Angular and temporal determinism of rotating machine signals: The diesel engine case

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Abstract

The aim of this work is to highlight theoretically and experimentally the effect of cyclic speed fluctuations on the temporal and angular deterministic parts of signals recorded on rotating machines operating in steady state conditions. The deterministic parts of such cyclostationary signals are defined by their periodic components, or their CS1 part (order 1 of cyclostationarity). It can be assessed by using cyclic averaging, using a time or angle sampling, leading to an estimation of the temporal or angular deterministic part. If the instantaneous speed of the machine is not purely periodic, the temporal and angular deterministic parts will be different. These differences are firstly theoretically considered, and then experimentally assessed in the case of a diesel engine.

Key words: Cyclostationarity, diesel engine noise, source separation
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Introduction

Acoustic or vibration signals acquired on rotating machines operating in steady-state conditions are said to be cyclostationary [1]. Cyclostationarity is the general framework to treat signals exhibiting periodical properties. The deterministic part of a cyclostationary signal is commonly defined by its purely periodic component, i.e. the expected value of the signal during a cycle, also called CS1 part (first order of cyclostationarity). This part can be estimated by averaging the signal over a large number of cycles [2]. The random part of the signal is defined relatively to the deterministic part : it is the residual

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part resulting from the subtraction of the deterministic part from the total signal. This definition of the deterministic / random decomposition of a cyclostationary signal is attractive because of its simplicity. It can however be ambiguous when dealing with signals acquired on rotating machines running in steady-state conditions. The determinism of vibration or acoustic signals acquired on rotating machines is dual : firstly the occurrence of mechanical events (mechanical shocks, combustions in reciprocating engines...) is guided by the position of the main shaft of the machine : this is an angular determinism. Secondly, the response of the structure to excitations results from a convolution product in the time domain : it is a temporal determinism. This construction of signals in temporal and angular domains brings out the concept of fuzzy cyclostationarity [3]. The duality of the determinism is not a problem if the instantaneous speed is purely periodic (or constant). In this case, a signal purely periodic in angle will be purely periodic in time [3,4]. Difficulties appear when the instantaneous rotation speed exhibits fluctuations from cycle to cycle, i.e. if the instantaneous speed is not purely periodic. In this case, the relation between temporal and angular domains is not deterministic. Cycle averaging operations, that are necessary to estimate deterministic parts of signals, will thus lead to different results in time or angle. The deterministic part of the signal in angle (resp. in time) will exhibit a random part in time (resp. in angle).
The aim of this paper is to assess theoretically and experimentally the effect of speed fluctuations on angular and temporal deterministic parts of signals acquired on rotating machines. Sections 1 and 2 concern respectively the temporal and angular deterministic part. Simple and general considerations about sources in rotating machines lead to the expression of low pass filters and damping effects on deterministic parts. The last section is an experimental illustration in the case of a diesel engine operating at cold idle.
In all the following, "time signals" or "angular signals" will stand for signals sampled with a constant step respectively in time or angle.

1 Extraction of the temporal deterministic part

1.1 synchronous averaging

The temporal deterministic part of a rotating machine signal corresponds to its expected value during one cycle in the time domain. It can be estimated by averaging it over a large number of cycles. The main difficulty of this averaging process is that consecutive cycles do not have the same number of samples because of cyclic speed variations. In fact, two cycle events separated by a constant value in angle will be separated by a varying delay in the time domain. It means that the resulting average does not correspond exactly to a cycle as defined relatively to the angle, but to a time portion of the signal corresponding approximately to the mean duration of a cycle. However, an angle reference corresponding to a given position of the main shaft is necessary to align those time portions before the averaging operation (position-locking in [2]). A given time portion will thus be defined as the m points preceding the chosen angle reference together with the p points following it, $T = (m+p)/f_s$ corresponding approximately to a cycle duration (with f_s the sampling frequency in Hz). Because of cyclic speed variations, the temporal deterministic part of a signal is necessarily defined together with a chosen angle reference. The choice of different angle references for the synchronization will lead to different results.

1.2 Effects of the synchronization error

Unfortunately, time portions corresponding to successive cycles cannot be exactly synchronized because of the discrete nature of acquired signals. The chosen angle reference is indeed localized between two time samples. Without resampling or interpolation techniques, a synchronization error is unavoidable, and corresponds to the delay between the reference and the nearest sample of the time portion. This error can be described by a random variable Δ uniformly distributed between $-1/(2f_s)$ and $1/(2f_s)$ (assuming the rotation speed and the sampling frequency are non commensurable). The effect of this error on the synchronous average can be obtained by considering its expected value. The Fourier decomposition of the deterministic part of the signal is expressed as follows

$$d(t) = \sum_n D_n e^{j2\pi f_n t}, \quad \text{with} \quad f_n = \frac{n}{T} = \frac{n}{m+p} f_s. \tag{1}$$

Considering the synchronization error, the expected value of the synchronous average is given by

$$\begin{aligned} \tilde{d}(t) = E[d(t+\Delta)] &= \sum_n D_n E[e^{j2\pi f_n (t+\Delta)}] \\ \tilde{d}(t) &= \sum_n D_n e^{j2\pi f_n t} E[e^{j2\pi f_n \Delta}] \\ \tilde{d}(t) &= \sum_n E[\cos(2\pi f_n \Delta)] D_n e^{j2\pi f_n t} \end{aligned} \tag{2}$$

Equation (2) brings out a frequency dependent bias factor, whose value is easily calculated considering the law of Δ :

$$b(f) = E[\cos(2\pi f \Delta)] = \frac{f_s}{\pi f} \sin\left(\frac{\pi f}{f_s}\right) \tag{3}$$

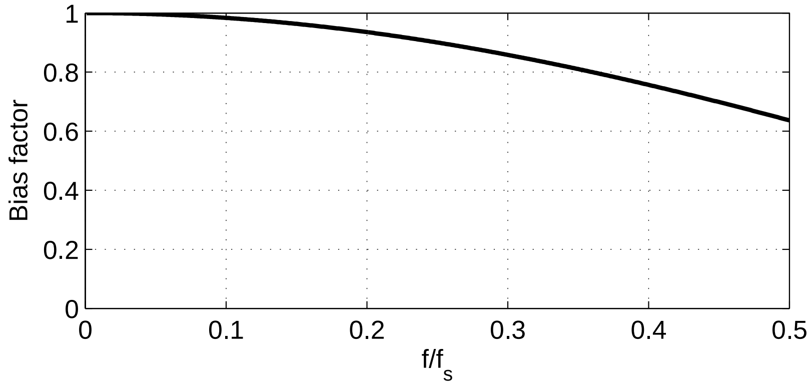

Fig. 1. Bias factor generated by the synchronization error

It means that the synchronous average will give a biased estimation of the deterministic part. The bias factor is a function of the ratio f/f_s, and varies between 1 (no bias) for $f = 0$ and 0.64 for $f/f_s = 1/2$ (Nyquist frequency). The bias factor is drawn in figure 1.

It can be noted that it is possible to use upsampling procedures to artificially increase f_s. No information is created : the frequency content of the upsampled signals will remain zero above the original Nyquist frequency. However, it permits to align more precisely time portions before the averaging process by decreasing the synchronization error Δ.

1.3 Effects of cyclic speed variations

The dynamic response of the structure and its acoustic radiation are defined in the time domain. Thus, the temporal synchronous averaging is theoretically optimal to extract the response of the structure to an impact excitation occurring in the vicinity of the cycle event chosen for the synchronization. However, the alignment of cycle events occurring at significantly different angles will be lost, as illustrated in figure 2.

Considering a cycle event occurring at θ (with $\theta = 0$ the synchronization angle), and a mean speed $\Omega(1+\nu)$ between 0 and θ (with ν the speed uncertainty), the time corresponding to θ is given by

$$t = \frac{\theta}{\Omega(1+\nu)} \tag{4}$$

In case of a moderated cyclic speed variation ($-0.1 < \nu < 0.1$), $1/(1+\nu)$ can

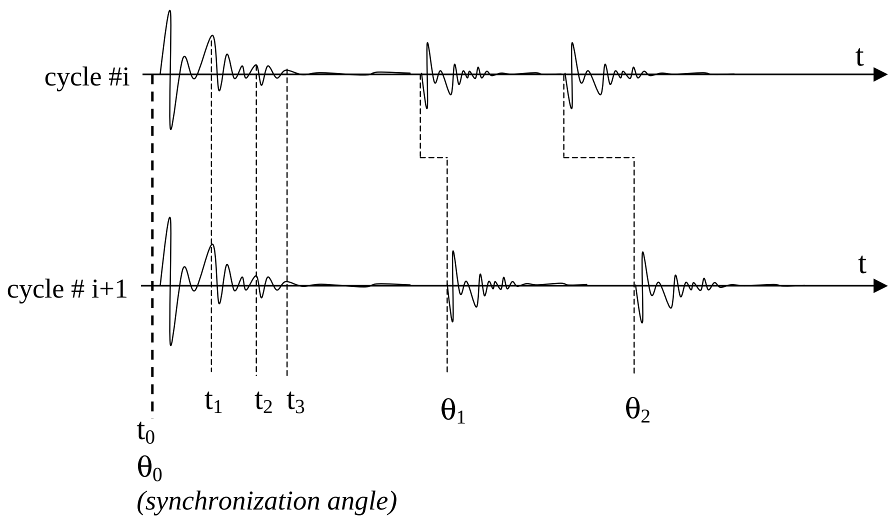

Fig. 2. Misalignment of synchronized consecutive cycles using a time sampling

be approximated by $(1-\nu)$:

$$t = \frac{\theta}{\Omega}(1-\nu) = \frac{\theta}{\Omega} - \nu\frac{\theta}{\Omega} = \frac{\theta}{\Omega} + \Delta \tag{5}$$

With $\Delta = -\nu\theta/\Omega$ the random delay induced by the cyclic speed variations. The bias error induced by this delay when averaging several cycles is obtained, as for the previous section, by calculating the expected value of $\cos(2\pi f\Delta)$ (see eq. (3)). Here, the difference is that the random delay is not uniformly distributed. Assuming that the cyclic speed uncertainty ν is a centered gaussian random variable with a standard deviation σ_ν, Δ is also a centered gaussian random variable, with a standard deviation $\sigma_\Delta = \theta\sigma_\nu/\Omega$. The expected value of the bias factor is finally given by

$$E[\cos 2\pi f\Delta] = \int_{-\infty}^{+\infty} \cos(2\pi f t)\frac{1}{\sigma_\Delta\sqrt{2\pi}}e^{-t^2/(2(\sigma_\Delta)^2)}dt \tag{6}$$

The integral in equation (6) is the expression of the Fourier transform of the gaussian function, which is also a gaussian function :

$$E[\cos 2\pi f\Delta] = \mathcal{F}\left[\frac{e^{-t^2/(2(\sigma_\Delta)^2)}}{\sigma_\Delta\sqrt{2\pi}}\right] = e^{-2\pi^2 f^2(\sigma_\Delta)^2} = e^{-2\pi^2 f^2\theta^2(\sigma_\nu)^2/\Omega^2} \tag{7}$$

The bias factor is drawn in figure 3 for several values of θ and for the speed fluctuation observed on a diesel engine at cold idle (see section 3.2). It can be seen that the bias error behaves as a low-pass filter, with for instance an attenuation of 20dB at 500 Hz for a cycle event occurring at π after the synchronization angle (with $\sigma_\nu = 4e-3$).

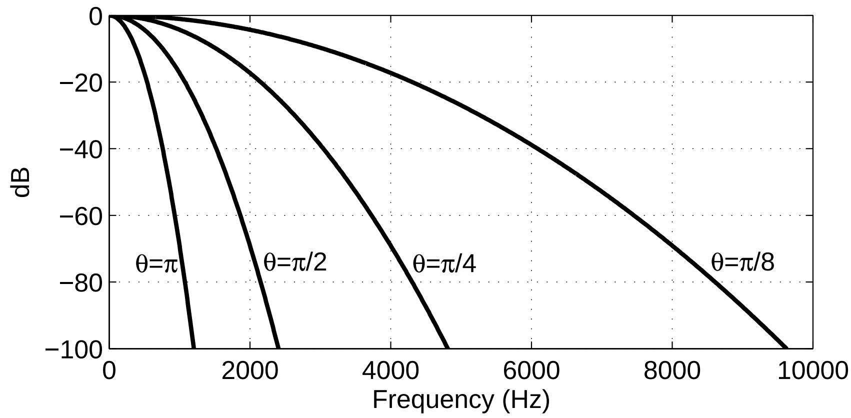

Fig. 3. Bias factors caused by cyclic speed variations on the synchronous average for events occurring at an angle θ from the synchronization angle. $\sigma_\nu = 4e - 3$, $\Omega = 850$rpm (cold idle)

2 Extraction of the angular deterministic part

2.1 Time to angle correspondence

There are two possibilities to obtain signals sampled respectively to the angle, and both are based on the use of an angular coder coupled to the main rotating shaft of the studied machine. This system provides a square signal with a period equal to the angular resolution. The first possibility is to drive the acquisition system with this clock signal, so as to obtain one sample by angle step. A difficulty of this method is that the anti-aliasing analogous filters must be adjusted on a varying sampling frequency. The second possibility is to record the angle signal, as well as other inputs, equally spaced in time, and to post process the time to angle transformation [5,3]. This second possibility has been chosen in this study, because it allows the processing of same signals sampled either in time or angle. The chosen sampling frequency must be adjusted in accordance with the angle resolution and the maximum instantaneous speed : the first harmonic of the signal of the angular coder must be lower than half the sampling frequency. A function $t(\theta)$ (time in function of the angle) is easily constructed from the clock signal, with a resolution that can be made higher than the resolution of the coder by upsampling the instantaneous speed, if needed. Then, signals are resampled in angle using an interpolation technique. The interpolation algorithm used for angular resampling is the *piecewise cubic spline* of the MATLAB software [6].

2.2 *Expression of the angular deterministic part in the time domain*

The assessment of the angular deterministic part of signals expressed in angle is much easier than for the temporal deterministic part : it is not necessary to synchronize signals. It is sufficient to average consecutive signal portions with a number of samples corresponding to the number of sample in one cycle. Then, it can be interesting to transform back this angular deterministic part in the time domain, for instance to compare it with the temporal deterministic part. This can be done by computing the cyclic average of the instantaneous rotation speed, the integration of which gives a cyclic average of the function $t(\theta)$, allowing to get back in time by using interpolation techniques.

2.3 *Effects of cyclic speed variations*

The occurrence of events generating vibration and noise in rotating machines is defined relatively to the rotation angle of the main shaft. Averaging in angle is theoretically advised if we are interested in the effective occurrence of events, for instance for diagnosis purpose or fault detection. Unfortunately, the response of the structure results from a convolution in the time domain. Thus, an averaging with respect to angle will bias the deterministic part of the response of the structure because of cyclic speed variations (see figure 4).

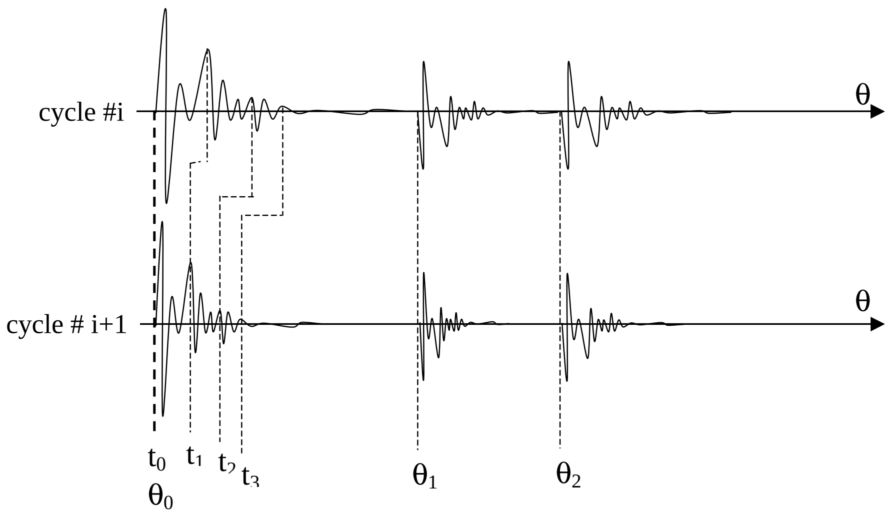

Fig. 4. Misalignment of synchronized consecutive cycles using an angular sampling

Let us consider the Fourier expansion of the (temporal) response of the structure to the deterministic part of an excitation occurring at an angle $\theta = 0$.

$$d(t) = \sum_n D_n e^{j2\pi f_n t} \tag{8}$$

The time - angle relation is expressed as in equation (5). The preceding expansion can thus be written in function of θ :

$$d(\theta) = \sum_n D_n e^{j2\pi f_n(\theta/\Omega+\Delta)}, \tag{9}$$

with $\Delta = -\nu\theta/\Omega$.

It is important to note that $\Omega(1+\nu)$ is the value during a given cycle realization of the instantaneous rotation speed average between 0 and θ. It is thus a function of θ, as well as the standard deviation of ν. However, σ_ν will be considered as constant in the following for the sake of simplicity.
The expected value of the angular cyclic average is thus given by

$$\tilde{d}(\theta) = E(d(\theta)) = \sum_n D_n e^{j2\pi f_n\theta/\Omega} E(e^{j2\pi f_n\Delta}) \tag{10}$$

The frequency dependent bias factor is identified in equation (10) :

$$b(\theta, f) = E(e^{j2\pi f\Delta}) = E[\cos(2\pi f\Delta)] \tag{11}$$

This expected value is explicitly written as follows (see eq. (7)):

$$b(\theta, f) = e^{-2\pi^2 f^2\theta^2(\sigma_\nu)^2/\Omega^2} \tag{12}$$

This attenuation is drawn in figure 5 in function of θ, for different values of frequency f, and considering Ω and σ_ν constant (this is of course an approximation, but experimental observations provided in section 3.2 show that this is realistic at least for reciprocating engines). It results in a kind of virtual damping, with a damping factor increasing with the speed uncertainty and the frequency. It is clear that the less the structure is damped, the more this bias will affect its response when averaging with respect to angle.

3 Experimental comparison between the angular and temporal determinism of diesel engine signals

3.1 experimental setup

The experimental illustration proposed in this paper focuses on a diesel engine (1.9L, common rail injection, four in-line cylinders) operating at cold idle (about 850rpm). The engine is mounted on a bench in a semi-anechoic room.

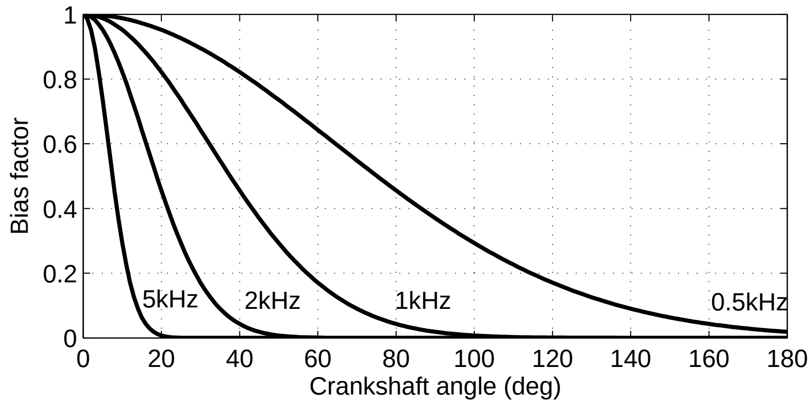

Fig. 5. Attenuation of the response of the system to an excitation occurring at an angle $\theta = 0$ caused by the averaging in the angular domain for different frequencies. $\sigma_\nu = 4e-3$

The instantaneous speed of the crankshaft is measured using an angle coder with a resolution of 0.5 deg. Several signals are recorded using a time sampling of 102400 Hz :

- 1 : cylinder pressure,
- 2 : accelerometer on a crankshaft bearing cap,
- 3 : acoustic pressure at about 1m from the engine,
- 4 : angle coder.

3.2 Cyclic speed variations of the studied engine

The cyclic speed incertitude is the cause of theoretical differences between temporal and angular determinisms. It is thus of prime interest to estimate its magnitude on the studied experimental case. The average speed of the engine running at cold idle is 854rpm. When studying the population of speeds averaged over a cycle (for 213 cycles), the normalized standard deviation is equal to 0.26%. But if we look at the population of speeds averaged over a course (quarters of cycles corresponding to the duration between two consecutive TDC), the normalized standard deviation is increased to 0.45%. The instantaneous cyclic speed averaged over 213 consecutive cycles is drawn in figure 6, together with the standard deviation. Angular decelerations and accelerations preceding and following combustions are visible, the instantaneous speed standing between 780 and 890 rpm, (between -8% and $+6\%$ of the nominal speed). The normalized standard deviation is between 0.4 and 0.5 %, with peaks at about 0.6 % in the vicinity of TDCs. It means that the dispersion of the instantaneous speed is of the same order of magnitude as the dispersion observed on the speeds averaged over a course. The instantaneous speed uncertainty thus seems principally generated by the uncertainty of the

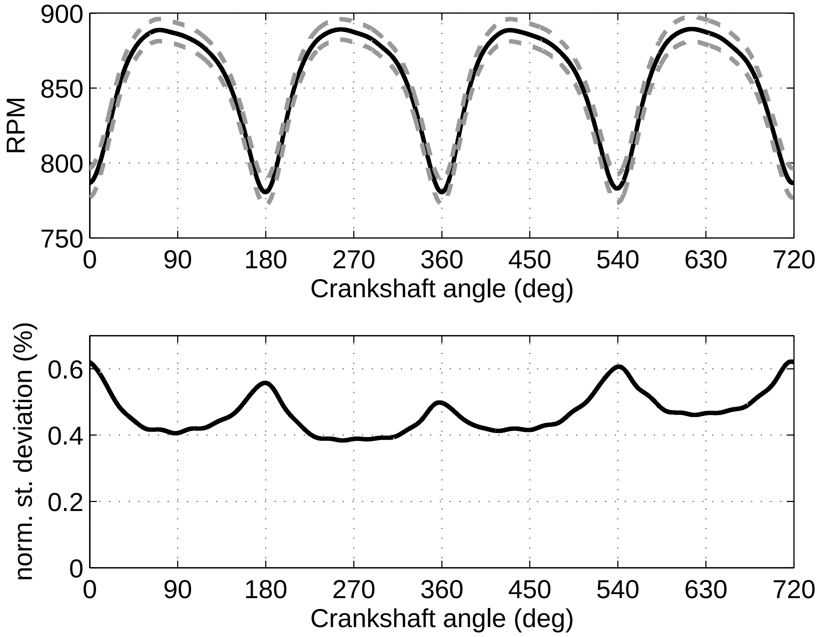

Fig. 6. Top : instantaneous cyclic speed averaged over 212 cycles, with confidence interval at $\pm 2\sigma$ in dashed red line. Bottom : normalized standard deviation.

averaged speed of a course, that can be explained by the combination of several physical unknowns like the exact fuel quantity injected in the cylinder or the auto-ignition delay. These observations legitimize assumptions made in section 2 : the dispertion of the cyclic speed is almost the same for the instantaneous speed and for averaged values over angle distances between 0 and π.

3.3 comparison of temporal and angular deterministic parts

Temporal and angular cyclic averages are computed using methods described in sections 1 and 2. Signals are recorded during 30s on the engine running at cold idle. Angular cyclic averages are transformed back into the time domain as described in section 2, to be compared to the time domain results. Angular and temporal deterministic parts of the microphone signal are drawn in figure 7 in function of time, the 0 corresponding to the TDC in the first cylinder, the cycle event chosen for the synchronization of the time averaging process.

Angular and temporal deterministic parts are clearly different. Four bursts are clearly identified, corresponding to the noise generated by the four combustions. For the angular deterministic part, the four bursts seem to have similar strengths. Concerning the temporal deterministic part, the burst following the TDC used for the synchronization (in the cylinder #1) is stronger than the

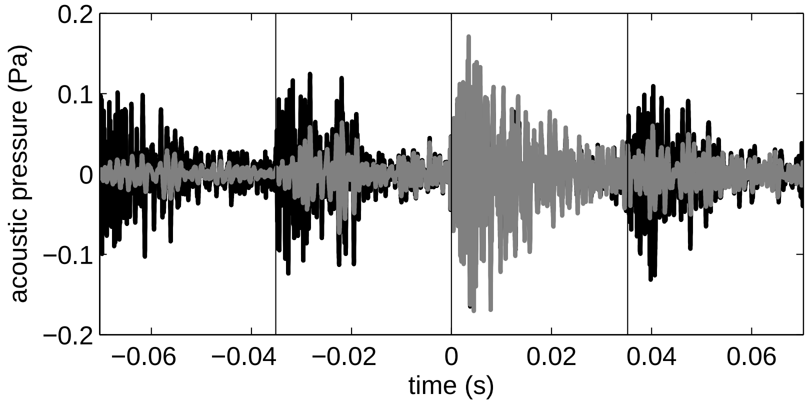

Fig. 7. Angular (black) and temporal (gray) cyclic averages (1m microphone signal). Vertical lines materialize TDCs, with the central one standing for the TDC used for the synchronization used for the time averaging.

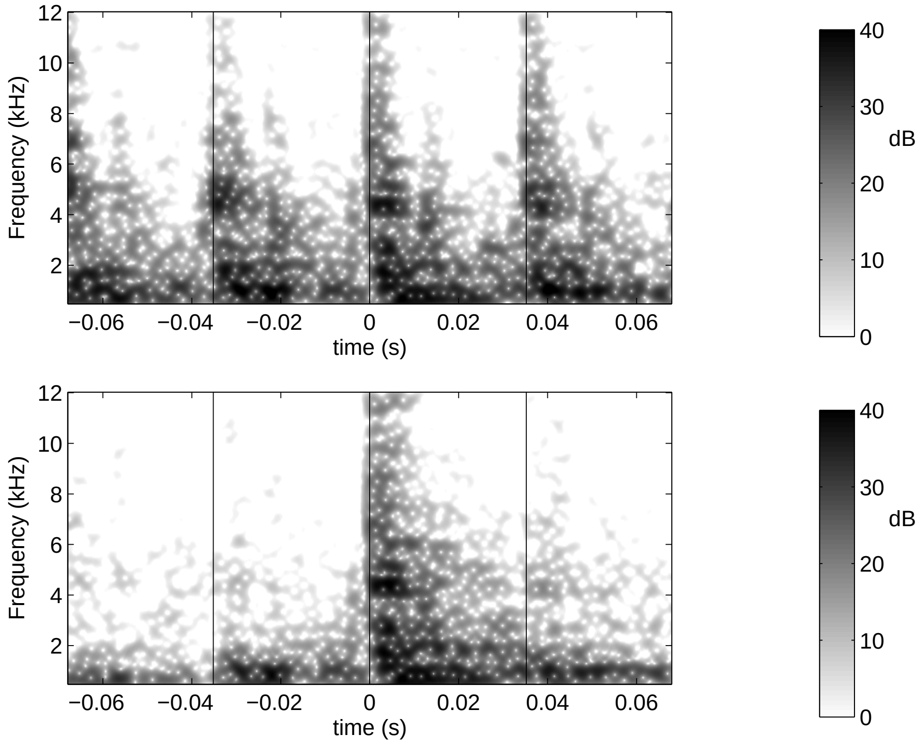

Fig. 8. Time-frequency representations of angular (top) and temporal (bottom) cyclic averages (1m microphone signal).

preceding and the following ones (respectively cylinders #2 #3). The burst corresponding to the TDC in cylinder #4 is even lower. These observations are conform to theoretical expectations found in section 1. Time-frequency representations of time and angular deterministic parts are drawn in figure 8. It is clear on the temporal deterministic part that the response of the engine to combustions #2, 3,and 4 is strongly attenuated over 1kHz, compared to the angular deterministic part. On the other side, the response to combustion in cylinder #1 (just after the TDC chosen for the synchronization), is clearly stronger on the temporal deterministic part, and its duration is longer. It confirms the results of section 2, forecasting a damping-like attenuation of the angular deterministic part, with a damping factor increasing with the frequency.

Quadratic values of temporal and angular deterministic parts of the pressure in cylinder #1, the bearing cap accelerometer and the microphone signals are drawn in figure 9, using an integration constant of 2ms. Signals are band-pass filtered between 0.5 and 12kHz. It can be seen on the cylinder pressure trace that temporal and angular deterministic parts are similar. The energy of this signal is indeed concentrated on a very short time corresponding to the combustion, the effect of cyclic speed fluctuations is thus negligible, the synchronization event used for the temporal cyclic averaging corresponding approximately to this combustion. It can be seen on the accelerometer and

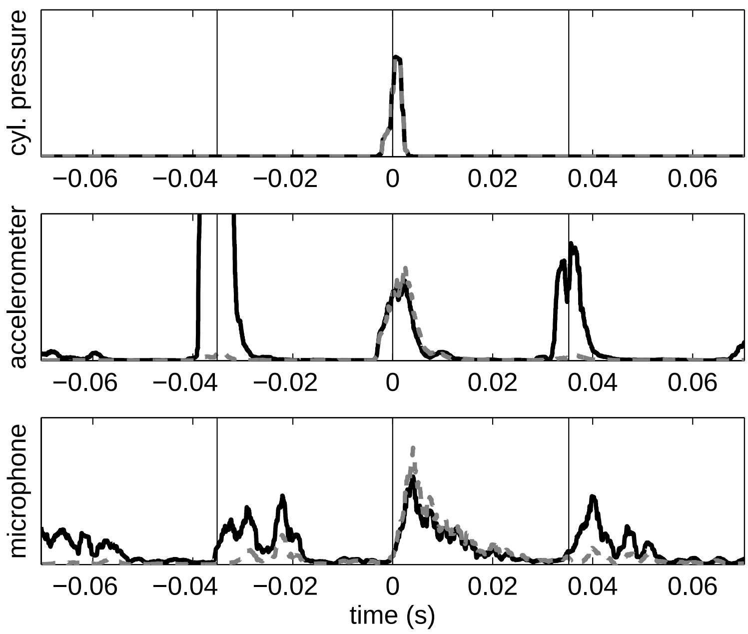

Fig. 9. Instantaneous quadratic values of angular (solid black) and temporal (dashed gray) cyclic averages for the cylinder pressure (top) accelerometer (center) and microphone (bottom) signals using a band-pass filter [0.5 12kHz], integration constant : 2ms .

microphone traces that the temporal deterministic part is a little more energetic during the response to the combustion in the cylinder # 1, and much less in other cylinders. The studied accelerometer is placed on the bearing cap between cylinders # 2 and 3, explaining why the energy of the angular deterministic part is more important just after the combustion in this two cylinders.

The study of the deterministic part of signals is of prime interest for the monitoring of the studied rotating machine, because it represents its operation at the first order. The random part of the signals is the difference between the original signal and its deterministic part. It is important to note at this stage that a light underestimation of the deterministic part can induce in some situations large overestimations of the random part. The study of the random part can be very interesting, for example in identification problems where some sources have to be separated. It is known that the correlation between noise and vibration sources of a rotating machine, that induces difficulties in the identification of their own contributions to the total noise, is mainly due to their determinism [7,8]. Removing the deterministic part of signals is thus a clever way to help in their separation. Several studies recommend to remove the temporal deterministic part of signals for system identification problems [9–11]. The underlying idea of these works is to consider the structure of the rotating machine as an invariant filter. In this case, the input-output relationship between sources and responses can be separated into two systems, the first one between the deterministic parts of excitation and responses, and the second one between random parts of excitation and responses. Coherent contributions of other sources are supposed to be deterministic, the system between random parts can thus be thought to be less disturbed than the system between deterministic parts.
The system identification has been implemented to the experimental case studied in this work. The aim is to identify the linear relationship between the pressure in cylinder #1 and the noise measured at 1m of the engine. The temporal deterministic / random separation of input and output signals has been realized as described in previous sections, and the transfer function has been estimated separately for the two systems using a H1 estimator (see [11] for details). The magnitude of obtained transfer functions are drawn in figure 10. It can be seen that the transfer function obtained using the deterministic part is globally stronger than the one obtained using random parts. It can be explained by the contributions of coherent mechanical sources that are still present on the deterministic part of the response signal, but that are efficiently suppressed by using the random parts. The engine was running at cold idle for this example, an operating condition for which the mechanical noise is particularly strong. However, it is quite difficult to objectively determine the best estimation of the transfer function, the real one being unknown. But a previous work [11] based on an important number of operating points brings out the superiority of the approach based on the suppression of the deterministic parts.

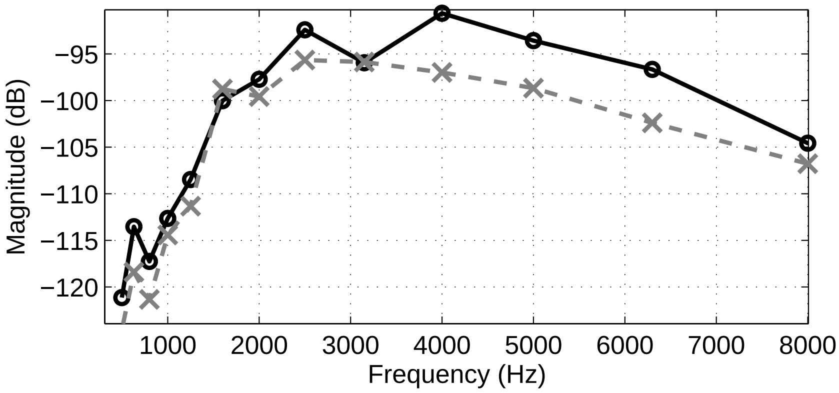

Fig. 10. Magnitude of transfer functions using deterministic parts (solid black line, o markers) and random parts (dashed gray line, × markers), averaged in third octave bands.

4 conclusion

The determinism of signals acquired on rotating machines is dual : mechanical events are determined in function of the angular position of the main shaft, and vibration and acoustic responses result from convolution operations in the time domain. If the cyclic rotation speed is purely periodic, this duality is transparent because the cyclic time-angle relationship is determined. But this situation is never perfectly achieved in real life, because the instantaneous rotation speed at a given cycle angle is never exactly the same from cycle to cycle, and this speed uncertainty is sometimes sufficient to induce different determinisms in time or angle. The effect of the speed uncertainty on the estimation of the deterministic part has been investigated in temporal and angular domains, and low pass filtering and damping effects have been quantified. The instantaneous speed uncertainty has been experimentally assessed in the case of a diesel engine, and the angular and temporal deterministic parts of vibration and acoustic signals have been assessed and compared, bringing out illustrations confirming theoretical expectations. An example of the application of the deterministic/random separation of signals has finally been proposed, illustrating the pertinence of such approaches for system identification problems.

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