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Arnaud Liefoghe, Laetitia Jourdan, El-Ghazali Talbi. Indicator-based approaches for multiobjective optimization in uncertain environments: An application to multiobjective scheduling with stochastic processing times. URPM 2010 - 25th Mini-EURO Conference: Uncertainty and Robustness in Planning and Decision Making, Apr 2010, Coimbra, Portugal. hal-00523181

HAL Id: hal-00523181

<https://hal.science/hal-00523181v1>

Submitted on 3 May 2023

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**Indicator-based approaches for multiobjective optimization in uncertain environments:
An application to multiobjective scheduling with stochastic processing times**

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ABSTRACT. Many real-world optimization problems have to face a lot of difficulties: they are often characterized by large and complex search spaces, multiple conflicting objective functions, and a host of uncertainties that have to be taken into account. Metaheuristics are natural candidates to solve those problems and make them preferable to classical optimization methods. We here propose a number of new evolutionary algorithms to find a set of non-dominated solutions from multiobjective optimization problems in uncertain environments. Experiments are conducted on multiobjective scheduling with stochastic processing times.

KEYWORDS. Multiobjective optimization problem; Uncertain environments; Evolutionary algorithms; Scheduling.

1. MOTIVATIONS

Nowadays, it has become clear that many practical optimization problems have to deal with multiple, usually conflicting, objective functions. In recent years, one of the most challenging issues related to multiobjective optimization is related to the identification of the Pareto set, or an approximation of it for large-size and difficult problems. For such a purpose, evolutionary algorithms have received a growing interest (Deb, 2001). However, a large part of multiobjective optimization problems are also subject to uncertainties coming from many sources, whether on decision variables and environmental parameters, or on an objective function by itself. There exists a growing demand for solving such real-world problems. In practice, existing approaches often consist in modeling the original, uncertain, problem in a deterministic way. However, a few adjustments in terms of resolution methods can be very useful to address such problems. This research area has received an increasing interest in recent years because of its difficulty, yet practical matter. By their inherent stochastic nature, conventional metaheuristics with sufficient adaptations present interesting mechanisms for solving optimization problems under uncertainty. Although they are commonly used to solve multiobjective problems on the one hand (Deb, 2001) and stochastic problems on the other hand (Jin and Branke, 2005), very few studies have been conducted simultaneously on both issues. For instance, scheduling problems are usually tackled in a single-objective and deterministic way. However, they are generally clearly multiobjective (T’Kindt and Billaut, 2002) and they are subject to many uncertainties (Billaut et al., 2008). Despite the importance of multiobjective optimization and optimization under uncertainty, a very limited number of studies devoted to the consideration of these two aspects exist to date.

To our knowledge, existing approaches for solving multiobjective optimization problems that are subject to uncertainties consist of the following ones. Firstly, Teich (2001) and Hughes (2001) independently

suggested to extend the concept of Pareto dominance for the stochastic case. They integrate this probabilistic dominance into the fitness assignment scheme of a multiobjective search method. But both studies make an assumption on the probability distribution the objective functions follow. Deb and Gupta (2006) propose to associate the average value for each dimension of the objective space to any feasible solution, determined over a given sample of objective vectors. A classical multiobjective search method, usually designed for deterministic multiobjective optimization problems, is then applied over these approximated objective vectors. More recently, Barrico and Antunes (2007) introduced a process to quantify the degree of robustness of a solution that is integrated into a multiobjective evolutionary algorithm. Additionally, Goh and Tan (2007) studied the impact of noisy objective functions on the performance of a set of evolutionary multiobjective optimization algorithms. According to the classification proposed by Jin and Branke (2005), all these methods deal either with noisy objective functions or with robustness on design variables, even if all of them can generally be applied in both cases. Although all the contributions of the paper can be applied to a large number of multiobjective optimization problems subject to uncertainties, experiments are here carried out on a multiobjective combinatorial optimization problem where stochastic environmental parameters are involved.

In many cases related to noisy objective functions, uncertainty on decision variables or on environmental parameters, the basic concept of uncertainty handling can be translated by the application of sets from the decision space to the objective space. Hence, taking the uncertainty into account often results in the comparison of sets. From a multiobjective standpoint, a solution is projected into a sample set of objective vectors, whose shape is generally not known in advance. The main challenge raised by such stochastic multiobjective problems can be summarized by the pair-wise comparison of objective vector sets, rather than a pair-wise comparison of single objective vectors in the deterministic case. Resolution methods and performance assessment protocols must then be adapted to deal with this specificity, either through the selection of representative objective vectors or by adjusting their internal mechanisms.

Here, we present a set of approaches devoted to stochastic multiobjective problem solving. Then, we introduce a bi-objective permutation flow-shop scheduling problem with stochastic processing times. To the best of our knowledge, this is the first time that such a stochastic scheduling problem is investigated in a multiobjective way.

2. INDICATOR-BASED METAHEURISTICS FOR MULTIOBJECTIVE OPTIMIZATION IN UNCERTAIN ENVIRONMENTS

The set of approaches proposed for the resolution of stochastic multiobjective optimization problems generalize the indicator-based fitness assignment strategy in order to take uncertainty into account. Hence, such schemes can take place in any indicator-based metaheuristic, and they are investigated here on evolutionary algorithms, in particular the Indicator-Based Evolutionary Algorithm (IBEA) proposed by Zitzler and Künzli (2004). To this end, we assume that a sample set of objective vectors is associated to any feasible solution. Sample sets are computed by means of a finite set of equally likely scenarios, each one corresponding to a realization of random variables (Fig. 1).

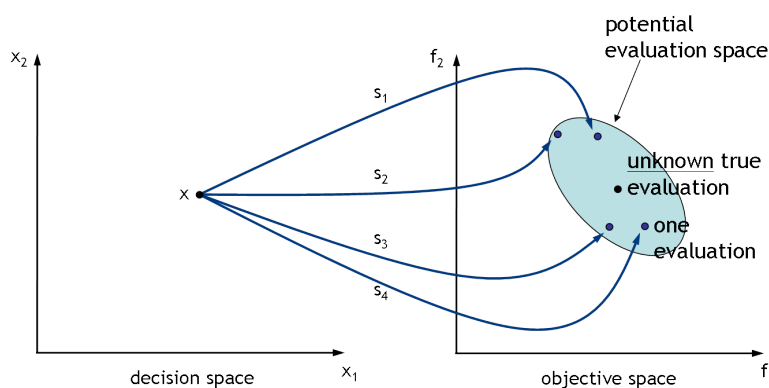


Figure 1. Multiple evaluations of a solution according to 4 scenarios (s_1, s_2, s_3, s_4). The true evaluation is unknown until the actual realization of the random variables.

Two kinds of approaches are here proposed (Fig. 2). The first ones are based on the scalarization of the objective vector values, while the second ones consist of adapting the fitness assignment strategy within IBEA in order to take uncertainty into account. For each case, four approaches, corresponding to a best-case, a worst-case, an average-case and a median-case strategy, are proposed. Following the work of Basseur and Zitzler (2006), these approaches can be seen as uncertainty-handling indicators to be used in the frame of IBEA.

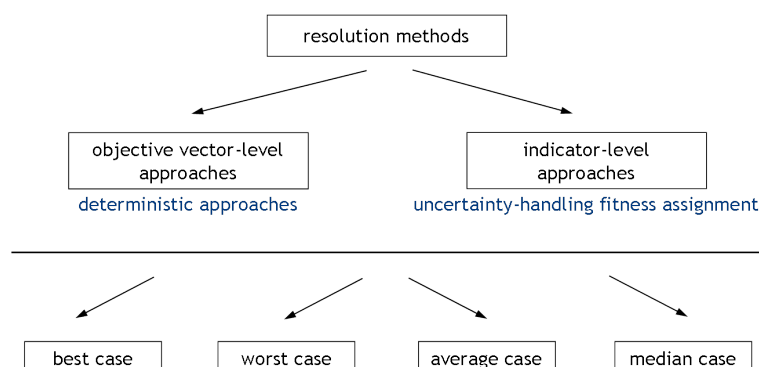


Figure 2. Approaches proposed to deal with multiobjective optimization problems under uncertainty.

2.1. Objective-vector level approaches

Firstly, objective vector-level approaches consist of reducing the objective vector sample set associated to each solution into a single representative objective vector, according to one of the four preference strategies (z^{best} for best-case, z^{worst} for worst-case, z^{avg} for average-case and z^{med} for median-case), as illustrated in Fig. 3. Such an objective vector can be seen as a deterministic one, and can then potentially be used into any multiobjective resolution method (and not only into IBEA as investigated in the paper).

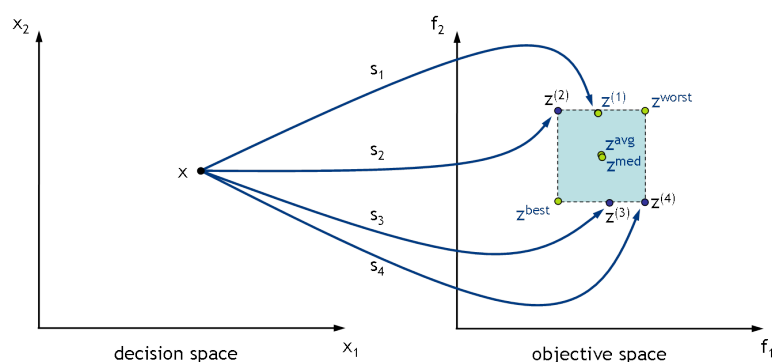


Figure 3. Illustration of the choice of a representative objective vector from a given sample set.

2.2. Indicator-level approaches

On the contrary, indicator-based approaches convert the set of indicator-values, obtained from the pair-wise comparison of solutions, into a single scalar value. Such strategies are then closely related to the indicator-based fitness assignment scheme based on a binary quality indicator (Zitzler and Künzli, 2004). It consists of introducing a total order between solutions by generalizing the Pareto dominance relation by means of a suitable binary quality indicator. It is based on a pair-wise comparison of solutions from the current population, based on a user-given indicator. To each solution is then assigned a fitness value measuring the “loss in quality” if it was removed from the current population (Zitzler and Künzli, 2004). However, contrary to the deterministic case where the comparison of two solutions in the decision space results in their comparison in the objective space (Fig.4 (a)), we here have to compute an indicator-value for each pair of objective vectors (Fig.4 (b)), and then convert these values into a single one by means of one of the four preference strategies (best-case, worst-case, average-case and for median-case). Let us

note that two cases may arise. If each evaluation of a given solution is independent from all the others performed until then, all the objective vectors from the samples associated to all solutions are strictly independent the one from the others (and the sample size can eventually differ from one solution to another). In the other case, a number of independent and equally probable scenarios is considered. Then, the solutions from the sample sets of two arbitrary solutions can be taken as paired: the two objective vectors corresponding to scenario s_1 , the two objective vectors corresponding to scenario s_2 , and so on. The latter case will be considered for our experiments.

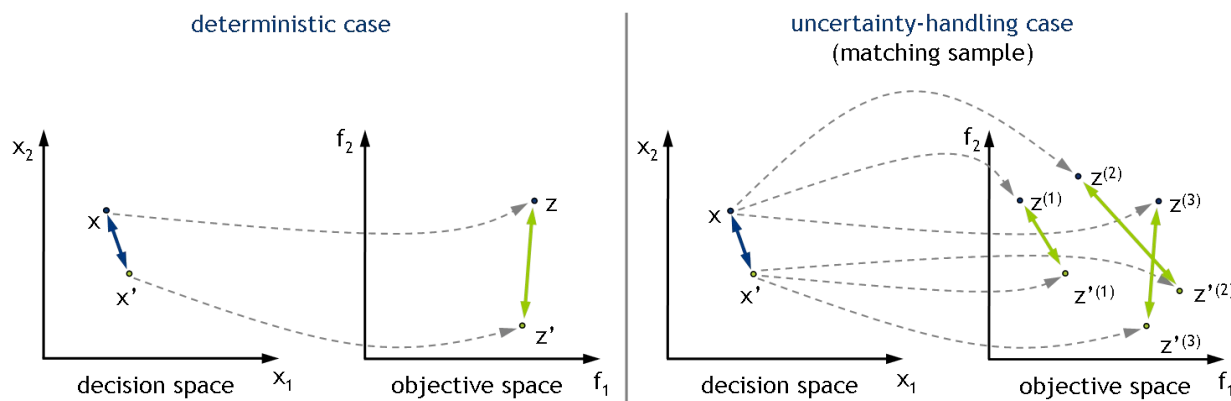


Figure 4. Illustration of the computation of indicator values for (left) the deterministic case and (right) the stochastic case.

3. APPLICATION TO STOCHASTIC MULTIOBJECTIVE SCHEDULING

3.1. A bi-objective permutation flow-shop scheduling problem with stochastic processing times

The permutation flow-shop scheduling problem is one of the most well-known scheduling problems and has been widely studied in the literature. It consists of scheduling a number of jobs on some machines. The majority of works devoted to this problem considers it on a single-objective deterministic form. However, many objective functions and many sources of uncertainty, depending on the particularities of the tackled problem. Here we consider a two-objective formulation where both the makespan (total completion time) and the total tardiness are to be minimized.

Our aim here is to find robust solutions with regards to stochastic environmental parameters. Indeed, in real-world situations, uncertainty may occur from many sources such as release or due date variations, machine breakdowns, unexpected arrival or cancellation of orders, variable processing times, and so on. For the particular case of the tackled problem, we decide to adopt a proactive stochastic approach where processing time values are regarded as uncertain and are represented by random variables. To this end, we propose a number of benchmark test instances generated according to four noise models a stochastic processing time may follow (uniform, exponential, normal, and log-normal probability distribution functions). Those instances are available at the URL: <http://www.lifl.fr/~liefooga/benchmarks/>.

3.2. Computation results

A set of eight benchmark test instances has been investigated, together with the assumption of five types of uncertainty on processing times (uniform, exponential, normal, log-normal, and various distributions), and two level of uncertainty ($\pm 10\%$, 20% of noise over processing times). The stopping condition is based on a maximum number of evaluations. The population size has been set to 100 individuals. In order to assess the performance of our algorithms, we first reevaluate the approximations on a new set of scenarios so that the influence of the search process is avoided. Then, we consider an assessment protocol based on the four preference strategies (best-case, worst-case, average-case and for median-case). Our different algorithms have also been compared to a naive, uncertainty-free approach that is based on a single (arbitrary) possible scenario, denoted by z^1 .

For an illustrative purpose, Table 1 and Table 2 present the results obtained with respect to the median-case objective vectors and to the median-case indicator-values, respectively. The best-, worst- and average-case results are not given due to space limitation. Each table gives the number of algorithms performing significantly better than the approach under consideration over a set of 10 experimental runs. The hypervolume difference indicator has been considered together with the Wilcoxon non-parametric statistical test. For each method, two different parameters have been investigated: a sample size of 10 scenarios, and a sample size of 20 scenarios.

First of all, the proposed approaches dealing with uncertainty were overall more efficient than the uncertainty-free approach, and this for almost all instances we experimented. These results prove that deterministic approaches can generally not compete with stochastic techniques, even the simplest ones. For the other approaches, the results are quite different from one preference type to another. We can still notice the very good performance of the approach based on the average objective vector when considering the average case. However, indicator-level approaches seem to appear more competitive than their objective vector-level counterpart in the other cases (best-, worst, and median-case).

Table 1. Comparison of median-case algorithms with respect to median-case objective vectors.

Instances	noise \pm 10%					noise \pm 20%				
	z^1	z^{med} (10)	z^{med} (20)	I^{med} (10)	I^{med} (20)	z^1	z^{med} (10)	z^{med} (20)	I^{med} (10)	I^{med} (20)
020 x 05 x 01	0	1	4	0	3	4	0	1	0	1
020 x 10 x 01	2	0	1	0	2	4	2	0	2	0
020 x 20 x 01	4	1	1	0	0	4	1	0	1	0
050 x 05 x 01	3	0	3	0	0	4	0	0	1	0
050 x 10 x 01	4	0	0	0	0	4	2	2	0	0
050 x 20 x 01	4	2	0	1	0	4	0	2	0	0

Table 2. Comparison of median-case algorithms with respect to median-case indicator-values.

Instances	noise \pm 10%					noise \pm 20%				
	z^1	z^{med} (10)	z^{med} (20)	I^{med} (10)	I^{med} (20)	z^1	z^{med} (10)	z^{med} (20)	I^{med} (10)	I^{med} (20)
020 x 05 x 01	1	1	4	0	3	4	0	2	0	0
020 x 10 x 01	4	0	0	0	2	4	2	0	3	1
020 x 20 x 01	4	2	2	0	0	4	3	1	1	0
050 x 05 x 01	3	0	3	0	1	4	0	0	2	0
050 x 10 x 01	4	0	0	0	0	4	2	2	1	0
050 x 20 x 01	4	2	0	2	0	4	1	3	0	0

4. CONCLUDING REMARKS

We proposed a set of new approaches to deal with multiobjective optimization problems in uncertain environments. We tested those methods on a combinatorial optimization problem from scheduling, while taking both the stochastic and the multiobjective aspects into account simultaneously. Although it is still relatively emerging in the context of multiobjective optimization, there exists a growing demand on taking uncertainty into account during the problem formulation and resolution approaches, given its undoubted practical importance. We hope to have raised the interest of this research domain and its impact on the resolution methods, especially from a metaheuristic point of view. We believe that metaheuristics have an obvious role to play in solving such problems, although many fundamental questions remain open.

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