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► **To cite this version:**

Y. Frostig, O.T. Thomsen. On the free vibration of sandwich panels with a transversely flexible and temperature dependent core material – Part II: numerical study. *Composites Science and Technology*, 2009, 69 (6), pp.863. 10.1016/j.compscitech.2008.04.013 . hal-00521261

**HAL Id: hal-00521261**

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Submitted on 27 Sep 2010

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## Accepted Manuscript

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PII: S0266-3538(08)00125-5  
DOI: [10.1016/j.compscitech.2008.04.013](https://doi.org/10.1016/j.compscitech.2008.04.013)  
Reference: CSTE 4027

To appear in: *Composites Science and Technology*

Received Date: 28 September 2007  
Revised Date: 28 March 2008  
Accepted Date: 4 April 2008

Please cite this article as: Frostig, Y., Thomsen, O.T., On the free vibration of sandwich panels with a transversely flexible and temperature dependent core material – Part II: numerical study, *Composites Science and Technology* (2008), doi: [10.1016/j.compscitech.2008.04.013](https://doi.org/10.1016/j.compscitech.2008.04.013)

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**ON THE FREE VIBRATION OF SANDWICH PANELS WITH A  
TRANSVERSELY FLEXIBLE AND TEMPERATURE DEPENDENT  
CORE MATERIAL – PART II: NUMERICAL STUDY**

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**ABSTRACT**

This paper, the second of two, presents a numerical study of a simply-supported sandwich panel that is based on the mathematical formulation that appears in part I. The solution of the unknowns in the case of a simply-supported panel is based on a trigonometric series solution and it converts the set of PDE's into an algebraic set of equations that are described by stiffness and mass matrices. In addition, it studies numerically, for a specific sandwich panel construction, the effects of the degradation of the mechanical properties of the core as a result of the thermal field on the free vibration response of the two computational models. The results of the mixed formulation model, denoted by MF, and the displacement formulation, denoted by DF, reveal a significant reduction of the eigenfrequencies as well as a shifting of the eigen-modes from higher modes to lower ones with increasing temperature.

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### Simply-Supported Panel –Mixed Formulation

The considered sandwich panel is simply-supported at the edges of the upper and the lower face sheets, and the vertical displacements of the core through its depth are prevented at the edges using a specific edge beam, see Fig. 1a. Furthermore, the face sheets are assumed to be isotropic or laminated composite with a symmetric layup. For this case an analytical closed-form solution exists which consists of a Fourier sine series in the space coordinates, which fully satisfies the boundary conditions of a simply-supported panel multiplied by a harmonic time function. In addition, since each coefficient of the Fourier series is independent, a one-term solution is used. The series solution reads ( $j=t,b$ ):

$$\begin{aligned} u_{oj}(x, t) &= \left( \sum_{m=1}^M C_{u_{oj_m}} \cos(\alpha_m x) \right) e^{I \omega t} \\ w_j(x, t) &= \left( \sum_{m=1}^M C_{w_{j_m}} \sin(\alpha_m x) \right) e^{I \omega t} \\ \tau(x, t) &= \left( \sum_{m=1}^M C_{\tau_m} \cos(\alpha_m x) \right) e^{I \omega t} \end{aligned} \quad (1)$$

where  $M$  are the number of terms in the truncated series;  $C_{f_m}$  ( $f_j = u_{ob}, w_b, u_{ob}, w_b, \vartheta$ ) are the constants of the series solution;  $\alpha_m = m\pi / L$  where  $m$  is the wave numbers,  $L$  is the length of the panel,  $I$  is the imaginary unit (complex notation) and  $\omega$  is the eigenfrequency of the panel.

The solution is determined through substitution of a general term of the series, Eqs. (1), into the equations of motion of the face sheets, see Eqs. (11) in first part, along with the force-displacements relations of isotropic face sheets, see Eqs. (13) in first part, and the compatibility equation, see Frostig and Thomsen (2007b). This yields a set of homogeneous algebraic equations for each wave number  $m$  that replaces the set of partial differential equations (PDEs). Thus, the solution of the PDE's is replaced by an eigenvalue problem, with

a mass and a stiffness matrix, where the eigenfrequency equals to the eigenvalue and the series constants for each wave number  $m$  are the corresponding eigenvectors as follows:

$$\left( \mathbf{K}_m - \omega_m^2 \mathbf{M}_m \right) \mathbf{C}_m = \mathbf{0} \quad (2)$$

where  $\mathbf{K}_m$  and  $\mathbf{M}_m$  are the stiffness and the mass matrices that correspond to the  $m$ 'th harmonic term in the series;  $\omega$  is the eigenfrequency that corresponds to the  $m$ 'th term,  $\mathbf{C}_m = [C_{w_t}, C_{w_b}, C_{u_{ot}}, C_{u_{ob}}, C_{\tau}]$  is the eigenvector, and  $\mathbf{0}$  is a null vector. The mass matrix is similar to the one that appears in Frostig and Baruch (1994). The mass and the stiffness matrices are not presented for the sake of brevity.

The dimension of the stiffness matrix is 5, and the dimension of the mass matrix is only 4 since no mass is related to the shear stress, but it has been extended to fit the dimension of the stiffness matrix. The matrices are not presented for the sake of brevity. The stiffness matrix consists of four equations for the face sheets that also include inertia terms, while the compatibility equations, which describe the perfect bonding condition in the longitudinal direction at the lower face-core interface under static or dynamic loadings, has no inertia loading. Hence, the eigenvalue problem yields only *four* eigenfrequencies for each value of  $m$ .

### Simply-Supported Panel - Displacement Formulation

For a simply supported sandwich panels with isotropic face sheets or symmetrical lamination lay-up a closed-form solution in the form of a series of trigonometric functions, which satisfies the boundary conditions exist. The solution can be expressed as:

$$u_{oj}(x, t) = \left( \sum_{m=1}^M C_{u_{ojm}} \cos(\alpha_m x) \right) e^{i\omega t} \quad (3)$$

$$w_j(x, t) = \left( \sum_{m=1}^M C_{w_j} \sin(\alpha_m x) \right) e^{I\omega t} \quad (j = t, b)$$

$$u_k(x, t) = \left( \sum_{m=1}^M C_{u_k} \cos(\alpha_m x) \right) e^{I\omega t} \quad (k = o, 1, 2, 3)$$

$$w_i(x, t) = \left( \sum_{m=1}^M C_{w_i} \sin(\alpha_m x) \right) e^{I\omega t} \quad (i = 0, 1, 2)$$

where  $C_{f_m}$  ( $f_i = u_{ob}, w_b, u_{ob}, w_b, u_{o,1,2,3}, w_{o,1,2}$ ) are the constants of the series solution.

The solution is determined through substitution of a general term (the  $m$ 'th wave number) of the series, see Eqs. (3) in Part I, into the equations of motion of the face sheets and the core, see Eqs. (16) to (18) in Part I, along with the force-displacements relations of isotropic face sheets, see Eqs. (13) in Part I, and the high-order stress resultants. This yields a set of homogeneous algebraic equations for each  $m$ -term that may be described by a mass and a stiffness matrix, see Eq. (2). Here it should be noticed that *eight* unknowns can be isolated through the solution of eight algebraic governing equations. These functions consist of four functions of the displacements within the core that are determined using the four compatibility equations, see Eqs. (18) in Part I, and the four Lagrange multipliers that are isolated analytically from the seven equations of the core, see Eqs. (17) in Part I. Hence, the number of unknowns reduces to seven, which corresponds to the real number of boundary conditions. Thus, although the full set of the governing equations consist of 15 equations, see Eqs. (16) to (18) in Part I, the actual number of eigenfrequencies is only 7. The condensed mass and the stiffness matrices of dimension 7, after substitution of the eight unknowns are not presented for the sake of brevity.

## NUMERICAL STUDY

The numerical study investigates the role of the degrading core properties as a result

of elevated temperature on the free vibration response of a simply-supported unidirectional sandwich panel using the two developed high-order sandwich models. A symmetric sandwich panel with a light PVC foam core has been considered, and the results include the non-dimensional eigenfrequencies for  $m=1$ , the first sinus halfwave along with the corresponding eigen-modes at room temperature level,  $T=20^{\circ}\text{C}$ , and at the highest elevated temperature. In addition, the degradation curves of the non-dimensional eigenfrequencies versus the temperature are presented for the various eigenmodes.

The considered unidirectional sandwich panel is assumed to be 60 mm wide, see Fig. 1a, and to consist of two identical laminated face sheets and a PVC foam core. The face sheets are laminated composites made of glass fibers with a thickness of 6 mm each and an equivalent modulus of elasticity of 18000 MPa and a density of  $2000\text{ kg/m}^3$ . The foam core of PVC HD 100, is 60 mm thick with a modulus of elasticity of 85 MPa and a shear modulus of 16 MPa at room temperature, and its density is  $100\text{ kg/m}^3$ . The temperature field within the core is assumed to vary linearly across the height, see Fig. 1a. The moduli of the core are temperature-dependent, and the  $E_{cz}(T)$ ,  $G_{xzc}(T)$  vs. temperature ( $T$ ) plots appear in Fig. 1b, in the temperature range of  $20\text{-}80^{\circ}\text{C}$ , based on the data in DIAB (2003). The through-thickness distribution of the core appears in Fig. 1c for the case of thermal gradient of  $\Delta T=50^{\circ}\text{C}$  across the core height. Two different types of temperature distributions have been considered, see Fig. 1a. In the first one the distribution is uniform and changing from  $20^{\circ}$  to  $75^{\circ}\text{C}$ , while in the second one the temperature distribution is linear with a gradient  $\Delta T$  across the core height, where the temperature at the lower face sheets remains at  $20^{\circ}\text{C}$  and the temperature at the upper face sheet changes from  $20^{\circ}$  to  $75^{\circ}\text{C}$ .

The non-dimensional eigenfrequencies that correspond to the first sinus half wave,  $m=1$ , at room temperature,  $T=20^{\circ}\text{C}$  and at  $T=75^{\circ}\text{C}$ , with a uniform and a linear temperature

field, of the two models appear in Table 1. The eigenfrequencies have been normalized with respect to the fundamental bending eigenfrequency of an equivalent single layer sandwich panel with a bending rigidity that equals the overall flexural rigidity of the sandwich panel. In Fig. 2, the first *four* eigenmodes predicted by the two computational models for the uniform temperature distribution at the low,  $T=20^{\circ}\text{C}$ , and the high temperature,  $T=75^{\circ}\text{C}$ , are presented.

The eigenfrequencies results for the first wave number, see Table 1, reveal that the values decrease significantly as a result of the elevated temperature that causes degradation of the mechanical properties of the core. In addition, the results of the two models coincide at the lower eigenfrequencies, and for the higher modes the results of the second model are always smaller than those of the first model, which reflects that model II is more flexible than the model I. The eigenmodes, see Fig. 2, are arranged in increasing order of the corresponding eigenfrequencies, see Table 1. In Fig. 2, MF stands for Mixed formulation (Model I), whereas DF stands for Displacement Formulation (Model II). In general, a typical sandwich panel with *uniform* mechanical properties has four typical eigenmodes. The first one is an overall mode where the two face sheets undergo a vertical displacement in phase, see mode no. 1 in Fig. 2a. The second mode corresponds to identical in-plane displacement of the two face sheets, see mode 2, denoted as axial mode. The third mode consists of in-plane displacements of the face sheets that are out of phase, see mode 3, denoted as out of phase axial mode. Finally, the fourth mode, see mode no. 4 in Fig. 2a, corresponds to vertical displacements of the face sheets that are in opposite directions, also denoted as the pumping mode. However, when the uniform temperature increases to  $75^{\circ}\text{C}$ , see Fig. 2b, the first eigen-mode coincides with the global mode, while the second eigen-mode corresponds to the pumping mode rather than to the axial mode, which reflects shifting of the higher mode to the lower ones as the temperature is increased and the vertical rigidity of the core degrades. The third and the fourth



modes correspond to the axial modes in phase and out of phase, respectively.

In Fig. 3, the eigenfrequency values versus the uniform temperature within the core, is presented for the first wave number. In the first mode, see mode no. 1, the results of the two computational models coincide. In the second mode, see mode no. 2 in Fig. 3, the eigenfrequency of the first model (Mixed Formulation – MF) remains almost unchanged up to 50°C after which it drops, while for the second model (Displacement Formulation – DF) the eigenfrequency reduces slightly up to 50°C. At temperatures above say 55 °C the two models coincide. In the third mode, see mode 3, there is a large difference between the results of the two models above 50°C. In the fourth mode the results almost coincide. The difference in the results is a result of the shifting of the higher eigenmodes to the lower ones.

The results for the case of linear temperature distribution across the core thickness, where the lower face remains at 20°C while the temperature at the upper face sheet equals  $20+\Delta T$ , appear in Table 1 and Figs. 4 and 5. Here, the eigenmodes are not pure modes as a result of the coordinate-dependent core properties (degradation of elastic moduli with increasing temperature, see Fig. 1c). Hence, the eigenmodes in each model are not exactly the same as for the case of uniform temperature. The reduction in the eigenfrequencies, see Table 1, between the thermal gradients of  $\Delta T=10^\circ\text{C}$  and  $\Delta T=55^\circ\text{C}$  are not as significant as observed for the uniform temperature case. Notice here that here the results of Model II in some cases are predicted to be higher than those predicted using Model I, although Model II models the sandwich panel as a more flexible structure. The difference is due to the fact that the eigenmodes of the corresponding eigenfrequencies in the two models are not exactly the same as in the case with the uniform temperature distribution. The corresponding eigenmodes, see Fig. 4, follows in general the same trends as those obtained for a uniform temperature distribution of 20°C with some changes. At the lower gradient, at  $\Delta T=10^\circ\text{C}$ , (see Fig. 4a) the results almost

coincide with those of the uniform distribution, see Fig. 2a, but with minor changes. In the case of the highest gradient ( $\Delta T=55^\circ\text{C}$ ), where the upper face sheets reach a temperature of  $75^\circ\text{C}$ , see Fig. 4b, there is a shift in the modes and the pumping mode again becomes the second one. However, as a result of the variable stiffness, the pumping mode of the second model does not correspond to the pure pumping mode, see mode no. 4 in Fig. 2a. Here, the maximum displacements is that of the in-plane displacement of the core,  $u_1(x,t)$ , rather than the face sheet displacements.

The eigenfrequencies versus the thermal gradient, for the various modes appear in Fig. 5. A significant reduction in the eigenfrequency of the first eigenmode is observed, but the reduction is than observed for the uniform temperature case. The reduction in eigenfrequencies for the second mode, see mode no. 2 in Fig. 5, are similar to those of the uniform temperature case but at different values. Also here the predicted eigenfrequency corresponding to Model I is almost constant up to a gradient of  $45^\circ\text{C}$  after which it drops steeply. The results obtained using Model II differ from Model I, and they change throughout the range of temperature. However the entire changes here are about 10% of the maximum values. For the other two modes of vibration, see modes 3 and 4, the results of the two models are quite similar. The differences between the results of the two models are mainly a result of the shifting from higher to lower vibration modes.

## SUMMARY AND CONCLUSIONS

A numerical study based on a rigorous systematic analysis of the problem of free vibrations of sandwich panels with compliant and temperature dependent core properties that appears in part I has been presented.

Two different temperature distributions have been considered in this part. The first one is a uniform distribution where the temperature variation through the core thickness is

uniform, and where (accordingly) the thermal degradation of the core properties is also uniform. Secondly a linear through-thickness temperature distribution is assumed. In the examples presented herein, it is assumed that the temperature at the lower face sheet remains constant at 20°C, while the temperature at the upper face sheet changes as a result of a thermal gradient. The linear distribution yields a core with coordinate dependent mechanical properties. The panel considered consists of a PVC foam core made by DIAB. The temperature range considered corresponds to a working range of temperature between 20°C to 80°C for the particular foam core considered.

The results are presented in terms of: the eigenfrequencies normalized with respect to the fundamental eigenfrequency (bending) of an equivalent single layer sandwich panel with the same flexural rigidity. More specifically, the first four eigen-modes; and the curves of the eigenfrequencies versus temperature or thermal gradient have been studied.

The elevated temperatures cause the modulus of elasticity and the shear modulus of the core to decrease. These two moduli affect the dynamic response of the overall panel in terms of eigenfrequencies and eigenmodes. The results of the two different structural models reveal that there is a decrease in the eigenfrequency as the temperature or thermal gradient increases. This decrease is significant, especially at the lowest vibration modes. Thus, the first eigenfrequency reduces to almost a one third at the upper temperature ( $T=75^{\circ}\text{C}$ ) compared with the first eigenfrequency at room temperature ( $T=20^{\circ}\text{C}$ ) for the case of uniform temperature. The first eigenfrequency reduces by about 50% for the linear thermal gradient case for a thermal gradient of  $\Delta T=50^{\circ}\text{C}$  across the core thickness and with a lower face temperature of 20 °C. These reductions are caused by the degradation of the core shear modulus with increasing temperature, which yields a flexible structure in the vertical direction as the temperature is raised.

In addition, the degrading core properties also affect the order of the eigenmodes.

More specifically, the higher mode at room temperature becomes the second mode as the temperature is elevated. Thus, as the temperature increases, the pumping mode (where the phase sheets move vertically out of phase), which is the fourth mode at room temperature, shifts into the second vibration mode at elevated temperatures. This shifting is a result of the reduction of the elastic modulus of the core as the temperature is increased.

In the case of a uniform temperature distribution, the basic eigenmodes of a sandwich panels, i.e. the overall mode, the axial mode, the axial out-of-phase mode and pumping mode, are preserved throughout the range of working temperatures. This situation is preserved to a certain extent when the temperature distribution is linear through the core thickness, except that the mentioned vibration modes are not pure any more (i.e. they interfere with each other). Also for this case, there is shifting of the higher modes to lower modes ones as the thermal gradient increases.

Finally, modern sandwich structures often utilize compliant polymer foam core materials that display significant degradation of their mechanical properties with increasing temperatures. The degradation effects may be significant even within the working range of temperatures for realistic sandwich structures, and the results presented herein demonstrates that thermal fields imposed on such sandwich structures may influence the vibration response significantly. Thus, the eigenfrequencies may be reduced significantly, and vibration mode shifts may be experienced as well. Hence, a reliable design must take into account the effects of degrading core properties as a result of elevated temperature on the static and the dynamic sandwich panel responses.

## **ACKNOWLEDGEMENT**

The work presented was sponsored by the US Navy, Office of Naval Research (ONR), Grant/Award No. N000140710227: "Influence of Local Effects in Sandwich Structures under

General Loading Conditions & Ballistic Impact on Advanced Composite and Sandwich Structures”. The ONR program manager was Dr. Yapa D. S. Rajapakse. The financial support received is gratefully acknowledged.

ACCEPTED MANUSCRIPT

Mode Number	Uniform Temp. Dist.		Linear Temp. Dist.	
	Model I	Model II	Model I	Model II
1	0.6479*	0.6479	0.6337*	0.6377
	<i>0.2202**</i>	<i>0.2202**</i>	<i>0.3246**</i>	<i>0.3288**</i>
2	11.5589*	11.4430*	11.5589*	11.4320*
	<i>6.2785**</i>	<i>6.2764**</i>	<i>9.6875**</i>	<i>10.5967**</i>
3	16.2939*	16.2701*	16.0428*	16.0221*
	<i>11.5590**</i>	<i>8.2739**</i>	<i>11.5590**</i>	<i>10.9097**</i>
4	24.2916*	24.2872*	23.3905*	23.3846*
	<i>12.7115**</i>	<i>12.4318**</i>	<i>13.1102**</i>	<i>13.1313**</i>
5		38.3922*		36.8862*
		<i>13.725**</i>		<i>21.7500**</i>
6		73.1858*		70.4385*
		<i>19.3116**</i>		<i>52.0288**</i>
7		87.6079*		84.2340*
		<i>22.6428**</i>		<i>61.5314**</i>

**Table No. 1:** Non-dimensional eigenfrequencies,  $\omega_{da}$ , of the two computational models with  $m=1$ , at various temperatures and gradients for a uniform and a linear through-thickness distribution.

Comments: Uniform temperature: \* at  $T=20^{\circ}\text{C}$ , \*\* at  $T=75^{\circ}\text{C}$  (in italic)

Linear through-thickness distribution: \* at  $\Delta T=10^{\circ}\text{C}$  ( $T_t=30, T_b=20$ ), \*\* at  $\Delta T=55^{\circ}\text{C}$  ( $T_t=75, T_b=20$ ).

## FIGURE CAPTIONS

**Fig. 1:** Layout of sandwich panel along with variable moduli of the core: (a) Panel layout and temperature distribution; (b) Moduli of core versus temperature; (c) Moduli distribution through the depth of core for a specific linear temperature distribution.

**Fig. 2:** Four eigen-modes of first sinus half waves at various temperatures for a uniform distribution of the various models: (a) At 20°C; (b) At 75°C. Legend: Top – thick line, Bot - thin line, black- model I, blue – model II,  $\blacklozenge$  \_\_\_  $w_t$  or  $w_b$ ,  $\blacksquare$  - - -  $u_{ot}$  or  $u_{ob}$ ,  $\square$  ....  $u_o$ ,  $\circ$  - - -  $u_1$ ,  $+$  \_ \_ \_  $w_o$ , *MF* – Mixed Formulation, *DF* – Displacement Formulation.

**Fig. 3:** Eigenfrequencies versus temperature of the uniform temperature distribution for the first four modes corresponding to the two computational models. Legend: *MF*—Mixed Formulation, *DF*—Displacement Formulation.

**Fig. 4:** Four eigen-modes of first sinus half waves at various temperature gradients for a linear thermal through-thickness distribution of the various models: (a) At  $\Delta T=10^\circ\text{C}$ ; (b) At  $\Delta T=55^\circ\text{C}$ . Legend: Top – thick line, Bot - thin line, black- model I, blue – model II,  $\blacklozenge$  \_\_\_  $w_t$  or  $w_b$ ,  $\blacksquare$  - - -  $u_{ot}$  or  $u_{ob}$ ,  $\square$  ....  $u_o$ ,  $\circ$  - - -  $u_1$ ,  $+$  \_ \_ \_  $w_o$ , *MF* – Mixed Formulation, *DF* – Displacement Formulation.

**Fig. 5:** Eigenfrequencies versus gradient of the linear temperature distribution for the first four modes and of the two computational models. Legend: *MF*—Mixed Formulation, *DF*—Displacement Formulation.

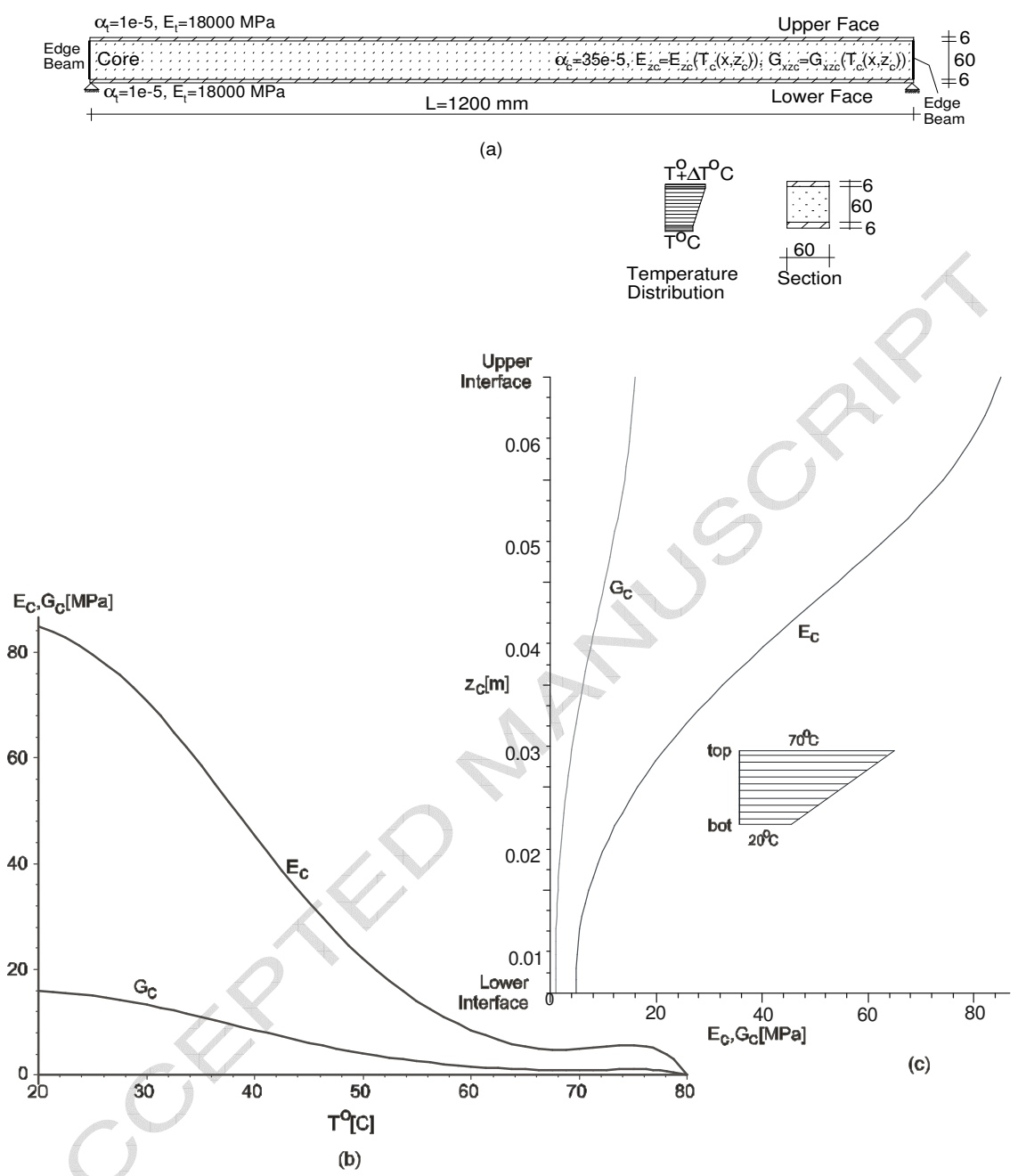


Fig. 1



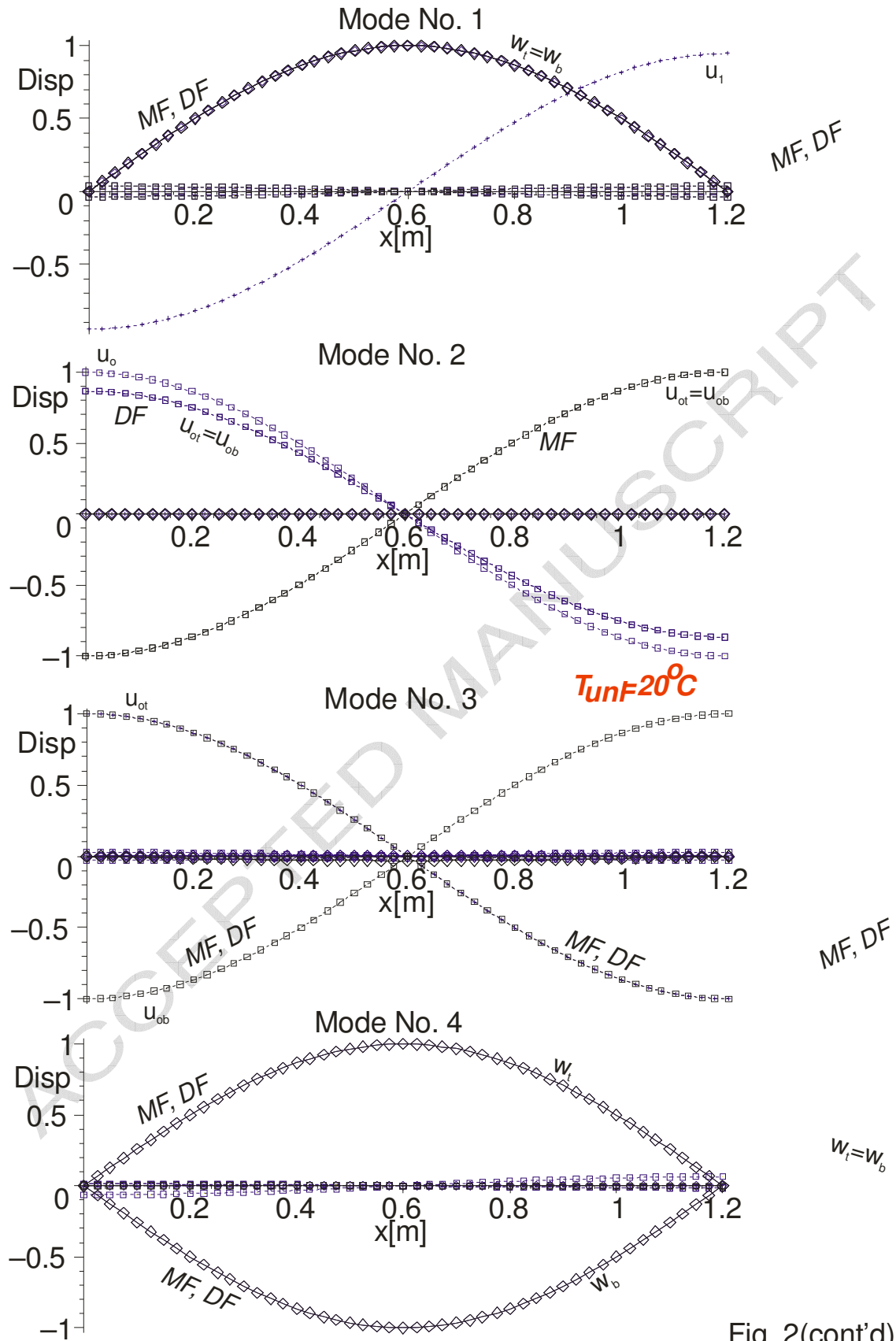


Fig. 2(cont'd)

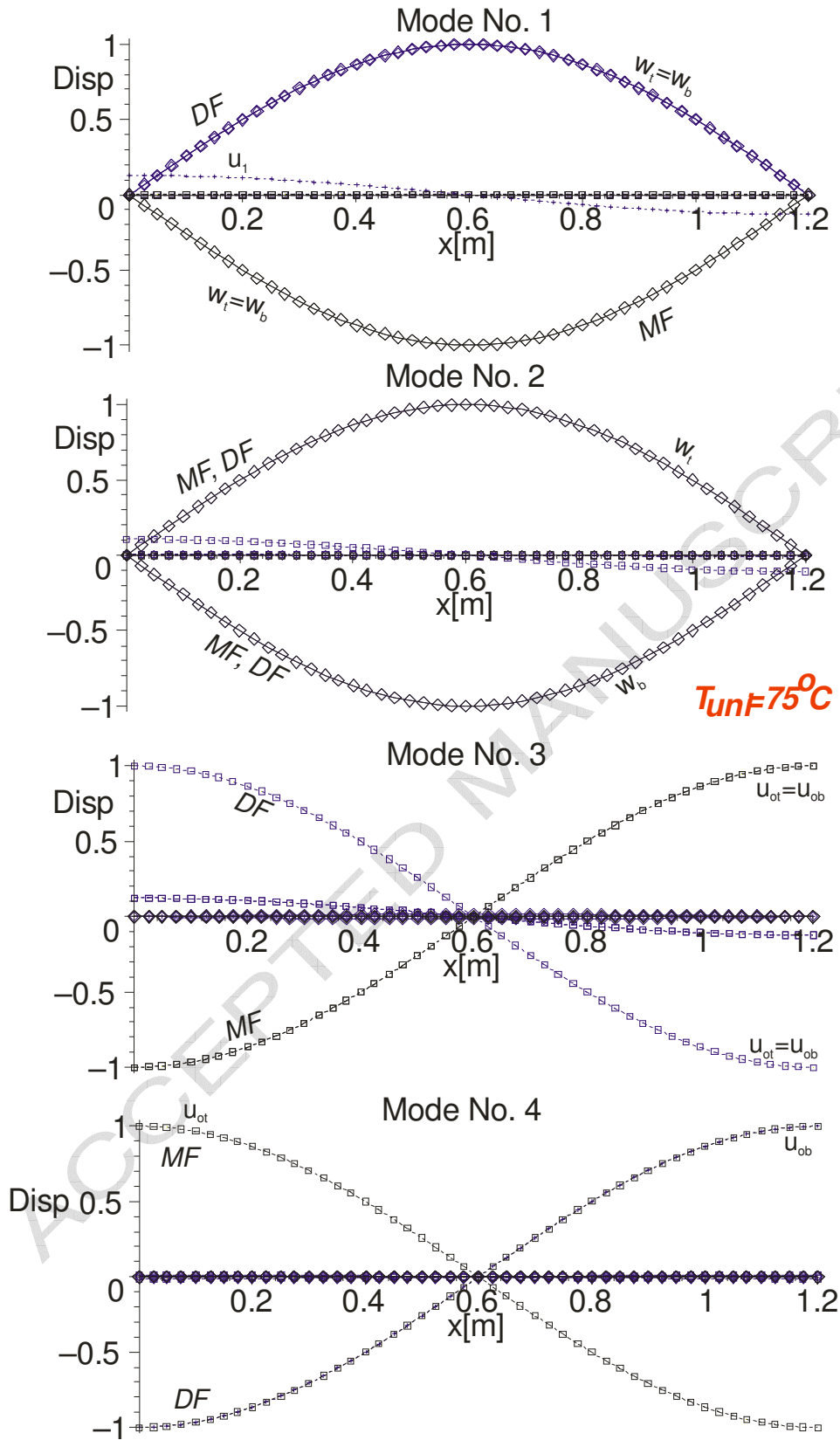


Fig. 2

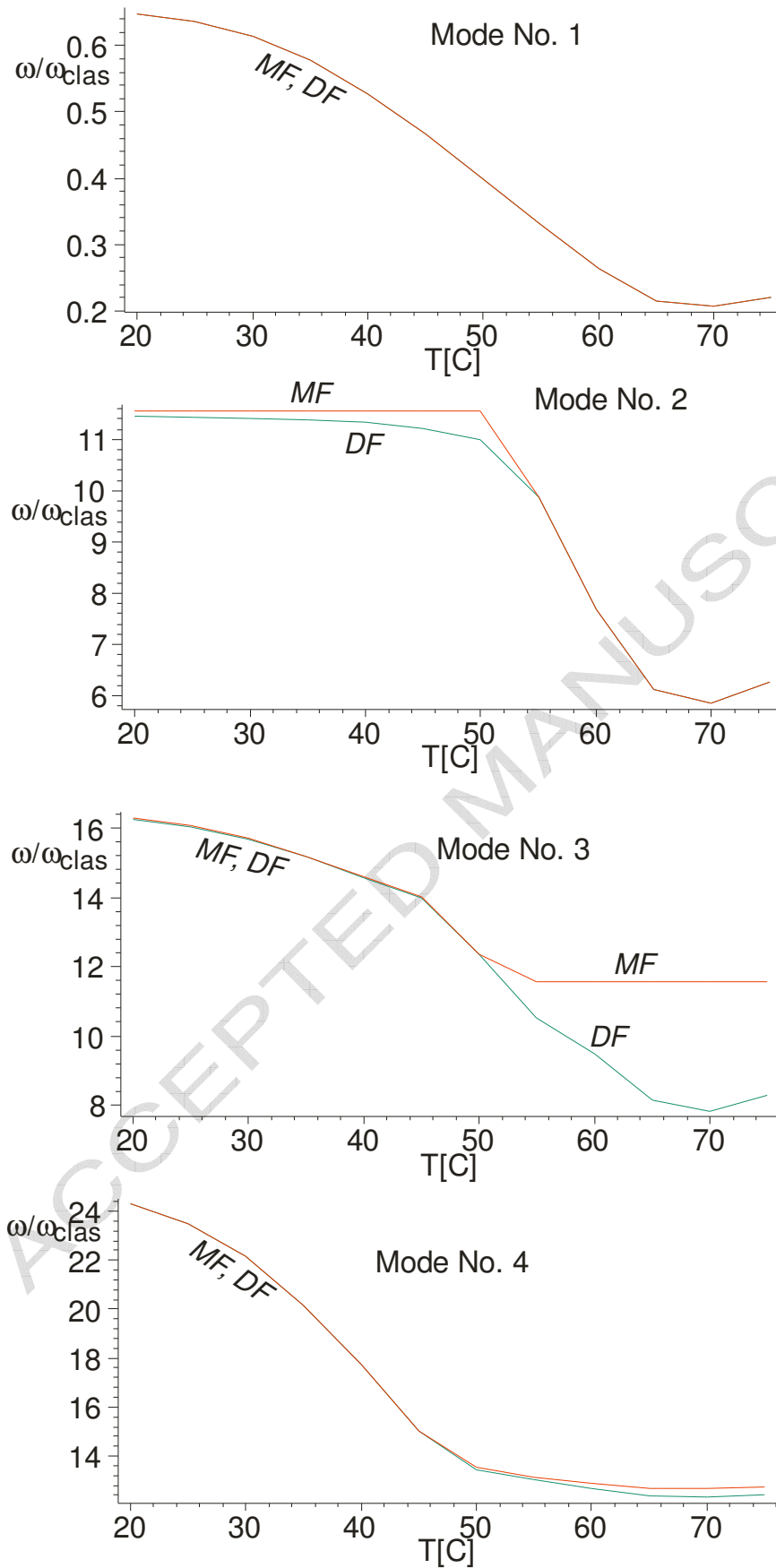
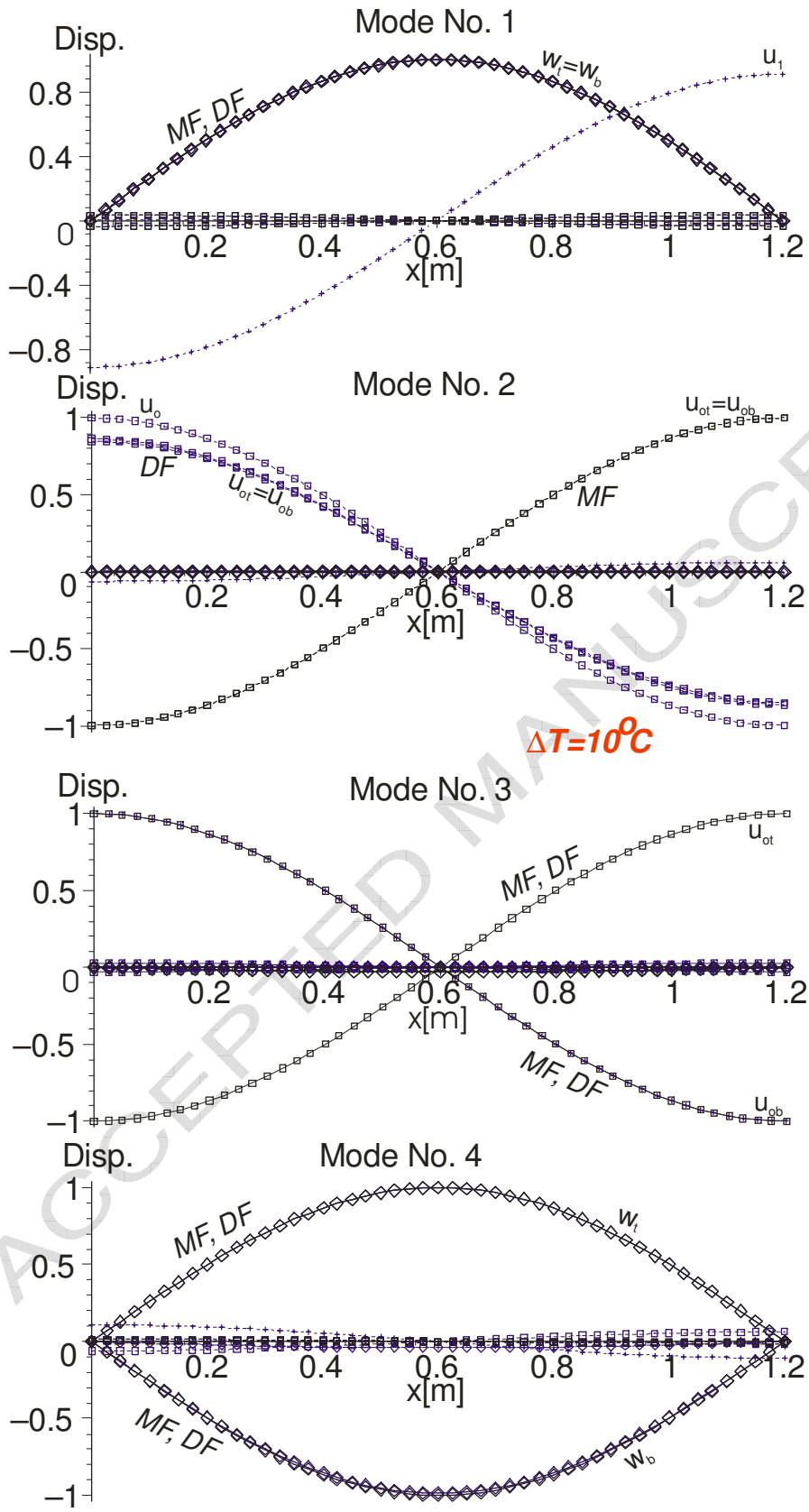


Fig. 3



(a)

Fig. 4 (cont'd)

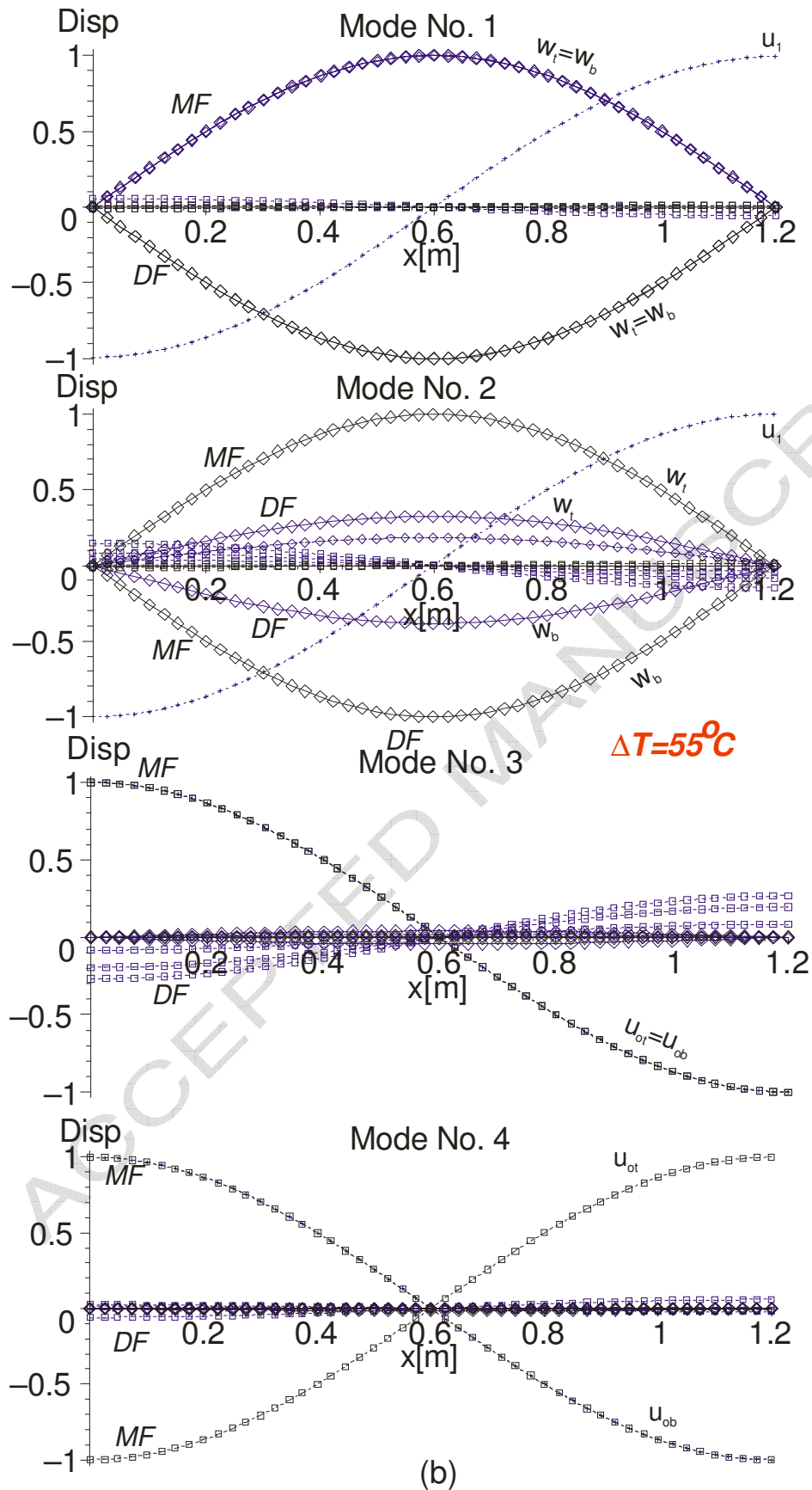


Fig. 4

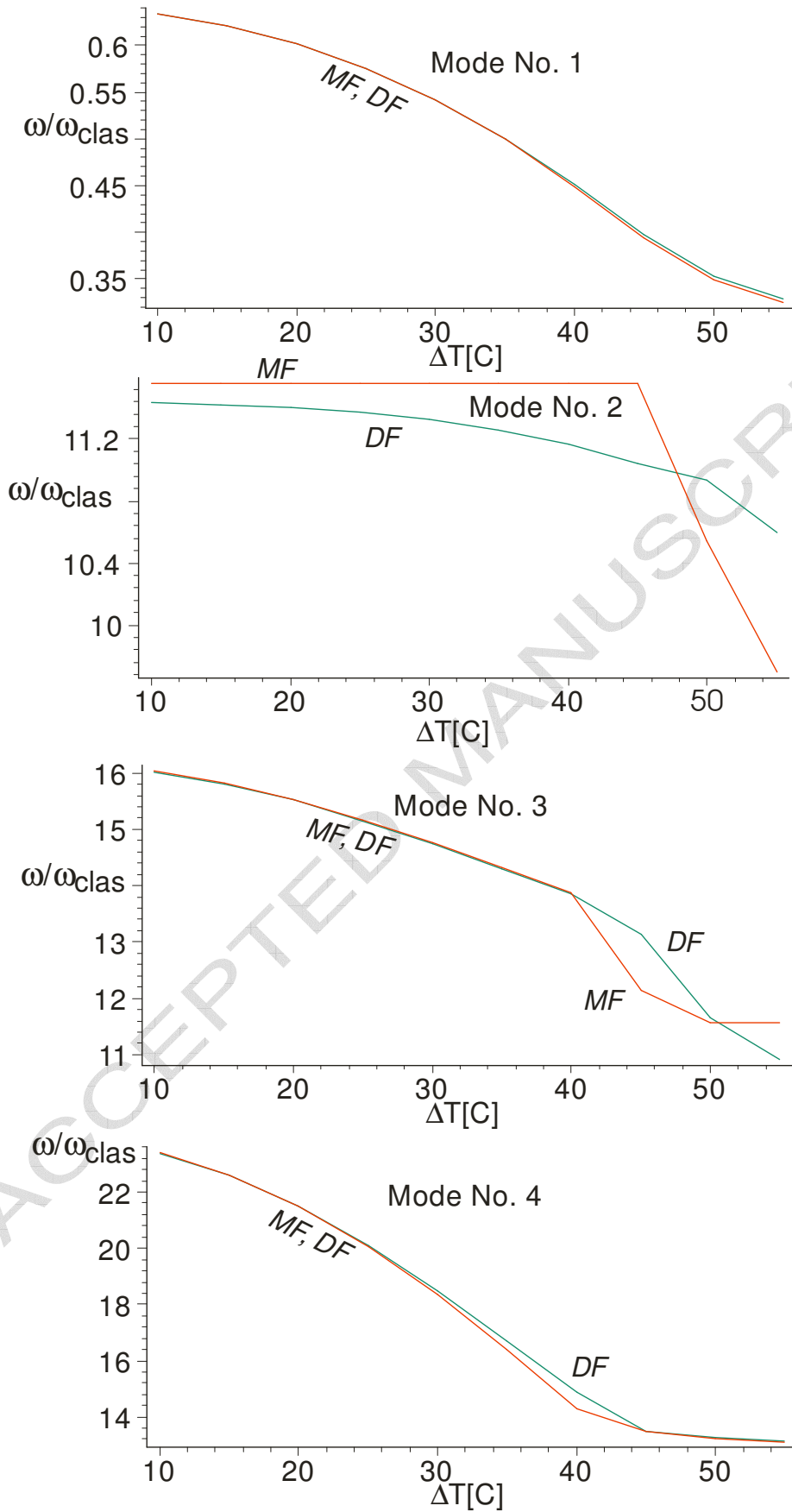


Fig. 5