# Interaction Potentials, Spectroscopy and Transport Properties of RG+/He (RG=Ar-Rn) 

Timothy Wright, Larry Alan Viehland, Ben R Gray

## To cite this version:

Timothy Wright, Larry Alan Viehland, Ben R Gray. Interaction Potentials, Spectroscopy and Transport Properties of RG+/He (RG=Ar-Rn). Molecular Physics, 2009, 107 (20), pp.2127-2139. 10.1080/00268970903183433 . hal-00521565

HAL Id: hal-00521565

## https://hal.science/hal-00521565

Submitted on 28 Sep 2010

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.


## Interaction Potentials, Spectroscopy and Transport Properties of RG+/He (RG=Ar-Rn)

| Journal: | Molecular Physics |  |
| ---: | :--- | :---: |
| Manuscript ID: | TMPH-2009-0172.R1 |  |
| Manuscript Type: | Full Paper |  |
| Date Submitted by the |  |  |
| Author: | 06-Jul-2009 |  |
| Complete List of Authors: | Wright, Timothy; University of Nottingham, School of Chemistry <br> Viehland, Larry <br> Gray, Ben; University ogf Nottingham |  |
| Keywords: | ionic complexes, ion mobility, spectroscopy |  |
|  |  |  |
|  |  |  |

URL: http://mc.manuscriptcentral.com/tandf/tmph

# Interaction Potentials, Spectroscopy and Transport Properties of $\mathbf{R G}^{+}-\mathbf{H e}$ ( $\mathbf{R G}=\mathbf{A r}-\mathbf{R n}$ ) 

Larry A. Viehland, Department of Science, Chatham University, Pittsburgh, PA 15232, USA<br>Benjamin R. Gray and Timothy G. Wright, School of Chemistry, University of Nottingham, University Park, Nottingham, NG7 2RD, UK


#### Abstract

High-level ab initio potential energy curves are calculated for the $\mathrm{RG}^{+}-$He complexes ( $\mathrm{RG}=\mathrm{Ar}-\mathrm{Rn}$ ). $\mathrm{RCCSD}(\mathrm{T})$ calculations are employed with large basis sets, and taking account of spin-orbit coupling. The calculated spectroscopic parameters are compared to experimentally-determined values, to other high-level ab initio results, and to results from potentials that were obtained by fitting to experimental data. The gas-phase mobilities of $\mathrm{RG}^{+}$ions in He are calculated from our potentials and compared, graphically and statistically, to the experimental mobilities as a function of $E / n_{0}$ at several temperatures. We conclude that more precise experimental data are required in order to discriminate between potentials with more certainty. In addition, we discuss previously reported, unexpectedly large drops in experimental mobility values for $\mathrm{RG}^{+}$in He at 4.35 K as $E / n_{0} \rightarrow 0$.


## I. Introduction

Recently, we presented ${ }^{1}$ high-quality $a b$ initio potential energy curves for the $\mathrm{Ne}^{+}-\mathrm{He}$ and $\mathrm{He}^{+}-\mathrm{Ne}$ complexes, and compared calculated spectroscopic parameters and transport coefficients with those previously available and with new experimental mobility data. In general, the agreement was excellent, the only exception being the comparison with some of the experimental spectroscopic data ${ }^{2}$ for $\mathrm{Ne}^{+}-\mathrm{He}$. In the present study, we extend our work to the heavier rare gas atomic cations interacting with He . As before, we aim to produce high quality $a b$ initio curves, to employ these in the derivation of spectroscopic and transport quantities, and to compare them to available experimental values. Mobilities are useful in understanding chemical reactions, such as those in plasmas. In addition, they provide a good experimental test of calculated potentials, over a wider range of internuclear separations than is generally possible using spectroscopic or scattering techniques. The latter is particularly important for open-shell species, which are often difficult to generate experimentally, and additionally there are questions as to which potentials to be used, depending on the spin-orbit states present in the experiment, which can depend on the experimental conditions. ${ }^{1}$ Reliable potentials, of course, are useful across a wide range of chemistry, but they are fo greater use if their reliability has been tested.

An early report of spectra by Tanaka et al. ${ }^{3}$ was extended by Dabrowski, Herzberg and Yoshino (DHY), ${ }^{4}$ who published a detailed UV absorption spectroscopic study of the $\mathrm{Ar}^{+}-\mathrm{He}$ complex. Since the ground state of $\mathrm{Ar}^{+}$is ${ }^{2} P$, interaction with He gives ${ }^{2} \Pi$ and ${ }^{2} \Sigma^{+}$electronic states in the absence of spin-orbit coupling, with the latter state expected to be lower in energy owing to less electron repulsion along the internuclear axis. Spin-orbit coupling leads to ${ }^{2} P_{3 / 2}$ and ${ }^{2} P_{1 / 2}$ states of $\mathrm{Ar}^{+}$which, upon interaction with He , yield ${ }^{2} \Sigma_{1 / 2}{ }^{+},{ }^{2} \Pi_{3 / 2}$ and ${ }^{2} \Pi_{1 / 2}$ molecular states; the first two correlate with the lower, ${ }^{2} P_{3 / 2}+$ He asymptote, and the third with the higher, ${ }^{2} P_{1 / 2}+$ He one. The molecular states, according to a Hund's case (a) analysis, are thus $X^{2} \Sigma_{1 / 2}{ }^{+}, A_{1}{ }^{2} \Pi_{3 / 2}$ and $A_{2}{ }^{2} \Pi_{1 / 2}$. Emission lines were observed by DHY, giving spectroscopic information on the $A_{2}{ }^{2} \Pi_{1 / 2}$ and $X^{2} \Sigma_{1 / 2}{ }^{+}$states, as discussed below.

Saito et al. ${ }^{5}$ have estimated an $\mathrm{Ar}^{+}-\mathrm{He}$ binding energy of $37 \mathrm{meV}\left(\sim 300 \mathrm{~cm}^{-1}\right)$ from mobility measurements. Carrington and coworkers ${ }^{6}$ have recorded microwave spectra of $\mathrm{Ar}^{+}-\mathrm{He}$ at energies close to the dissociation asymptote. These transitions are in regions where the Born-Oppenheimer approximation was found to break down, and a detailed treatment of the energy levels was presented. The transitions observed in that work, together with those from DHY, were employed to generate fitted potential energy curves for all three states mentioned above; this set of potentials was designated MAL1.

In addition to this fitted potential, other theoretical results are available for $\mathrm{Ar}^{+}-\mathrm{He}$. Early work focused on charge exchange, mainly concerned with nascent $\mathrm{He}^{+}$ions in Ar. ${ }^{7,8}$ In 1978, Olson and Liu ${ }^{9}$ calculated CI curves for the $X^{2} \Sigma^{+}$and $A^{2} \Pi$ states of $\mathrm{Ar}^{+}-\mathrm{He}$ in the absence of spin-orbit coupling. In 1985, Hausamann and Morgner ${ }^{10}$ made use of theoretical and experimental data available at the time and a model to obtain potential energy curves for $\mathrm{Ar}^{+}-\mathrm{He}, \mathrm{Kr}^{+}-\mathrm{He}$ and $\mathrm{Xe}^{+}-\mathrm{He}$ that included spinorbit coupling. Soon afterwards, Siska ${ }^{11}$ reported the results of a modified TangToennies model for the potential energy curves for $\mathrm{Ar}^{+}-\mathrm{He}$, but in the absence of spin-orbit coupling. An MRDCI study by Stärk and Peyerimhoff ${ }^{12}$ in 1986
considered a number of low-lying states of the [Ar-He] ${ }^{+}$system, with the lowest ${ }^{2} \Sigma^{+}$and ${ }^{2} \Pi$ states corresponding to $\mathrm{Ar}^{+}-\mathrm{He}$; owing to emphasis being placed on the excited states, they did not achieve an optimal description of the lower states that are pertinent to the present study. Potential curves were also obtained by Brunetti et al. ${ }^{13}$ by fitting an expression to the spectroscopic data and assuming the spin-orbit coupling was equal to that of the separated atoms and independent of the internuclear separation. Balasubramanian et al. ${ }^{14}$ reported non-spin-orbit curves for $\mathrm{Ar}^{+}-\mathrm{He} .{ }^{15}$ In 1990, Staemmler ${ }^{16}$ reported ab initio calculations at the SCF and CEPA levels using large Gaussian basis sets for the lowest states of $\mathrm{Ar}^{+}-\mathrm{He}$; spin-orbit effects were added using a semi-empirical treatment. Gemein and Peyerimhoff ${ }^{17}$ used MRD-CI calculations, both with and without spin-orbit coupling, to calculate potential energy curves, reporting equilibrium bond lengths, dissociation energies and harmonic vibrational frequencies. In 1991, Viehland, Viggiano and Mason (VVM) ${ }^{18}$ combined the SCF and CI potentials of Olson and Liu ${ }^{9}$ with the known long-range behaviour to obtain potentials (without spin-orbit effects) for the ${ }^{2} \Sigma^{+}$and ${ }^{2} \Pi$ states of $\mathrm{Ar}^{+}-\mathrm{He}$; the Siska ${ }^{11}$ potentials were also employed, but in a modified form. Ionization energies for RG-He and the associated spectroscopic parameters for $\mathrm{RG}^{+}-\mathrm{He}$ have been reported by Ma, Li and $\mathrm{Ng}^{19}$; no spin-orbit coupling was included. In 2001, Murrell et al. ${ }^{20}$ reported non-SO and SO potential energy curves for $\mathrm{Ar}^{+}-\mathrm{He}$ over a limited range using MRCI+Q energies and an aug-cc-pVQZ basis set. Most recently, to our knowledge, Bera et al. ${ }^{21}$ have reported UCCSD(T)/aug-cc-pVTZ spectroscopic and potential parameters for $\mathrm{Ar}^{+}-\mathrm{He}$. In the present work, we shall compare the above results with our potential energy curves, and additionally assess them against other experimental data.

The large body of data on $\mathrm{Ar}^{+}-\mathrm{He}$ is not matched for the $\mathrm{Kr}^{+}-\mathrm{He}$ ion. Early experimental work comprises the UV emission spectra from Tanaka et al. ${ }^{3}$ where the assignment was only tentative, and scattering studies of $\mathrm{He}^{+}$and Kr by Weise. ${ }^{22}$ The transport studies of Saito et al. ${ }^{5}$ suggested that the binding energy of the $\mathrm{Kr}^{+}-\mathrm{He}$ ground state was $\sim 29 \mathrm{meV}\left(\sim 230 \mathrm{~cm}^{-1}\right)$. There are model potential results for $\mathrm{Kr}^{+}-\mathrm{He}$ from Hausamann and Morgner ${ }^{10}$ and relativistic CI calculations by Balasubramanian et al. ${ }^{23}$, where spin-orbit coupling was included using basis sets of approximately double- $\zeta$ quality. ${ }^{15}$ Ma et al. ${ }^{19}$ also reported spectroscopic and potential parameters for $\mathrm{Kr}^{+}-\mathrm{He}$. The most detailed information, however has come from a joint microwave and theoretical study by Carrington and coworkers. ${ }^{24}$ At present, the fitted (MAL1) interaction potential reported in that work (similar to that obtained for $\mathrm{Ar}^{+}-$ He , mentioned above) represents most of the information that is known for this species.

To our knowledge, there is no experimental or theoretical spectroscopic information available for $\mathrm{Xe}^{+}-\mathrm{He}$ or $\mathrm{Rn}^{+}-\mathrm{He}$, except for the model potential calculations for $\mathrm{Xe}^{+}-$ He, by Hausamann and Morgner. ${ }^{10}$

Experimental studies have been performed on the transport of $\mathrm{RG}^{+}$ions through He . We have already mentioned our previous work on $\mathrm{Ne}^{+}$in $\mathrm{He}^{1}$, but all of the other $\mathrm{RG}^{+}$ ions (except Rn ) have also been studied. The mobility of $\mathrm{Ar}^{+}$through He has been studied at a number of temperatures: at 4.35 K by Saito et al. ${ }^{5}$; at 82 K by Koizumi et al. ${ }^{25}$; at 92, 170, 293, 428 and 552 K by Viehland et al. ${ }^{18}$ and at 300 K by Lindinger and Albritton. ${ }^{26}$ By calculating theoretical transport coefficients and comparing to the experimental data, it is possible to test the potential (or conversely the experimental
data) over a wide range of internuclear separations. ${ }^{27}$ Generally, such tests are more stringent than comparison with spectroscopic data, as the latter usually only sample a very small region of internuclear separation. One of the present species is an exception, however, since spectroscopic data close to the minimum, ${ }^{4}$ as well as at very long range ${ }^{6}$ are available for $\mathrm{Ar}^{+}-\mathrm{He}$.

For $\mathrm{Kr}^{+}$in He , the mobilities having been measured at: 4.35 K by Saito et al. ${ }^{5}$; 82 K by Koizumi et al. ${ }^{28}$; 294 K by Jones et al. ${ }^{29}$; and 300 K by two groups, Johnsen et al. ${ }^{30}$ and Kaneko et al. ${ }^{31}$ For $\mathrm{Xe}^{+}$in He , only two sets of data have been reported: at 82 K by Koizumi et al. ${ }^{28}$, and at 295 K by Johnsen et al. ${ }^{30}$

## II. Calculations

## A. Ab initio Potentials

The calculations reported here were performed at the $\operatorname{RCCSD}(T)$ level of theory, employing d-aug-cc-pV5Z quality basis sets for $\mathrm{He}-\mathrm{Ar}$; the frozen core approximation was used throughout. The basis sets were doubly-augmented to allow a good description of the long-range interactions, with the double-diffuse augmentation being achieved by extending the basis set by one set of functions of each angular momentum type; the $\zeta$ values were obtained by even-tempered extrapolations of the two most diffuse functions of the same type in the standard basis set. For the heavier atoms, small-core relativistic effective core potentials (ECPs) were employed: ECP10MDF, ECP28MDF and ECP60MDF for Kr , Xe and Rn , respectively; the standard aug-cc-pV5Z valence basis sets ${ }^{32}$ were then used with these ECPs, again extending the basis set by one diffuse function of each type, as described above. The potential energy curves of the ${ }^{2} \Sigma^{+}$and ${ }^{2} \Pi$ electronic states were calculated point by point using the MOLPRO ${ }^{33}$ suite of programs. A full counterpoise correction was made at each point to correct for basis set superposition error.

Since the interaction between He and $\mathrm{RG}^{+}$is expected to be weak, the spin-orbit coupling is expected to be localized on $\mathrm{Ar}^{+}$, as was assumed in ref. 6. We therefore derived the SO potentials analytically, using expressions arising from the well-known atomic model ${ }^{34}$ that connects the SO-potentials, $X^{2} \Sigma_{1 / 2}{ }^{+}, A_{1}{ }^{2} \Pi_{3 / 2}$ and $A_{2}{ }^{2} \Pi_{1 / 2}$ (denoted $V_{X}, V_{A_{1}}$ and $V_{A_{2}}$ ), to the ${ }^{2} \Sigma^{+}$and ${ }^{2} \Pi$ potentials (denoted $V_{\Sigma}$ and $V_{\Pi}$ ). These expressions are
$V_{X}=\frac{1}{2}\left(V_{\Sigma}+V_{\Pi}+\Delta-\delta\right)$
$V_{A_{1}}=V_{\Pi}$
and
$V_{A_{2}}=\frac{1}{2}\left(V_{\Sigma}+V_{\Pi}+\Delta+\delta\right)$,
where

$$
\begin{equation*}
\delta^{2}=\left(V_{\Sigma}-V_{\Pi}\right)^{2}-\frac{2 \Delta}{3}\left(V_{\Sigma}-V_{\Pi}\right)+\Delta^{2} \tag{4}
\end{equation*}
$$

and $\Delta$ is the SO splitting between the $\mathrm{RG}^{+}$atomic ${ }^{2} P_{3 / 2}$ and ${ }^{2} P_{1 / 2}$ states. The values of $\Delta$ are 1431.41, 5371.0, 10537.01 and $30895.1 \mathrm{~cm}^{-1}$ for $\mathrm{RG}=\mathrm{Ar}-\mathrm{Rn}$, respectively. ${ }^{35}$

## B. Spectroscopic Constants

Once obtained, the potential energy data set was used as input into the LEVEL program ${ }^{36}$ in order to obtain spectroscopic and other data, in particular the rovibrational energy levels. From the latter, fits of the lowest few levels to standard spectroscopic formulae were employed to obtain equilibrium values.

## C. Transport Properties

To obtain transport coefficients, the calculated potentials were first employed in the QVALUES program ${ }^{37,38}$ to obtain transport cross sections with a precision of $0.1 \%$. Transport coefficients were obtained by employing these cross sections in the GC program, ${ }^{39}$ a modified version of the older GRAMCHAR program. ${ }^{40}$ The modifications were made to cope with the increased demands when the ion mobility has both a maximum and a minimum (as was the case for systems involving the coinage metal cations moving through the rare gases ${ }^{39}$ ), to allow higher-order approximations to be employed, and finally to take advantage of faster, modern computers.

The overall precision for the calculated mobilities is $0.1 \%$ at small $E / n_{0}$ (the ratio of the electric field strength to the gas number density). In the worst cases, it is $1 \%$ near and beyond the mobility maximum, but only $3 \%$ in the region between the minima and maximum, where the mobility is increasing rapidly with increasing $E / n_{0}$.

Calculated values for the mobilities and other transport coefficients for each one of the naturally-occurring $\mathrm{RG}^{+}$isotopes in He have been obtained over wide ranges of $E / n_{0}$ at gas temperatures of $100,200,300,400$ and 500 K , and for gas temperatures that have been used in transport experiments (see above). All of these results have been entered into the database maintained at Chatham University. ${ }^{41}$

## III. Results and Discussion

## A. Potential Curves and Spectroscopy

In Figure 1, we show our ab initio calculated potential energy curves for $\mathrm{Ar}^{+}-\mathrm{He}$, along with the MAL1 potentials ${ }^{6}$. As may be seen, the two sets of curves are in very good agreement, and although there are differences at the minima, the directions of these differences are not consistent for the three SO states. (The data points for our curves are given as Supplementary Material.)

We start by commenting on the ab initio results that did not include SO coupling, although such studies cannot be assessed against experimental data directly. Results prior to the work in ref. 6 have been commented on previously ${ }^{6,24}$. The more recent work of Murrell et al. ${ }^{20}$ reports the results of MRCI + Q calculations employing an
aug-cc-pVQZ basis set. We have noted previously ${ }^{1}$ that for $\mathrm{Ne}^{+}-\mathrm{He}$ a singlereference method appears to be satisfactory and we assume the same is true for the species considered herein. For $\mathrm{Ar}^{+}-\mathrm{He}$, this certainly seems to the case since the derived $R_{\mathrm{e}}$ and $D_{\mathrm{e}}$ values in Table 1 for the $\mathrm{MRCI}^{20}$ and the present $\operatorname{RCCSD}(\mathrm{T})$ potentials are very close, both for the ${ }^{2} \Sigma^{+}$and the ${ }^{2} \Pi$ states. The most recent study by Bera et al. ${ }^{21}$ reports $\operatorname{CCSD}(\mathrm{T})$ calculations employing aug-cc-pVTZ basis sets for the ${ }^{2} \Sigma^{+}$state. Since the Gaussian program ${ }^{42}$ was employed therein, we assume that unrestricted wave functions were used. A number of calculations were employed, including the DFT hybrid method, B3LYP, as well as MP2 and $\operatorname{CCSD}(\mathrm{T})$. The B3LYP values indicated a large overestimate of the interaction, something that has been noted previously by one of us for $\mathrm{NO}^{+}-\mathrm{Ar}^{43}$ The MP2 and $\operatorname{CCSD}(\mathrm{T})$ results ${ }^{21}$ are much closer to each other, with the $\operatorname{CCSD}(\mathrm{T})$ values being closer to the results of the present study (see Table 1). We note that results were also reported where all electrons were correlated, at both the MP2 and $\operatorname{CCSD}(\mathrm{T})$ levels; however, since no tight functions were included to describe the core-valence correlation, those results must be viewed with caution. We only include the frozen core $\operatorname{CCSD}(\mathrm{T})$ results in Table 1, for which good agreement is seen for $R_{\mathrm{e}}$, but the well depths are shallower than those found herein.

Moving on to studies which have included SO coupling, we only consider the most recent and reliable potentials, the key aspects of which are presented in Table 2. (The other potentials are commented upon in refs. 6 and 18.) We first note that the model potential results of Siska ${ }^{11}$ and a modified version of these ${ }^{18}$ were mainly fitted to the UV results of Dabrowski et al. ${ }^{4}$, whereas the MAL1 potentials also have input from the microwave spectrum that helps characterize the long-range region better. As noted before, ${ }^{6}$ there are significant differences between the three aforementioned potentials, with the differences being largest for the $A_{1}$ state, for which there were no observed transitions in the UV study. Comparing these to the present results and to the MAL1 potential, we see that the present $R_{\mathrm{e}}$ value is slightly larger, with the $D_{\mathrm{e}}$ value being slightly smaller; interestingly the present value falls between the previously determined values. For the $A_{2}$ state, we obtain excellent agreement with the $D_{\mathrm{e}}$ value from the MAL1 potential, ${ }^{6}$ and indeed with the value from Siska. ${ }^{11}$ All of these values lie significantly above those of $\mathrm{VVM}^{18}$ and the value from DHY. ${ }^{4}$ In the latter case, dissociation energies were obtained from a combination of extrapolation and energy differences, so perhaps it is not surprising that the value is not so accurate. Lastly, the most dramatic differences in $D_{\mathrm{e}}$ values occur for the $A_{1}$ state, for which there were no experimental data; however, since the three states here are interconnected via spin-orbit coupling, knowing two of them defines the third. It may be seen from Table 2 that the present work is in excellent agreement with the MAL1 potentials for both $R_{\mathrm{e}}$ and $D_{\mathrm{e}}$. In conclusion, the MAL1 potentials, having included long-range spectroscopic data in the fit, appear to be much more reliable potentials than either of the two earlier ones, which were based purely on the UV spectra. The MAL1 potentials are in very good agreement with the curves obtained in the present work. We note that there is also very good agreement with the SO results reported in ref. 20 for the $X$ and $A_{2}$ states (no parameters were reported for the $A_{1}$ state).

For $\mathrm{Kr}^{+}-\mathrm{He}$, there is no assigned UV spectrum. Hence, all of the information on this species comes from the microwave study of Carrington and coworkers ${ }^{24}$ on levels that are only a few $\mathrm{cm}^{-1}$ from the dissociation limit. Thus, these experiments sample only
the long-range portion of the interaction potential. Since microwave spectroscopy is such a precise technique, the whole potential curve should be obtainable, in principle, if enough transitions are observed. In fact, sufficient transitions were observed ${ }^{24}$ to generate MAL1 potentials, similar in type to those for $\mathrm{Ar}^{+}-\mathrm{He}$, but now having no input from UV spectra. These MAL1 potential are, therefore, only defined in terms of transitions between levels very close to the dissociation limits.

As may be seen in Figure 2, the agreement between our $\mathrm{Kr}^{+}-\mathrm{He}$ potentials and the MAL1 potentials looks very good, with the MAL1 potentials being very slightly deeper in all cases, especially for the $X$ state. These features are evident in the data presented in Table 2. Given that the MAL1 potentials have been obtained by fitting only to transitions between high-lying levels, the agreement between the two sets of curves is remarkable.

We note in passing that the previous study by Balasubramanian et al. ${ }^{15}$ has been noted as having an error ${ }^{24}$, as the ground state is designated $X^{2} \Pi_{3 / 2}$, and the dissociation energies are far from those obtained here or from the MAL1 potentials. However, the estimate for the binding energy of $29 \mathrm{meV}\left(\sim 230 \mathrm{~cm}^{-1}\right)$ from mobility studies ${ }^{5}$ is of the correct magnitude.

There are no reliable data to which to compare our results for either $\mathrm{Xe}^{+}-\mathrm{He}$ or $\mathrm{Rn}^{+}-$ He , although in the former case there are experimental mobility results, and these will be considered below.

## B. Transport Results for $\mathrm{Ar}^{+}$in He

As noted in the Introduction, there have been a number of studies measuring the mobility of $\mathrm{Ar}^{+}$in He at a variety of temperatures. Such studies can, if there are enough data points, and if the accuracy and precision are high enough, provide stringent tests of potentials across a very wide range of internuclear separation. ${ }^{27}$ To this end, we have calculated mobilities using both the MAL1 and the present potentials, and have compared the results to experimental data. In Figure 3, the results at 300 K are given, both those calculated from the present potentials and the experimental data points.

For the calculations, we have employed a (110) weighting of the cross sections arising from the $X, A_{1}$ and $A_{2}$ potentials, which is akin to assuming only the ${ }^{2} P_{3 / 2}$ atomic state of $\mathrm{Ar}^{+}$is present. We have also used a (001) weighing, which is for the ${ }^{2} P_{1 / 2}$ state. We have performed such calculations for both the MAL1 and present potentials. Unfortunately, the cited experimental error bars are large enough that they encompass both of the calculated curves. If it is assumed that the error bars are pessimistic, then by eye it appears that the (110) weighting of the potentials is in better agreement with the actual experimental data points than the (001) one. In agreement with our conclusion for $\mathrm{Ne}^{+}$in $\mathrm{He}^{1}$, this would suggest that only the ${ }^{2} P_{3 / 2}$ ground state of $\mathrm{Ar}^{+}$ were present in the experiments, i.e., any ions created in the higher SO state were efficiently quenched to the lower state before they reached the drift region of the apparatus.

Another way of comparing data is to employ the statistical quantities $\delta$ and $\chi .^{38}$ The dimensionless quantity $\delta$ is a measure of the relative differences compared to the combined experimental and calculational errors, while $\chi$ is a measure of the relative standard deviation to the sum of the squared error estimates. Values of $|\delta|$ that are substantially lower (alternatively, higher) than 1 indicate that there is substantial agreement (disagreement) between the calculated and measured values, on average. Values of $\chi$ that are not much larger than $|\delta|$ indicate that there is little scatter in the experimental data and that the agreement between the calculated and measured values is uniform over all values of $E / n_{0}$, while values of $\chi$ substantially greater than $|\delta|$ indicate that at least one of these factors is not true. In Table 3, we report these quantities for the present potential and smoothed versions ${ }^{44,45,46}$ of the experimental data for ${ }^{40} \mathrm{Ar}^{+}$in He , but not including the experimental values for $E / n_{0}<2 \mathrm{Td}$ (see below). What is immediately clear is that for the data at the higher temperatures ( $\geq 293$ K) there is agreement between calculated and experimental data for either state, which is a result of the large experimental error bars, as can be seen in Figure 3. These statistical quantities also indicate that, with regards to the mobility data, there is no statistical difference between the results obtained using the present potentials and those obtained using the MAL1 potentials.

The mobility data from $\mathrm{VVM}^{23}$ for $\mathrm{RG}^{+}$in He have been suspected ${ }^{47,48}$ for many years of being less accurate than claimed by the experimenters. Hence it is not surprising to find in Table 3 that these data are in significant disagreement with the present calculations, as they are in similar disagreement with results calculated from all of the other available potentials.

The data which have the smallest error bars are those at $4.35 \mathrm{~K} .{ }^{5}$ Interestingly, the experimental data show a distinct drop in the mobility of $\mathrm{RG}^{+}$in He as $E / n_{0} \rightarrow 0-$ see Figure 4. Similar results had been reported previously for $\mathrm{He}^{+}$in $\mathrm{He},{ }^{49}$ a system in which resonant charge transfer (RCT) occurs. One explanation that has been put forward ${ }^{5}$ is quantum effects, as previously predicted by Dickinson ${ }^{50}$ and Heiche and Mason. ${ }^{51}$ The same unusual mobility behaviour was found in $\mathrm{N}_{2}{ }^{+}$and $\mathrm{CO}^{+}$in He by Sanderson et al. ${ }^{52}$ in which RCT cannot occur, and in this case orbiting resonances and/or collision-induced rotational excitation were proposed. Other experiments have also observed such unusual low- $E / n_{0}$ mobilities, in He for $\mathrm{Ne}^{+}, 53 \mathrm{O}_{2}{ }^{+},{ }^{54} \mathrm{NO}^{+55}$ and other ions. ${ }^{56}$ A first theoretical account of the low $E / n_{0}$ anomalous mobilities of $\mathrm{N}_{2}{ }^{+}$and $\mathrm{O}_{2}{ }^{+}$in He at 4.35 K was presented by Ohtsuki et al. ${ }^{57}$ They calculated mobilities from MRCI potentials, and demonstrated that the anomalous behavior was due to Feshbach-like resonances, where rotational excitation of the molecular ions occurred as a result of collisions with He. This explanation cannot apply to atomic ions such as $\mathrm{RG}^{+}$in He , of course, nor for other atomic ions, $\mathrm{O}^{+}$and $\mathrm{N}^{+},{ }^{58}$ for which similar behaviour has been found. The suggestion that quantum effects might be the source of the low $E / n_{0}$ behaviour was tested explicitly by Viehland and Hurly in $1996^{48}$ and was found not to explain the observed results. As such, the anomalous low $E / n_{0}$ behaviour of atomic ions in He at 4.35 K so far remains unexplained.

The fact that the present results, as was the case for $\mathrm{Ne}^{+}$in $\mathrm{He},{ }^{1}$ also show that the theoretical mobilities do not show the dramatic fall in values as $E / n_{0} \rightarrow 0$ has led us to consider this issue further. We note that, in ref. 5 , the ions were provided with a constant injection energy of 20 eV . Consequently, the value of $E / n_{0}$ is controlled by varying the pressure, and hence low $E / n_{0}$ values are achieved at higher pressures. At
4.35 K , there is a possibility for $\mathrm{He}_{2}$ (and possibly higher-order clusters) to form, and since the formation would be a termolecular process (with a third body being required to stabilize the $\mathrm{He}_{2}$ species) it would be highly pressure dependent. The effect of He dimer formation would be to increase the effective reduced mass of collisions, and hence lower the mobility. This hypothesis could be tested by performing measurements at $E / n_{0}$ values where both $E$ and the pressure were changed, but yielding the same resulting $E / n_{0}$ value.

## B. Other Transport Results

Figures 5 and 6 compare calculated and experimental mobilities for ${ }^{84} \mathrm{Kr}^{+}$in He at 300 and 4.35 K , respectively. In Table 4, we report the statistical quantities from a comparison of the present potential for ${ }^{84} \mathrm{Kr}^{+}$in He and smoothed versions ${ }^{40,41,44}$ of the experimental data in which neither the isotope nor the state was specified. The calculated mobilities for the five naturally occurring isotopes of Kr are very similar. This is consistent with the theoretical prediction (from the first approximation of the two-temperature kinetic theory ${ }^{59}$ ) that the mobility is inversely proportional to the square root of the reduced mass. Hence any differences between theory and experiment shown in Table 4 are not due to which isotope was used.

Again we note that calculated mobilities involving the MAL1 potentials are not statistically different from those using the present potentials - more evidence for their close agreement. Table 4 shows that at 4.35 K there is agreement between calculated and experimental data for either state. Note that again we do not include the experimental values for $E / n_{0}<2 \mathrm{Td}$ at 4.35 K , for the reasons we have discussed for $\mathrm{Ar}^{+}$in He above. The data at 82 K are in satisfactory agreement with the calculations for both states, owing to our use of $4 \%$ as the experimental accuracy; the experimenters did not state the accuracy of their results, but examination of the raw data ${ }^{28}$ shows rapid variations by as much as $4 \%$. Finally, the data near room temperature are in good to excellent agreement with the calculations for the ${ }^{2} P_{1 / 2}$ excited state but not with those for the ${ }^{2} P_{3 / 2}$ ground state.

As shown in Figures 4 and 6, a small mobility minimum occurs at low gas temperatures. Such minima arise frequently with open-shell ions, because the $R^{-6}$ component of their long-range interactions with rare gas atoms is large, compared to the $R^{-4}$ component that dominates at the very largest separations. More extensive comments about mobility minimum have been given previously. ${ }^{39}$

The calculated mobilities for ${ }^{132} \mathrm{Xe}^{+}$in He at 300 K are compared with experimental values in Figure 7 and Table 5. As before, the calculated mobilities for different isotopes are very similar, so difference between theory and experiment are not due to which isotope was used. The data at 82 K are in reasonable agreement with the calculations for both states, owing to our choice again to use $4 \%$ for the accuracy, a value that was not reported by the experimenters. ${ }^{28}$ The data near room temperature are not in good agreement with the calculations, suggesting that those data may be even less accurate than the estimate of $5 \%$ made by the experimenters ${ }^{27}$.

There are no experimental values for $\mathrm{Rn}^{+}$in He , and so the quantities calculated here and placed into the database ${ }^{41}$ represent the only values available.

A word of caution is in order, however, as we previously ${ }^{1,60}$ noted problems arising from injection effects when a heavy ion is passed into a drift tube containing a light gas; this can lead to a measured mobility that is too high and careful checking of experimental values is required.

## IV. Conclusions

The favourable comparison of the present, completely $a b$ initio, potentials with previous experimental $\left(\mathrm{DHY}^{4}\right)$ results and fitted MAL1 ${ }^{6,24}$ results for $\mathrm{Ar}^{+}-\mathrm{He}$ and $\mathrm{Kr}^{+}-\mathrm{He}$, suggests that the present results are reliable. It is noteworthy that, for the most part, there is agreement with the MAL1 potentials of Carrington et al. ${ }^{6,24}$ and the non-SO potentials of Siska ${ }^{11}$ and VVM. ${ }^{18}$ The discrepancies between those results and ours are likely in part attributable to different parameterizations and the different experimental data sets employed. The good agreement with the $\mathrm{Kr}^{+}-\mathrm{He}$ MAL1 potentials ${ }^{24}$ is particularly noteworthy, since the latter was obtained by fitting only to experimental spectroscopic data obtained from transitions between levels within a few $\mathrm{cm}^{-1}$ of the dissociation limit.

The agreement in Table 2 of our calculated spectroscopic quantities with those obtained from UV absorption spectra ${ }^{4}$ is good, in the main. However, independent checking of some values is desirable, particularly for the dissociation energies.

Transport coefficients are known to be good tests of interaction potentials across a very wide range of interatomic (or intermolecular) separation, but such tests are reliant on the experimental precision being sufficiently good. The results above have shown that, near room temperature, the experimental errors of $5-7 \%$ are too large to be discriminatory for $\mathrm{Ar}^{+}$in He . We noted, however, that there appears to be a better agreement between the reported data points and the values calculated by assuming only $\mathrm{Ar}^{+}\left({ }^{2} \mathrm{P}_{3 / 2}\right)$ was present-a conclusion that would be consistent with our previous findings for $\mathrm{Ne}^{+}$in $\mathrm{He}^{1}$, but somewhat contrary to comments made previously ${ }^{5}$ regarding populations of the spin-orbit states. We have also noted that the distinct fall in the mobility values as $E / n_{0} \rightarrow 0$ at 4.35 K is not in agreement with the theoretical predictions; this has been discussed previously, ${ }^{48}$ where quantum chemical effects were ruled out. Since constant injection energies were employed in the mobility measurements and the low $E / n_{0}$ values were determined by increases in the pressure, we have conjectured that the presence of dimers (and other clusters) at 4.35 K could be the cause of the anomalous experimental results. We have suggested that these results should be independently checked in experiments where both $E$ and $n_{0}$ are varied.

Finally, we note that the comments above regarding $\mathrm{Ar}^{+}-\mathrm{He}$ and $\mathrm{Kr}^{+}-\mathrm{He}$ suggest that the $\mathrm{Xe}^{+}-\mathrm{He}$ and $\mathrm{Rn}^{+}-\mathrm{He}$ potentials should also be reliable. In these cases, however, higher precision experimental results are required in order to test our potentials more definitively.

## Acknowledgements

The work of L. A.V. was supported by the National Science Foundation under Grant CHE-0718024. T. G. W. is grateful to the EPSRC for the provision of computing time under the auspices of the NSCCS. B.R.G thanks the EPSRC and University of

Nottingham for an EPSRC DTA studentship. Dr. Mark Law (Aberdeen) and Prof. Jeremy Hutson (Durham) are thanked for providing the MAL1 potentials.

Table 1: Spectroscopic Parameters for He-RG ${ }^{+}$- Non-Spin-Orbit Curves. Isotopes employed are: ${ }^{4} \mathrm{He},{ }^{40} \mathrm{Ar},{ }^{84} \mathrm{Kr}$, ${ }^{131} \mathrm{Xe}$ and ${ }^{222} \mathrm{Rn}$. Here, the symbols represent the usual spectroscopic quantities: $R_{\mathrm{e}}$ is the equlibrium bond length, $D_{\mathrm{e}}$ is the depth of the potential;, $D_{0}$ is the energy between the zero-point and the asymptote, $\omega_{\mathrm{e}}$ is the harmonic vibrational frequency, $\omega_{\mathrm{e}} \mathrm{X}_{\mathrm{e}}$ is the anharmonicity constant, $B_{0}$ is the equilibrium rotational constant of the $v=0$ level, $D_{0, J}$ is the first distortion constant for the $v=0$ level, $\sigma$ is the crossing point of the potential, when the potential $=0$, and $\Delta G_{1 / 2}$ and $\Delta G_{3 / 2}$ are the first two vibrational term values.

| System | State <br> [Ref] | $\sigma / \AA$ | $R_{\mathrm{e}} / \AA$ | $D_{\mathrm{e}} / \mathrm{cm}^{-1}$ | $D_{0} / \mathrm{cm}^{-1}$ | $\omega_{\mathrm{e}} / \mathrm{cm}^{-1}$ | $\omega_{\mathrm{e}} \mathrm{Xe}_{\mathrm{e}} / \mathrm{cm}^{-1}$ | $B_{0} / \mathrm{cm}^{-1}$ | $D_{0 J} / 10^{-5} \mathrm{~cm}^{-1}$ | $\Delta G_{1 / 2} / \mathrm{cm}^{-1}$ | $\Delta G_{3 / 2} / \mathrm{cm}^{-1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{He}-\mathrm{Ar}^{+2} \Sigma^{+}$ | This Work | 2.05 | 2.41 | 420.1 | 341.2 | 164.3 | 17.31 | 0.754 | 8.33 | 130.2 | 94.1 |
|  | $\mathrm{BBD}^{21 \mathrm{a}}$ |  | 2.45 | 331 |  | 155.7 | 32.44 |  | 7.50 | 97.86 | 50.30 |
|  | $\mathrm{GP}^{17}$ |  | 2.47 | 274 |  | 186 |  |  |  |  |  |
|  | $\mathrm{MNG}^{20}$ |  | 2.42 | 403 |  |  |  |  |  |  |  |
| ${ }^{2} \Pi$ | This Work | 2.52 | 3.00 | 142.0 | 102.0 | 75.6 | 10.79 | 0.468 | 9.22 | 55.6 | 29.3 |
|  | $\mathrm{GP}^{17}$ |  | 3.12 | 113 |  | 140 |  |  |  |  |  |
|  | $\mathrm{MNG}^{20}$ |  | 3.02 | 137 |  |  |  |  |  |  |  |
| $\mathrm{He}-\mathrm{Kr}^{+2} \Sigma^{+}$ | This Work | 2.30 | 2.68 | 301.7 | 240.0 | 127.9 | 14.57 | 0.578 | 6.28 | 99.4 | 68.4 |
| ${ }^{2} \Pi$ | This Work | 2.82 | 3.22 | 115.6 | 81.8 | 63.2 | 9.23 | 0.387 | 7.42 | 46.2 | 23.3 |
| $\mathrm{He}-\mathrm{Xe}^{+2} \Sigma^{+}$ | This Work | 2.63 | 3.03 | 203.6 | 156.9 | 95.6 | 11.97 | 0.441 | 5.07 | 72.4 | 46.1 |
| ${ }^{2} \Pi$ | This Work | 3.11 | 3.52 | 87.8 | 60.2 | 58.0 | 9.31 | 0.316 | 5.87 | 39.4 | 20.8 |
| $\mathrm{He}-\mathrm{Rn}^{+2} \Sigma^{+}$ | This Work | 2.73 | 3.15 | 184.0 | 141.0 | 87.7 | 11.15 | 0.406 | 4.71 | 66.2 | 41.6 |
| ${ }^{2} \Pi$ | This Work | 3.22 | 3.65 | 79.2 | 53.9 | 44.8 | 6.71 | 0.291 | 5.93 | 32.9 | 14.8 |

${ }^{\mathrm{a}} \operatorname{CCSD}(\mathrm{T}) / \mathrm{FC}$ values only cited from that work.

URL: http://mc.manuscriptcentral.com/tandf/tmph

Table 2: Spectroscopic Parameters for $\mathrm{He}-\mathrm{RG}^{+}$—Spin-Orbit Curves. Isotopes employed are: ${ }^{4} \mathrm{He},{ }^{40} \mathrm{Ar},{ }^{84} \mathrm{Kr},{ }^{131} \mathrm{Xe}$ and ${ }^{222} \mathrm{Rn}$. Quantities are defined in Table 1.

| System | Ref | $\sigma / \AA$ | $R_{\mathrm{e}} / \AA$ | $D_{\mathrm{e}} / \mathrm{cm}^{-1}$ | $D_{0} / \mathrm{cm}^{-1}$ | $\omega_{\mathrm{e}} / \mathrm{cm}^{-1}$ | $\begin{aligned} & \omega_{\mathrm{e}} \mathrm{X}_{\mathrm{X}} \mathrm{f} \\ & \mathrm{~cm}^{-1} \end{aligned}$ | $B_{0} / \mathrm{cm}^{-1}$ | $\begin{gathered} D_{0, /} / 10^{-5} \\ \mathrm{~cm}^{-1} \end{gathered}$ | $\begin{gathered} \Delta G_{1 / 2 /} / \\ \mathrm{cm}^{-1} \end{gathered}$ | $\begin{gathered} \Delta G_{3 / 2} / \\ \mathrm{cm}^{-1} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{{ }^{2} \Sigma_{1 / 2}{ }^{+}}{ }$ | This work | 2.18 | 2.60 | 271.1 | 215.5 | 117.6 | 13.60 | 0.643 | 10.4 | 90.9 | 62.3 |
|  | $\mathrm{DHY}^{4}$ |  | 2.585 | 262 |  | 120.3 | 13.60 | 0.659 |  |  |  |
|  | $\mathrm{Sis}^{11}$ | 2.19 | 2.564 | 282.6 |  |  |  |  |  |  |  |
|  | $\mathrm{Sta}^{16}$ |  | 2.66 | 242 | 190 | 116.9 | 17.7 |  |  | 83.9 | 55.2 |
|  | $\mathrm{GP}^{17}$ |  | 2.77 | 177 |  | 105 |  |  |  |  |  |
|  | $\mathrm{MNG}^{20}$ |  | 2.62 | 258 | ( |  |  |  |  |  |  |
|  | MAL1 ${ }^{6}$ |  | 2.573 | 281.6 |  |  |  | 0.651 |  | 93.2 | 64.1 |
|  | $\mathrm{HM}^{10}$ | 2.17 | 2.65 | 269.4 | 215.3 | , |  |  |  |  |  |
| ${ }^{2} \Pi_{3 / 2}$ | This Work | 2.52 | 3.00 | 142.0 | 102.0 | 75.6 | 10.79 | 0.468 | 9.22 | 55.6 | 29.3 |
|  | Sta ${ }^{16}$ |  | 3.03 | 133 | 94 | 89.3 | 21.2 |  |  | 52.9 | 26.9 |
|  | $\mathrm{GP}^{17}$ |  | 3.10 | 105 |  | 80 |  |  |  |  |  |
|  | MAL1 ${ }^{6}$ |  | 2.98 | 143.4 |  |  | ( |  |  |  |  |
|  | $\mathrm{HM}^{10}$ | 2.59 | 2.96 | 161.3 | 114.5 |  | , |  |  |  |  |
| ${ }^{2} \Pi_{1 / 2}$ | This Work | 2.52 | 2.88 | 178.1 | 131.7 | 91.6 | 12.58 | 0.512 | 8.33 | 68.0 | 38.2 |
|  | Sta ${ }^{16}$ |  | 2.91 | 166 | 121 | 102.2 | 21.7 |  |  | 64.0 | 35.1 |
|  | $\mathrm{GP}^{17}$ |  | 3.00 | 137 |  | 93 |  |  |  |  |  |
|  | $\mathrm{MNG}^{20}$ |  | 2.89 | 169 |  |  |  |  | 人 |  |  |
|  | DHY ${ }^{4}$ |  | 2.872 | 154 |  |  |  | 0.5146 |  | 69.2 |  |
|  | $\mathrm{Sis}^{11}$ | 2.48 | 2.872 | 185.4 |  |  |  |  |  |  |  |
|  | MAL1 ${ }^{6}$ |  | 2.863 | 181.6 |  |  |  | 0.5160 |  | 68.9 |  |
|  | $\mathrm{HM}^{10}$ | 2.51 | 2.86 | 192.8 | 140.3 |  |  |  |  |  |  |

URL: http://mc.manuscriptcentral.com/tandf/tmph

| $\begin{aligned} & {\mathrm{He}-\mathrm{Kr}^{+}}^{2}{ }^{2} \Sigma_{1 / 2}{ }^{+} \\ & \hline \end{aligned}$ | This Work | 2.51 | 2.90 | 199.8 | 152.4 | 96.0 | 12.33 | 0.487 | 6.75 | 72.3 | 44.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{HM}^{10}$ | 2.51 | 2.91 | 240.4 | 183.1 |  |  |  |  |  |  |
|  | MAL1 ${ }^{6}$ |  | 2.87 | 208 |  |  |  |  |  |  |  |
| ${ }^{2} \Pi_{3 / 2}$ | This Work | 2.82 | 3.22 | 115.6 | 81.8 | 63.2 | 9.23 | 0.387 | 7.42 | 46.2 | 23.3 |
|  | $\mathrm{HM}^{10}$ | 2.78 | 3.18 | 161.3 |  |  |  |  |  |  |  |
|  | MAL1 ${ }^{6}$ |  | 3.20 | 116 |  |  |  |  |  |  |  |
| ${ }^{2} \Pi_{1 / 2}$ | This Work | 2.70 | 3.09 | 145.9 | 106.4 | 76.4 | 10.68 | 0.424 | 6.87 | 56.4 | 30.9 |
|  | $\mathrm{HM}^{10}$ | 2.65 | 3.04 | 192.8 | 140.3 |  |  |  |  |  |  |
|  | MAL1 ${ }^{6}$ |  | 3.06 | 149 |  |  |  |  |  |  |  |
| $\begin{gathered} \mathrm{He}-\mathrm{Xe}^{+} \\ { }^{2} \Sigma_{1 / 2}{ }^{+} \end{gathered}$ | This Work | 2.83 | 3.24 | 143.5 | 106.0 | 75.8 | 10.59 | 0.383 | 5.45 | 55.1 | 32.5 |
|  | $\mathrm{HM}^{10}$ | 2.25 | 2.65 | 403.3 | 327.5 |  |  |  |  |  |  |
| ${ }^{2} \Pi_{3 / 2}$ | This Work | 3.11 | 3.52 | 87.8 | 60.1 | 58.0 | 9.31 | 0.316 | 5.87 | 39.4 | 20.8 |
|  | $\mathrm{HM}^{10}$ | 2.78 | 3.18 | 149.2 | 104.9 |  |  |  |  |  |  |
| ${ }^{2} \Pi_{1 / 2}$ | This Work | 2.98 | 3.40 | 109.3 | 77.5 | 62.6 | 9.38 | 0.343 | 5.71 | 45.1 | 22.7 |
|  | $\mathrm{HM}^{10}$ | 2.59 | 2.59 | 22.6 | 167.8 |  |  |  |  |  |  |
| $\begin{gathered} \mathrm{He}-\mathrm{Rn}^{+} \\ { }^{2} \Sigma_{1 / 2}{ }^{+} \end{gathered}$ | This Work | 2.93 | 3.35 | 129.2 | 94.7 | 67.9 | 9.46 | 0.352 | 5.08 | 50.0 | 27.9 |
| ${ }^{2} \Pi_{3 / 2}$ | This Work | 3.22 | 3.65 | 79.2 | 53.9 | 44.8 | 6.71 | 0.291 | 5.93 | 32.9 | 14.8 |
| ${ }^{2} \Pi_{1 / 2}$ | This Work | 3.09 | 3.52 | 98.6 | 69.5 | 55.0 | 8.11 | 0.316 | 5.45 | 40.0 | 19.9 |

URL: http://mc.manuscriptcentral.com/tandf/tmph

Table 3: Statistical Comparison between Calculated and Experimental Mobility Data for $\mathrm{Ar}^{+}$in He . See text for definitions of $\delta$ and $\chi$. $N$ is the number of data points, and \%Expt and \%Calc are the experimental and calculational precisions. When $N$ is small, over-interpretation of the results should be avoided.

| T/K | Data Source | State | $E / n_{0}$ <br> Td | $N$ | \%Expt | \%Calc | $\delta$ | $\chi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.35 | Ref. 39 | ${ }^{2} P_{3 / 2}$ | $2.0-11.5$ | 7 | 2.0 | 0.1 | 0.25 | 0.34 |
|  |  |  | $115-35.5$ | 6 | 2.0 | 0.1 | -1.23 | 1.39 |
|  |  | ${ }^{2} P_{1 / 2}$ | $2.0-11.4$ | 7 | 2.0 | 0.1 | -0.16 | 0.22 |
|  |  |  | $12.4-50.0$ | 6 | 2.0 | 1.0 | -0.79 | 1.33 |
| 82 | Ref. 40 | ${ }^{2} P_{3 / 2}$ | $5.0-32.0$ | 5 | 4.0 | 0.1 | 1.45 | 1.47 |
|  |  |  | $32.0-70.0$ | 4 | 4.0 | 0.5 | 0.79 | 1.05 |
|  |  | ${ }^{2} P_{1 / 2}$ | $5.0-32.0$ | 5 | 4.0 | 0.1 | 1.58 | 1.70 |
|  |  |  | $32.0-70.0$ | 4 | 4.0 | 0.5 | 2.30 | 2.38 |
| 92 | Ref. 39 | ${ }^{2} P_{3 / 2}$ | $1.0-15.0$ | 10 | 5.0 | 0.1 | 1.73 | 1.75 |
|  |  | ${ }^{2} P_{1 / 2}$ | $1.0-15.0$ | 10 | 5.0 | 0.1 | 1.62 | 1.65 |
| 170 | Ref. 39 | ${ }^{2} P_{3 / 2}$ | $4.0-30.0$ | 10 | 5.0 | 0.1 | 1.09 | 1.10 |
|  |  | ${ }^{2} P_{1 / 2}$ | $4.0-30.0$ | 10 | 5.0 | 0.1 | 1.29 | 1.35 |
| 293 | Ref. 39 | ${ }^{2} P_{3 / 2}$ | $6.0-40.0$ | 10 | 5.0 | 0.1 | -0.62 | 0.62 |
|  |  | ${ }^{2} P_{1 / 2}$ | $6.0-40.0$ | 10 | 5.0 | 0.1 | 0.14 | 0.33 |
| 300 | Ref. 41 | ${ }^{2} P_{3 / 2}$ | $5.0-50.0$ | 10 | 7.0 | 0.1 | -0.03 | 0.11 |
|  |  |  | $50.0-130.0$ | 8 | 7.0 | 0.1 | -0.57 | 0.62 |
|  |  | ${ }^{2} P_{1 / 2}$ | $5.0-50.0$ | 10 | 7.0 | 0.1 | 0.67 | 0.70 |
|  |  | ${ }^{2} P_{3 / 2}$ | $50.0-130.0$ | 8 | 7.0 | 0.5 | 0.58 | 0.61 |
| 428 | Ref. 39 | ${ }^{2} P_{1 / 2}$ | $80.0-50.0$ | 10 | 5.0 | 0.1 | -0.64 | 0.72 |
|  |  | ${ }^{2}$ | 5.0 | 0.1 | 0.48 | 0.75 |  |  |
| 552 | Ref. 39 | ${ }^{2} P_{3 / 2}$ | $10.0-50.0$ | 9 | 5.0 | 0.1 | -0.46 | 0.53 |
|  |  | ${ }^{2} P_{1 / 2}$ | $10.0-50.0$ | 9 | 5.0 | 0.1 | 0.88 | 0.97 |

Table 4: Statistical Comparison between Calculated and Experimental Mobility Data for ${ }^{84} \mathrm{Kr}^{+}$in He . See text for definitions of $\delta$ and $\chi . N$ is the number of data points, and \%Expt and \%Calc are the experimental and calculational precisions. When $N$ is small, over-interpretation of the results should be avoided.

| T/K | Data Source | State | $E / n_{0}$ <br> Td | N | \%Expt | \%Calc | $\delta$ | $\chi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4.35 | Ref. 39 | ${ }^{2} P_{3 / 2}$ | $2.0-11.5$ | 7 | 2.0 | 0.1 | 0.07 | 0.19 |
|  |  |  | $11.5-35.5$ | 8 | 2.0 | 0.1 | -0.34 | 0.54 |
|  |  | ${ }^{2} P_{1 / 2}$ | $2.0-11.5$ | 7 | 2.0 | 0.1 | 0.23 | 0.28 |
|  |  |  | $11.5-50.0$ | 6 | 2.0 | 0.3 | -0.27 | 0.63 |
| 82 | Ref. 39 | ${ }^{2} P_{3 / 2}$ | $4.0-32.0$ | 10 | 4.0 | 0.1 | 0.64 | 0.65 |
|  |  |  | $32.0-70.0$ | 4 | 4.0 | 0.1 | 0.46 | 0.47 |
|  |  | ${ }^{2} P_{1 / 2}$ | $4.0-32.0$ | 10 | 4.0 | 0.1 | 0.54 | 0.57 |
|  |  |  | $32.0-70.0$ | 4 | 4.0 | 0.1 | 0.82 | 0.82 |
| 294 | Ref. 40 | ${ }^{2} P_{3 / 2}$ | $3.0-45.0$ | 5 | 4.0 | 0.1 | -0.79 | 0.87 |
|  |  |  | $45.0-250.0$ | 5 | 4.0 | 0.1 | -0.89 | 0.91 |
|  |  | ${ }^{2} P_{1 / 2}$ | $3.0-45.0$ | 5 | 4.0 | 0.1 | -0.66 | 0.71 |
|  |  |  | $45.0-250.0$ | 5 | 4.0 | 0.1 | 0.02 | 0.53 |
| 300 | Ref. 41 | ${ }^{2} P_{3 / 2}$ | $5.0-35.0$ | 9 | 7.0 | 0.1 | -1.87 | 1.93 |
|  |  |  | $35.0-100.0$ | 5 | 7.0 | 0.1 | -1.73 | 1.87 |
|  |  | ${ }^{2} P_{1 / 2}$ | $5.0-35.0$ | 9 | 7.0 | 0.1 | -0.49 | 0.53 |
|  |  |  | $35.0-100.0$ | 5 | 7.0 | 0.1 | -0.09 | 0.25 |

Table 5: Statistical Comparison between Calculated and Experimental Mobility Data for ${ }^{132} \mathrm{Xe}^{+}$in He. See text for definitions of $\delta$ and $\chi . N$ is the number of data points, and \%Expt and \%Calc are the experimental and calculational precisions. When $N$ is small, over-interpretation of the results should be avoided.

| T/K | Data Source | State | $E / n_{0}$ <br> Td | N | \%Expt | \%Calc | $\delta$ | $\chi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 82 | Ref. 39 | ${ }^{2} P_{3 / 2}$ | $17.0-32.0$ | 3 | 4.0 | 0.1 | 0.55 | 0.56 |
|  |  |  | $32.0-70.0$ | 5 | 4.0 | 0.1 | 0.20 | 0.43 |
|  |  | ${ }^{2} P_{1 / 2}$ | $17.0-32.0$ | 3 | 4.0 | 0.1 | 0.41 | 0.44 |
|  |  |  | $32.0-70.0$ | 4 | 4.0 | 0.1 | 0.33 | 0.46 |
| 295 | Ref. 55 | ${ }^{2} P_{3 / 2}$ | $3.0-32.0$ | 11 | 5.0 | 0.1 | -1.24 | 1.28 |
|  |  |  | $32.0-80.0$ | 4 | 5.0 | 0.1 | 0.54 | 1.04 |
|  |  | ${ }^{2} P_{1 / 2}$ | $3.0-32.0$ | 11 | 5.0 | 0.1 | -1.24 | 1.30 |
|  |  |  | $32.0-80.0$ | 4 | 5.0 | 0.1 | -0.72 | 1.27 |

## Figure Captions

## Figure 1

Calculated potential energy curves for $\mathrm{Ar}^{+}-\mathrm{He}$. The solid black curves represent the present potential, calculated at the RCCSD(T)/d-aug-cc-pV5Z level of theory, while the red dashed ones represent the MAL1 potentials from ref. 6.

## Figure 2

Calculated interaction energy curves for $\mathrm{Kr}^{+}-\mathrm{He}$. The black curves are the present ones, calculated at the RCCSD(T)/d-aug-cc-pV5Z level of theory - see text for details. The red dashed ones are the MAL1 potentials from ref. 24.

## Figure 3

Standard mobilities for $\mathrm{Ar}^{+}$in He at 300 K . The points, with error bars are the experimental (smoothed) data points from refs. 44, 45, 46. The solid line is the calculated mobility curve assuming the ${ }^{2} P_{3 / 2}$ state only is present; the dashed line is the calculated mobility curve assuming the ${ }^{2} P_{1 / 2}$ state only is present. The ordinate is the standard mobility in $\mathrm{cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$, and the abscissa is the ratio of the electric field to the number density, in units of Townsend (Td).

## Figure 4

Standard mobilities for $\mathrm{Ar}^{+}$in He at 4.35 K . The points, with error bars are the experimental data points from ref. 5 . The solid line is the calculated mobility curve assuming the ${ }^{2} P_{3 / 2}$ state only is present; the dashed line is the calculated mobility curve assuming the ${ }^{2} P_{1 / 2}$ state only is present. The ordinate is the standard mobility in $\mathrm{cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$, and the abscissa is the ratio of the electric field to the number density, in units of Townsend (Td).

## Figure 5

Standard mobilities for $\mathrm{Kr}^{+}$in He at 300 K . The points, with error bars are the experimental (smoothed) data points from refs. 44, 45, 46. The solid line is the calculated mobility curve assuming the ${ }^{2} P_{3 / 2}$ state only is present; the dashed line is the calculated mobility curve assuming the ${ }^{2} P_{1 / 2}$ state only is present. The ordinate is the standard mobility in $\mathrm{cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$, and the abscissa is the ratio of the electric field to the number density, in units of Townsend (Td).

## Figure 6

Standard mobilities for $\mathrm{Kr}^{+}$in He at 4.35 K . The points, with error bars are the experimental data points from ref. 5. The solid line is the calculated mobility curve assuming the ${ }^{2} P_{3 / 2}$ state only is present; the dashed line is the calculated mobility curve assuming the ${ }^{2} P_{1 / 2}$ state only is present. The ordinate is the standard mobility in
$\mathrm{cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$, and the abscissa is the ratio of the electric field to the number density, in units of townsend (Td).

## Figure 7

Standard mobilities for $\mathrm{Xe}^{+}$in He at 300 K . The points, with error bars are the experimental (smoothed) data points from refs. 44, 45, 46. The solid line is the calculated mobility curve assuming the ${ }^{2} P_{3 / 2}$ state only is present; the dashed line is the calculated mobility curve assuming the ${ }^{2} P_{1 / 2}$ state only is present. The ordinate is the standard mobility in $\mathrm{cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$, and the abscissa is the ratio of the electric field to the number density, in units of townsend (Td).

Figure 1


Figure 2


Figure 3


Figure 4


Figure 5


Figure 6


Figure 7


## References

${ }^{1}$ T. G. Wright, B. R. Gray, L. A. Viehland and R. J. Johnsen, J. Chem. Phys. 129, 184307 (2008).
${ }^{2}$ I. Dabrowski and G. Herzberg, J. Molec. Spectrosc. 73, 183 (1978).
${ }^{3}$ Y. Tanaka, K. Yoshino and D. E. Freeman, J. Chem. Phys. 62, 4484 (1975).
${ }^{4}$ I. Dabrowski, G. Herzberg and K. Yoshino, J. Molec. Spectrosc.. 89, 491 (1981).
${ }^{5}$ N. Saito, T. M. Kojima, N. Kobayashi and Y. Kaneko, J. Chem. Phys. 100, 5726 (1994).
${ }^{6}$ A. Carrington, C. A. Leach, A. J. Marr, A. M. Shaw, M. R. Viant, J. M. Hutson and M. M. Law, J. Chem. Phys. 102, 2379 (1995).
${ }^{7}$ R. Albat and B. Wirsam, J. Phys. B. At. Molec. Phys. 10, 81 (1977).
${ }^{8}$ V. Sidris, J. C. Brenot, J. Pommier, M. Barat, O. Bernardini, D. C. Lorents and F. T. Smith, J. Phys. B. At. Molec. Phys. 10, 2431 (1977).
${ }^{9}$ R. E. Olson and B. Liu, Chem. Phys. Lett., 56, 537 (1978).
${ }^{10}$ D. Hausamann and H. Morgner, Mol. Phys. 54, 1085 (1985).
${ }^{11}$ P. E. Siska, J. Chem. Phys. 85, 7497 (1986).
${ }^{12}$ D. Stärk and S. D. Peyerimhoff, Molec. Phys. 59, 1241 (1986).
${ }^{13}$ B. Brunetti, F. Vecchiocattivi, A. Aguilar-Navarro and A. Solé, Chem. Phys. Lett. 126, 245 (1986).
${ }^{14}$ K. Balasubramanian, M. Z. Liao and S. H. Lin, Chem. Phys. Lett. 138, 49 (1987).
${ }^{15}$ M. Z. Liao, K. Balasubramanian, D. Chapman and S. H. Lin, Chem. Phys. 111, 423 (1987).
${ }^{16}$ V. Staemmler, Z. Phys. D 16, 167 (1990).
${ }^{17}$ B. Gemein and S. D. Peyerimhoff, Chem. Phys. Lett. 173, 7 (1990).
${ }^{18}$ L. A. Viehland, A. A. Viggiano and E. A. Mason, J. Chem. Phys. 95, 7286 (1991).
${ }^{19}$ N. L. Ma, W.-K. Li and C. Y. Ng, J. Chem. Phys. 99, 3617 (1993).
${ }^{20}$ J. N. Murrell, F. Y. Naumkin and C. R. Griffiths, Molec. Phys. 99, 115 (2001).
${ }^{21}$ N. C. Bera, I. Bhattacharyya and A. K. Das, Int. J. Quant. Chem. 107, 1067 (2006).
${ }^{22}$ H.-P. Weise, Ber. Bunsenges. Phys. Chem. 77, 579 (1973).
${ }^{23}$ K. Balasubramanian, M. Z. Liao and S. H. Lin, Chem. Phys. Lett. 138, 49 (1987).
${ }^{24}$ A. Carrington, C. H. Pyne, A. M. Shaw, S. M. Taylor, J. M. Hutson and M. M. Law, J. Chem. Phys. 105, 8602 (1996).
${ }^{25}$ T. Koizumi, N. Kobayashi and Y. Kaneko, J. Phys. Soc. Jpn, 48, 1678 (1980).
${ }^{26}$ W. Lindinger and D. L. Albritton, J. Chem. Phys. 62, 3517 (1975).
${ }^{27}$ M. J. Hogan, J. Chem. Phys. 125, 164325 (2006)
${ }^{28}$ T. Koizumi, K. Okuno, N. Kobayashi and Y. Kaneko, J. Phys. Soc. Jpn. 51, 2650 (1982).
${ }^{29}$ T. T. C. Jones, J. D. C. Jones, K. Birkinshaw and N. D. Twiddy, Chem. Phys. Lett. 86, 503 (1982).
${ }^{30}$ R. Johnsen, M. T. Leu and M. A. Biondi, Phys. Rev. A 8, 2557 (1973).
${ }^{31}$ Y. Kaneko, T. Koizumi and N. Kobayashi, Mass Spectrom. (Jpn) 26, 35 (1978).
${ }^{32}$ K.A. Peterson, D. Figgen, E. Goll, H. Stoll and M. Dolg, J. Chem. Phys. 119, 11113 (2003).
${ }^{33}$ MOLPRO is a package of ab initio programs written by H.-J. Werner, P. J. Knowles and others.
${ }^{34}$ See, for example, V. Aquilanti, D. Cappelletti, V. Lorent, E. Luzzatti, and F. Piranti, J. Phys. Chem. 97, 2063 (1993), and references therein.
${ }^{35}$ NIST-JANAF Thermochemical Tables, Ed. M. W. Chase, Jr. (AIP, New York, 1998).
${ }^{36}$ R. J. LeRoy, Level 7.2 - A computer program for solving the radial Schrödinger equation for bound and quasibound levels, and calculating various values and matrix elements. University of Waterloo Chemical Physics Research Program Report CP555R, 2000.
${ }^{37}$ L. A. Viehland, Chem. Phys. 70, 149 (1982).
${ }^{38}$ L. A. Viehland, Chem. Phys. 85, 291 (1984).
${ }^{39}$ A. Yousef, S. Shrestha, L. A. Viehland, E. P. F. Lee, B. R. Gray, V. L. Ayles, T. G. Wright and W. H. Breckenridge, J. Chem Phys. 127, 154309 (2007).
${ }^{40}$ L. A. Viehland, Chem. Phys. 179, 71 (1994).
${ }^{41}$ To access this database you must telnet to the computer named sassafrass.chatham.edu and logon as gastrans. The required password will be provided upon request by email to viehland@chatham.edu.
${ }^{42}$ M. J. Frisch, G. W. Trucks, H. B. Schlegel, G. E. Scuseria, M. A. Robb, J. R.
Cheeseman, J. A. Montgomery, Jr., T. Vreven, K. N. Kudin, J. C. Burant, J. M.
Millam, S. S. Iyengar, J. Tomasi, V. Barone, B. Mennucci, M. Cossi, G. Scalmani, N.
Rega, G. A. Petersson, H. Nakatsuji, M. Hada, M. Ehara, K. Toyota, R. Fukuda, J.
Hasegawa, M. Ishida, T. Nakajima, Y. Honda, O. Kitao, H. Nakai, M. Klene, X. Li, J.
E. Knox, H. P. Hratchian, J. B. Cross, V. Bakken, C. Adamo, J. Jaramillo, R.

Gomperts, R. E. Stratmann, O. Yazyev, A. J. Austin, R. Cammi, C. Pomelli, J.
Ochterski, P. Y. Ayala, K. Morokuma, G. A. Voth, P. Salvador, J. J. Dannenberg, V.
G. Zakrzewski, S. Dapprich, A. D. Daniels, M. C. Strain, O. Farkas, D. K. Malick, A.
D. Rabuck, K. Raghavachari, J. B. Foresman, J. V. Ortiz, Q. Cui, A. G. Baboul, S.

Clifford, J. Cioslowski, B. B. Stefanov, G. Liu, A. Liashenko, P. Piskorz, I.
Komaromi, R. L. Martin, D. J. Fox, T. Keith, M. A. Al-Laham, C. Y. Peng, A.
Nanayakkara, M. Challacombe, P. M. W. Gill, B. G. Johnson, W. Chen, M. W.
Wong, C. Gonzalez and J. A. Pople, GAUSSIAN 03 (Revision B.03), Gaussian, Inc., Wallingford, CT, 2003.
${ }^{43}$ T. G. Wright, J. Chem. Phys. 105, 7579 (1996).
${ }^{44}$ L. A. Viehland and E. A. Mason, At. Data Nucl. Tables 60, 37 (1995).
${ }^{45}$ H. W. Ellis, M. G. Thackston, E. W. McDaniel and E. A. Mason, At. Data Nucl. Data Tables 31, 113 (1984).
${ }^{46}$ H. W. Ellis, R. Y. Pai, E. W. McDaniel, E. A. Mason and L. A. Viehland, At. Data Nucl. Data Tables 17, 177 (1976).
${ }^{47}$ L. A. Viehland and E. A. Mason, J. Chem. Phys. 99, 1457 (1993).
${ }^{48}$ L. A. Viehland and J. J. Hurly, J. Chem. Phys. 105, 11143 (1996).
${ }^{49}$ T. M. Kojima, N. Saito, N. Kobayashi and Y. Kaneko, J. Phys. Soc. Jpn. 61, 6 (1992).
${ }^{50}$ A. S. Dickinson, J. Phys. B 1, 4687 (1968).
${ }^{51}$ G. Heiche and E. A. Mason, J. Chem. Phys. 53, 387 (1970).
${ }^{52}$ J. Sanderson, H. Tanuma, N. Kobayashi and Y. Kaneko J. Phys. B 26, L465 (1993).
${ }^{53}$ S. Jinno, H. Hidaka, H. Tanuma and N. Kobayashi, XXII ICPEAC, Santa Fe, NM USA (2001).
${ }^{54}$ J. Sanderson, H. Tanuma, N. Kobayashi and Y. Kaneko, J. Phys. B 27, L433 (1994).
${ }^{55}$ H. Hidaka, S. Jinno, H. Tanuma and N. Kobayashi, J. Phys. B 36, 1515 (2003).
${ }^{56}$ H. Hidaka, PhD Thesis, Tokyo Metropolitan University (2003).
${ }^{57}$ K. Ohtsuki, M. Hananoe and M. Matsuzawa, Phys. Rev. Lett. 95, 213201 (2005).
${ }^{58}$ J. Sanderson, H. Tanuma, N. Kobayashi and Y. Kaneko, J. Chem. Phys. 103, 7098 (1995).
${ }^{59}$ H. W. Ellis, E. W. McDaniel, D. L. Albritton, L. A. Viehland, S. L. Lin and E. A. Mason, At. Data Nucl. Data Tables 22, 179 (1978).
${ }^{60}$ D. M. Danailov, L. A. Viehland, R. Johnsen, T. G. Wright, and E. P. F. Lee, J. Chem. Phys. 127, 084303 (2007).

The Supplementary Material consists of an Excel spreadsheet with four worksheets. There is one sheet for each of $\mathrm{Ar}^{+}-\mathrm{He}, \mathrm{Kr}^{+}-\mathrm{He}, \mathrm{Xe}^{+}-\mathrm{He}$ and $\mathrm{Rn}^{+}-\mathrm{He}$. In each sheet, the non-spin-orbit curves for the ${ }^{2} \Pi$ and ${ }^{2} \Sigma^{+}$states are given, as well as the three spin orbit potentials (see text). Note the units given at the tops of the columns!

| R (Bohr) | SIG | (Hartr | GMA | (cm-1) | R (Bohr) | X (cm-1) | V1 (cm-1) | $\mathrm{V} 2(\mathrm{~cm}-1)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.5 | 0.971199 |  |  |  | 1.5 |  |  |  |
| 1.6 | 0.773494 |  |  |  | . 6 |  |  |  |
| 1.7 | 0.620759 | 1.04432 | 136240.7 | 229201.7 | 1.7 | 7136713 | 229201.7 | 230160.8 |
| 1.8 | 0.500106 | 0.879911 | 109760.6 | 193118.2 | 1.8 | 8110232.3 | 193118.2 | 194077.9 |
| 1.9 | 0.403194 | 0.759758 | 88490.76 | 166747.6 | 1.9 | 988962.11 | 166747.6 | 167707.7 |
| 2 | 0.324521 | 0.645709 | 71224.14 | 141716.7 |  | 271694.86 | 141716.7 | 142677.4 |
| 2.1 | 0.260337 | 0.546381 | 57137.38 | 119916.7 | 2.1 | 157607.32 | 119916.7 | 120878.2 |
| 2.2 | 0.207943 | 0.460252 | 45638.31 | 101013.7 | 2.2 | 246107.3 | 101013.7 | 101976.1 |
| 2.3 | 0.165275 | 0.385949 | 36273.63 | 84705.97 | 2.3 | 36741.46 | 84705.97 | 85669.55 |
| 2.4 | 0.130665 | 0.322222 | 28677.68 | 70719.46 | 2.4 | 29144.11 | 70719.46 | 71684.44 |
| 2.5 | 0.102725 | 0.267891 | 22545.54 | 58795.17 | 2.5 | 23010.29 | 58795.17 | 59761.83 |
| 2.6 | 0.080281 | 0.221833 | 17619.63 | 48686.74 | 2.6 | 18082.34 | 48686.74 | 49655.44 |
| 2.7 | 0.062339 | 0.182994 | 13681.83 | 40162.48 | 2.7 | 14142.09 | 40162.48 | 41133.64 |
| 2.8 | 0.048062 | 0.150397 | 10548.48 | 33008.36 | 2.8 | 11005.78 | 33008.36 | 33982.47 |
| 2.9 | 0.036752 | 0.123159 | 8066.125 | 27030.19 | 2.9 | 8519.87 | 27030.19 | 28007.85 |
| 3 | 0.027828 | 0.100488 | 6107.604 | 22054.57 |  | 36557.064 | 22054.57 | 23036.52 |
| 3.1 | 0.020816 | 0.081689 | 4568.529 | 17928.69 | 3.1 | 5012.838 | 17928.69 | 18915.79 |
| 3.2 | 0.015327 | 0.066155 | 3363.89 | 14519.31 | 3.2 | 3802.016 | 14519.31 | 15512.6 |
| 3.3 | 0.011049 | 0.053361 | 2424.964 | 11711.4 | 3.3 | 2855.687 | 11711.4 | 12712.08 |
| 3.4 | 0.00773 | 0.042858 | 1696.532 | 9406.266 | 3.4 | 2118.426 | 9406.266 | 10415.78 |
| 3.5 | 0.005169 | 0.034263 | 1134.456 | 7519.894 | 3.5 | 1545.874 | 7519.894 | 8539.886 |
| 3.6 | 0.003206 | 0.027252 | 703.594 | 5981.096 | 3.6 | 61102.668 | 5981.096 | 7013.433 |
| 3.7 | 0.001713 | 0.021551 | 376.0369 | 4729.858 | 3.7 | 760.6937 | 4729.858 | 5776.611 |
| 3.8 | 0.000591 | 0.01693 | 129.6568 | 3715.798 | - 3.8 | 497.6672 | 3715.798 | 4779.197 |
| 3.9 | -0.000242 | 0.013199 | -53.07774 | 2896.791 | 3.9 | 9295.9834 | 2896.791 | 3979.14 |
| 4 | -0.000848 | 0.010196 | -186.0574 | 2237.748 |  | 4141.805 | 2237.748 | 3341.296 |
| 4.1 | -0.001277 | 0.007789 | -280.2779 | 1709.536 | 4.1 | 24.35397 | 1709.536 | 2836.314 |
| 4.2 | -0.001569 | 0.005869 | -344.461 | 1288.015 | 4.2 | $2-64.6902$ | 1288.015 | 2439.655 |
| 4.3 | -0.001757 | 0.004343 | -385.516 | 953.2551 | 4.3 | 3-131.6633 | 953.2551 | 2130.812 |
| 4.4 | -0.001863 | 0.003138 | -408.912 | 688.8058 | 4.4 | -181.3439 | 688.8058 | 1892.648 |
| 4.5 | -0.001909 | 0.002192 | -418.9749 | 481.152 | 4.5 | 5-217.3291 | 481.152 | 1710.916 |
| 4.6 | -0.00191 | 0.001454 | -419.1175 | 319.2061 | 4.6 | $6-242.3557$ | 319.2061 | 1573.854 |
| 4.7 | -0.001877 | 0.000884 | -412.0219 | 193.9102 | . 7 | 7 -258.555 | 193.9102 | 1471.853 |
| 4.8 | -0.001822 | 0.000446 | -399.7928 | 97.87471 | 4.8 | 8-267.6479 | 97.87471 | 1397.14 |
| 4.9 | -0.00175 | 0.000114 | -384.0521 | 25.1057 | 4.9 | $9-271.0465$ | 25.1057 | 1343.51 |
| 5 | -0.001668 | -0.000133 | -366.0464 | -29.24938 |  | 5 -269.9408 | -29.24938 | 1306.055 |
| 5.1 | -0.00158 | -0.000315 | -346.7611 | -69.10378 | 5.1 | $1-265.3687$ | -69.10378 | 1280.914 |
| 5.2 | -0.001489 | -0.000445 | -326.9009 | -97.60037 | 5.2 | 2 -258.1769 | -97.60037 | 1265.086 |
| 5.3 | -0.001399 | -0.000534 | -307.0055 | -117.2587 | 5.3 | $3-249.0905$ | -117.2587 | 1256.236 |
| 5.4 | -0.00131 | -0.000593 | -287.4591 | -130.0848 | 5.4 | 4-238.7008 | -130.0848 | 1252.567 |
| 5.5 | -0.001224 | -0.000627 | -268.536 | -137.672 | 5.5 | 5 -227.4902 | -137.672 | 1252.692 |
| 5.6 | -0.001141 | -0.000644 | -250.4227 | -141.2846 | 5.6 | $6-215.8445$ | -141.2846 | 1255.547 |
| 5.7 | -0.001063 | -0.000647 | -233.2335 | -141.9189 | 5.7 | 7 -204.0617 | -141.9189 | 1260.319 |
| 5.8 | -0.000989 | -0.00064 | -217.0341 | -140.354 | 5.8 | 8-192.3703 | -140.354 | 1266.392 |
| 5.9 | -0.00092 | -0.000625 | -201.8508 | -137.2068 | 5.9 | 9-180.9417 | -137.2068 | 1273.294 |
| 6 | -0.000855 | -0.000606 | -187.6793 | -132.9511 |  | 6 -169.8956 | -132.9511 | 1280.675 |
| 6.1 | -0.000795 | -0.000583 | -174.4933 | -127.9559 | 6.1 | 1-159.3134 | -127.9559 | 1288.274 |
| 6.2 | -0.000739 | -0.000558 | -162.2598 | -122.502 | 6.2 | 2-149.2503 | -122.502 | 1295.899 |
| 6.3 | -0.000688 | -0.000532 | -150.9327 | -116.8066 | 6.3 | 3-139.7367 | -116.8066 | 1303.407 |
| 6.4 | -0.00064 | -0.000506 | -140.4638 | -111.0256 | 6.4 | - 4 -130.7847 | -111.0256 | 1310.705 |
| 6.5 | -0.000596 | -0.00048 | -130.7937 | -105.2798 | 6.5 | 5-122.3895 | -105.2798 | 1317.726 |

$6.6-0.000555-0.000454-121.8721-99.65246$
$6.7-0.000518-0.000429-113.6396-94.2029$
$6.8-0.000483-0.000405-106.0633-88.97063$
$6.9-0.000451-0.000383-99.05987-83.97977$
$7-0.000422-0.000361-92.60732-79.24132$
$\begin{array}{llllll}7.1 & -0.000395 & -0.000341 & -86.65297 & -74.75964\end{array}$
$\begin{array}{lllllll}7.2 & -0.00037 & -0.000321 & -81.15513 & -70.53256\end{array}$
$7.3-0.000347-0.000303-76.07649-66.55349$
$7.4-0.000325-0.000286-71.38632-62.81583$
$7.5-0.000305-0.00027-67.04511-59.30643$
$\begin{array}{llllll}7.6 & -0.000287 & -0.000255 & -63.03311 & -56.01651\end{array}$
$\begin{array}{llllll}7.7 & -0.00027 & -0.000241 & -59.31082 & -52.93509\end{array}$
$7.8-0.000255-0.000228-55.86727-50.0468$
$\begin{array}{llllll}7.9 & -0.00024 & -0.000216 & -52.66733 & -47.34068\end{array}$
$8-0.000226-0.000204-49.69784-44.80794$
$8.1-0.000214-0.000193-46.93684-42.43761$
$8.2-0.000202-0.000183-44.3668-40.21653$
$8.3-0.000191-0.000174-41.97233-38.13811$
$8.5-0.000172-0.000157-37.65965-34.36095$
$8.6-0.000163-0.000149-35.71291-32.64905$
$8.7-0.000154-0.000141-33.89347-31.0403$
$8.9-0.000139-0.000128-30.58868-28.11199$
$9-0.000133-0.000122-29.09874-26.77769$
$9.1-0.000126-0.000116-27.69588 \quad-25.5221$
$9.3-0.000115-0.000106-25.13737-23.22511$
$9.4-0.000109-0.000101-23.97063-22.17405$
$9.5-0.000104-9.65 \mathrm{E}-05-22.87042-21.18226$
9.7 -9.5E-05 -8.82E-05 -20.85945 -19.36094
9.8 -9.08E-05 -8.44E-05 -19.93276 -18.52451
$9.9-8.69 \mathrm{E}-05-8.08 \mathrm{E}-05-19.06834-17.73361$
$10.1-7.95 \mathrm{E}-05 \quad-7.42 \mathrm{E}-05-17.45437-16.27675$
10.2 -7.62E-05 -7.11E-05 -16.72981-15.60581
$10.3 \quad-7.3 \mathrm{E}-05 \quad-6.82 \mathrm{E}-05-16.02108-14.97019$
10.6 -6.45E-05 -6.04E-05 -14.15249-13.25399

11 -5.49E-05 -5.17E-05 -12.05844 -11.34338
$13-2.7 \mathrm{E}-05-2.59 \mathrm{E}-05-5.922238-5.688253$
$15-1.48 \mathrm{E}-05-1.44 \mathrm{E}-05-3.243847-3.154161$
17 -8.86E-06 -8.6E-06 -1.945034 -1.888348
$20-4.57 \mathrm{E}-06-4.44 \mathrm{E}-06-1.003242-0.975325$
$25-1.84 \mathrm{E}-06-1.81 \mathrm{E}-06-0.403401-0.397678$
$30-8.65 \mathrm{E}-07-8.79 \mathrm{E}-07-0.189884-0.192901$
$40 \quad-3.3 \mathrm{E}-07 \quad-2.89 \mathrm{E}-07-0.072404-0.063373$

| 6.6 | -114.5418 | -99.65246 | 1324.427 |
| ---: | ---: | ---: | ---: |
| 6.7 | -107.2191 | -94.2029 | 1330.787 |
| 6.8 | -100.4109 | -88.97063 | 1336.787 |
| 6.9 | -94.06835 | -83.97977 | 1342.439 |
| 7 | -88.17963 | -79.24132 | 1347.741 |
| 7.1 | -82.71043 | -74.75964 | 1352.708 |
| 7.2 | -77.63175 | -70.53256 | 1357.354 |
| 7.3 | -72.9162 | -66.55349 | 1361.696 |
| 7.4 | -68.54087 | -62.81583 | 1365.749 |
| 7.5 | -64.47483 | -59.30643 | 1369.533 |
| 7.6 | -60.70188 | -56.01651 | 1373.062 |
| 7.7 | -57.19188 | -52.93509 | 1376.356 |
| 7.8 | -53.93236 | -50.0468 | 1379.428 |
| 7.9 | -50.89618 | -47.34068 | 1382.298 |
| 8 | -48.07158 | -44.80794 | 1384.976 |
| 8.1 | -45.44024 | -42.43761 | 1387.476 |
| 8.2 | -42.98605 | -40.21653 | 1389.813 |
| 8.3 | -40.69653 | -38.13811 | 1391.996 |
| 8.5 | -36.56177 | -34.36095 | 1395.951 |
| 8.6 | -34.69308 | -32.64905 | 1397.741 |
| 8.7 | -32.94367 | -31.0403 | 1399.42 |
| 8.9 | -29.76407 | -28.11199 | 1402.473 |
| 9 | -28.3259 | -26.77769 | 1403.859 |
| 9.1 | -26.97202 | -25.5221 | 1405.164 |
| 9.3 | -24.50052 | -23.22511 | 1407.548 |
| 9.4 | -23.37227 | -22.17405 | 1408.638 |
| 9.5 | -22.30814 | -21.18226 | 1409.665 |
| 9.7 | -20.3603 | -19.36094 | 1411.55 |
| 9.8 | -19.46365 | -18.52451 | 1412.416 |
| 9.9 | -18.62371 | -17.73361 | 1413.232 |
| 10.1 | -17.06205 | -16.27675 | 1414.741 |
| 10.2 | -16.35534 | -15.60581 | 1415.43 |
| 10.3 | -15.67095 | -14.97019 | 1416.09 |
| 10.6 | -13.85312 | -13.25399 | 1417.857 |
| 11 | -11.82016 | -11.34338 | 1419.828 |
| 13 | -5.844251 | -5.688253 | 1425.644 |
| 15 | -3.213953 | -3.154161 | 1428.226 |
| 17 | -1.926139 | -1.888348 | 1429.503 |
| 20 | -0.993936 | -0.975325 | 1430.425 |
| 25 | -0.401494 | -0.397678 | 1431.01 |
| 30 | -0.190889 | -0.192901 | 1431.218 |
| 40 | -0.069393 | -0.063373 | 1431.344 |


| R (Bohr) | S | PI (Hartree | SIGMA (cr | -1) | R (Bohr) | X (cm-1) | V1 (cm-1) | 1) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.5 | 1.53087 |  |  |  | 1.5 |  |  |  |
| 1.6 | 1.206861 |  |  |  | 1.6 |  |  |  |
| 1.7 | 0.95986 |  |  |  | . 7 |  |  |  |
| 1.8 | 0.768867 | 0.984674 | 168746.9 | 216111 | 1.8 | 8170407.1 | 216111 | 219821.8 |
| 1.9 | 0.619182 |  |  |  | 1.9 |  |  |  |
| 2 | 0.500441 | 0.824559 | 109834.1 | 180969.8 |  | 2111536.6 | 180969.8 | 184638.2 |
| 2.1 | 0.405279 | 0.700283 | 88948.51 | 153694.3 | 2.1 | 190642.64 | 153694.3 | 157371.2 |
| 2.2 | 0.328413 | 0.594032 | 72078.34 | 130375 | 2.2 | 273762.18 | 130375 | 134062.1 |
| 2.3 | 0.265989 | 0.503048 | 58377.88 | 110406.2 | 2.3 | 360049.36 | 110406.2 | 114105.8 |
| 2.4 | 0.215135 | 0.425042 | 47216.68 | 93285.89 | 2.4 | 448873.44 | 93285.89 | 97000.12 |
| 2.5 | 0.173653 | 0.358196 | 38112.37 | 78614.97 | 2.5 | 539751.67 | 78614.97 | 82346.68 |
| 2.6 | 0.139818 | 0.301016 | 30686.51 | 66065.28 | 2.6 | 632305.17 | 66065.28 | 69817.63 |
| 2.7 | 0.112249 | 0.252227 | 24635.72 | 55357.41 | 2.7 | 726230.05 | 55357.41 | 59134.08 |
| 2.8 | 0.08982 | 0.210721 | 19713.16 | 46247.97 | 2.8 | 821278.95 | 46247.97 | 50053.18 |
| 2.9 | 0.071608 | 0.175521 | 15716.03 | 38522.48 | 2.9 | 917248.44 | 38522.48 | 42361.07 |
| 3 | 0.05685 | 0.145763 | 12477.03 | 31991.28 |  | 13970.59 | 31991.28 | 35868.71 |
| 3.1 | 0.044916 | 0.120683 | 9857.914 | 26486.86 | 3.1 | 11306.55 | 26486.86 | 30409.22 |
| 3.2 | 0.035288 | 0.09961 | 7744.711 | 21861.95 | 3.2 | 29141.777 | 21861.95 | 25835.88 |
| 3.3 | 0.027536 | 0.081957 | 6043.543 | 17987.54 | 3.3 | 37381.988 | 17987.54 | 22020.09 |
| 3.4 | 0.021312 | 0.067212 | 4677.384 | 14751.37 | . | 5949.985 | 14751.37 | 18849.77 |
| 3.5 | 0.016326 | 0.054931 | 3583.123 | 12056.02 | 3.5 | 54782.829 | 12056.02 | 16227.32 |
| 3.6 | 0.012344 | 0.044732 | 2709.258 | 9817.506 | 3.6 | 63829.627 | 9817.506 | 14068.14 |
| 3.7 | 0.009175 | 0.036285 | 2013.781 | 7963.655 | . 7 | 3049.47 | 7963.655 | 12298.97 |
| 3.8 | 0.006664 | 0.02931 | 1462.537 | 6432.731 | 3.8 | 82409.774 | 6432.731 | 10856.49 |
| 3.9 | 0.004683 | 0.023566 | 1027.725 | 5172.152 | 3.9 | 91884.678 | 5172.152 | 9686.199 |
| 4 | 0.003129 | 0.018851 | 686.7932 | 4137.261 |  | 1453.754 | 4137.261 | 8741.301 |
| 4.1 | 0.00192 | 0.014992 | 421.4088 | 3290.259 | 4.1 | 11100.755 | 3290.259 | 7981.913 |
| 4.2 | 0.000987 | 0.011843 | 216.6851 | 2599.227 | 4.2 | 2812.6317 | 2599.227 | 7374.281 |
| 4.3 | 0.000276 | 0.009283 | 60.54647 | 2037.365 | 4.3 | 578.7552 | 2037.365 | 6890.156 |
| 4.4 | -0.000259 | 0.007209 | -56.8132 | 1582.116 | 4.4 | 4390.2849 | 1582.116 | 6506.018 |
| 4.5 | -0.000653 | 0.005534 | -143.3477 | 1214.681 | 4.5 | $5 \quad 239.7856$ | 1214.681 | 6202.548 |
| 4.6 | -0.000936 | 0.004189 | -205.4941 | 919.3112 | 4.6 | 120.92 | 919.3112 | 5963.897 |
| 4.7 | -0.001132 | 0.003112 | -248.465 | 682.964 | 4.7 | 728.27831 | 682.964 | 5777.221 |
| 4.8 | -0.00126 | 0.002254 | -276.4722 | 494.7704 | 4.8 | 8-42.78013 | 494.7704 | 5632.078 |
| 4.9 | -0.001335 | 0.001576 | -292.8977 | 345.7833 | 4.9 | 9 -96.19083 | 345.7833 | 5520.076 |
| 5 | -0.001369 | 0.001042 | -300.4674 | 228.5894 |  | $5-135.3051$ | 228.5894 | 5434.427 |
| 5.1 | -0.001373 | 0.000625 | -301.3518 | 137.1255 | 5.1 | . -162.9259 | 137.1255 | 5369.7 |
| 5.2 | -0.001355 | 0.000302 | -297.2806 | 66.38669 | 5.2 | 2-181.4041 | 66.38669 | 5321.51 |
| 5.3 | -0.00132 | $5.6 \mathrm{E}-05$ | -289.6209 | 12.29758 | 5.3 | $3-192.681$ | 12.29758 | 5286.358 |
| 5.4 | -0.001273 | -0.00013 | -279.4483 | -28.47903 | 5.4 | 4-198.3567 | -28.47903 | 5261.429 |
| 5.5 | -0.001219 | -0.000267 | -267.6032 | -58.66518 | 5.5 | 5-199.7397 | -58.66518 | 5244.471 |
| 5.6 | -0.001161 | -0.000367 | -254.7332 | -80.46818 | 5.6 | 6-197.8876 | -80.46818 | 5233.686 |
| 5.7 | -0.0011 | -0.000436 | -241.3343 | -95.68004 | 5.7 | 7-193.6526 | -95.68004 | 5227.638 |
| 5.8 | -0.001038 | -0.000482 | -227.7817 | -105.7451 | 5.8 | 8-187.7143 | -105.7451 | 5225.188 |
| 5.9 | -0.000977 | -0.00051 | -214.3521 | -111.8272 | 5.9 | 9-180.6092 | -111.8272 | 5225.43 |
|  | -0.000917 | -0.000523 | -201.2473 | -114.8555 |  | 6 -172.7571 | -114.8555 | 5227.654 |
| 6.1 | -0.000859 | -0.000527 | -188.6077 | -115.5715 | 6.1 | 1 -164.482 | -115.5715 | 5231.303 |
| 6.2 | -0.000804 | -0.000522 | -176.5388 | -114.5636 | 6.2 | $2-156.0387$ | -114.5636 | 5235.936 |
| 6.3 | -0.000752 | -0.000512 | -165.0713 | -112.2902 | 6.3 | 3-147.5925 | -112.2902 | 5241.231 |
| 6.4 | -0.000703 | -0.000497 | -154.2534 | -109.1184 | 6.4 | 4-139.2924 | -109.1184 | 5246.921 |
| 6.5 | -0.000657 | -0.00048 | -144.0917 | -105.3279 | 6.5 | 5-131.2324 | -105.3279 | 5252.813 |

$6.6-0.000613-0.000461-134.5796-101.1383$
$6.7-0.000573-0.000441-125.7019-96.71571$
$6.8-0.000535-0.00042-117.4365-92.18812$
$6.9-0.0005-0.000399-109.7505-87.65122$
$7-0.000468-0.000379-102.6154-83.1743$
$\begin{array}{lllllllll}7.1 & -0.000437 & -0.000359 & -95.99381 & -78.80966\end{array}$
$7.2-0.000409-0.00034 \quad-89.8595-74.59284$
$\begin{array}{llllll}7.3 & -0.000384 & -0.000321 & -84.17291 & -70.54542\end{array}$
$\begin{array}{llllll}7.4 & -0.00036 & -0.000304 & -78.90113 & -66.68517\end{array}$
$7.5-0.000337-0.000287-74.02001-63.01775$
$7.6-0.000317-0.000271-69.49225-59.54786$
$7.7-0.000298-0.000256-65.2959-56.27549$
$\begin{array}{lllllll}7.8 & -0.00028 & -0.000242 & -61.40461 & -53.19187\end{array}$
$7.9-0.000263-0.000229-57.79206-50.29481$
$8-0.000248-0.000217-54.43849-47.57552$
$8.1-0.000234-0.000205-51.32195-45.02522$
$8.2-0.000221-0.000194-48.42488-42.63514$
$8.3-0.000208-0.000184-45.72973-40.39431$
$8.5-0.000186-0.000166-40.87934-36.33183$
$8.6-0.000176-0.000157-38.69777-34.48824$
$8.7-0.000167-0.000149-36.66324-32.75878$
$8.8-0.000158-0.000142-34.76039-31.13687$
$8.9-0.00015-0.000135-32.98202-29.61468$
$9-0.000143-0.000128-31.31858-28.18304$
$9.1-0.000136-0.000122-29.75857-26.83698$
$9.3-0.000123-0.000111-26.92826-24.37648$
$9.4-0.000117-0.000106-25.64143-23.25118$
$9.5-0.000111-0.000101-24.43234-22.1909$
$9.7-0.000101-9.22 \mathrm{E}-05-22.22482-20.24481$
$9.8-9.67 \mathrm{E}-05-8.82 \mathrm{E}-05-21.21598-19.35168$
9.9 -9.23E-05 -8.43E-05 -20.26603 -18.50816
$10.1-8.44 \mathrm{E}-05-7.73 \mathrm{E}-05-18.52235-16.95562$
10.2 -8.07E-05 $-7.4 \mathrm{E}-05-17.72206-16.24097$
$10.3-7.73 \mathrm{E}-05-7.09 \mathrm{E}-05-16.96646-15.56471$
$10.6 \quad-6.8 \mathrm{E}-05-6.26 \mathrm{E}-05-14.92992-13.7406$
11 -5.77E-05 $-5.34 \mathrm{E}-05-12.6743-11.71487$
$13-2.8 \mathrm{E}-05-2.64 \mathrm{E}-05-6.135025-5.799921$
$15-1.52 \mathrm{E}-05-1.46 \mathrm{E}-05-3.340628-3.20365$
17 -9.02E-06 -8.73E-06 -1.979413 -1.914992
$20-4.61 \mathrm{E}-06-4.51 \mathrm{E}-06-1.012619-0.988985$
$25-1.87 \mathrm{E}-06-1.83 \mathrm{E}-06-0.409457-0.402579$
$30-9.03 \mathrm{E}-07$-8.92E-07 -0.198253 -0.195697
$40-3.01 \mathrm{E}-07-2.99 \mathrm{E}-07-0.066036-0.065542$

| 6.6 | -123.4787 | -101.1383 | 5258.761 |
| ---: | ---: | ---: | ---: |
| 6.7 | -116.0745 | -96.71571 | 5264.657 |
| 6.8 | -109.0467 | -92.18812 | 5270.422 |
| 6.9 | -102.4042 | -87.65122 | 5276.003 |
| 7 | -96.15063 | -83.1743 | 5281.361 |
| 7.1 | -90.27797 | -78.80966 | 5286.474 |
| 7.2 | -84.78025 | -74.59284 | 5291.328 |
| 7.3 | -79.63809 | -70.54542 | 5295.92 |
| 7.4 | -74.83531 | -66.68517 | 5300.249 |
| 7.5 | -70.3576 | -63.01775 | 5304.32 |
| 7.6 | -66.18154 | -59.54786 | 5308.141 |
| 7.7 | -62.29246 | -56.27549 | 5311.721 |
| 7.8 | -58.66982 | -53.19187 | 5315.073 |
| 7.9 | -55.2953 | -50.29481 | 5318.208 |
| 8 | -52.15278 | -47.57552 | 5321.139 |
| 8.1 | -49.22468 | -45.02522 | 5323.878 |
| 8.2 | -46.49636 | -42.63514 | 5326.436 |
| 8.3 | -43.95244 | -40.39431 | 5328.828 |
| 8.5 | -39.36436 | -36.33183 | 5333.153 |
| 8.6 | -37.29533 | -34.48824 | 5335.109 |
| 8.7 | -35.36238 | -32.75878 | 5336.94 |
| 8.8 | -33.55309 | -31.13687 | 5338.656 |
| 8.9 | -31.86004 | -29.61468 | 5340.263 |
| 9 | -30.27381 | -28.18304 | 5341.772 |
| 9.1 | -28.78506 | -26.83698 | 5343.19 |
| 9.3 | -26.07793 | -24.37648 | 5345.773 |
| 9.4 | -24.84492 | -23.25118 | 5346.952 |
| 9.5 | -23.6854 | -22.1909 | 5348.062 |
| 9.7 | -21.56498 | -20.24481 | 5350.095 |
| 9.8 | -20.59699 | -19.3568 | 5351.027 |
| 9.9 | -19.6802 | -18.50816 | 5351.906 |
| 10.1 | -18.00021 | -16.95562 | 5353.522 |
| 10.2 | -17.22845 | -16.24097 | 5354.265 |
| 10.3 | -16.49929 | -15.56471 | 5354.968 |
| 10.6 | -14.53354 | -13.7406 | 5356.863 |
| 11 | -12.35453 | -11.71487 | 5358.965 |
| 13 | -6.023328 | -5.799921 | 5365.088 |
| 15 | -3.294969 | -3.20365 | 5367.751 |
| 17 | -1.95794 | -1.914992 | 5369.064 |
| 20 | -1.004741 | -0.988985 | 5370.003 |
| 25 | -0.407164 | -0.402579 | 5370.595 |
| 30 | -0.197401 | -0.195697 | 5370.803 |
| 40 | -0.065871 | -0.065542 | 5370.934 |
|  |  |  |  |


| (Bohr) | S |  |  | - | Bohr) | X (cm-1) | 1) | 1) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1.027011 |  | 225402.9 | 0 |  |  |  |  |
| 2.1 | 0.835373 | 1.0853 | 183343.2 | 238195.7 | 2.1 | 190534.1 | 238195.7 | 261899.9 |
| 2.2 | 0.681153 | 0.889625 | 149495.8 | 195250 | 2.2 | 156235.9 | 195250 | 219405 |
| 2.3 | 0.556505 | 0.781772 | 122138.7 | 171579 | 2.3 | 129075.5 | 171579 | 195537.3 |
| 2.4 | 0.455442 | 0.664318 | 99957.9 | 145801.1 | 2.4 | 106703 | 145801.1 | 169951.1 |
| 2.5 | 0.373281 | 0.565291 | 81925.79 | 124067.1 | 2.5 | 88450.78 | 124067.1 | 148437.2 |
| 2.6 | 0.306301 | 0.48144 | 67225.22 | 105663.9 | 2.6 | 73503.07 | 105663.9 | 130281.2 |
| 2.7 | 0.251533 | 0.410143 | 55205.1 | 90015.97 | 2.7 | 61209.93 | 90015.97 | 114906.2 |
| 2.8 | 0.206624 | 0.349311 | 45348.76 | 76664.83 | 2.8 | 51056.45 | 76664.83 | 101852.2 |
| 2.9 | 0.169712 | 0.297283 | 37247.56 | 65245.98 | 2.9 | 42636.6 | 65245.98 | 90752.04 |
| 3 | 0.139321 | 0.252725 | 30577.32 | 55466.71 | 3 | 35629.74 | 55466.71 | 81309.38 |
| 3.1 | 0.114268 | 0.214548 | 25078.92 | 47087.81 | 3.1 | 29781.15 | 47087.81 | 73280.69 |
| 3.2 | 0.093603 | 0.181845 | 20543.49 | 39910.33 | 3.2 | 24887 | 39910.33 | 66461.93 |
| 3.3 | 0.076552 | 0.153851 | 16801.31 | 33766.37 | 3.3 | 20783.05 | 33766.37 | 60679.73 |
| 3.4 | 0.062485 | 0.129914 | 13713.78 | 28512.82 | 3.4 | 17336.32 | 28512.82 | 55785.39 |
| 3.5 | 0.050881 | 0.109474 | 11167.15 | 24026.83 | 3.5 | 14438.45 | 24026.83 | 51650.63 |
| 3.6 |  | 0.092049 | 0 | 20202.36 |  |  |  |  |
| 3.7 | 0.033437 | 0.077219 | 7338.626 | 16947.63 | 3.7 | 9950.14 | 16947.63 | 45231.21 |
| 3.8 | 0.026954 | 0.064622 | 5915.675 | 14182.96 | 3.8 | 8225.98 | 14182.96 | 42767.76 |
| 3.9 | 0.021625 | 0.053944 | 4746.156 | 11839.26 | 3.9 | 6777.697 | 11839.26 | 40702.82 |
| 4 | 0.017252 | 0.044909 | 3786.352 | 9856.492 | 4 | 5562.844 | 9856.492 | 38975.1 |
| 4.1 | 0.013669 | 0.037283 | 3000.018 | 8182.594 | 4.1 | 4545.597 | 8182.594 | 37532.12 |
| 4.2 | 0.01074 | 0.030858 | 2357.131 | 6772.491 | 2 | 3695.617 | 6772.491 | 36329.11 |
| 4.3 | 0.008351 | 0.025457 | 1832.806 | 5587.236 | 4.3 | 2987.136 | 5587.236 | 35328.01 |
| 4.4 | 0.006408 | 0.020928 | 1406.391 | 4593.227 | 4.4 | 2398.204 | 4593.227 | 34496.51 |
| 4.5 | 0.004833 | 0.017139 | 1060.813 | 3761.563 | - 4.5 | 1910.165 | 3761.563 | 33807.31 |
| 4.6 | 0.003562 | 0.013976 | 781.8476 | 3067.408 | 4.6 | 1507.074 | 3067.408 | 33237.28 |
| 4.7 | 0.002541 | 0.011343 | 557.7772 | 2489.499 | 4.7 | 1175.414 | 2489.499 | 32766.96 |
| 4.8 | 0.001726 | 0.009157 | 378.7825 | 2009.635 | 4.8 | 903.6114 | 2009.635 | 32379.91 |
| 4.9 | 0.001079 | 0.007346 | 236.8043 | 1612.3 | 4.9 | 681.899 | 1612.3 | 32062.31 |
| 5 | 0.00057 | 0.005851 | 125.094 | 1284.24 | 5 | 501.934 | 1284.24 | 31802.5 |
| 5.1 | 0.000174 | 0.004621 | 38.0986 | 1014.269 | 5.1 | 356.7076 | 1014.269 | 31590.76 |
| 5.2 | -0.000131 | 0.003612 | -28.79946 | 792.8104 | 5.2 | 240.2585 | 792.8104 | 31418.85 |
| 5.3 | -0.000362 | 0.002788 | -79.45201 | 611.825 | 5.3 | 147.5623 | 611.825 | 31279.91 |
| 5.4 | -0.000533 | 0.002116 | -116.9493 | 464.4895 | 5.4 | 74.44734 | 464.4895 | 31168.19 |
| 5.5 | -0.000656 | 0.001572 | -143.9183 | 345.0887 | 5.5 | 17.37317 | 345.0887 | 31078.9 |
| 5.6 | -0.00074 | 0.001134 | -162.4968 | 248.7921 | 5.6 | -26.61183 | 248.7921 | 31008.01 |
| 5.7 | -0.000795 | 0.000782 | -174.434 | 171.5685 | 5.7 | -59.95773 | 171.5685 | 30952.19 |
| 5.8 | -0.000825 | 0.000501 | -181.1631 | 110.038 | 5.8 | -84.70411 | 110.038 | 30908.68 |
| 5.9 | -0.000838 | 0.00028 | -183.8495 | 61.37461 | 5.9 | -102.5395 | 61.37461 | 30875.16 |
| 6 | -0.000836 | 0.000106 | -183.4391 | 23.24017 | 6 | -114.8526 | 23.24017 | 30849.75 |
| 6.1 | -0.000823 | -2.88E-05 | -180.6913 | -6.31656 | 6.1 | -122.7847 | -6.31656 | 30830.88 |
| 6.2 | -0.000803 | -0.000132 | -176.225 | -28.90481 | 6.2 | -127.2741 | -28.90481 | 30817.24 |
| 6.3 | -0.000777 | -0.000209 | -170.5274 | -45.8733 | 6.3 | -129.0876 | -45.8733 | 30807.79 |
| 6.4 | -0.000747 | -0.000266 | -163.9871 | -58.30782 | 6.4 | -128.8409 | -58.30782 | 30801.65 |
| 6.5 | -0.000715 | -0.000306 | -156.9112 | -67.13289 | 6.5 | -127.043 | -67.13289 | 30798.1 |
| 6.6 | -0.000681 | -0.000333 | -149.5346 | -73.07188 | 6.6 | -124.0891 | -73.07188 | 30796.58 |
| 6.7 | -0.000647 | -0.00035 | -142.0418 | -76.75128 | 6.7 | -120.3089 | -76.75128 | 30796.62 |
| 6.8 | -0.000613 | -0.000358 | -134.5731 | -78.64873 | 6.8 | -115.9541 | -78.64873 | 30797.83 |
| 6.9 | -0.00058 | -0.000361 | -127.2316 | -79.18001 | 6.9 | -111.231 | -79.18001 | 30799.92 |
|  | -0.000547 | -0.000358 | -120.0943 | -78.65751 | 7 | -106.2944 | -78.65751 | 30802.64 |


| 7.1 | -0.000516 | -0.000352 | -113.216 | -77.35134 |
| ---: | ---: | ---: | ---: | ---: |
| 7.2 | -0.000486 | -0.000344 | -106.6296 | -75.46635 |
| 7.3 | -0.00045 | -0.000333 | -100.3592 | -73.1702 |
| 7.4 | -0.00043 | -0.000322 | -94.4136 | -70.59401 |
| 7.5 | -0.000405 | -0.000309 | -88.79944 | -67.83833 |
| 7.6 | -0.000381 | -0.000296 | -83.5101 | -64.98644 |
| 7.7 | -0.000358 | -0.000283 | -78.539 | -62.09781 |
| 7.8 | -0.000337 | -0.00027 | -73.87736 | -59.22084 |
| 7.9 | -0.000317 | -0.000257 | -69.50981 | -56.38781 |
| 8 | -0.000298 | -0.000244 | -65.42539 | -53.62643 |
| 8.1 | -0.000281 | -0.000232 | -61.60653 | -50.95542 |
| 8.2 | -0.000264 | -0.00022 | -58.04007 | -48.38757 |
| 8.3 | -0.000249 | -0.000209 | -54.70844 | -45.92726 |
| 8.5 | -0.000222 | -0.000188 | -48.69703 | -41.34902 |
| 8.6 | -0.00021 | -0.000179 | -45.98652 | -39.23548 |
| 8.7 | -0.000198 | -0.00017 | -43.45598 | -37.23168 |
| 8.9 | -0.000177 | -0.000153 | -38.88663 | -33.55652 |
| 9 | -0.000168 | -0.000145 | -36.82221 | -31.87047 |
| 9.1 |  | -0.000138 |  | 0 |
| 9.30 .28544 |  |  |  |  |
| 9.3 | -0.000143 | -0.000125 | -31.39849 | -27.38375 |
| 9.4 | -0.000136 | -0.000119 | -29.81403 | -26.05726 |
| 9.5 | -0.000129 | -0.000113 | -28.33109 | -24.81229 |
| 9.8 | -0.000111 | -0.000103 | -24.40349 | -22.53321 |
| 9.9 | -0.000106 | $-9.35 \mathrm{E}-05$ | -23.25056 | -20.51112 |
| 10.1 | $-9.63 \mathrm{E}-05$ | $-8.53 \mathrm{E}-05$ | -21.14294 | -18.7137 |
| 10.2 | $-9.19 \mathrm{E}-05$ | $-8.15 \mathrm{E}-05$ | -20.18049 | -17.8889 |
| 10.3 | $-8.78 \mathrm{E}-05$ | $-7.8 \mathrm{E}-05$ | -19.27317 | -17.1128 |
| 10.6 | $-7.68 \mathrm{E}-05$ | $-6.85 \mathrm{E}-05$ | -16.84621 | -15.02519 |
| 11 | $-6.46 \mathrm{E}-05$ | $-5.8 \mathrm{E}-05$ | -14.18441 | -12.72261 |
| 13 | $-3.03 \mathrm{E}-05$ | $-2.79 \mathrm{E}-05$ | -6.65946 | -6.113761 |
| 15 | $-1.62 \mathrm{E}-05$ | $-1.52 \mathrm{E}-05$ | -3.556331 | -3.337808 |
| 17 | $-9.48 \mathrm{E}-06$ | $-9.02 \mathrm{E}-06$ | -2.080157 | -1.980562 |
| 20 | $-4.79 \mathrm{E}-06$ | $-4.62 \mathrm{E}-06$ | -1.051391 | -1.014397 |
| 25 | $-1.92 \mathrm{E}-06$ | $-1.87 \mathrm{E}-06$ | -0.421503 | -0.411398 |
| 30 | $-9.3 \mathrm{E}-07$ | $-9.13 \mathrm{E}-07$ | -0.204094 | -0.200303 |
| 40 | $-3.15 \mathrm{E}-07$ | $-3.12 \mathrm{E}-07$ | -0.069182 | -0.068469 |


| 7.1 | -101.2704 | -77.35134 | 30805.8 |
| ---: | ---: | ---: | ---: |
| 7.2 | -96.2488 | -75.46635 | 30809.25 |
| 7.3 | -91.30149 | -73.1702 | 30812.87 |
| 7.4 | -86.47782 | -70.59401 | 30816.57 |
| 7.5 | -81.81556 | -67.83833 | 30820.28 |
| 7.6 | -77.33801 | -64.98644 | 30823.94 |
| 7.7 | -73.06054 | -62.09781 | 30827.52 |
| 7.8 | -68.99339 | -59.22084 | 30831 |
| 7.9 | -65.13705 | -56.38781 | 30834.34 |
| 8 | -61.4934 | -53.62643 | 30837.54 |
| 8.1 | -58.05698 | -50.95542 | 30840.6 |
| 8.2 | -54.82324 | -48.38757 | 30843.5 |
| 8.3 | -51.78194 | -45.92726 | 30846.25 |
| 8.5 | -46.24808 | -41.34902 | 30851.3 |
| 8.6 | -43.7365 | -39.23548 | 30853.61 |
| 8.7 | -41.38149 | -37.23168 | 30855.79 |
| 8.9 | -37.11013 | -33.55652 | 30859.77 |
| 9 | -35.17181 | -31.87047 | 30861.58 |
| 9.1 |  |  |  |
| 9.3 | -30.06036 | -27.38375 | 30866.38 |
| 9.4 | -28.56187 | -26.05726 | 30867.79 |
| 9.5 | -27.15825 | -24.81229 | 30869.11 |
| 9.8 | -23.78009 | -22.53321 | 30871.94 |
| 9.9 | -22.33746 | -20.51112 | 30873.68 |
| 10.1 | -20.33323 | -18.7137 | 30875.58 |
| 10.2 | -19.41667 | -17.8889 | 30876.45 |
| 10.3 | -18.55308 | -17.128 | 30877.27 |
| 10.6 | -16.23923 | -15.02519 | 30879.47 |
| 11 | -13.69716 | -12.72261 | 30881.89 |
| 13 | -6.477563 | -6.113761 | 30888.8 |
| 15 | -3.48349 | -3.337808 | 30891.69 |
| 17 | -2.046959 | -1.980562 | 30893.09 |
| 20 | -1.039059 | -1.014397 | 30894.07 |
| 25 | -0.418135 | -0.411398 | 30894.69 |
| 30 | -0.20283 | -0.200303 | 30894.9 |
| 40 | -0.068944 | -0.068469 | 30895.03 |


| R (Bohr) | SIGMA (H) | (Hartre | IGMA | (cm-1) | R (Bohr) | $X(\mathrm{~cm}-1)$ | V1 (cm-1) | $\mathrm{V} 2(\mathrm{~cm}-1)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.9 | 0.99675 |  | 218761.2 | 0 |  |  |  |  |
| 2 | 0.804579 | 0.998666 | 176584.8 | 219181.8 | 2 | 179568.1 | 219181.8 | 226735.5 |
| 2.1 | 0.652882 | 0.91379 | 143291 | 200553.8 | 2.1 | 146400 | 200553.8 | 207981.8 |
| 2.2 | 0.532183 | 0.755944 | 116800.7 | 165910.6 | 2.2 | 119848.3 | 165910.6 | 173400.1 |
| 2.3 | 0.435403 | 0.663809 | 95560.02 | 145689.3 | 2.3 | 98616.27 | 145689.3 | 153170 |
| 2.4 | 0.357231 | 0.567218 | 78403.05 | 124489.9 | 2.4 | 81422.83 | 124489.9 | 132007.1 |
| 2.5 | 0.293662 | 0.484993 | 64451.4 | 106443.7 | 2.5 | 67427.84 | 106443.7 | 114004.2 |
| 2.6 | 0.241669 | 0.414581 | 53040.32 | 90989.98 | 2.6 | 55965.88 | 90989.98 | 98601.42 |
| 2.7 | 0.198946 | 0.354039 | 43663.6 | 77702.48 | 2.7 | 46530 | 77702.48 | 85373.09 |
| 2.8 | 0.16372 | 0.301868 | 35932.31 | 66252.33 | 2.8 | 38730.45 | 66252.33 | 73991.2 |
| 2.9 | 0.13461 | 0.256884 | 29543.51 | 56379.42 | 2.9 | 32263.53 | 56379.42 | 64196.41 |
| 3 | 0.110526 | 0.218119 | 24257.64 | 47871.52 | 3 | 26889.03 | 47871.52 | 55777.15 |
| 3.1 | 0.09059 | 0.184761 | 19882.25 | 40550.24 | 3.1 | 22413.98 | 40550.24 | 48555.53 |
| 3.2 | 0.07409 | 0.15611 | 16260.86 | 34262.22 | 3.2 | 18681.72 | 34262.22 | 42378.37 |
| 3.3 | 0.060439 | 0.131558 | 13264.82 | 28873.74 | 3.3 | 15563.81 | 28873.74 | 37111.76 |
| 3.4 | 0.049155 | 0.11057 | 10788.17 | 24267.37 | 3.4 | 12954.98 | 24267.37 | 32637.57 |
| 3.5 | 0.039836 | 0.092675 | 8742.893 | 20339.72 | 3.5 | 10768.53 | 20339.72 | 28851.09 |
| 3.6 | 0.03215 | 0.077456 | 7056.107 | 16999.6 | 3.6 | 8933.474 | 16999.6 | 25659.25 |
| 3.7 | 0.025821 | 0.064548 | 5667.063 | 14166.73 | 3.7 | 7391.488 | 14166.73 | 22979.31 |
| 3.8 | 0.020619 | 0.05363 | 4525.301 | 11770.45 | 3.8 | 6094.912 | 11770.45 | 20737.85 |
| 3.9 | 0.016351 | 0.044419 | 3588.689 | 9748.929 | 3.9 | 5004.587 | 9748.929 | 18870.04 |
| 4 | 0.012859 | 0.03667 | 2822.237 | 8048.078 | 4 | 4088.381 | 8048.078 | 17318.94 |
| 4.1 | 0.010009 | 0.030167 | 2196.744 | 6620.83 | 4.1 | 3319.641 | 6620.83 | 16034.94 |
| 4.2 | 0.007691 | 0.024724 | 1687.935 | 5426.372 | 4.2 | 2676.151 | 5426.372 | 14975.17 |
| 4.3 | 0.005812 | 0.020182 | 1275.567 | 4429.426 | 4.3 | 2139.149 | 4429.426 | 14102.85 |
| 4.4 | 0.004296 | 0.016401 | 942.8367 | 3599.612 | 4.4 | 1692.721 | 3599.612 | 13386.74 |
| 4.5 | 0.003079 | 0.013263 | 675.7141 | 2910.855 | 4.5 | 1323.201 | 2910.855 | 12800.38 |
| 4.6 | 0.002108 | 0.010666 | 462.5603 | 2340.833 | 4.6 | 1018.858 | 2340.833 | 12321.55 |
| 4.7 | 0.001338 | 0.008523 | 293.6746 | 1870.523 | 4.7 | 769.5673 | 1870.523 | 11931.64 |
| 4.8 | 0.000734 | 0.00676 | 161.0044 | 1483.695 | 4.8 | 566.5999 | 1483.695 | 11615.11 |
| 4.9 | 0.000264 | 0.005316 | 57.86449 | 1166.639 | 4.9 | 402.4647 | 1166.639 | 11359.05 |
| 5 | -9.7E-05 | 0.004136 | -21.29089 | 907.6746 | 5 | 270.7127 | 907.6746 | 11152.68 |
| 5.1 | -0.000369 | 0.003176 | -81.04759 | 697.0075 | 5.1 | 165.8581 | 697.0075 | 10987.11 |
| 5.2 | -0.00057 | 0.002398 | -125.1927 | 526.3287 | 5.2 | 83.21686 | 526.3287 | 10854.93 |
| 5.3 | -0.000715 | 0.001771 | -156.8585 | 388.7005 | 5.3 | 18.82752 | 388.7005 | 10750.02 |
| 5.4 | -0.000814 | 0.001268 | -178.5975 | 278.2851 | 5.4 | -30.64112 | 278.2851 | 10667.34 |
| 5.5 | -0.000877 | 0.000867 | -192.5319 | 190.2296 | 5.5 | -67.99667 | 190.2296 | 10602.7 |
| 5.6 | -0.000913 | 0.000549 | -200.3386 | 120.4718 | 5.6 | -95.55011 | 120.4718 | 10552.69 |
| 5.7 | -0.000927 | 0.000299 | -203.4201 | 65.65024 | 5.7 | -115.2437 | 65.65024 | 10514.48 |
| 5.8 | -0.000925 | 0.000105 | -202.9065 | 22.9746 | 5.8 | -128.6811 | 22.9746 | 10485.76 |
| 5.9 | -0.00091 | -4.49E-05 | -199.6978 | -9.864025 | 5.9 | -137.1753 | -9.864025 | 10464.62 |
| 6 | -0.000886 | -0.000158 | -194.5226 | -34.75161 | 6 | -141.8012 | -34.75161 | 10449.54 |
| 6.1 | -0.000856 | -0.000243 | -187.9537 | -53.26328 | 6.1 | -143.4378 | -53.26328 | 10439.23 |
| 6.2 | -0.000822 | -0.000304 | -180.4411 | -66.66761 | 6.2 | -142.7886 | -66.66761 | 10432.69 |
| 6.3 | -0.000785 | -0.000346 | -172.3425 | -76.02079 | 6.3 | -140.4303 | -76.02079 | 10429.08 |
| 6.4 | -0.000747 | -0.000374 | -163.9322 | -82.1713 | 6.4 | -136.8192 | -82.1713 | 10427.73 |
| 6.5 | -0.000708 | -0.000391 | -155.421 | -85.81822 | 6.5 | -132.322 | -85.81822 | 10428.09 |
| 6.6 | -0.00067 | -0.000399 | -146.9646 | -87.52209 | 6.6 | -127.2248 | -87.52209 | 10429.75 |
| 6.7 | -0.000632 | -0.0004 | -138.6794 | -87.73975 | 6.7 | -121.7542 | -87.73975 | 10432.34 |
| 6.8 | -0.000595 | -0.000396 | -130.6511 | -86.83514 | 6.8 | -116.0862 | -86.83514 | 10435.61 |
| 6.9 | -0.00056 | -0.000388 | -122.9387 | -85.09981 | 6.9 | -110.3559 | -85.09981 | 10439.33 |

$7-0.000527-0.000377-115.5753-82.76608$ $7.1-0.000495-0.000365-108.5873-80.01674$ $7.2-0.000465-0.000351-101.9811-76.99389$ $7.3-0.000436-0.000336-95.75678-73.80932$ $\begin{array}{lllllll}7.4 & -0.00041 & -0.000321 & -89.91217 & -70.54793\end{array}$ $7.5-0.000385-0.000307-84.43408-67.27556$ $7.6-0.000361-0.000292-79.30935-64.03611$ $\begin{array}{llllll}7.7 & -0.00034 & -0.000277 & -74.51822 & -60.86632\end{array}$ $7.8-0.000319-0.000263-70.04972-57.79206$ $\begin{array}{lllll}7.9 & -0.0003 & -0.00025 & -65.8797 & -54.83135\end{array}$ $8-0.000282-0.000237-61.9928-51.99354$ $8.1-0.000266-0.000225-58.36928-49.28522$
$8.2-0.000251-0.000213-54.99376-46.70859$
$8.3-0.000236-0.000202-51.84869-44.26803$
$8.5-0.00021-0.000181-46.18185-39.77758$
$8.6-0.000199-0.000172-43.63375-37.7211$
$8.7-0.000188-0.000163-41.25465-35.78534$
$\begin{array}{llll}8.9 & -0.000168 & -36.9625 & 0\end{array}$
$9-0.00016-0.00014-35.02376-30.63646$
9.1 $-0.000151-0.000133-33.21324-29.11989$
$\begin{array}{lllll}9.3 & -0.000136 & -0.00012 & -29.93007 & -26.3589\end{array}$
$9.4-0.00013-0.000114-28.44178-25.09994$
$\begin{array}{lllll}9.7 & -0.000112 & -24.50737 & 0\end{array}$
$9.8-0.000106-9.46 \mathrm{E}-05-23.35099-20.7647$
$9.9-0.000101-9.04 \mathrm{E}-05-22.26387-19.83246$
$10.1-9.24 \mathrm{E}-05-8.26 \mathrm{E}-05-20.27583-18.12317$
10.2 -8.82E-05 -7.9E-05 -19.36693-17.33908
$10.3-8.43 \mathrm{E}-05 \quad-7.56 \mathrm{E}-05-18.5094-16.59691$
$10.6-7.39 \mathrm{E}-05-6.65 \mathrm{E}-05-16.2118-14.60033$
$11-6.23 \mathrm{E}-05-5.64 \mathrm{E}-05-13.68413-12.38556$
$13-2.95 \mathrm{E}-05-2.73 \mathrm{E}-05-6.484545-5.982144$ $15-1.59 \mathrm{E}-05-1.49 \mathrm{E}-05-3.48376-3.279535$ 17 -9.32E-06 -8.89E-06 -2.045469 -1.951725 $20-4.73 \mathrm{E}-06-4.58 \mathrm{E}-06-1.037196-1.004546$ $25-1.9 \mathrm{E}-06-1.86 \mathrm{E}-06-0.416213-0.407331$ $30-9.15 \mathrm{E}-07-8.99 \mathrm{E}-07-0.200726-0.197343$ $40-3.04 \mathrm{E}-07 \quad-3.01 \mathrm{E}-07-0.066779-0.066163$

| 7 | -104.6616 | -82.76608 | 10443.33 |
| ---: | ---: | ---: | ---: |
| 7.1 | -99.08096 | -8.01674 | 10447.49 |
| 7.2 | -93.66518 | -76.99389 | 10451.7 |
| 7.3 | -88.45111 | -73.80932 | 10455.9 |
| 7.4 | -83.46533 | -70.54793 | 10460.02 |
| 7.5 | -78.72078 | -67.27556 | 10464.02 |
| 7.6 | -74.22319 | -64.03611 | 10467.89 |
| 7.7 | -69.97152 | -60.86632 | 10471.6 |
| 7.8 | -65.967 | -57.79206 | 10475.14 |
| 7.9 | -62.19949 | -54.83135 | 10478.5 |
| 8 | -58.66182 | -51.99354 | 10481.69 |
| 8.1 | -55.343 | -49.28522 | 10484.7 |
| 8.2 | -52.23348 | -46.70859 | 10487.54 |
| 8.3 | -49.32301 | -44.26803 | 10490.22 |
| 8.5 | -44.04796 | -39.77758 | 10495.1 |
| 8.6 | -41.66361 | -37.7211 | 10497.32 |
| 8.7 | -39.43217 | -35.78534 | 10499.4 |
| 8.9 | -24.67045 | 0 | 10524.72 |
| 9 | -33.56173 | -30.63646 | 10504.91 |
| 9.1 | -31.84914 | -29.11989 | 10506.53 |
| 9.3 | -28.73995 | -26.3589 | 10509.46 |
| 9.4 | -27.32807 | -25.09994 | 10510.8 |
| 9.7 | -16.3509 | 0 | 10528.85 |
| 9.8 | -22.48903 | -20.7647 | 10515.38 |
| 9.9 | -21.45353 | -19.83246 | 10516.37 |
| 10.1 | -19.55837 | -18.12317 | 10518.17 |
| 10.2 | -18.69107 | -17.33908 | 10519 |
| 10.3 | -17.87198 | -16.59691 | 10519.78 |
| 10.6 | -15.6747 | -14.60033 | 10521.87 |
| 11 | -13.25131 | -12.38556 | 10524.19 |
| 13 | -6.31083 | -5.982144 | 10530.86 |
| 15 | -3.415686 | -3.279535 | 10533.66 |
| 17 | -2.014221 | -1.951725 | 10535.03 |
| 20 | -1.026313 | -1.004546 | 10535.99 |
| 25 | -0.413252 | -0.407331 | 10536.6 |
| 30 | -0.199598 | -0.197343 | 10536.81 |
| 40 | -0.066574 | -0.066163 | 10536.94 |

