{1} - \alpha_{3}\sqrt{\rho g}\frac{\sqrt{x_{1}+l}}{s_{1}}u_{3} - \alpha_{3}\sqrt{\rho g}\frac{\sqrt{x_{1}+l}}{s_{1}}v_{1}$$
(30)

$$\dot{x}_{2} = \frac{\alpha_{2}\sqrt{P}}{s_{2}}u_{2} - \alpha_{4}\sqrt{\rho g}\frac{\sqrt{x_{2}+l}}{s_{2}}u_{4} - \alpha_{4}\sqrt{\rho g}\frac{\sqrt{x_{2}+l}}{s_{2}}v_{2}$$
(31)

$$\dot{x}_{3} = \alpha_{3}\sqrt{\rho g} \frac{\sqrt{x_{1}+l}}{s_{3}} u_{3} + \alpha_{4}\sqrt{\rho g} \frac{\sqrt{x_{2}+l}}{s_{3}} u_{4} - \alpha_{5}\sqrt{\rho g} \frac{\sqrt{x_{3}}}{s_{3}} u_{5} \qquad (32)$$

$$+\alpha_3\sqrt{\rho g}\frac{\sqrt{x_1+l}}{s_1}v_1 + \alpha_4\sqrt{\rho g}\frac{\sqrt{x_2+l}}{s_3}v_2$$

The output measurements are $y_1 = x_2$ and $y_2 = x_3$ $(y = h(x) = (x_2, x_3))$. In order to achieve the FDI w.r.t. v_1 , we will apply theorem 4 to system

(30), (31), (32) with output y_1, y_2 : Set f = 0, $g_1 = \frac{\alpha_1 \sqrt{P}}{s_1} \frac{\partial}{\partial x_1}$, $g_2 = \frac{\alpha_2 \sqrt{P}}{s_2} \frac{\partial}{\partial x_2}$, $g_3 = -\alpha_3 \sqrt{\rho g} \frac{\sqrt{x_1+l}}{s_1} \frac{\partial}{\partial x_1} + \alpha_3 \sqrt{\rho g} \frac{\sqrt{x_1+l}}{s_3} \frac{\partial}{\partial x_3}$, $g_4 = -\alpha_4 \sqrt{\rho g} \frac{\sqrt{x_2+l}}{s_2} \frac{\partial}{\partial x_2} + \alpha_4 \sqrt{\rho g} \frac{\sqrt{x_2+l}}{s_3} \frac{\partial}{\partial x_3}$, $g_5 = 0$, $\tilde{f} = 0$, $\tilde{g}_1 = \tilde{g}_2 = \tilde{g}_3 = \tilde{g}_4 = 0$, $\tilde{g}_5 = -\alpha_5 \sqrt{\rho g} \frac{\sqrt{x_3}}{s_3} \frac{\partial}{\partial x_3}$, $e_1 = g_3$ and $e_2 = g_4$. Under the above notations, system (30), (31), (32) takes the form (11), where the domain $x_3 < 0.5$ and the output $(y_1, y_2) = h(x) = (x_2, x_3)$.

In order to apply theorem 4, let us calculate Δ^* and ker $dO_{\varphi \circ h}$. Calculation of Δ^* :

$$\Delta_1 = span\{e_2\} = span\{-\frac{1}{s_2}\frac{\partial}{\partial x_2} + \frac{1}{s_3}\frac{\partial}{\partial x_3}\}$$
$$[f, e_2] = [g_1, e_2] = [g_2, e_2] = [g_3, e_2] = [g_4, e_2] = [g_5, e_2] = 0 . \text{ Thus, } \Delta^* =$$

 $\begin{array}{l} \Delta_1 = span\{-\frac{1}{s_2}\frac{\partial}{\partial x_2} + \frac{1}{s_3}\frac{\partial}{\partial x_3}\}.\\ \text{Consequently}, \varphi \text{ is given by } L_{e_2}(\varphi oh) = 0. \text{ This leads to: } \varphi oh(x) = \psi(s_2x_2 + \frac{1}{s_2}\frac{\partial}{\partial x_2}) + \frac{1}{s_2}\frac{\partial}{\partial x_2} + \frac{1}{s_2}\frac{\partial}{\partial x_2}$ s_3x_3) and a particular solution is given by:

$$\varphi oh(x) = s_2 x_2 + s_3 x_3$$

A simple computation shows that ker $dO_{\varphi \circ h}$ is spanned by $\{s_2 dx_2 + s_3 dx_3, dx_1\}$; Now, let us check conditions i) and ii) of theorem 4.

Condition i): since $L_{e_1}(\varphi oh) = L_{g_3}(s_2x_2 + s_3x_3) = \alpha_3\sqrt{\rho g}\sqrt{x_1 + l} \neq 0$, then $e_1 \notin \ker dO_{\varphi \circ h}.$

To show condition ii), we transform system (30), (31), (32) with output $(y_1, y_2) = (x_2, x_3)$ into the form (13), (14), (15). To do so, we perform a similar change of coordinates given by formula (15): $\xi_1 = \bar{y} = \varphi \circ h(x) = s_2 x_2 - s_3 x_3$, $\xi_2 = L_{q_3}(\varphi \circ h)(x) = \alpha_3 \sqrt{\rho g} \sqrt{x_1 + l}$ and $\xi_3 = x_2$. Hence system (30), (31), (32) with output \bar{y} , takes the following form:

$$\dot{\xi}^{1} = u_{3}A\xi^{1} + \gamma(u, y, \xi^{1}) + v_{1}E_{1}^{1}(u, \xi^{1})$$
(33)

$$\dot{\xi}^2 = \omega(u,\xi) + v_2 E_2^2(u,\xi^1) \tag{34}$$

$$\bar{y} = C\xi^1 \tag{35}$$

where $\xi^{1} = \begin{bmatrix} \xi_{1} & \xi_{2} \end{bmatrix}^{T}, \xi^{2} = \xi_{3}$ and,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\gamma(u, y, \xi^{1}) = \begin{bmatrix} \alpha_{2}\sqrt{P}u_{2} - \alpha_{5}\sqrt{\rho g}\sqrt{y_{2}}u_{5} & \frac{\alpha_{1}\alpha_{3}^{2}\rho g\sqrt{P}}{2s_{1}}\frac{1}{\xi_{2}}u_{1} - \frac{\alpha_{3}^{2}\rho g}{2s_{1}}u_{3} \end{bmatrix}^{T}$$

$$\omega(u, \xi) = \frac{\alpha_{2}\sqrt{P}}{s_{2}}u_{2} - \frac{\alpha_{4}\sqrt{\rho g}}{s_{2}}\sqrt{\xi_{3} + l}u_{4}$$

$$E_{1}^{1}(u, \xi^{1}) = \begin{bmatrix} \xi_{2} & -\frac{\alpha_{3}^{2}\rho g}{2s_{1}} \end{bmatrix}^{T}$$

$$E_{2}^{2}(u, \xi^{1}) = -\frac{\alpha_{4}\sqrt{\rho g}}{s_{2}}\sqrt{\xi_{3} + l}$$

To verify condition ii) of theorem 4, it suffices to show that subsystem (33), (35) in which $v_1 = 0$ admits an asymptotic observer. To achieve this objective, we only need to show that system (33), (35) satisfies hypotheses H2), H3) stated in subsection 3.1

Hypothesis H2) is obviously satisfied.

To obtain H3), we extend $\gamma(u, y, \xi^1)$ to a global Lipschitz function $\tilde{\gamma}$ so that the obtained nonlinear term becomes a global Lipschitz function. According to the physical domain given by (29), $\tilde{\gamma}$ can be obtained by replacing $\frac{1}{\xi_2}$ by the function:

$$\begin{cases} \frac{1}{\xi_2} & \text{if } \xi_2 = \alpha_3 \frac{s_2}{s_3} \sqrt{\rho g} \sqrt{x_1 + l} \ge \alpha_3 \frac{s_2}{s_3} \sqrt{\rho g} \sqrt{l} \\ & \sqrt{l} & \text{if } \xi_2 \le \sqrt{l} \end{cases}$$

Consequently, the obtained nonlinear term becomes a global Lipschitz function. Finally, the residual generator takes the form:

$$\begin{cases} \dot{\zeta} = u_3 A \zeta + \tilde{\gamma}(u, y, \zeta) - \rho S_{\theta}^{-1} C^{\top} (C \zeta - \bar{y}) \\ r = C \zeta - \bar{y} \end{cases}$$
(36)
$$= \begin{bmatrix} \zeta_1 & \zeta_2 \end{bmatrix}^T, S_{\theta} = \begin{bmatrix} \frac{1}{\theta} & -\frac{1}{\theta^2} \\ -\frac{1}{\theta^2} & \frac{2}{\theta^3} \end{bmatrix}.$$

4.3 Detection and Isolation of v_2

where ζ

The failure model that we will consider takes the form :

$$\dot{x}_2 = \frac{\alpha_2 \sqrt{p}}{s_2} u_2 - u_4 \alpha_4 \sqrt{\rho g} \frac{\sqrt{x_2 + l}}{s_2} + v_2 \alpha_4 \sqrt{\rho g} \frac{\sqrt{x_2 + l}}{s_2}$$

and the output measurement is $y_2 = x_2$. Obviously, the following system:

$$\begin{cases} \dot{\hat{x}}_2 = \frac{\alpha_2 \sqrt{p}}{s_2} u_2 - u_4 \alpha_4 \sqrt{\rho g} \frac{\sqrt{y_2 + l}}{s_2} - k(\hat{x}_2 - x_2) \\ r_2 = \hat{x}_2 - y_2 \end{cases}$$
(37)

is a residual filter which detects and isolates v_2 , where k > 0 is a constant.

4.4 Detection of v_3

In this case, we only consider the detection of v_3 . More precisely, the residual filter that we will consider only detects v_3 in the absence of v_1, v_2 .

The on-line measurements are $y_1 = x_2$, $y_2 = x_3$ and $y_3 = x_5$. Using equations (24), (26), (27) and (28), we obtain the following reduced model:

$$\begin{cases} \dot{x}_{1} = \frac{\alpha_{1}\sqrt{P}}{s_{1}}u_{1} - \alpha_{3}\sqrt{\rho g}\frac{\sqrt{x_{1}+l}}{s_{1}}u_{3} \\ \dot{x}_{3} = \alpha_{3}\sqrt{\rho g}\frac{\sqrt{x_{1}+l}}{s_{3}}u_{3} + \alpha_{4}\sqrt{\rho g}\frac{\sqrt{y_{1}+l}}{s_{3}}u_{4} - \alpha_{5}\sqrt{\rho g}\frac{\sqrt{x_{3}}}{s_{3}}u_{5} \\ \dot{x}_{4} = \frac{\alpha_{2}\sqrt{P}}{y_{1}s_{2}}(\tau - x_{4})u_{2} + \frac{u_{6}}{\rho C_{p}s_{2}y_{1}} + \left[\frac{h_{2}+h_{2}'}{\rho C_{P}y_{1}} + \frac{2h_{2}'}{\rho C_{P}r_{2}}\right](\tau' - x_{4}) \\ \dot{x}_{5} = \frac{u_{3}\alpha_{3}}{x_{3}s_{3}}\sqrt{\rho g}\sqrt{x_{1}+l}(\tau - x_{5}) + \frac{u_{4}\alpha_{4}}{x_{3}s_{3}}\sqrt{\rho g}\sqrt{y_{1}+l}(x_{4} - x_{5}) \end{cases}$$
(38)

First, we show that such system admits an exponential observer. Next, we give a residual filter which detect the failure v_3 in the absence of v_1 and v_2 .

To show that system (38) with output $\bar{y} = (y_2, y_3) = (x_3, x_5)$ admits an exponential observer, we transform this system into the cascade form given in subsection 3.2 and we apply theorem 7.

To do so, consider the change of coordinates $z_1 = x_3$, $z_2 = \sqrt{x_1 + l}$, $z_3 = x_5$, $z_4 = x_4$ and set $z^1 = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$, $z^2 = \begin{pmatrix} z_3 \\ z_4 \end{pmatrix}$, we obtain the following cascade system:

$$\dot{z}^{1} = a_{1}(s, u)A_{1}z^{1} + \gamma^{1}(u, s, z^{1})$$

$$\dot{z}^{2} = a_{2}(s, u, z^{1})A_{2}z^{2} + \gamma^{2}(u, s, z^{1}, z^{2})$$

$$\bar{y} = (\bar{y}_{1}, \bar{y}_{2}) = (z_{1}, z_{3})$$
(39)

where, $\boldsymbol{u}=(u_1,...,u_6)$, $\boldsymbol{s}=y_1=x_2$ the known measurement and,

$$A_{1} = A_{2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$a_{1}(s, u) = \frac{\alpha_{3}\sqrt{\rho g}}{s_{3}}u_{3}$$

$$a_{2}(s, u, z^{1}) = \frac{u_{4}\alpha_{4}}{z_{1}s_{3}}\sqrt{\rho g}\sqrt{y_{1}+l}$$

$$\gamma^{1}(u, s, z^{1}) = \begin{bmatrix} \alpha_{4}\sqrt{\rho g}\frac{\sqrt{y_{1}+l}}{s_{3}}u_{4} - \alpha_{5}\sqrt{\rho g}\frac{\sqrt{z_{1}}}{s_{3}}u_{5} \\ \frac{1}{2z_{2}}\frac{\alpha_{1}\sqrt{P}}{s_{1}}u_{1} - \frac{\alpha_{3}}{2s_{1}}\sqrt{\rho g}u_{3} \end{bmatrix}$$

$$\gamma^{2}(u, s, z^{1}, z^{2}) = \begin{bmatrix} \frac{u_{3}\alpha_{3}}{z_{1}s_{3}}\sqrt{\rho g}z_{2}(\tau - z_{3}) - \frac{u_{4}\alpha_{4}}{z_{1}s_{3}}\sqrt{\rho g}\sqrt{y_{1}+l}z_{3} \\ \frac{\alpha_{2}\sqrt{P}}{y_{1}s_{2}}(\tau - z_{4})u_{2} + \frac{u_{6}}{\rho C_{P}sy_{1}} + \begin{bmatrix} \frac{h_{2}+h_{2}'}{\rho C_{P}r_{2}} \end{bmatrix} (\tau' - z_{4}) \end{bmatrix}$$

Clearly, H_4) and H_5) (see subsection 3.2 are satisfied. Using similar notations as in section 3, the observer takes the form:

$$\begin{cases} \dot{\hat{z}}^{1} = a_{1}(s, u)A_{1}\hat{z}^{1} + \breve{\gamma}^{1}(u, s, \breve{z}^{1}) - \lambda_{1}(S_{\theta}^{1})^{-1}(\hat{z}_{1} - \bar{y}_{1}) \\ \dot{\hat{z}}^{2} = a_{2}(s, u, \breve{z}^{1})A_{2}\hat{z}^{2} + \gamma^{2}(u, s, \breve{z}^{1}, \breve{z}^{2}) - \lambda_{2}(S_{\theta}^{2})^{-1}(\hat{z}_{3} - \bar{y}_{2}) \end{cases}$$
(40)

Here,

$$\breve{z}^i = \left(\begin{array}{c} \bar{y}_i \\ \hat{z}_2^i \end{array}
ight)$$

$$\breve{\gamma}^{1}(u,s,\breve{z}^{1}) = \begin{bmatrix} \alpha_{4}\sqrt{\rho g} \frac{\sqrt{y_{1}+l}}{s_{3}} u_{4} - \alpha_{5}\sqrt{\rho g} \frac{\sqrt{y_{1}}}{s_{3}} u_{5} \\ \frac{1}{2\alpha(\hat{z}_{2})} \frac{\alpha_{1}\sqrt{P}}{s_{1}} u_{1} - \frac{\alpha_{3}}{2s_{1}}\sqrt{\rho g} u_{3} \end{bmatrix}$$

$$(41)$$

$$\alpha(\hat{z}_2) = \begin{cases} z_2 \text{ if } z_2 > \sqrt{l} \\ \sqrt{l} \text{ if } \hat{z}_2 < \sqrt{l} \end{cases}$$

$$(42)$$

$$a_2(s, u, \breve{z}^1) = \frac{u_4 \alpha_4}{\bar{y}_1 s_3} \sqrt{\rho g} \sqrt{y_1 + l}$$
(43)

$$\breve{\gamma}^{2}(u,s,\breve{z}^{1},\breve{z}^{2}) = \begin{bmatrix} \frac{u_{3}\alpha_{3}}{\bar{y}_{1}s_{3}}\sqrt{\rho g}\hat{z}_{2}(\tau-\bar{y}_{2}) - \frac{u_{4}\alpha_{4}}{\bar{y}_{1}s_{3}}\sqrt{\rho g}\sqrt{y_{1}+l}\bar{y}_{2} \\ \frac{\alpha_{2}\sqrt{P}}{y_{1}s_{2}}(\tau-\hat{z}_{4})u_{2} + \frac{u_{6}}{\rho C_{p}s_{2}y_{1}} + \begin{bmatrix} \frac{h_{2}+h_{2}'}{\rho C_{P}y_{1}} + \frac{2h_{2}'}{\rho C_{P}r_{2}} \end{bmatrix} (\tau'-\hat{z}_{4}) \end{bmatrix}$$

Note that if we initialize systems (39) and (40) at the same initial state belonging to the physical domain corresponding to the physical constraints (29), then the two trajectories z(t), $\hat{z}(t)$ coincide. Moreover, we have:

$$\begin{array}{lcl} \gamma^{1}(u(t),s(t),z^{1}(t)) & = & \check{\gamma}^{1}(u(t),s(t),\check{z}^{1}(t)) \\ \gamma^{2}(u(t),s(t),z^{1}(t),z^{2}(t)) & = & \check{\gamma}^{2}(u(t),s(t),\check{z}^{1}(t),\check{z}^{2}(t)) \\ & \text{for every } t & \geq & 0 \end{array}$$

Since $\check{\gamma}^1(u, s, \check{z}^1)$ and $\check{\gamma}^2(u, s, \check{z}^1, \check{z}^2)$ are global Lipschitz w.r.t. (\hat{z}_2, \hat{z}_4) , it follows that for an adequate choice of θ, λ_1 and λ_2 (see theorem 9) system (40) becomes an exponential observer.

Finally, the residual filter of v_3 is given by (40) and the following system:

$$r_3 = \hat{z}_3 - y_3$$

4.5 Experimental results

To show the performance of the residual generators proposed above, we did the following experiment:

From t = 150s until t = 200s (resp. from t = 450s until t = 500s), we varied the aperture of the valve v_3 (resp. v_4), see figure (3.a). The corresponding residual generators are given by the dynamical systems (36) and (37). The gain parameters are $\rho = 10$, $\theta = 0.002$, k = 0.04. Clearly, figures (3.b) and (3.c) show that residuals r_1 and r_2 detect and isolate their corresponding failures.

From t = 900s until t = 950s we varied the power of the heating resistance (see figure 4.a), as it was shown in section 4.4 the residual given by (40) is sensitive to all faults v_1, v_2 and v_3 . The parameters of filter (40) are $\lambda_1 =$ 0.2, $\lambda_2 = 0.1$, $\theta = 0.4$. In this case, the residual generator r_3 only detects v_3 . However, a logical decision permits to isolate the fault v_3 .

5 Conclusion

In this paper, we presented a nonlinear approach to solve the problem of failure detection and isolation. The proposed approach deals with a nonlinear extension of the linear geometric approach proposed by Massoumnia *et al.*,[27]. We have shown that our contribution corresponds to a sufficient condition and provides a design procedure to solve the problem. We illustrated the methodology through a hydraulic process. Three kinds of failures are considered and the design of residual generators is given, these residual generators allows us to detect and isolate two failure modes, the third one can be detected by the third residual generator and isolated by a logical decision.

The performance of the residual generators is shown and gives satisfying results.

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figure 1



figure 2.c



