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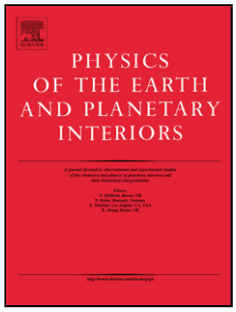

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Analysis of the variability of the axial dipole moment of a numerical geodynamo model

J. Kuipers [a], P. Hoyng [b,*], J. Wicht [c], G.T. Barkema [a,†]

[a] Theoretical Physics, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands;
[b] SRON Netherlands Institute for Space Research, Sorbonnelaan 2, 3584 CA Utrecht, The Netherlands;
[c] Max Planck Institut für Sonnensystemforschung, Lindau, Germany

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Abstract

We have analysed the time evolution of the Axial Dipole Moments (ADMs) from three numerical geodynamo models by relating it to the Fokker-Planck equation governing the systematic and random ADM motion. We have determined the effective growth rate of the ADM and the diffusion coefficient D characterising its random fluctuations. We find that the numerical ADM data exhibit a nonlinear quenching that is not significantly different from that of the Sint-2000 data. The quenching is only partly due to a reduction of the r.m.s. convective flow speed with increasing ADM. Our results suggest that in these numerical models similar mechanisms may be at work as in the earth's core, and that the results of Brendel et al. (2007) are unlikely to be an artifact caused by the restricted length of the dataset. They also suggest that the dynamics of the ADM is that of a Brownian particle (i.e. driven by additive noise) in a bistable potential, and we illustrate some consequences of this idea.

Keywords: Geodynamo, Hydromagnetic dynamos, Reversals, Secular variation, Turbulent convection, Stochastic processes.

1 Introduction

The strength of the geomagnetic dipole moment is variable on all time scales from a few 100 years and longer. The most spectacular manifestation of this variability are the occasional sudden reversals of the sign of the dipole moment. It is now generally accepted that the geomagnetic field is the result of inductive processes in the Earth's liquid metallic outer core. Several groups have confirmed this idea with the help of numerical simulations (Glatzmaier and Roberts, 1995; Kuang and Bloxham, 1997; Christensen et al., 1999).

The geomagnetic field is the result of many processes taking place in the convecting outer core that mutually interact in a complicated way. It is therefore not surprising that a statistical modelling of (the lowest multipole moments of) the field yields useful results. Constable and Parker (1988) were the first to give a complete characterization of the statistical properties of the geomagnetic field in terms of its spherical harmonic expansion coefficients. The authors showed that the distribution of the axial dipole consists of two

*Corresponding author.
†E-mail addresses: j.kuipers@uu.nl (J. Kuipers), p.hoyng@sron.nl (P. Hoyng), wicht@linmpi.mpg.de (J. Wicht), g.t.barkema@uu.nl (G.T. Barkema).

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Table 1: Model characteristics †

model	Δt (10^6 yr)	number of data points	Ra/Ra_c	mean Rm	reversals
$T2$	4.70	27982	6.7	325	none
$T4$	8.67	51492	8.9	430	~ 10
$T5$	2.80	14530	13.4	590	many

† The Ekman number is $E = 10^{-3}$, Prandtl number $P = 1$, magnetic Prandtl number $Pm = 10$. The magnetic diffusion time $d^2/\eta = 1.22 \times 10^5$ yr. Further details in Wicht et al. (2008).

gaussians shifted to the peak position of their polarity state, and that the expansion coefficients of the non-dipole field may, after appropriate scaling, be regarded as statistically independent samples of a single normal distribution with zero mean. Hulot and Le Mouël (1994) have extended this so-called GGP (Giant Gaussian Process) approach by considering the evolution of the statistical properties with time, and Bouligand et al. (2005) have tested the GGP modelling technique on hydromagnetic geodynamo simulations.

The statistical approach may be advanced further by modelling the *time evolution* of the multipole moments of the geomagnetic field over long time scales as a stochastic process. A first step in this direction was taken by Brendel et al. (2007), who analysed the time evolution of the Virtual Axial Dipole Moment (VADM), a measure of the geomagnetic dipole of which several records have been published. These authors used the Sint-2000 data of Valet et al. (2005), a time series of 2000 VADM values spanning a period of 2 Myr and several reversals.

An attractive feature of this method is that it allows us to measure certain dynamical properties of the geodynamo from the data. In this way, Brendel et al. (2007) were able to infer that the geomagnetic dipole has a linear growth time of 20^{+13}_{-7} kyr, and that the nonlinear quenching of the growth rate follows a quadratic function of the type $[1 - (x/x_0)^2]$, where x is the magnitude of the dipole moment. From the diffusive motion of the VADM Brendel et al. (2007) inferred a diffusion coefficient that depends only very weakly on the dipole strength, and this may indicate that the magnetic field quenches the magnitude of the turbulent velocity in the Earth's outer core.

The Sint-2000 record has 2000 data points, and comprises only a few reversals and few small VADM values. This prevented a determination of the diffusion coefficient for small VADM. It would be desirable to have a longer data set to put the conclusions of Brendel et al. (2007) on a firmer basis. Unfortunately, a significantly longer data set with comparable time resolution is not available for the geodynamo, and therefore we resort to numerical geodynamo models. We select time series of the axial dipole moment of three numerical geodynamo models, and repeat the analysis of Brendel et al. (2007) using these much longer data sets as input. Our aim is to see to what extent numerical models confirm the main conclusions of Brendel et al. (2007) and to show that they are not affected by the restricted length of the Sint-2000 data. Furthermore, we explore whether the results differ between the three numerical models that show very distinct reversal behaviours.

In the next section we recapitulate some basics of a stochastic process and we summarise our data analysis technique. In section 3 we describe the numerical model that we employ. Our results are presented in section 4, and some of the implications are discussed in section 5. A summary and our conclusions appear in section 6.

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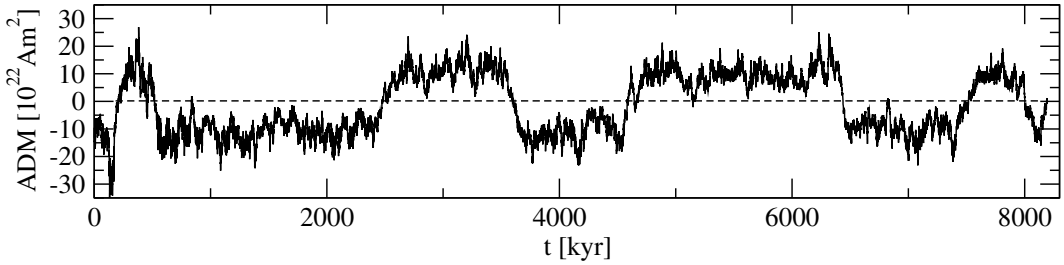

Figure 1: Axial dipole moment (ADM) as a function of time of the geodynamo model $T4$.

2 Theoretical background and data analysis

Consider a stochastic equation of the type

$$\dot{x} = v(x) + F(x)L(t) \ . \tag{1}$$

Here x is the value of the parameter studied, for example the axial dipole moment. The function $v(x)$ represents the effective growth rate of x, and is sometimes called the drift velocity. The fluctuations are embodied in the term $F(x)L(t)$ and they induce an additional diffusive motion of x.[1] Here $L(t)$ is a stationary random function with zero mean and a short correlation time $\tau_{\rm c}$:

$$\langle L(t)\rangle = 0, \quad \langle L(t)L(t-\tau)\rangle = L^2_{\rm r.m.s.}\tau_{\rm c}\,\delta(\tau) \ . \tag{2}$$

A short correlation time means that the duration $\tau_{\rm c}$ of the memory of $L(t)$ is much shorter than all other time scales in the process. Under these circumstances the autocorrelation function of $L(t)$ behaves as a δ-function of time. The probability distribution $\rho(x,t)$ of $x(t)$ determined by Eq. (1) obeys the Fokker-Planck equation (Van Kampen, 1992; Gardiner, 1990):

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(v\rho) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\,(D\rho) \ . \tag{3}$$

Here t is time, and v is again the effective growth rate of x. The diffusion coefficient is equal to

$$D \simeq 2F^2\int_0^\infty \langle L(t)L(t-\tau)\rangle\, d\tau \ \simeq \ F^2 L^2_{\rm r.m.s.}\tau_{\rm c} \ . \tag{4}$$

The Fokker-Planck equation is a simple and versatile tool for modelling the dynamics of a stochastic process. The statistical properties of a wide variety of different stochastic processes can be described by a Fokker-Planck equation (3), among which the time evolution of the geomagnetic dipole moment, see for example Hoyng et al. (2002).

Retrieval of $v(x)$ and $D(x)$

The question is now: given a realisation (a time profile) of the stochastic process $x(t)$, how may we determine the effective growth rate $v(x)$ and the diffusion coefficient $D(x)$? This issue has been addressed by Brendel et al. (2007), and we briefly summarize the idea here. We begin by discretizing the variable x in the Fokker-Planck equation (3):

$$\frac{\partial \rho_i}{\partial t} \ = \ \sum_j M_{ij}\rho_j \ . \tag{5}$$

Here M is a tridiagonal matrix and the indices i and j label the bins on the x-axis. The elements of M can be expressed in terms of the drift velocity v_i and diffusion coefficient D_i.

[1] The noise is called additive if F is constant, and multiplicative if $F \propto x$.

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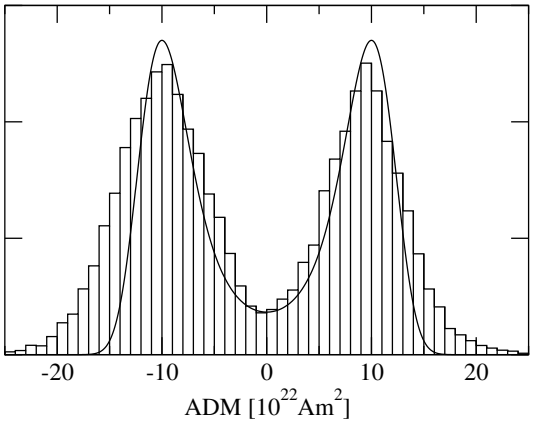

Figure 2: Amplitude distribution of the axial dipole moment of the $T4$ run in Fig. 1. The distribution happens to be rather symmetrical with respect to $ADM = 0$. But this is a coincidence as there are only some 10 reversals. Other runs of comparable length have noticeably asymmetric peaks. The drawn line is the theoretical distibution (9) for $\lambda_0 x_0^2/2D = 2$ and $x_0 = 10 \cdot 10^{22}\,\mathrm{Am}^2$, see section 5.

Next, we construct the matrix T whose elements T_{ij} contain the transition probabilities for a system in bin j at some time t to move to bin i at a later time $t + \tau$. To this end we construct from the data $x(t)$ a histogram of all sets $\{x(t),\, x(t + \tau)\}$. In the present case, where $x(t)$ is a sequence of axial dipole moments, there is no sign preference and the sequences $x(t)$ and $-x(t)$ are equally likely realisations. Therefore we add to the histogram all sets $\{-x(t),\, -x(t+\tau)\}$. In this way we effectively symmetrise the data. Then we count the number of times that the system is located in bin j at some time t and in bin i at time $t + \tau$. The time lag τ must be chosen larger than the correlation time of the randomly fluctuating part of the system, and smaller than the time scale on which the data changes systematically.

It follows from Eq. (5) that $\rho_i(t+\tau) = \sum_j [\exp(\tau M)]_{ij} \rho_j(t)$. The theoretical transition matrix T_{th} is therefore

$$T_{\mathrm{th}} \;=\; \exp(\tau M)\ . \tag{6}$$

Finally, we minimize the difference $||T - T_{\mathrm{th}}||$ between the measured and the theoretical matrix by fitting the elements of M. In this way we obtain M and hence also v and D. Details may be found in Brendel et al. (2007) and Newman and Barkema (1999).

3 Numerical dynamo model

We have selected three dynamo models that represent three different dynamo regimes in terms of reversal behaviour. In model T2 the magnetic field is dominated by a strong, relatively stable dipole component that never reverses. Model T4 shows Earth-like rare reversals and excursions, see Fig. 1. The dipole is once more strong and dominant during the stable polarity epochs. In model T5 the dipole is no longer stronger than the other field contributions and continuously changes polarity. Fig. 2 shows the resulting bimodal distribution of the axial dipole moment (ADM) in model T4. The distribution for T2 shows a single peak centered around a mean ADM of $-18 \times 10^{22}\,\mathrm{Am}^2$. For T5 the distributions for positive and negative ADMs have merged into a single peak centered around zero.

The selected models are full 3D numerical simulations of convection and magnetic field generation in a rotating spherical shell with an insulating outer boundary and a conducting inner core that is allowed to rotate (Wicht et al., 2008). Inner and outer core have the same magnetic resistivity η. For details on the numerical method see Christensen and Wicht (2007). The convection is driven by an imposed temperature difference characterised by the Rayleigh number Ra. The reversal behaviour depends on how strongly the system

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is driven, and we increase Ra from the stable-dipole regime to the regime of continuous reversals (Kutzner and Christensen, 2002; Wicht et al., 2008).

In Table 1 we list the ratio of Ra and the critical Rayleigh number Ra_c, along with other characterizing properties. The Ekman number ($E = \nu/\Omega d^2 = 10^{-3}$), which measures the relative magnitude of viscous and Coriolis forces, the Prandtl number ($P = \nu/\kappa = 1$), and the magnetic Prandtl number ($Pm = \nu/\eta = 10$) are identical in all models. Here, d is the shell thickness, ν the kinematic viscosity, and κ the thermal diffusivity. The magnetic Reynolds number is defined as $Rm = u_{\mathrm{r.m.s.}} d/\eta$ where $u_{\mathrm{r.m.s.}}$ is the magnitude of the flow averaged over the core. Rm measures the relative importance of magnetic field generation to resistive magnetic field decay, and is estimated to $Rm \approx 500$ for the Earth (Christensen and Tilgner, 2004). The success of numerical simulations in modeling the geodynamo is often attributed to the fact that both Rm and the Elsasser number ($\equiv$ ratio of the magnitudes of the Lorentz force and Coriolis force in the Navier-Stokes equation) assume realistic values.

The main reason for choosing these models is that the relatively large Ekman number permits longer numerical runs which are necessary for the statistical analysis presented here. While the adopted Ekman number is larger than in other contemporary dynamo simulations and much too large when compared to Earth, Wicht et al. (2008) have shown that the models represent the typical behaviour also found at lower Ekman number.

Assuming a magnetic diffusion time of $\tau_\eta = d^2/\eta = 1.22 \times 10^5\,\mathrm{yr}$, the typical flow time scale can be estimated as $\tau_c = d/u_{\mathrm{r.m.s.}} = \tau_\eta/Rm$, which amounts to roughly 250 yr for $Rm = 500$. We may identify τ_c with the correlation time of the convection, since the simulations confirm that the azimuthally averaged (mean) flow components are small compared to the flow in the individual convective cells. Though the numerical simulations run at a much smaller time step, τ_c is about the time resolution we use for our analysis, as a higher resolution would not further improve the statistical accuracy. The total number of data point used for analyzing each of the three models is given in Table 1.

4 Results

We have analysed the time series of axial dipole moments of model $T2$, $T4$ and $T5$ as outlined in Section 2, using a time lag of $\tau = 10^4$ yr, considerably larger that the turnover time of 250 yr. The resulting values for v and D as a function of ADM are shown in Fig. 3. Model $T2$ is weakly forced and its ADM hardly ever attains values smaller than $\sim 5 \times 10^{22}$ Am^2. Therefore we cannot infer its v and D at these small ADM, and the corresponding part of the horizontal axis has been left out in the top panel of Fig. 3.

The distribution $\rho(x)$ of the ADM displayed in Fig. 2 can be argued to obey the Fokker-Planck equation (3) with (Hoyng, 2009, in preparation):

$$v(x) \;=\; \lambda_0\, q(x/x_0)\, x \;, \tag{7}$$

$$D(x) \;\simeq\; \frac{\beta}{R^2}\,\frac{x^2}{N} + \text{const.} \tag{8}$$

Here x is the ADM, x_0 its value in nonlinear equilibrium; λ_0 is the linear growth rate of the axial dipole mode, and $q(u)$ is the quenching function, of which we know that it is even and monotonously decreasing with $q(0) = 1$ and $q(1) = 0$. On this ground $q(u) = 1 - u^2$ is believed to be a fair approximation, but the actual form is not known. Furthermore, β is the coefficient of turbulent diffusion, R the radius of the outer core (so β/R^2 is the inverse turbulent diffusion time) and N is a measure of the number of convection cells in the outer core. The small constant in (8) is due to the overtones and is important at small x, but for the present discussion we may ignore it.

The scaling $D\,(:)\,x^2$ may be understood as follows. According to the induction equation of MHD, the random (diffusive) motion of the magnetic field is governed by $\partial \boldsymbol{B}/\partial t =$

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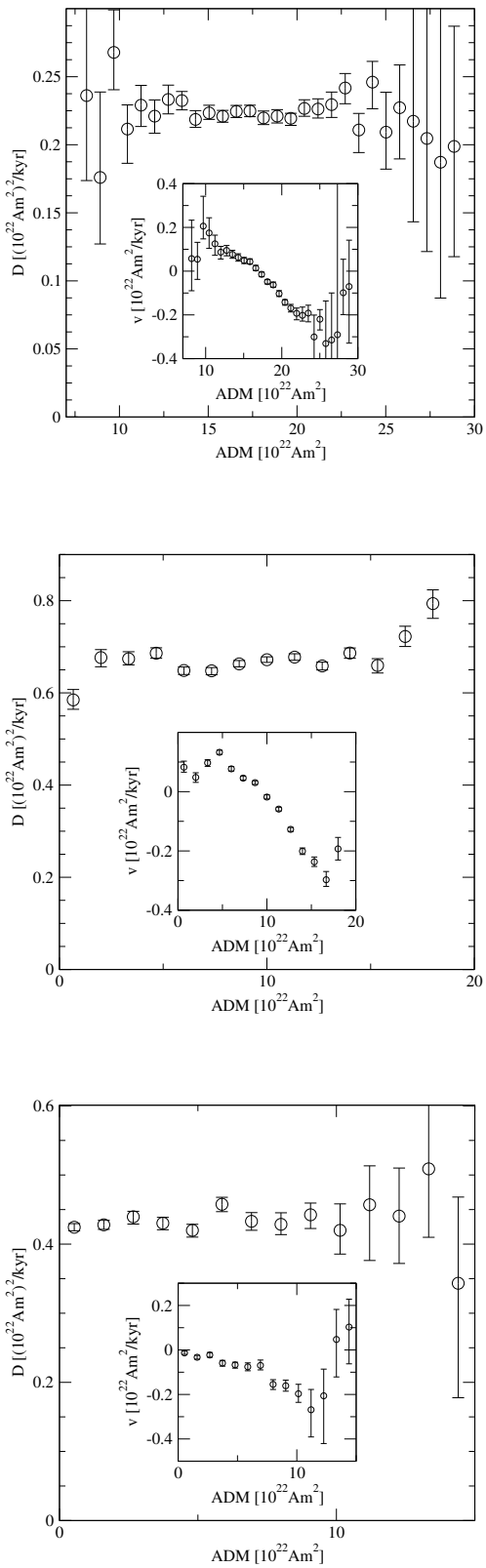

Figure 3: Diffusion coefficient D and effective growth rate v (inset) as a function of the axial dipole moment (ADM) for the three models $T2$ (top panel), $T4$ (middle panel) and $T5$ (bottom panel). The error bars indicate the range that contains 80% of the inferred values. Because the data have been symmetrised we display D and v for positive ADM only.

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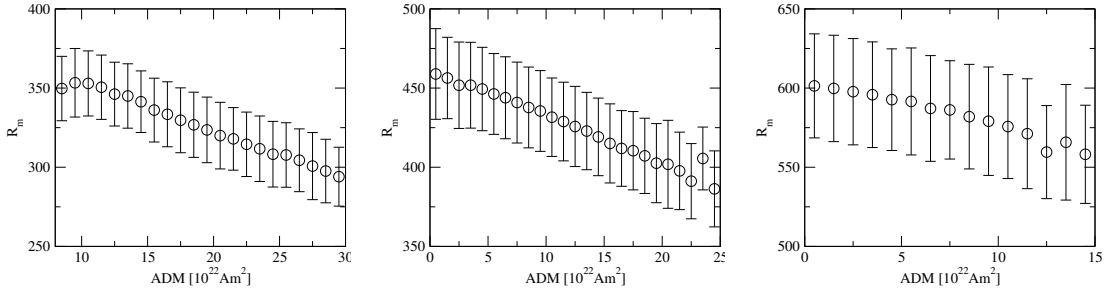

Figure 4: The magnetic Reynolds number $Rm = u_{\mathrm{r.m.s.}} d/\eta$ (vertical axis) as a function of the axial dipole moment (horizontal axis), for model $T2$, $T4$ and $T5$ (from left to right). The bars indicate one sigma standard deviations.

$\nabla \times (\boldsymbol{u} \times \boldsymbol{B})$, where $\boldsymbol{u}$ is the convective part of the flow in the outer core. On replacing $\boldsymbol{B}$ by the axial dipole moment x we find $\dot{x} \simeq \mathrm{const} \cdot u \cdot x$. This is of the type (1) with $F(x) = \mathrm{const} \cdot x$ and $L(t) = u(t)$. With the help of (4) we obtain: $D\,(:)\; x^2 u^2_{\mathrm{r.m.s.}} \tau_{\mathrm{c}} \simeq \beta\, x^2$, as in (8). The argument is linear and treats u as autonomous, not influenced by x. The bottom line is that the change in x in a time δt is $\delta x \simeq \mathrm{const} \cdot u\, \delta t \cdot x$, which is larger for given u if x is larger. This explains why D should scale as x^2 in the absence of nonlinear feedback.

Quenching

The shape of the effective growth rate $v(x)$ of run $T4$ (middle panel of Fig. 3) agrees qualitatively with the ad-hoc formula $v(x) = \lambda_0[1-(x/x_0)^2]x$, and we find a linear growth time of $\lambda_0^{-1} \sim 30$ kyr (with considerable uncertainty). However, in the other two runs $T2$ and $T5$ we cannot clearly recognise the expected shape of $v(x)$, though we do observe that $v(x)$ becomes negative for large x.

From their analysis of the Sint-2000 data Brendel et al. (2007) found that $\lambda_0^{-1} \sim 20$ kyr, and that D does not scale as x^2 in the geodynamo, but is virtually independent of x. They cited magnetic quenching of the convective flow as the most likely cause. For the numerical models with their much longer data set we arrive at the same conclusion: in all three models $D(x)$ is constant over the x-range where we could measure it. This suggests that in the geodynamo and the numerical models a similar kind of quenching is at work, despite the vast difference in the parameter values.

The advantage of a numerical model is that the nature of the quenching is open to scrutiny. Here we shall consider one aspect: quenching of the amplitude of the convective flow. This is facilitated by the fact that the code outputs the magnetic Reynolds number $Rm = u_{\mathrm{r.m.s.}} d/\eta$ at each timestep, see Fig. 4. Although there is a considerable spread in the data, the trend that Rm decreases with ADM is unmistakable. On average, the r.m.s. magnitude of the convective flow becomes smaller for increasing magnetic dipole moment. While this is clear evidence of quenching, the range of the decrease of Rm (about 10%) is insufficient to explain the constancy of D. In order that D is independent of x, the autocorrelation of the convective flow (which determines β) and/or its correlation length must also be affected. The value of D behaves non-monotonously with increasing Ra. From Fig. 3 we infer $D \simeq 0.22$ for $T2$, $D \simeq 0.67$ for $T4$ and then down to $D \simeq 0.45$ for $T5$, in units of $(10^{22}\,\mathrm{Am}^2)^2\,\mathrm{kyr}^{-1}$. The increase from T2 to T4 likely reflects the increasing time dependence and growing complexity of the flow (Wicht et al. 2008). We speculate that the smaller value for T5 may go back to the regime change to non-dipole dominated dynamos, an aspect that needs to be explored.

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5 Internal consistency and perspective

These results suggest that the dynamics of the axial dipole can to a large extent be described by a simple Langevin equation (1) with $F =$ constant. The noise would thus be additive, i.e., after inclusion of nonlinear feedback the axial dipole would behave according to the well-known paradigm of a Brownian particle in a symmetric bistable potential. This is what the Sint-2000 data and the numerical models we studied seem to be telling us. We briefly address the internal consistency and potential of this idea with the help of model $T4$.

Assuming that $v(x) = \lambda_0[1 - (x/x_0)^2]x$ and $D =$ constant, the stationary ADM distribution is readily found from eq. (3):

$$\rho(x) \;\propto\; \exp\left[-\frac{\lambda_0 x_0^2}{2D}\left\{1-\left(\frac{x}{x_0}\right)^2\right\}^2\right] . \tag{9}$$

The parameter x_0 is now seen to correspond to the position of the maxima of the ADM distribution. Fig. 2 shows that (9) reproduces the measured distribution reasonably well in $|x| \leq x_0$ for $\lambda_0 x_0^2/2D = 2$ (we did not try to obtain a best fit). For large $|x|$ formula (9) predicts a much smaller distribution than measured. The discrepancy can be traced to the shape of $v(x)$: for large x, the theory assumes a very large damping rate $v \propto -x^3$, but in reality v decreases much less rapidly (Fig. 3, middle panel). A minor effect is that D increases slightly for large x. We shall not pursue this any further - we merely mention that due allowance of these points is able to repair the conflict. From $\lambda_0 x_0^2/2D = 2$ and $x_0 = 10 \cdot 10^{22}\,\mathrm{Am}^2$, $D = 0.67$ (dimensionless units) we deduce $\lambda_0^{-1} \simeq 37$ kyr, in fair accordance with $\lambda_0^{-1} \simeq 30$ kyr from the previous section.

For $D =$ constant the mean time $\langle T_{\mathrm{rev}}\rangle$ between reversals may be computed as twice the inverse Kramers escape rate (Gardiner, 1990):

$$\langle T_{\mathrm{rev}}\rangle \;\approx\; \pi\lambda_0^{-1}\sqrt{2}\;\exp\left(\frac{\lambda_0 x_0^2}{2D}\right) . \tag{10}$$

This should be a robust result, since $\langle T_{\mathrm{rev}}\rangle$ is insensitive to the shape of the tails of $\rho(x)$, and indeed, (10) yields $\langle T_{\mathrm{rev}}\rangle \simeq 1\,\mathrm{Myr}$, approximately what we measure in Fig. 1.

The *secular variation*, defined for example as the full-width at half-maximum of the ADM distribution between reversals, is proportional to $(D/\lambda_0)^{1/2}$, so that

$$\log\langle T_{\mathrm{rev}}\rangle \;\propto\; (\text{secular variation})^{-2} . \tag{11}$$

Hence we predict a very strong scaling of the mean time between reversals with the inverse of the secular variation. Although derived for $\rho(x)$ given by (9), we expect that this is also a robust result, applicable to a wide class of geodynamo models.

We conclude that by and large the numbers seem to match. Before we advance further along this track, it seems better to wait until theoretical studies of quenching mechanisms yield reliable expressions for the effective growth rate $v(x)$ and confirm the notion of a constant diffusion coefficient D.

6 Conclusions and summary

We have analyzed time series of axial dipole moments taken from the hydromagnetic dynamo models T2, T4 and T5 of Wicht et al. (2008), and we determined the effective growth rate of the ADM and the diffusion coefficient characterizing the random fluctuations of the ADM. In the analysis of Brendel et al. (2007) of the Sint-2000 data it was concluded that this effective diffusion coefficient is independent of the strength of the ADM, corresponding to additive noise. Since this conclusion is in contrast with the expected behaviour corresponding to multiplicative noise, and since this analysis was hampered by the limited

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number of data points (2000), we repeated their analysis on numerical ADM data which span larger effective time ranges.

The main conclusion is that in the range where we could measure it, the numerical ADM data of all three models exhibit a nonlinear quenching that is not significantly different from that of the Sint-2000 data. The fact that D is independent of the ADM for all three models suggests that similar quenching mechanisms are at work irrespective of the reversal behaviour or the dominance of the dipole moment. In the numerical model the quenching of the diffusion coefficient D is partly due to a reduction of the r.m.s. convective flow speed with increasing ADM. But other properties of the flow (such as autocorrelation time and correlation length) must also be affected.

Our results offer the perspective that the dynamics of the axial dipole can be described by a simple Langevin equation (1) with $F =$ constant, i.e. with effectively additive noise. A theoretical basis for this idea would greatly benefit further understanding of the behaviour of the geodynamo.

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