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Transport hub flow modelling

Wilfried Despaigne * Emmanuel Frénod †

October 3, 2010

Abstract - Purpose: The purpose of this paper is to investigate the road freight haulage activity. Using the physical and data flow information from a freight forwarder, we intend to model the flow of inbound and outbound goods in a freight transport hub.

Approach: This paper presents the operation of a road haulage group. To deliver goods within two days to any location in France, a haulage contractor needs to be part of a network. This network handles the processing of both physical goods and data. We will also explore the ways in which goods and data flows are connected. We then build a first model based on Ordinary Differential Equations which decrypt the flow of goods inside the hub and which is consistent with available data. This first model is designed to work at a fine-scale level. A second model which aggregates factors of the fine scale one is also built and a way to couple hub models to build a hub network is depicted. Tests are carried out to show the accuracy of the models. Finally, an explanation on how to use the models for industrial process optimizing is given.

Keywords - Mathematical Modelling, Ordinary Differential Equations, Haulage Activity, Hub, Express, Supply Chain Modelling

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1 Introduction

This paper is part of a research programme whose target is to build a model describing the workings of a fresh and frozen product transportation network. In our vision, the nodes of the network are the hubs where merchandise is brought in, offloaded, handled, and loaded in trucks. The hubs are connected by the comings and goings of trucks.

Research has been done to reduce transport cost. Efforts have brought out algorithms optimizing transport plans [3, 20, 18, 17] and based on these algorithms, software has been developed (see for instance [24, 15]). On the other hand, very little seems to have been done on the description of hubs and of the way hubs interact with the network they are part of. These seem to be the two topics in which improvement can be made to bring about new productivity gains. The goal of this paper is to contribute to the understanding of the internal working of hubs, building a model and using it. We model it at a fine-scale and at a larger scale. We also build a simple model of interaction between hubs to be able to build simplified network models.

The paper is organized as follows. In sections 2, 3, and 4, we explain how a fresh and frozen transport network works and how information on it is presented and organized. This helps one understand how a TMS (Transport Management System) is organized and why it is organised in that specific way. In section 5, we build a hub model using ODE (Ordinary Differential Equations) type tools. We first build a fine-scale model and then, aggregating parts of it, we obtain a model, which is valid at a larger scale. We also show how these hub models may interact. In section 6, we build an implementation of the fine-scale model and we simulate the working of a very simplified hub. This shows that the model fits hub qualitative functioning well. Then, the aggregated model is also implemented. Tests show a good accuracy with fine scale results. In section 7, we give solutions to possible industrial problems via optimization questions involving the fine-scale model or the aggregated one.

2 Road freight haulage

The transport domain is a customer service. It is one of the rare domains in which the user is to be found at both ends of the supply chain (both upstream and downstream). A manufacturer or a retailer has suppliers he can influence, and customers he does his best to satisfy. The freight carrier, on the other hand, must satisfy both the sender and the receiver of the goods, who, at either end of the chain, demand quality conformity to current norms and regulations, punctuality, hygiene, and so on. The carrier is the partner of industrial and retailing firms [11]. He organizes the circulation between the various players of the supply chain (all the companies sending, transferring, or receiving goods). To get a clear notion of the carrier's activity, we will attempt to describe the process of freight haulage, and the activities it generates.

The Just-In-Time policy [16], also supported by the supermarket distribution sector, has led to the implementation of the new concept of rolling stock - meaning goods are loaded into trailers instead of being stored in warehouses. Rolling stock implies a massive decrease of storage costs, as stocks are transferred to the transport resources. To limit storage, deliveries are more frequent, which generates a significantly faster rotation of vehicles. The absence of stock generates re-ordering in real-time, and forces haulage contractors to invest heavily in equipment and data systems, constantly optimizing their working methods to satisfy the customers' requirements [22].

Since the goal is to reduce the resources needed to carry goods, the transport domain is being organized into freight forwarder networks. These networks consist in several haulage contractors, belonging or not to the same group, pooling their resources and working together to satisfy, as well as possible, the customers' requirements. The strength of a freight forwarder network resides in the players' ability to coordinate their strengths in the user's service and then realizing the "Strategic Management of Supply and Logistics" [2]. Consignments are grouped and sorted at a dispatching hub, managed by a

haulage contractor. A carrier houses an average of 8 departments: the hub, packaging, operations, data capture and logging, trade, management control, after-sales service, and truck park. Due to the heavy investments required, almost all the freight forwarders have evolved out of family businesses which have been, since the mid 1990s, passing into the control of international groups [12].

The creation of freight forwarding networks makes it possible to offer users nationwide delivery in less than 48 hours. The operation of such a network requires strict piloting through a set of procedures and software for the management of data flow, physical streams and interfaces between the various players. This set of tools is known as Supply Chain Management (SCM). To achieve control of the flows (both goods and data), the process must be industrialized: the transport plans optimized and the flows placed in synergy through computerized data processing. The SCM tools have considerably helped carriers to improve their productivity. The carrier's key tool is the TMS (Transport Management System) [14]. This is the equivalent of the WMS (Warehouse Management System) used for managing storage facilities. The TMS makes it possible to schedule and optimize routes, carry out day-to-day operations, dialogue with users and, through the reporting function, to analyze the carrier's capacities.

The issue of carrying the greatest volume of goods at the lowest cost has been researched. Teaching and business software, based on the algorithms of numerical and optimization methods, enable the carrier to optimize his transport schedule while satisfying the great majority of customer requirements. The comprehension and modelling of a freight forwarding hub, however, remains a little-explored domain. As explained in the introduction, the goal of this article is to bring a significant contribution to this issue. As we want to make models compatible with data structures in use within the large companies, the first step is to describe the physical and data flows involved in goods transportation. This information will enable us to model the flows, both inbound and outbound, revolving around the freight forwarder hub.

3 Description of the flows resulting from express transport

Hauling goods follows a succession of standard steps. We will begin by introducing the three main players of any transport operation. The first link of the chain is the sender. Often a manufacturer wants to ship his goods to a customer. He is also the prime contractor, as well as the initial loading hub. By issuing a way bill, he calls upon a carrier who will pick up the goods and forward them to the final receiver, who is also the final unloading point. This leads us to the notion of **stopovers**, or geographical points to which are associated an operation and a date. Stopovers are loading and unloading points, but they can also include other events (a relay point for drivers or trailers, for example).

It is usual to discriminate between two types of haulage: **full-load** and **express**. A full load designates a full truck load commissioned by a customer for a single recipient. A full truckload (FTL) is the usual term for a full vehicle carrying goods directly from the sender to the final receiver, without transiting through a hub - a type of operation justifying the use of an exclusive vehicle. Express involves the set of operations (pick-up, transit through the hub, shipping, transit through the hub, distribution) required for the routing of goods through the hub-and-spoke network. We will return later, in more detail, to these various operations.

Whether the haulage follows the full-load formula or is express, it is performed via an orderly succession of **routes**. A route is the trip between at least two stopovers, the loading and unloading point. A route requires a certain number of resources. These include the manpower and equipment (driver, trailer, tractor, handlers, sub-contractors) allotted to the route by the carrier so that it can be achieved in the best conditions.

A route is broken down into **stages** (Fig.1). This term designates a continuous trip during a route, performed with constant resources (same tractor, same driver, same trailer). Depending on its location in the supply chain, the stage will be called **pick-up**, **shipping** or **distribution**.

An express centre's role is to regroup consignments shipped by several senders and intended for several

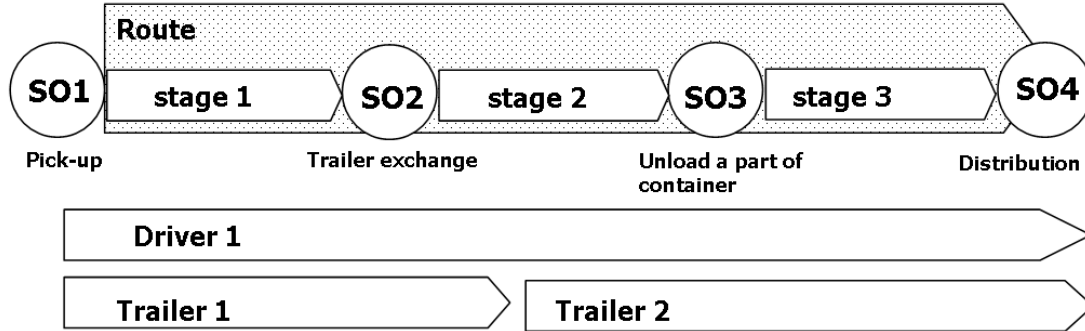


Figure 1: Exemple of breakdown of a route into stages.

customers. A given centre (A) can play three roles according to the origin and destination of goods transiting through his hub. If the centre is responsible for picking up the goods within its **catchment area**, it will be referred to as the **pick-up centre**. If the goods are being handed on by another centre further upstream in the network, and centre (A) is responsible for delivering it to the final receiver, it will be referred to as the **distribution centre**. Finally, if the goods are handed on from centre (B) and are to be forwarded to centre (C) within the same network, centre (A) is referred to as the **transit** centre.

Pick-up is the action consisting in taking a vehicle to the sender's quay to load the goods and carry them to the hub. There are three major pick-up categories:

- *Multi-customer pick-up*: upon leaving the centre's hub, the full truck performs a round of distribution. Once empty, it performs a pick-up round, visiting several senders before returning to the centre hub.
- *Single-customer pick-up*: at the end of its distribution circuit, the truck visits a single sender, loads its consignment, and returns to the centre to deliver it to the hub.
- *Two-way single-customer pick-up*: implies that a specific, exclusive vehicle performs a two-way trip between the customer and the centre hub. There is no previous distribution tour.

Shipping is the transport operation consisting in sending a consignment between two carriers.

Distribution is the transport from the dispatching centre to the final receiver.

The **catchment area** is the geographical area in which the carrier's pick-up and delivery customers are located.

To be manageable, the physical flow of goods must be accompanied by a flow of data. Every package, every transfer or handling operation is recorded in databases. The sum of this data makes up the Information and Communication System (ICS), which is designed to make information both reliable and relevant for operation and decision management [21, 23]. The purpose is to give deciders the tool they

need to pilot their activity [5]. An ICS also enables the users to share data in real time. Let us suppose that contractor (A) picks up goods intended for a receiver located 600 miles away. This operation requires the involvement of at least two transit centres (B) and (C) before arriving at its destination. Shipping Data Tracking (SDT) [6] is one of the ICS functions. The ICS makes it possible, for all the players involved, to access the data they need as soon as they receive contractor (A)'s transit order. Thus, they are able to plan for the processing of the consignment when it reaches them. The generic transport data system, called TMS, has to track the flow of goods in real time, and also to reduce costs and delivery times by planning and optimizing haulage circuits.

The information source is the computer version of the way bill. This is called the **position**. The position is the transport request made by the prime contractor. A position displays data on:

- the prime contractor, who pays for transportation,
- the sender, who ships the consignment,
- the loading point: the actual place where the goods are to be collected, for example the warehouse,
- the final receiver of the goods, for example the retailer,
- the unloading point: the actual place where the goods are to be unloaded, for example a retailer's dispatching base,
- the refrigeration status: fresh foods, frozen foods, non-refrigerated goods,
- the nature of the goods, for example cold meats or dairy products, etc.
- miscellaneous data: delivery constraints, description of packaging, etc.

The fulfilment of the contract agreed on with the customer gives rise to a succession of steps, the first of which is the creation of the position, and the last, the sum of trips necessary to carry it out. The position is broken down into **segments**. A segment is the part of a position covering the haulage between a loading point and an unloading point. Each segment belongs to the contractor allocating its resources to perform it. The segments' names vary with the nature of the stopovers.

A pick-up segment, for instance, links an outside entity, the sender, with a haulage contractor (Fig.2). The main information related to the segment is the contractor in charge of the segment, the sending entity, the unloading hub, and the type of segment. In most cases, a position is broken down into three segments, pick-up to show the goods were collected from the sender, transit to indicate the route taken between two haulage contractors, and delivery, to leave a trace of the handing over of the consignment to the final receiver. A segment begins and ends with an **event**; a pick-up segment, for instance, includes a pick-up event at the beginning and a handing-over event upon arrival at the hub.

All the segments being hauled together are regrouped on a **voucher** (Fig.2). The voucher covers the position segments being hauled together and corresponding to the same criteria (same refrigeration status, same loading hub, same date). There are, therefore, as many types of vouchers as there are types of segments (pick-up, shipping, delivery. . .). The information given by the voucher refers to type, sequence, departure date, arrival date, as well as sending and receiving entities.

To describe the resources used to transfer a consignment between two stopovers, the ICS uses the above-defined notion of **stage**. A driver, his truck and his trailer move one or several vouchers. The position segments regrouped on a voucher are assigned to a route. The route regroups one or several stages. A set of routes makes up a **schedule**.

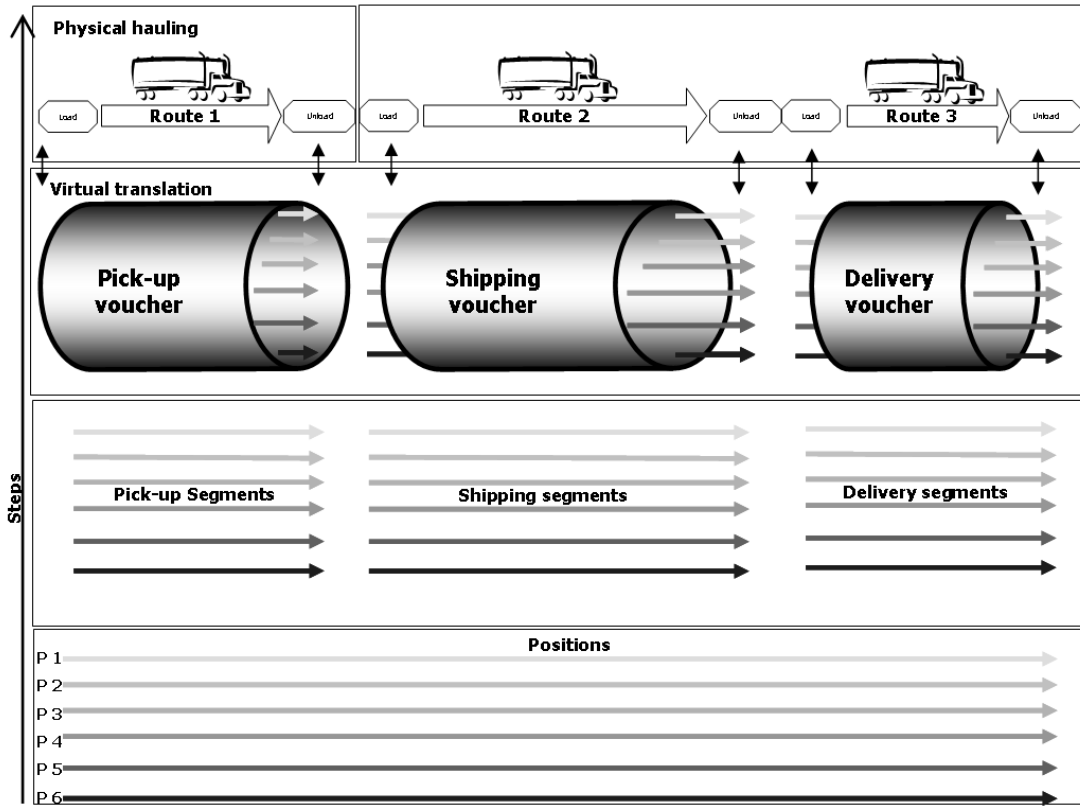


Figure 2: Breakdown of positions.

Roughly speaking, an event is located at the pick-up and distribution hub, but also at the dispatching hub. A hub is an important component of haulage activity; we will now describe how it generically functions.

4 The Hub

Let us look at an example used by [2]. According to Fig. 3, suppliers F1, F2, etc. can each provide a different product. At the other end of the line, customers C1, C2, etc. can all want various quantities of each product. For instance, customer C1 can want 2 units of product F1, 3 units of F2, 5 units of F4, while customer C2 can want 5 units of F1, 2 units of F2 1 unit of F4 and 3 units of F5. The use of the hub enables the contractor to reduce the number of trips from 5×5 to $5 + 5$ (Fig.3). Besides, thanks to the hub, the contractor can offer new services, such as the processing of orders from various suppliers. Though it is not the hub's primary vocation, it can be used as a temporary storage area to offer customers extra services, such as more fluid shipping and distribution.

A hub's size varies with the importance of the flow to be accommodated. The overall surface, including hub, office space and access, and parking area covers an average of 1 ha. [13]. On both sides of the hub are a series of loading bays, used to load and unload trailers. A good organization of the hub is of primary importance to optimize the flows. The greatest challenges are managing available space, deadlines,

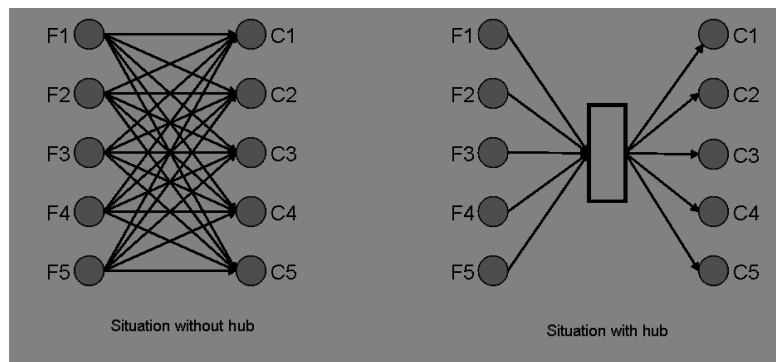


Figure 3: The economical effect of a middleman [2].

quality of delivery, and safety. The hub includes several areas (Fig.4). The “Load” and “Unload” areas face each other along the outer walls. Between them is a traffic area in which the packages are stacked on pallets and moved with forklifts and various handling apparatus. When a truck arrives, the driver

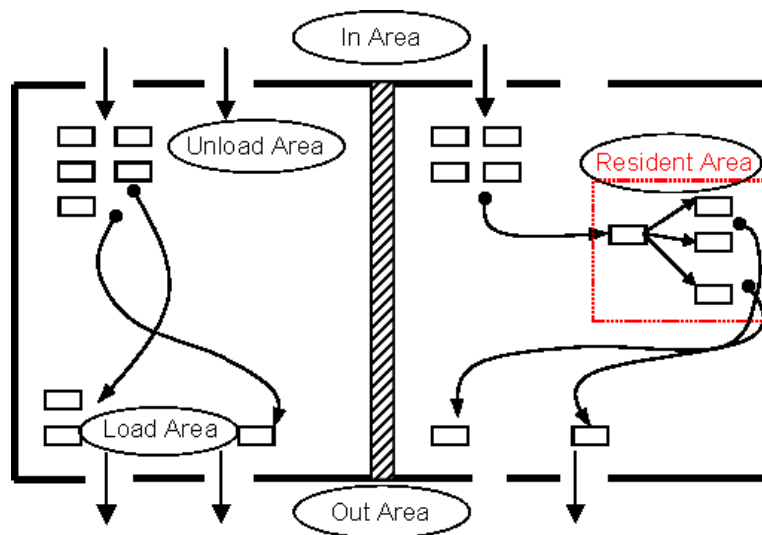


Figure 4: Flow of goods within a hub.

hands his paperwork in to the operations department. This service assigns him an unloading bay on the “In Area” side. If the bay is already in use, the driver parks in a waiting zone. The data capturing of the transport documents is transmitted to the hub, generating the printing of labels. These labels show the consignment’s origin, destination, quantity, nature, and especially the entry and exit bays it will use - and, if relevant, the “transit area” in which it is to be placed. This means that upon receipt of a consignment, an employee checks the provenance and quantity of the unloaded goods and labels them. Referring to these labels, the handlers know what route the consignment must take through the hub, and its movements are limited and controlled. In the background, the ICS regroups the packages according to their destination. If the packages do not require any sorting, they are directly taken to an area facing their loading bay. This process increases the available storage space and makes it possible,

at any time, to know where the goods are, thus limiting losses. If the unloaded goods need to be sorted before departure, they are taken to the sorting zone. The dispatching operations must not slow down the traffic of other packages.

To conform to the customer’s required deadline, the absolute must is to have the truck depart on time. The **schedule** defined by operations assigns arrival and departure times for each route. If these times are disregarded, the whole day’s organization suffers. This is why the trucks leave on time even when they are not fully loaded. When everything is going as planned, the loading or unloading of a truck takes about 20 minutes.

The hub’s productivity is calculated on a ton-per-hour index. This index also makes it possible to schedule the hub’s human resources for each day of the week. Based on a forecast expressed in tons [8, 9, 7]), the week’s planned manpower is:

- *PI_d*: productivity index of day *d*, with *d* being Monday, Tuesday, . . . , Saturday,
- *SH_d*: number of hours scheduled for day *d*,
- *TF_d*: forecasts in tons for day *d*,
- *SH_d* = ratio *TF_d* on moving average of the 6 previous *PI_d*.

This description done, a model of hub functioning taking the above explanation into account must be set out.

5 Modelling of the flows transiting through the hub

This section is devoted to model building.

5.1 Modelling procedure

We chose to follow the following modelling protocol which yields flexible models, which are well adapted to data at disposal. In a first step, we consider that the hub may be well described by a multi-compartment model between which mass of goods is transferred. Considering a given position, we assume that the transfer of mass of goods of this position is well quantified by transfer functions involved in an ordinary differential equation system [19, 1]. This system makes up what we call the fine-scale model working at the position scale. Then summing over the positions quantities involved in the fine-scale model, and making assumptions to deduce average transfer functions, we set out what we call the aggregated model. Finally, we build up a hub network by linking hubs by stage times.

5.2 Fine-scale model

We model hub activity by a five compartment model and four transfer functions. Schematically, a hub is made of five areas. The first one, we call “**In Area**”, or “In” in short, is not physically located. It takes into account that when a lorry arrives at a given hub, several tasks need to be done between registration and unloading (platform reaching, trailer opening, unloading). Goods are unloaded on an “**Unload Area**” or “Ulo”. After unloading, goods are handled. They may be taken into a “**Reserve**” or to a “**Storage Area**” if they are scheduled to remain at the hub for a long period of time. We call the set of both areas “**Residence Area**” or “Res”. After residing in the “Residence Area”, goods are brought to a “**Load Area**” or “Loa” in short, before going outside which is modelled by an “**Out Area**”

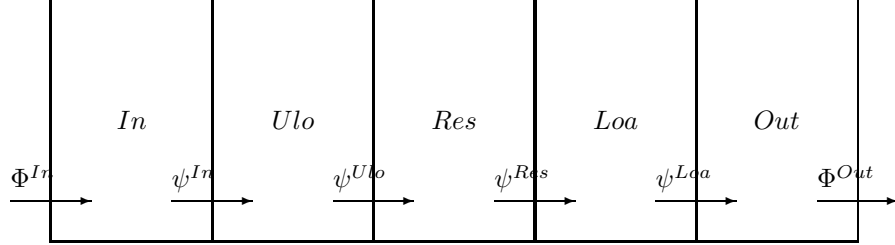


Figure 5: The five compartments and their transfer functions.

or “Out” in short.

The velocity at which goods are transferred between areas depends on good packaging and on the workforce allocated to this task. Moreover, the following important facts also need to be taken into account. First, the above schematic description does not fit all the situations. For instance, goods may be directly transferred from the “Unload Area” to the “Load Area”. Secondly, in big hubs, specific personnel is allocated to unloading goods, while others are allocated to handling goods within the hub, and others to loading. On the other hand, in smaller hubs, the same personnel may be allocated to any task when necessity arises.

The model we have created takes all of the previous facts into account. Moreover, it has to be valid at a time scale of several minutes. Indeed, the applications for which we want to use it need discrete time models with time steps ranging from a minimum time step Δt_{\min} which is about 15 min to a maximum time step Δt_{\max} which is about 3 hours. Hence, we build an ordinary differential equation model from which discrete models will be deduced by approximation methods for applications.

For $A \in \{In, Ulo, Res, Loa, Out\}$, p being in the set Pos of positions and $t \in \mathbb{R}$, we define $S^{A,(p)}(t)$ as the mass of goods belonging to position p at time t in area A . Strictly speaking, taking into account the time scale considerations $S^{A,(p)}(t)$ is the mean value of this mass of goods over an interval with length Δt_{\min} and centred in t .

We also define $\Phi^{In,(p)}(t)$ which is the mass flow of goods of position p entering “In Area” at time t . Its precise definition consists in saying that for a given time step $\Delta t > \Delta t_{\min}$ the mass entering “In Area” between t and $t + \Delta t$ is

$$\int_t^{t+\Delta t} \Phi^{In,(p)}(s) ds. \quad (5.1)$$

Similarly, we define $\Phi^{Out,(p)}(t)$ as the flow of mass of goods of position p leaving “Out Area” at time t . Considering quantities $S^A(t) = \sum_{p \in Pos} S^{A,(p)}(t)$ as the total mass of goods being in area A at time

t , we may define flow of mass between areas using a transfer function. We consider transfer function $\psi^{In,(p)}(t)$ which is the mass flow of goods of position p going from “In Area” to “Unload Area” at t . We also define $\psi^{Ulo,(p)}(t)$ as the transfer function from “Residence Area” to “Load Area” and $\psi^{Loa,(p)}(t)$ as the transfer function from “Load Area” to “Out Area”. It is reasonable to consider that

$$\begin{cases} \psi^{In,(p)}(t) = \tilde{\psi}^{In}(\Pi(p, t), S^{Ulo}(t), n^{In,(p)}(t)) \chi(S^{In,(p)}(t)), \\ \text{where } \chi(S) = 1 \text{ if } S > 0 \text{ and } 0 \text{ if } S \leq 0. \end{cases} \quad (5.2)$$

Equation (5.2) means that $\psi^{In,(p)}(t)$ is a function of the position. The way that $\tilde{\psi}^{In}$ depends on p has to take into account the packaging features of the goods of position p and possibly other information concerning p . Then $\Pi(p, t)$ stands for a list of information concerning p at time t . $\tilde{\psi}^{In}$ is also a function of S^{Ulo} . This allows one to take into account that S^{Ulo} may be overloaded. $\tilde{\psi}^{In}$ also depends on the number of employees $n^{In,(p)}(t)$ allocated to the transfer task. $\chi(S^{In,(p)}(t))$ says that when no more goods of position p are present in “In Are” then the transfer stops. In the same way, we also consider

$$\psi^{Ulo,(p)}(t) = \tilde{\psi}^{Ulo}(\Pi(p, t), S^{Res}(t), n^{Ulo}(t))\chi(S^{Ulo,(p)}(t)), \quad (5.3)$$

$$\psi^{Res,(p)}(t) = \tilde{\psi}^{Res}(\Pi(p, t), \nu, S^{Loa}(t), n^{Res}(t))\chi(S^{Res,(p)}(t)), \quad (5.4)$$

$$\psi^{Loa,(p)}(t) = \tilde{\psi}^{Loa}(\Pi(p, t), S^{Out}(t), n^{Loa}(t))\chi(S^{Loa,(p)}(t)). \quad (5.5)$$

Having those quantities and transfer functions, we can write the ordinary differential equation system modelling the transfer of mass of goods of position p within the hub.

$$\frac{dS^{In,(p)}}{dt} = \Phi^{In,(p)} - \psi^{In,(p)}, \quad (5.6)$$

$$\frac{dS^{Ulo,(p)}}{dt} = \psi^{In,(p)} - \psi^{Ulo,(p)}, \quad (5.7)$$

$$\frac{dS^{Res,(p)}}{dt} = \psi^{Ulo,(p)} - \psi^{Res,(p)}, \quad (5.8)$$

$$\frac{dS^{Loa,(p)}}{dt} = \psi^{Res,(p)} - \psi^{Loa,(p)}, \quad (5.9)$$

$$\frac{dS^{Out,(p)}}{dt} = \psi^{Loa,(p)} - \Phi^{Out,(p)}, \quad (5.10)$$

translating that in each area, good mass variation is the result of an in-flow and an out-flow.

Once this model is set out, we have to show that it is well adapted to the working of big hubs by fixing functions $\tilde{\psi}^A$ and Φ^A . We also have to show that it is flexible enough to take into account smaller hubs. For this, we present what we call “area overlapping” and adapted workforces $n^A(t)$.

The information lot $\Pi(p, t)$ is the following

$$\Pi(p, t) = (p, m(p), N_p(p, t), T_{In}(p), T_{Out}(p), Pub(p)). \quad (5.11)$$

Where $m(p)$ is the mass of goods constituting position p and $N_p(p, t)$ its pallet number. We consider that this pallet number depends on time t to be able to take into account repackaging that may be applied to positions within the hub. $T_{In}(p)$ is the time at which the truck containing position p is ready to be unloaded and $T_{Out}(p)$ is the time at which position p has to leave the hub. $Pub(p)$ is the list of positions which are in the same truck as p and that have to be unloaded before p . In big hubs, the following form may be chosen for ψ^{In} :

$$\tilde{\psi}^{In}(\Pi(p, t), S^{Ulo}(t), n^{In,(p)}(t)) = \frac{1}{\tau^{In}} \frac{m(p)}{N_p(p, t)} n^{In,(p)}(t) \chi(S_{\max}^{Ulo} - S^{Ulo}(t)). \quad (5.12)$$

In this equation, τ^{In} stands for the time needed to transfer a pallet from “In Area” to “Unload Area” and S_{\max}^{Ulo} for a maximal capacity. Hence, since $\frac{m(p)}{N_p(p, t)}$ is the mean mass per pallet, expression (5.12) says that the mass per pallet from “In Area” to “Unload Area” is in direct proportion to the mass of each pallet and to the workforce allocated to unload position p , and that it is in inverse proportion to the time needed to unload one pallet. Factor $\chi(S_{\max}^{Ulo} - S^{Ulo})$ makes the mass flow stop when S_{\max}^{Ulo} is reached. Notice that S_{\max}^{Ulo} may be set to $+\infty$ if this threshold effect is not necessary.

Workforce $n^{In,(p)}(t)$ allocated to unload position p may be defined as a proportion, linked with the mass fraction of position p in the total mass to unloaded modulated by a priority index, of the total workforce $n^{In}(t)$ allocated to unloaded task by the following formula:

$$n^{In,(p)}(t) = \frac{\int_{t-\Delta t_{\min}}^t \gamma(p, s) S^{In,(p)}(s) ds}{\int_{t-\Delta t_{\min}}^t \sum_{p' \in Pos} \gamma(p', s) S^{In,(p')}(s) ds} n^{In}(t). \quad (5.13)$$

Where $\gamma(p, t) \in [0, +\infty]$ is a priority index. It causes the enforcement of the workforce allocated to a position with a high value of priority index. $\gamma(p, t)$ may have several definitions. A simple one consists in setting

$$\gamma(p, t) = \gamma(p) = \frac{\tilde{\gamma}}{T_{Out}(p) - T_{In}(p)}, \text{ for any } t \in [T_{In}(p), T_{Out}(p)] \text{ and for a given constant } \tilde{\gamma}. \quad (5.14)$$

A second reasonable definition consists in setting a time dependant function

$$\gamma(p, t) = \min(\tilde{\gamma}_1 + \frac{\tilde{\gamma}_2}{T_{Out}(p) - t}; \tilde{\gamma}_3), \text{ for any } t \in [T_{In}(p), T_{Out}(p)] \quad (5.15)$$

for three given constants $\tilde{\gamma}_1, \tilde{\gamma}_2$ and $\tilde{\gamma}_3$. This choice causes an enforcement of the priority index when time increases. Lastly $\gamma(p, t)$ may be considered as an unknown function that has to be set via an optimization process. This idea will be explored later on in this paper.

A formula of this kind is preferable to a formula of the type

$$n^{In,(p)}(t) = \frac{\gamma(p, t) S^{In,(p)}(t)}{\sum_{p' \in Pos} \gamma(p', t) S^{In,(p')}(t)} n^{In}(t), \quad (5.16)$$

which does not take into account the time needed to transfer workforce from an unloading area to another. Moreover, using (5.16) artificially accelerates the beginning of unloading of a position and lengthens its end by allocating without discernment a large workforce when the mass to unload is large and a small workforce at the end of the process when the mass which remains to unload is small. Taking averages, as in (5.15), helps keep the principle which consists in allocating a large workforce to a large mass, but only in mean. The time interval length Δt_{\min} on which the values are averaged corresponds to an adaptation time or a relaxation time of workforce.

Before defining the other functions $\tilde{\psi}^A$ for $A \in \{Ulo, Res, Loa\}$, a notation generalization needs to be set: τ^{Ulo} stands for the time needed to transfer a pallet from ‘‘Unload Area’’ to ‘‘Residence Area’’, τ^{Res} stands for the time needed to transfer a pallet from ‘‘Residence Area’’ to ‘‘Load Area’’ and τ^{Loa} stands for the time needed to transfer a pallet from ‘‘Load Area’’ to ‘‘Out Area’’. S_{\max}^A is the maximal capacity of area A and $n^A(t)$ and $n^{A,(p)}(t)$, linked by

$$n^{A,(p)}(t) = \frac{\int_{t-\Delta t_{\min}}^t \gamma(p, s) S^{A,(p)}(s) ds}{\int_{t-\Delta t_{\min}}^t \sum_{p' \in Pos} \gamma(p', s) S^{A,(p')}(s) ds} n^A(t), \quad (5.17)$$

are the workforces allocated to area A and the workforce allocated to position p in area A . Then, the following forms may be chosen for $\tilde{\psi}^{Ulo}$ and $\tilde{\psi}^{Loa}$:

$$\tilde{\psi}^{Ulo}(\Pi(p, t), S^{Res}(t), n^{Ulo,(p)}(t)) = \frac{1}{\tau^{Ulo}} \frac{m(p)}{N_p(p, t)} n^{Ulo,(p)}(t) \chi(S_{\max}^{Res} - S^{Res}(t)), \quad (5.18)$$

$$\tilde{\psi}^{Loa}(\Pi(p, t), S^{Out}(t), n^{Loa,(p)}(t)) = \frac{1}{\tau^{Loa}} \frac{m(p)}{N_p(p, t)} n^{Loa,(p)}(t) \chi(S_{\max}^{Out} - S^{Out}(t)). \quad (5.19)$$

Function $\tilde{\psi}^{Res,(p)}$ depends on a supplementary variable ν which is the time at which the transfer of goods of position p , from “Residence Area” to “Load Area”, has to begin. This variable helps take into account that goods may be scheduled to remain a long period of time within a given hub. The definition of $\tilde{\psi}^{Res,(p)}$ is then:

$$\begin{cases} \tilde{\psi}^{Res,(p)}(\Pi(p, t), \nu(p), S^{Loa}(t), n^{Res,(p)}(t)) &= \frac{1}{\tau^{Res}} \frac{m(p)}{N_p(p, t)} n^{Res,(p)}(t) \chi(S_{\max}^{Loa} - S^{Loa}(t)) & \text{if } t \geq \nu(p), \\ &= 0 & \text{otherwise.} \end{cases} \quad (5.20)$$

For the definition of $\gamma(p, t)$ which is needed in $n^{Res,(p)}$ computation, if before arriving in “Residence Area”, it was defined by (5.14), it is necessary to set its value to

$$\gamma(p, t) = \frac{\tilde{\gamma}}{T_{Out}(p) - \nu(p)} \text{ for any } t \in [\nu(p), T_{Out}(p)]. \quad (5.21)$$

If formula (5.15) were chosen, this form remains valid. Concerning $\nu(p)$ several choices may be carried out. The simplest one consists in setting

$$\nu(p) = T_{Out}(p) - \tilde{\nu}, \quad (5.22)$$

for constant $\tilde{\nu}$ which is an estimated time for a given position to go from “Residence Area” to “Out Area”. Of course, constant $\tilde{\nu}(\Pi(p, t))$ may depend on $\Pi(p, t)$ with the idea that the more pallets p has, the longer $\tilde{\nu}(\Pi(p, t))$ has to be. $\nu(p)$ may also consist in an unknown that has to be set by an optimization process.

Considering $\Phi^{In,(p)}$ and $\Phi^{Out,(p)}$, denoting by $\delta_{[t=T]}$ the Dirac mass at time T and by $\tilde{\delta}^{\Delta t, T}(t)$ its following regularization:

$$\begin{cases} \tilde{\delta}^{\Delta t, T}(t) &= 0 & \text{if } t < T - \Delta t, \\ &= \frac{t - T + \Delta t}{\Delta t^2} & \text{if } T - \Delta t \leq t < T, \\ &= \frac{-t + T + \Delta t}{\Delta t^2} & \text{if } T \leq t < T + \Delta t, \\ &= 0 & \text{if } t \geq T + \Delta t, \end{cases} \quad (5.23)$$

we may set

$$\Phi^{Out,(p)} = m(p) \delta_{[t=T_{Out}(p)]}, \quad (5.24)$$

which says that the total mass of position p is transferred out of “Out Area” at time $T_{Out}(p)$, or

$$\Phi^{Out,(p)} = m(p) \delta^{\frac{\Delta t}{4}, T_{Out}(p)}(t), \quad (5.25)$$

which is a regularization of (5.24).

Concerning $\Phi^{In,(p)}$, we introduce a time $\tilde{T}(p)$ needed for unloading positions in $Pub(p)$ before reaching p in the truck, and then we define

$$\Phi^{In,(p)} = m(p) \delta_{[t=T_{In}(p)+\tilde{T}(p)]}, \quad (5.26)$$

or

$$\Phi^{In,(p)} = m(p)\tilde{\delta}^{\frac{\Delta t_{\min}}{4}, T_{In}(p)+\tilde{T}(p)}(t). \quad (5.27)$$

In those definitions, $\tilde{T}(p)$ may be chosen as

$$\tilde{T}(p) = \tau^{In} \sum_{p' \in Pub(p)} N_p(p', T_{In}(p)). \quad (5.28)$$

It may also be set via an optimization process that we will see below.

The model we have just built is well adapted to big hubs. For smaller hubs, or for hubs which do not work exactly as described above, modifications and simplifications of the model may be done in order to adapt it. Some of those modifications will now be explained.

First, the workforce may be not allocated by area, but by area group. In this situation, \mathcal{A} stands for the area group. For instance, \mathcal{A} may be $\{In, Ulo\}$ or $\{Res, Loa\}$ or $\{In, Ulo, Res, Loa\}$. Then, for a given A which is in group \mathcal{A} , the following formula

$$n^{A,(p)}(t) = \frac{\int_{t-\Delta t_{\min}}^t \gamma(p, s) S^{A,(p)}(s) ds}{\int_{t-\Delta t_{\min}}^t \sum_{p' \in Pos} \sum_{A' \in \mathcal{A}} \gamma(p', s) S^{A',(p')}(s) ds} n^{\mathcal{A}}(t), \quad (5.29)$$

where $n^{\mathcal{A}}(t)$ stands for the workforce allocated to group \mathcal{A} a time t , is used in place of (5.17). It is possible to mix formula (5.17) and one of the (5.29) types in the following way: For given times, (5.17) is chosen and for other times (5.29) is chosen.

If in a given hub the ‘‘Residence Area’’ does not exist or if a given position is scheduled to be directly transferred from ‘‘Unload Area’’ to ‘‘Load Area’’, then we can use the model above, introducing what we call ‘‘area overlapping’’. This ‘‘area overlapping’’ simply consists in replacing equation (5.6) by:

$$S^{Res,(p)}(t) = S^{Ulo,(p)}(t), \text{ for any time } t. \quad (5.30)$$

Notice that it is possible to overlap more than two areas and that this ‘‘area overlapping’’ is compatible with any workforce distribution of type (5.29) or (5.17).

5.3 Aggregated Models

We will now build an aggregated model which is valid regarding flows more macroscopically and at a larger time scale. (Typically, the minimum time step of a discrete time model deduced from the aggregated model is about $4\Delta t_{\min} \simeq 1h$). The aggregated model involves less variables and data and is easier to implement. Hence, having it at hand may be important for the first implementations and for network modelling. The main characteristic of this model is that it is not as precise as the fine-scale one defined by equations (5.1)-(5.28). In particular, it aggregates flows from every position.

We first consider that function $\Phi^{In}(t)$ gives the flow entering ‘‘In Area’’ at time t and $\Phi^{Out}(t)$ gives the flow leaving ‘‘Out Area’’ at time t . Then the quantities involved are:

$$S^A(t) = \sum_{p \in Pos} S^{A,(p)}(t) \text{ for } A \in \{In, Ulo, Res, Loa, Out\}, \quad (5.31)$$

where the sum may be reduced to $p \in Pos$ such that $T_{In}(p) \leq t \leq T_{Out}(p)$.

We deduce the aggregated model from equation (5.1)-(5.28). In equation (5.12),(5.18),(5.19), and (5.20) we find factors:

$$\frac{1}{\tau^A} \frac{m(p)}{N_p(p,t)} \text{ for } A\{In, Ulo, Res, Loa, Out\}. \quad (5.32)$$

Those factors may be averaged from history or using data or running simulation using the fine-scale model in order to deduce averaged coefficients

$$\omega^A \text{ for } A\{In, Ulo, Res, Loa, Out\}. \quad (5.33)$$

Then, we define transfer function $\psi^{In}(t)$ which is the mass flow of goods going from “In Area” to “Unload Area” at time t by

$$\psi^{In}(t) = \bar{\psi}^{In}(S^{Ulo}(t), n^{In}(t))\chi(S^{In}(t)), \quad (5.34)$$

with $\bar{\psi}^{In}$ defined by

$$\bar{\psi}^{In}(S^{Ulo}, n^{In}) = \omega^{In} n^{In} \chi(S_{\max}^{Ulo} - S^{Ulo}). \quad (5.35)$$

In a similar way, we define

$$\psi^{Ulo}(t) = \bar{\psi}^{Ulo}(S^{Res}(t), n^{Ulo}(t))\chi(S^{Ulo}(t)), \quad (5.36)$$

$$\psi^{Loa}(t) = \bar{\psi}^{Loa}(S^{Out}(t), n^{Loa}(t))\chi(S^{Loa}(t)), \quad (5.37)$$

where

$$\bar{\psi}^{Ulo}(S^{Res}, n^{Ulo}) = \omega^{Ulo} n^{Ulo} \chi(S_{\max}^{Res} - S^{Res}), \quad (5.38)$$

$$\bar{\psi}^{Loa}(S^{Out}, n^{Loa}) = \omega^{Loa} n^{Loa} \chi(S_{\max}^{Out} - S^{Out}). \quad (5.39)$$

Concerning transfer function $\psi^{Res}(t)$, which is the mass flow of goods from “Residence Area” to “Load Area”, we consider that the fact that goods may remain a long time in “Residence Area” can be described by a known function S_{Rem}^{Res} which gives, at any time, the mass of goods which is scheduled to remain in “Residence Area”. Then setting

$$\bar{\psi}^{Res}(S^{Loa}, n^{Res}) = \omega^{Res} n^{Res} \chi(S_{\max}^{Loa} - S^{Loa}), \quad (5.40)$$

$\psi^{Res}(t)$ is defined by

$$\psi^{Res}(t) = \bar{\psi}^{Res}(S^{Loa}(t), n^{Res}(t))\chi(S^{Res}(t) - S_{Rem}^{Res}(t)). \quad (5.41)$$

The ordinary differential equation system finally reads

$$\frac{dS^{In}}{dt} = \Phi^{In} - \psi^{In}, \quad (5.42)$$

$$\frac{dS^{Ulo}}{dt} = \psi^{In} - \psi^{Ulo}, \quad (5.43)$$

$$\frac{dS^{Res}}{dt} = \psi^{Ulo} - \psi^{Res}, \quad (5.44)$$

$$\frac{dS^{Loa}}{dt} = \psi^{Res} - \psi^{Loa}, \quad (5.45)$$

$$\frac{dS^{Out}}{dt} = \psi^{Loa} - \Phi^{Out}. \quad (5.46)$$

Model (5.31)-(5.42) is built to be conform to the functioning of big hubs. Nonetheless, for smaller hubs “area overlapping” in the same spirit of the process described by, (5.30) may be done.

5.4 Hub network models

To model a hub network, we build connexions between instantiations of model (5.1)-(5.28) or (5.31)-(5.42). We then consider a network made of n hubs $\{h_1, \dots, h_n\} = \mathcal{H}$ and that the set \mathcal{H} is divided into two subsets \mathcal{H}_{mod} and \mathcal{H}_{unm} ($\mathcal{H}_{mod} \cap \mathcal{H}_{unm} = \emptyset$, $\mathcal{H}_{mod} \cup \mathcal{H}_{unm} = \mathcal{H}$). Subset \mathcal{H}_{mod} contains hubs for which we have on our disposal an instantiation of model (5.1)-(5.16) or (5.31)-(5.42) and \mathcal{H}_{unm} contains hubs for which a model of the (5.1)-(5.16) or (5.31)-(5.42) kind cannot be accessed. For instance, \mathcal{H}_{unm} may contain hubs of subcontractors. Then, we consider the sets $K_{i,j}$, for $i = 1, \dots, n$ and $j = 1, \dots, n$ including all the possible routes from hub #i to hub #j. Notice that if hub #i is not connected with hub #j we have $K_{i,j} = \emptyset$. It also seems reasonable to write $K_{i,j}$ in the following way:

$$K_{i,j} = \{k_{i,j}^1, k_{i,j}^2, \dots, k_{i,j}^l, \dots, k_{i,j}^{L_{i,j}}\}. \quad (5.47)$$

Where $k_{i,j}^l$ stands for a possible route from hub #i to #j. To each route $k_{i,j}^l$ is attributed a time $T_{i,j}^l$ which is the time required to travel $k_{i,j}^l$.

Concerning the mass of goods flow of position p entering ‘‘In Area’’ of hub #i, $\Phi_i^{In,(p)}$ (see formula 5.1), we introduce, $R_i^{\lambda,(p)}(t)$ which is the mass flow of good of position p coming from factory or depot λ , $B_i^{\lambda,(p)}(t)$ which is the mass flow directly brought by factory λ at hub #i, and $F_i^{j,(p)}(t)$ which is the mass flow of goods of position p issued from hub #j. Then we have:

$$\Phi_i^{In,(p)}(t) = \sum_{\lambda \in \Lambda_i} R_i^{\lambda,(p)}(t) + \sum_{\lambda \in \Lambda_i} B_i^{\lambda,(p)}(t) + \sum_{j=1}^n F_i^{j,(p)}(t), \quad (5.48)$$

where Λ_i is the set of factories working with hub #i.

Concerning the flow of mass of goods of position p leaving ‘‘Out Area’’ of hub #i, we introduce $D_i^{\alpha,(p)}(t)$ which is the flow of mass of goods leaving hub #i towards retailer α , $P_i^{\alpha,(p)}(t)$ which is the flow of mass of goods directly brought out by retailer α at hub #i, and $E_i^{j,(p)}(t)$ which is the flow of goods going to hub #j. We have:

$$\Phi_i^{Out,(p)}(t) = \sum_{\alpha \in \mathcal{A}_i} D_i^{\alpha,(p)}(t) + \sum_{\alpha \in \mathcal{A}_i} P_i^{\alpha,(p)}(t) + \sum_{j=1}^n E_i^{j,(p)}(t), \quad (5.49)$$

where \mathcal{A}_i is the set of retailers working with hub #i.

The way to connect every instantiation of model (5.1)-(5.28) (indexed by $i = 1, \dots, n$) consists in translating that goods going out of a given hub towards a second one will later be goods coming from the first hub to the second one. In other words:

$$F_i^{j,(p)}(t) = \sum_{l=1}^{L_{i,j}} a(k_{i,j,p}^l) E_j^{i,(p)}(t - T_{i,j}^l), \quad (5.50)$$

where $a(k_{i,j,p}^l)$ stands for the proportion of mass of goods of position p going from hub #i to #j by travel $k_{i,j}^l$. In particular,

$$\sum_{l=1}^{L_{i,j}} a(k_{i,j,p}^l) = 1 \text{ for any } p \in Pos. \quad (5.51)$$

The way to connect every instantiation of model (5.31)-(5.42) indexed by $i = 1, \dots, n$ consists in setting:

$$\begin{aligned} R_i^\lambda(t) &= \sum_{p \in Pos} R_i^{\lambda,(p)}(t) \quad , \quad D_i^\alpha(t) = \sum_{p \in Pos} D_i^{\alpha,(p)}(t), \\ B_i^\lambda(t) &= \sum_{p \in Pos} B_i^{\lambda,(p)}(t) \quad , \quad P_i^\alpha(t) = \sum_{p \in Pos} P_i^{\alpha,(p)}(t), \\ F_i^j(t) &= \sum_{p \in Pos} F_i^{j,(p)}(t) \quad , \quad E_i^j(t) = \sum_{p \in Pos} E_i^{j,(p)}(t), \end{aligned} \quad (5.52)$$

and,

$$\begin{aligned} \Phi_i^{In}(t) &= \sum_{\lambda \in \Lambda_i} R_i^\lambda(t) + \sum_{\lambda \in \Lambda_i} B_i^\lambda(t) + \sum_{j=1}^n F_i^j(t), \\ \Phi_i^{Out}(t) &= \sum_{\alpha \in \mathcal{A}_i} D_i^\alpha(t) + \sum_{\alpha \in \mathcal{A}_i} P_i^\alpha(t) + \sum_{j=1}^n E_i^j(t), \end{aligned} \quad (5.53)$$

and to write,

$$F_i^j(t) = \sum_{l=1}^{L_{i,j}} \bar{a}(k_{i,j}^l) E_j^i(t - T_{i,j}^l), \quad (5.54)$$

where $\bar{a}(k_{i,j}^l)$, satisfying

$$\sum_{l=1}^{L_{i,j}} \bar{a}(k_{i,j}^l) = 1, \quad (5.55)$$

are the proportions of mass of goods going from hub #i to #j via travel $k_{i,j}^l$. \bar{a} may be deduced by the history or from data.

Notice that we can consider meandered versions of (5.50) and (5.54)

$$\begin{aligned} \int_t^{\Delta t} F_i^{j,(p)}(s) ds &= \sum_{l=1}^{L_{i,j}} \bar{a}(k_{i,j}^l) \int_{t-T_{i,j}^l}^{t-T_{i,j}^l+\Delta t} E_j^{i,(p)}(s) ds, \\ \int_t^{\Delta t} F_i^j(s) ds &= \sum_{l=1}^{L_{i,j}} \bar{a}(k_{i,j}^l) \int_{t-T_{i,j}^l}^{t-T_{i,j}^l+\Delta t} E_j^i(s) ds. \end{aligned} \quad (5.56)$$

6 Validation tests

6.1 Fine-scale model results

The model described in section 5.2 was programmed in environment R [10]. The numerical method used for computations is the Runge-Kutta 4 methods, [4].

To show the model's performances, we simulated the activity of a hub from a simple example. We had two positions come within the space of an hour. They stayed at the hub for ten hours. We wanted to observe how their mass filled the different hub areas during these ten hours.

To do so, the above-described model requires set parameters for positions, available workforce at the hub, and the size of the hub. Four scenarios will help us understand how a model behaves according

to the varied parameters. The first scenario sets the parameters as indicated in the following tables. Two positions of 5 tons and 20 pallets each come into the hub, respectively at 1:00 and 2:00. They are programmed to leave at 11:00 and 12:00. Their parameters to determine the priority index with formula (5.14) are the same. The positions can be unloaded at the same time. The different hub areas are large enough to store the total mass of both positions. The amount of manpower is equally divided between each position. The transfer time of a pallet from one area to another is estimated at one minute. To estimate the position transfer time from the “Residence Area” to the “Out Area”, we ran the model once to see the time necessary for unloading. This amount of time is allotted to parameter ν .

Position number: p	Mass in tons: $m(p)$	Number of pallets: N_p	Positions ahead: $Pub(p)$	Unloading date: $T_{In}(p)$	Outbound date: $T_{Out}(p)$
0R4080329341	5	20	<i>Null</i>	1	10
0R4080328864	5	20	<i>Null</i>	2	11

Table 1: *Position parameters.*

Position p transfer time between “Residence Area” and “Out Area”: ν	Parameter 1 of the priority index: γ_1	Parameter 2 of the priority index: γ_2	Parameter 3 of the priority index: γ_3
8	1	1	5
8	1	1	5

Table 2: *Position parameters, continued.*

Max. weight in “In Area”, in tons: S_{\max}^{In}	Max. weight in “Unload Area”, in tons: S_{\max}^{Ulo}	Max. weight in “Residence Area”, in tons: S_{\max}^{Res}	Max. weight in “Load Area”, in tons: S_{\max}^{Loa}	Max. weight in “Out Area”, in tons: S_{\max}^{Out}
20	20	20	20	20

Table 3: *Hub parameters.*

Figure 6 shows the results obtained by running the model with the above parameters. The first graph in Fig. 6 shows the evolution of the sum of mass of goods from both positions, in each area. It also shows the evolution of the total mass stored in the hub. The second and third graphs in Fig. 6 show the evolution of the mass in each area, for each position. The “In” curve represents the evolution of mass of goods in the “In Area”. The “Ulo” curve represents the evolution of mass of goods in the “Unload Area”. The “Res” curve represents the mass of goods in the “Residence area”. The “Loa” curve represents the evolution of the mass of goods in the “Load Area”. The “Out” curve represents the evolution of mass of goods in the “Out Area”. While the “Unload”, “Residence”, and “Load” areas are physically marked areas in the hub, the “In” and “Out” areas represent the sum of the areas in the trailers which are docked at the hub for unloading and loading. Indeed, a truck contains goods from a series of positions. The truck and its trailer go to a dock to be unloaded at a specific time. After being handled and moved from the “Unload Area” to the “Residence Area”,

Transfer time for a pallet from the “In Area” to the “Unload Area” in fractions of hours: τ^{In}	Transfer time for a package from the “Unload Area” to the “Residence Area” in fraction of hours: τ^{Ulo}	Transfer time for a package from the “Residence Area” to the “Load Area” in fraction of hours: τ^{Res}	Transfer time for a package from the “Load Area” to the “Out Area” in fraction of hours: τ^{Loa}
1/60	1/60	1/60	1/60

Table 4: *Hub team parameters.*

Manpower available for unloading: n^{In}	Manpower available for transfer to the “Residence Area”: n^{Ulo}	Manpower available for transfer to the “Load Area”: n^{Res}	Manpower available for loading: n^{Loa}
0.1	0.1	0.1	0.1

Table 5: *Hub team parameters, continued.*

then from the “Residence Area” to the “Load Area”, the goods from the same position are loaded in the trailer for departure at a scheduled time. This is the date of departure or the date at which the goods leave the “Out Area”.

The “In” curve from the first graph in Fig. 6 shows that the start date for the unloading of position n°0R4080329341 is 1:00. The second graph shows that the unloading continues to approximately 7:30. This graph also shows that the transfer of position n°0R4080329341 mass is slowed down from the “In Area” to the “Unload Area”. This is simply due to the coming into the hub of the second position. The evolution of the distribution of second position (n°0R4080328864) mass throughout the hub can be seen in the third graph in Fig. 6. The second position comes in for unloading at 2:00 and leaves the “Out Area” at 11:00.

As previously stated, the manpower available for unloading is equal to the manpower available for transferring goods from the “Unload Area” to the “Residence Area”. Thus, as soon as the goods are unloaded, they are transferred to the “Residence Area”. This is why, as can be seen in the three graphs from Fig. 6, the “Unload Area” does not keep stock. While the “In Area” is being cleared, the “Residence” and “Out” areas are filling up (see “Res” and “Out” curves). The “Out Area” starts filling up at 2:00 because we set parameter ν at 8:00. The “Load Area” does not fill up. The reason for this is that the goods leaving the “Resident Area” are immediately transferred to the “Out Area”. Position n°0R4080329341 leaves, as scheduled, at 10:00 and position n°0R4080328864 leaves at 11:00. When the goods go to the “Out Area”, the “In” and “Resident” areas empty out. The “Tot” curve shows the goods mass present in the hub at each moment.

The second scenario uses the same parameters as the previous one except that the available manpower to transfer the goods from “Unload Area” to “Resident Area” is reduced. The amount of manpower goes from 0.1 to 0.08. Fig. 7 gives the results obtained.

As before, the “In” curve has two peaks which correspond to both positions coming into the “In Area”. The drops in the curve between 1:00 and 2:00 and then between 2:00 and 7:00 correspond to the unloading times of the trucks. As the team to transfer the unloaded goods from the “Unload Area” to the “Resident Area” is smaller than the unloading team, the goods progressively accumulate in the “Unload Area” (see “Ulo” curve). This is also why the “Resident Area” does not fill up as much as in the previous situation. It can also be noted that the “Res” curve starts dropping at 3:00. This is due to the fact that

Δt_{\min}
1/4

Table 6: *Model parameters.*

the goods transferred to the “Resident Area” are transferred to the “Load Area” right away, to be moved to the “Out Area”. There is a break in the “Out” curve slope at 3:00, being the time at which the second positions start being loaded into the trucks. The “Out” curves reach the level of 10 tons at 9:00. This is the sign that the goods in both positions are loaded and ready to leave the “Out Area”. At 10:00, the “Out” curve drops for the first time to show the departure of position n°0R4080329341. Then, the curve stagnates at 5 tons at 1 hour and drops again at 11:00 to 0 ton. Position n°0R4080328864 is left. Similarly, when the amount of available manpower decreases for loading (see parameter n^{Loa}), goods stock pile up in the “Load Area”, therefore causing the “Loa” curve to be positive.

A third scenario sets the parameters as they were for the first simulation, considering that the “Residence Area” is not large enough to handle more than one ton of goods ($S_{\max}^{Res} = 1$). The results illustrated in Fig. 8 indicate that the consequence of this constraint is the stocking of surplus goods in the “Unload Area”. Fig. 8 perfectly shows that the “Resident Area” (see “Res” curve) starts filling up at 1:00 with the arrival of the first position. At 1:30, it reaches its maximum set capacity of one ton. At the same time, the “Unload Area” begins to stock pile goods. The break in the “Ulo” curve at 2:00 indicates that the goods are coming from the first and second positions. As expected, the “Unload Area” empties out before the “Resident Area”.

The last scenario considers that six and one half hours to unload two positions of 5 tons is long. Six and one half hours is how long the “In” curve stays positive, thus between 1:00 and 7:30. The first idea that comes to mind is to decrease the unloading time and to increase the amount of manpower allotted to unloading. We set all of the parameters as they were in the first scenario, but we increased the value of parameter n^{In} from 0.1 to 0.5.

The “In” curve in Fig. 9 indicates that the positions were unloaded faster than in the first scenario. Both were unloaded in less than an hour. However, we did not increase the amount of manpower for transferring the goods from the “Unload Area” to the “Resident Area”. This is why it can be noted that the “Ulo” curve grows, which means that the goods stay in the “Unload Area” longer. Again, the “Unload Area” becomes empty long before the “Resident Area”, which indicates that the goods are constantly being moved. Furthermore, the sum of the masses in the hub is 5 tons between 1:00 and 2:00, and between 10:00 and 11:00. It is 10 tons between 2:00 and 10:00. This curve is an indicator of the validity of the model.

These four scenarios show that, for these simple cases, the model behaves in accordance with the express hub. They also show that the model, as it was programmed, allows one to simulate several scenarios. The model shows, through irregular values (negative ones, for example) that a given scenario is not possible. For example, Fig. 10 shows the result when we use the first scenario and have the first position leave at 6:00. The model tells us, through negative values for the “Out Area”, that this objective is not feasible with the means used. To achieve it, the capacity of manpower allotted to this position would have to be increased throughout the various hub areas. The distribution of weight throughout the hub can be monitored position by position, therefore allowing us to easily detect any problems.

As has been shown, the model which was programmed on software R functions. However, R does not allow it to do so with a large number of positions. The above examples show the management of two

positions in the hub. To do this, our laptop, having a 1.8GHz dual processor and 2.5 Go of RAM required a little less than 5 minutes. Given that a transport agency manages between 1,000 and 2,000 positions a day, our computer, which is certainly not powerful, would need more than three and a half days to simulate a real life scenario. We believe that, to significantly increase execution performances, one solution would be to program the mathematical model in a compiled language such as C++. Another solution would be to aggregate the model. The results from aggregated model simulations are given in the following section (section 6.2).

6.2 Results obtained for the aggregated model

Like the fine-scale model, the aggregated model was developed in an R environment. The goal was not to program it for industrial use, but to show the credibility of the mathematical model. We would also like to show that the response times of the aggregated model are faster than those of the fine-scale model. To demonstrate the similarities of results obtained from both models, we ran the aggregated model with parameters identical to the above-described scenarios (section 6.1), and others which are the results of the aggregation of parameters of the fine-scale model. Finally, we superposed the curves from both models.

The aggregated model requires fewer parameters than the fine-scale model. It does not give the details of the distribution of goods throughout the hub, position by position. To keep the same scenario as the first one given in section 6.1, the aggregated model requires the following parameters:

Position Number	Mass in tons: $m(p)$	Unloading date: $T_{In}(p)$	Outbound date: $T_{Out}(p)$	Position p transfer time between “Resi- dence Area” and “Out Area”: ν
0R4080329341	5	1	10	8
0R4080328864	5	2	11	8

Table 7: *Position parameters.*

Max. weight in “In Area”, in tons: S_{\max}^{In}	Max. weight in “Unload Area”, in tons: S_{\max}^{Ulo}	Max. weight in “Residence Area”, in tons: S_{\max}^{Res}	Max. weight in “Load Area”, in tons: S_{\max}^{Loa}	Max. weight in “Out Area”, in tons: S_{\max}^{Out}
20	20	20	20	20

Table 8: *Hub parameters.*

Figure 11 superposes the results obtained by both models studied. It shows that the models empty out the “In Area” at the same speed. The transfer of the goods from the “Unload Area” to the “Resident Area” also takes place in the same way for both models. Both models start loading at the same time. However, the fine-scale model loads the “Out Area” a little more slowly than the aggregated model. This is why the “Resident Area” does not fill up as much with the aggregated model. This phenomenon could come from function $n^{A,(p)}(t)$ (5.17) which does not exist for the aggregated model. As expected, the “Out Area” is cleared at 10:00 and 11:00. In reality, this is the time at which the trucks leave.

Mean hourly human cost to transfer a unity of mass between the “In Area” and the “Unload Area”: ω^{In}	Mean hourly human cost to transfer a unity of mass between the “Unload Area” and the “Residence Area”: ω^{Ulo}	Mean hourly human cost to transfer a unity of mass between the “Residence Area” and the “Load Area”: ω^{Res}	Mean hourly human cost to transfer a unity of mass between the “Load Area” and the “Out Area”: ω^{Out}
15	15	15	15

Table 9: *Hub parameters, continued.*

Manpower available for unloading: n^{In}	Manpower available for transfer to the “Residence Area”: n^{Ulo}	Manpower available for transfer to the “Load Area”: n^{Res}	Manpower available for loading: n^{Loa}
0.1	0.1	0.1	0.1

Table 10: *Hub team parameters.*

Δt_{\min}
1/4

Table 11: *Model parameters.*

In the same conditions, the run time of the first scenario with the fine-scale model was 4:47min while the run time of the aggregated model was 1:37min, three times less than the former.

7 Towards the resolution of industrial problems via optimization questions

In this section, we give examples of how the models just built may be used in order to translate industrial problems linked with hub working into optimization questions. The first example consists in setting the question of determining the necessary workforce and its division between areas in terms of a constrained minimization problem involving the aggregated model. The second one concerns the allocation of workforce to positions which will be seen as an optimization process on the priority index $\gamma(p, t)$ involved in equation (5.14). The third example consists in looking at the setting of times $\tilde{\nu}(p)$ as a constrained minimization problem.

7.1 Necessary workforce determining

Here, we consider the aggregated model (5.31)-(5.42) with known “In” and “Out” flows $\Phi^{In}(t)$ and $\Phi^{Out}(t)$. Inner flows ψ^{In} , ψ^{Res} and ψ^{Loa} are given by (5.34)-(5.41) considering that workforces $n^{In}(t)$, $n^{Ulo}(t)$, $n^{Res}(t)$ and $n^{Loa}(t)$ are unknown to be determined.

At a given time t_i , we consider that $S^{In}(t_i)$, $S^{Res}(t_i)$, $S^{Loa}(t_i)$ and $S^{Out}(t_i)$ are known and, a final time t_f being given, we want to find function n , n^{In} , n^{Res} , n^{Loa} and n^{Out} defined on $[t_i, t_f]$, which are linked

with:

$$n^{In}(t) + n^{Ulo}(t) + n^{Res}(t) + n^{Loa}(t) = n(t), \forall t \in [t_i, t_f], \quad (7.1)$$

and which minimizes

$$\int_{t_i}^{t_f} n(t) ds, \quad (7.2)$$

under the constraint

$$\frac{dS^{Out}}{dt} = \psi^{Loa} - \psi^{Out} \geq 0. \quad (7.3)$$

Additional constraints may be added. For instance, in order to impose a time stability of the workforce distribution we may impose

$$\left| \frac{dn^A}{dt}(t) \right| \leq n_{\max}, \forall t \in [t_i, t_f], \quad (7.4)$$

or

$$\frac{1}{n^A(t) + 1} \left| \frac{dn^A}{dt}(t) \right| \leq n_{\max}, \forall t \in [t_i, t_f], \quad (7.5)$$

for a constant n_{\max} . Moreover, we may also impose that n be constant on subintervals $[t_i, t_1]$, $[t_1, t_2]$, \dots , $[t_k, t_f]$.

7.2 Allocation of workforce to positions

Here, we consider the fine-scale model (5.6)-(5.11) for every position p that passes by the hub between an initial time t_i , where every $S^{A,(p)}(t_i)$ are known for every concerned position and area $A \in \{In, Ulo, Res, Loa, Out\}$ and a final time t_f . We consider that $n^{In}(t)$, $n^{Ulo}(t)$, $n^{Res}(t)$, $n^{Loa}(t)$ are given for any time $t \in [t_i, t_f]$. For instance, they may have been computed by the procedure just described. We also consider that $\Phi^{Out,(p)}(t)$ are known for each of the considered positions, for instance given by (5.24) or (5.25). Concerning $\Phi^{In,(p)}(t)$, we consider that it is given by (5.26) in which $\tilde{T}(p)$ is an unknown to be determined. Concerning the inner flows, we consider that they are given by formula (5.2)-(5.6), (5.12), (5.13), (5.16)-(5.18) where $\nu(p) = \tilde{\nu}(\Pi(p, t))$ are given and $n^{A,(p)}$ given by (5.16) and (5.17), for $A \in \{In, Ulo, Res, Loa, Out\}$ with $\gamma(p, t)$ being unknown functions to determine.

As a matter of fact, we are looking for constants $\tilde{T}(p)$ and function $\gamma(p, t)$ such that

$$\sum_{p/S^{A,(p)}(t) > 0} n^{A,(p)}(t) \leq n^A(t), \text{ for } A \in \{In, Ulo, Res, Loa, Out\}, \quad (7.6)$$

$$\gamma(p, t) = 0 \text{ for all } t \in [T_{In}(p), T_{In}(p) + \tilde{T}(p)], \quad (7.7)$$

where,

$$\tilde{T}(p) = \tau^{In} \sum_{p' \in Pub(p)} \frac{N_p(p', T_{In}(p))}{n^{In,(p')}(t)}, \quad (7.8)$$

and

$$\frac{dS^{Out,(p)}}{dt}(t) = \psi^{Loa,(p)}(t) - \Phi^{Out,(p)}(t) \geq 0 \text{ for any, } t \in [t_i, t_f], \quad (7.9)$$

minimizing

$$\int_{t_i}^{t_f} \sum_{\substack{p/S^{A,(p)}(t) > 0 \\ A \in \{In, Ulo, Res, Loa, Out\}}} \left| \frac{dn^{A,(p)}}{dt}(t) \right| dt. \quad (7.10)$$

Constraint (7.6) means that the sum over all the positions between which workforce $n^{A,(p)}(t)$ is distributed cannot exceed $n^A(t)$. $\tilde{T}(p)$ is defined as the time needed to unload all the goods in the truck containing position p , before reaching goods belonging to p . Then (7.8) seems to be the natural definition of $\tilde{T}(p)$ and (7.7) means that the priority index of given position p remains 0 until goods of p are reachable. Constraint (7.9) only means that the “Out flow” needs to be a real “Out flow”, or in other words, that the whole of position p has to be ready to get out of the hub at the scheduled time.

The fitness function (7.10) is set to minimize the time variation of the workforce distribution between positions.

7.3 Optimizing time $\tilde{\nu}(p)$

The last optimization problem, linked with the models built in the previous section, concerns the value of $\tilde{\nu}(p)$. The value of $\tilde{\nu}(p)$ gives an estimation of the time needed for position p to go from “Residence Area” to “Out Area”. We assume that the number of pallets $N_p(p, t)$ of position p remains the same from $\nu(p)$ to $T_{out}(p)$ and we consider that $\nu(p)$ is given (5.22) with $\tilde{\nu}(p)$ given by:

$$\tilde{\nu}(p) = \tilde{\nu}(\Pi(p, t)) = aN_p(p, T_{out}(p)) + b, \quad (7.11)$$

for every position being in the hub between initial time t_i and final time t_f , for two constants a and b to set. The minimization problem to compute a and b consists in considering model (5.6)-(5.10) between times t_i and t_f with known values of $S^{A,(p)}(t_i)$ for all positions passing by the hub between time t_i and t_f and $A \in \{In, Res, Loa, Out\}$, and with known functions $\Phi^{In,(p)}$ and $\Phi^{Out,(p)}$ for every $t \in [t_i, t_f]$. We then consider that inner flows are given by (5.18)-(5.20), (5.24)-(5.27) and then a and b must minimize

$$\sum_{p/T_{In}(p) > t_i \text{ and } T_{Out}(p) < t_f} |\tilde{\nu}(p)|, \quad (7.12)$$

under the constraint $a \geq 0$, $b \geq 0$ and

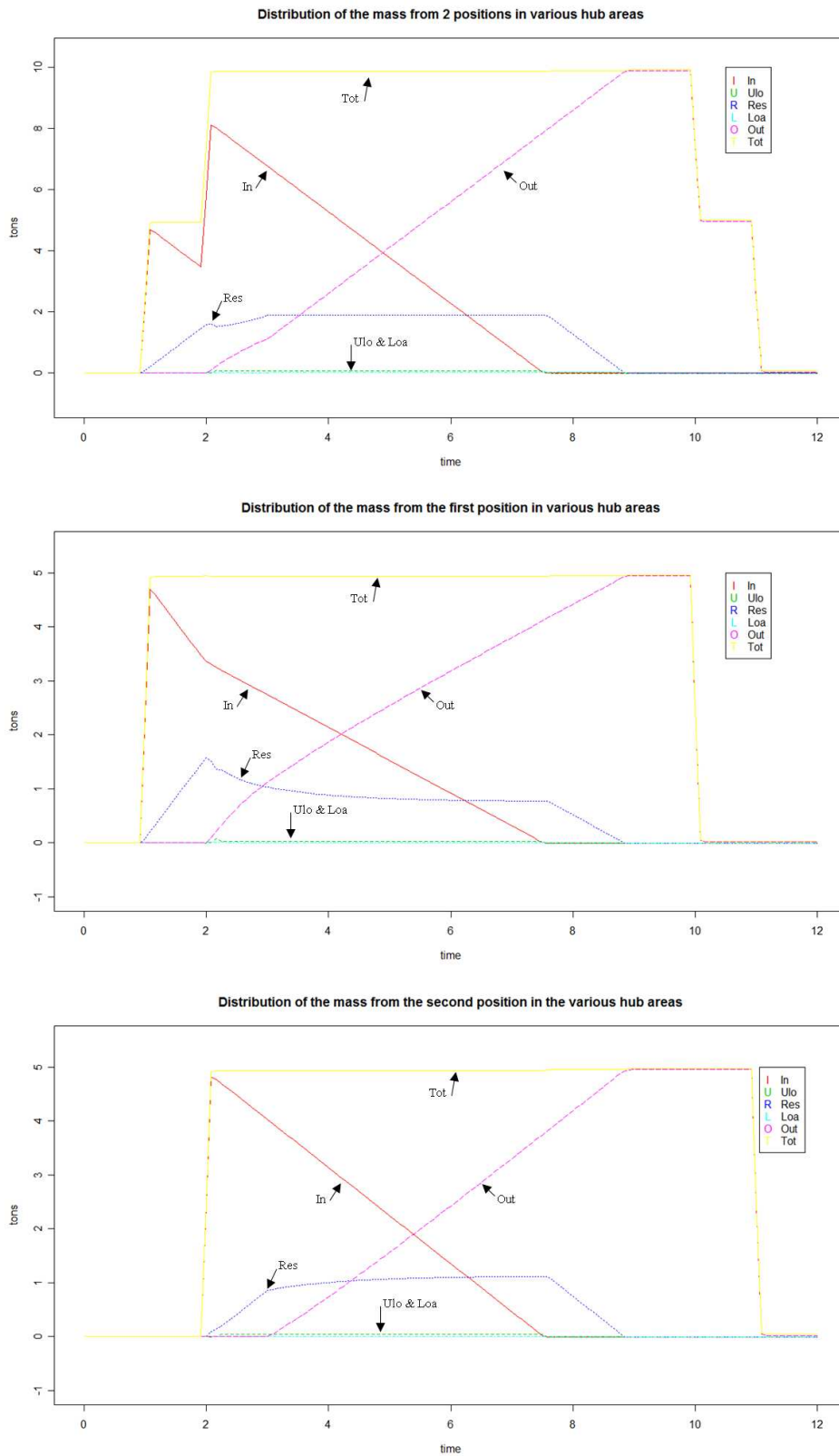
$$\frac{dS^{Out,(p)}}{dt}(t) = \psi^{Loa,(p)}(t) - \Phi^{Out,(p)}(t) \geq 0 \text{ for any } t \in [t_i, t_f]. \quad (7.13)$$

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 Figure 6: Goods mass distribution, come from 2 positions, in the various hub areas, followed by details for each position.

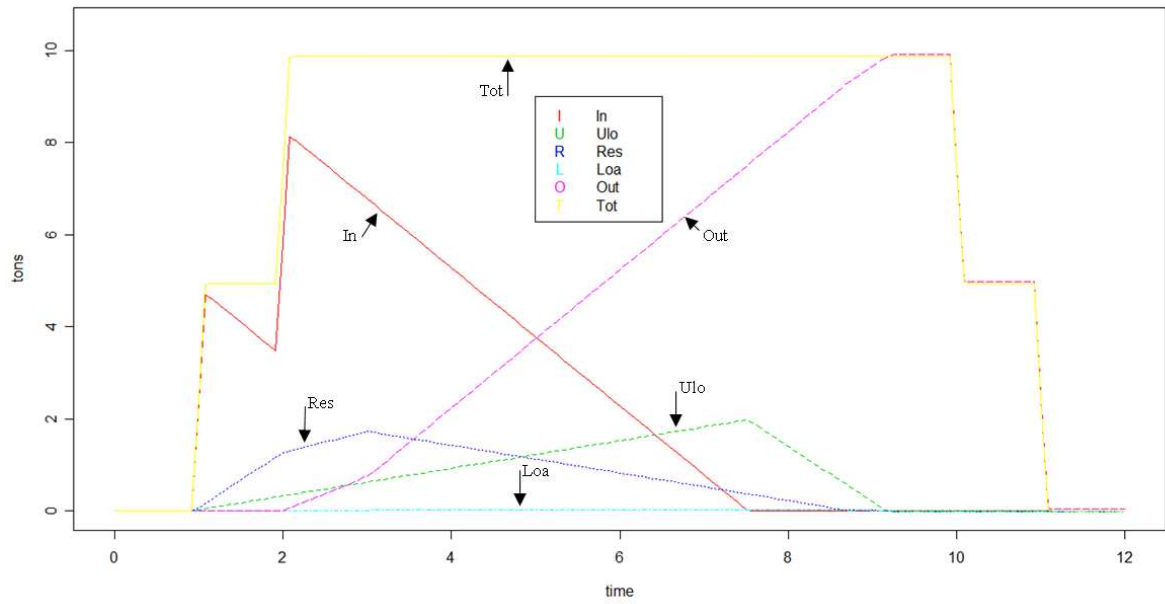


Figure 7: Goods mass distribution in the various hub areas, with $n^{Res} = 0,08$.

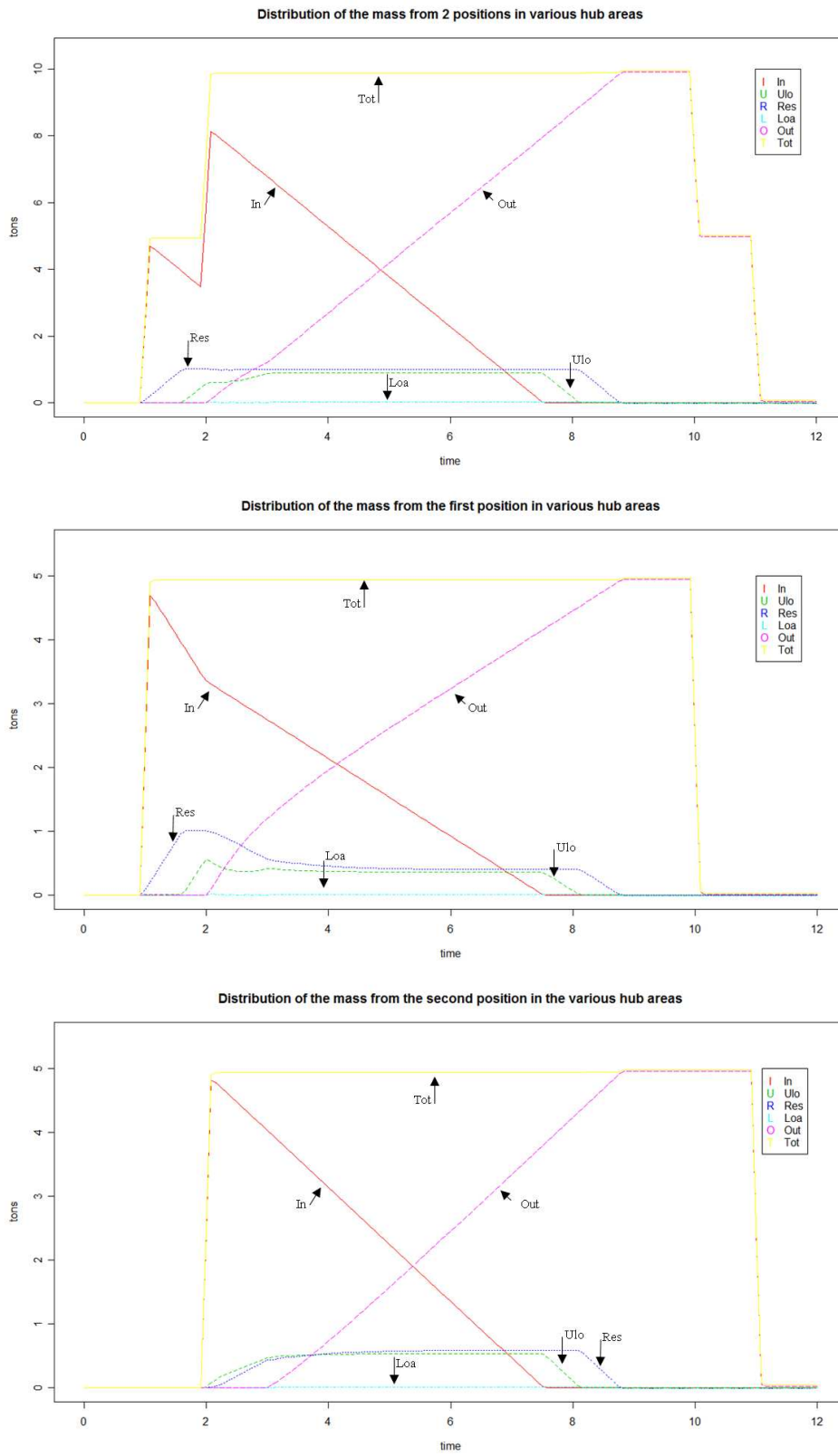


Figure 8: Goods mass distribution in the various hub areas, with $S_{\max}^{Res} = 1$.

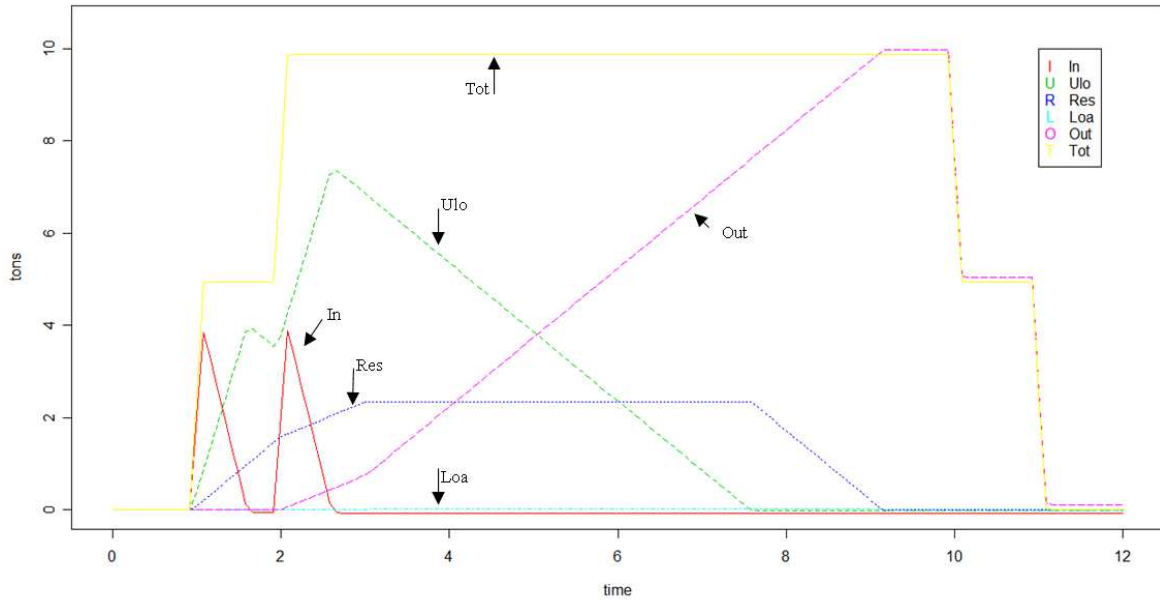


Figure 9: Goods mass distribution in the various hub areas, with $n^{In} = 0, 5$.

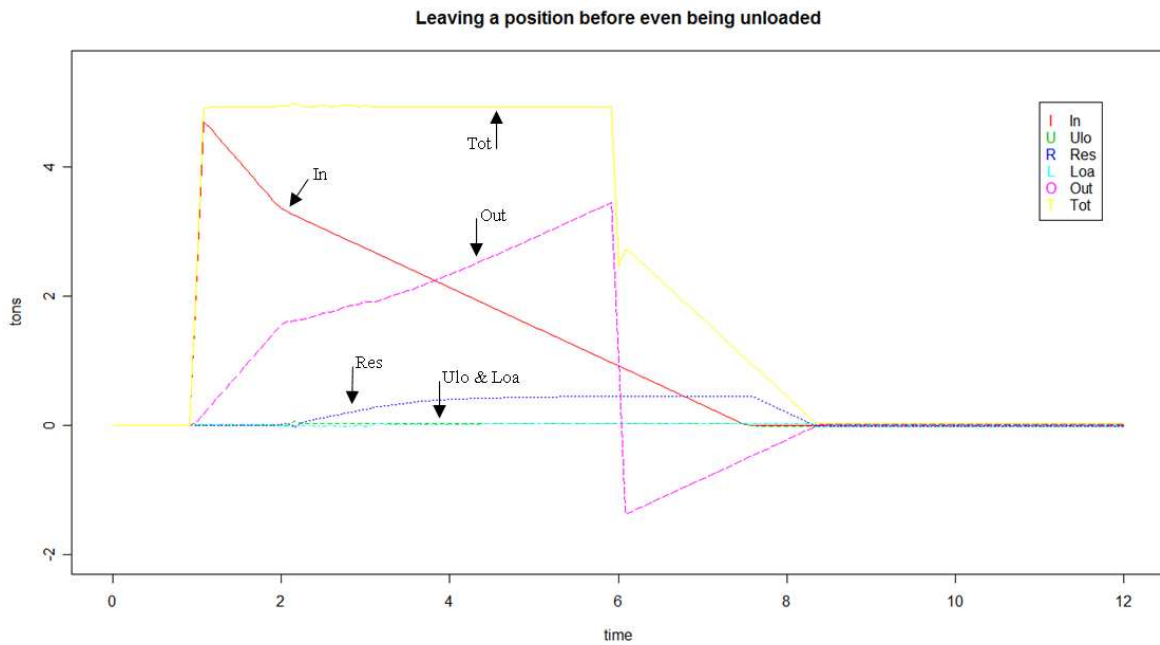


Figure 10: Incoherence in goods mass distribution in the various hub areas.

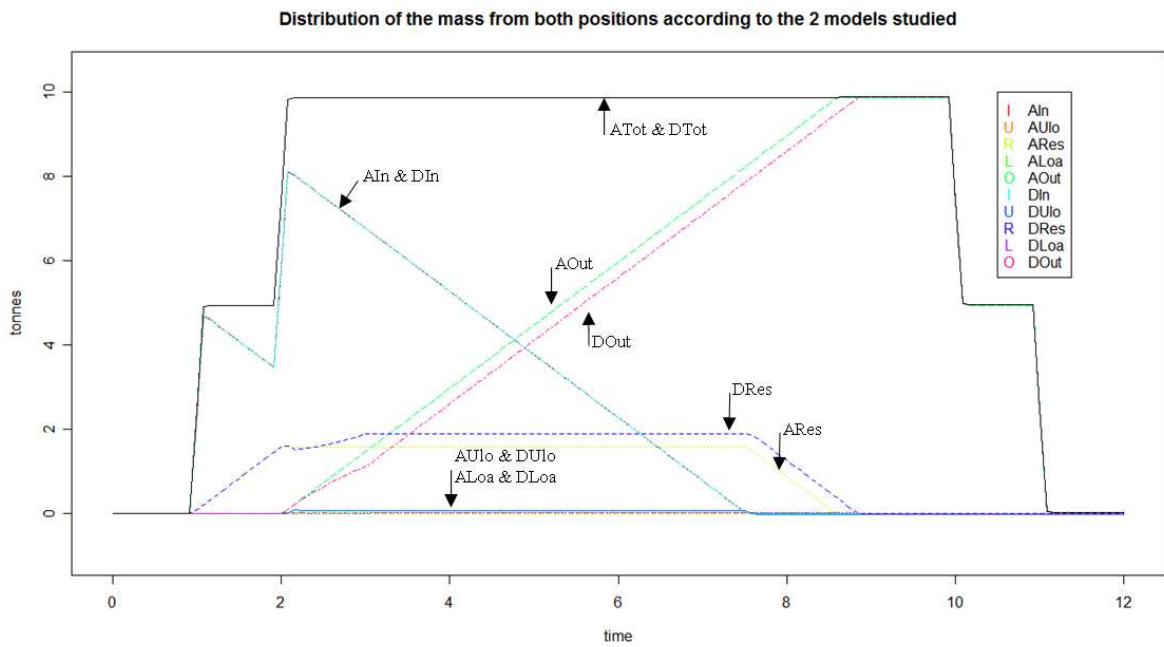


Figure 11: *Superposition of the fine-scale model results with those of the aggregated model.*