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Rainfall overland flow simulations and real events in Niger

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Abstract

We are interested in simulating rainfall overland flows on agricultural fields, because they may have some undesirable effects such as soil erosion and pollutant transport. The model used for these simulations is the system of Shallow Water (SW) or Saint-Venant system as in [7, 8]. In this context, we can meet some numerical difficulties due to dry/wet transitions and to steady states.

This work is divided into two parts. First we present the numerical method for the resolution of the Shallow Water equations integrated in FullSWOF_2D (Full Shallow Water for Overland Flow : an object oriented code in C++). This method is based on a hydrostatic reconstruction scheme [1, 2] to cope with dry/wet transitions and steady states, coupled with a semi-implicit friction term treatment [3]. In the second part, FullSWOF_2D is tested and by comparison with experimental field observations validated on several real events measured by IRD on two different runoff plots in Niger (West Africa).

1 The model

1.1 The shallow water equations

As in [8] and [7], we consider the 2D Shallow Water equations (SW2D) which write (see figure 1a)

$$\left\{\begin{array}{l}\partial_t h+\partial_x(hu)+\partial_y(hv)=R-I\\ \partial_t(hu)+\partial_x\left(hu^2+gh^2/2\right)+\partial_y(huv)=gh\left(S_{0_x}-S_{f_x}\right)\\ \partial_t(hv)+\partial_x(huv)+\partial_y\left(hv^2+gh^2/2\right)=gh\left(S_{0_y}-S_{f_y}\right)\end{array}\right. \tag{1}$$

where the unknowns are the velocities $u(x,y,t)$ and $v(x,y,t)$ [m/s] and the water height $h(x,y,t)$ [m]. The subscript x (respectively y) stands for the x-direction (resp. the y-direction) : $S_{0_x}=-\partial_x z(x,y)$ and $S_{0_y}=-\partial_y z(x,y)$) are the ground slopes, S_{f_x} and S_{f_y} are the friction terms. $R(x,y,t)$ [m/s] is the rainfall intensity and $I(x,y,t)$ [m/s] is the infiltration rate.

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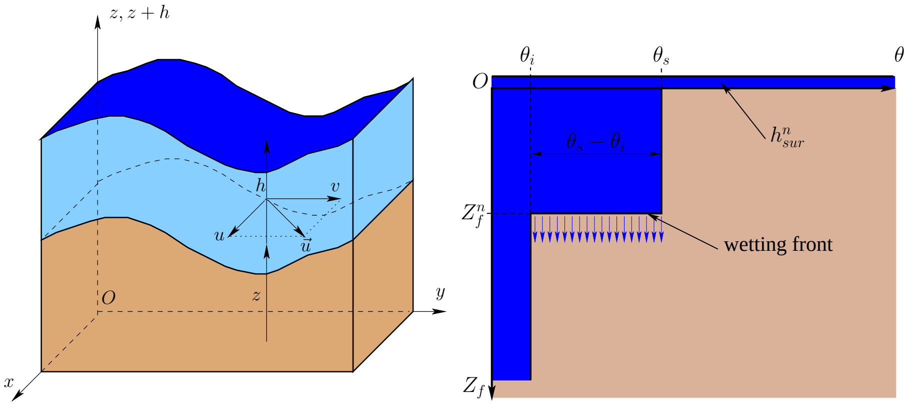

(a) (b)

Figure 1: Illustration of (a) SW2D and (b) Green-Ampt models.

As in [7], we use the Darcy-Weisbach friction law which writes

$$S_{f_x} = f\frac{u.\sqrt{u^2+v^2}}{8gh} \quad S_{f_y} = f\frac{v.\sqrt{u^2+v^2}}{8gh} \tag{2}$$

1.2 The infiltration model

Infiltration is computed at each cell using a Green-Ampt model [9]. With this model, the movement of water in soil is assumed to be in the form of an advancing wetting front (located at Z^n_f [m]) that separates a zone still at the initial soil moisture θ_i and a fully satured zone at soil moisture θ_s (see figure 1b). At the moment $t = t_n$, the infiltration capacity I^n_C [m/s] is calculated thanks to

$$I^n_C = K_s\left(1 + \frac{h_f - h^n_{sur}}{Z^n_f}\right) \text{ where } Z^n_f = \frac{V^n_{inf}}{\theta_s - \theta_i} \tag{3}$$

where h_f is the wetting front capillary pressure head, K_s the hydraulic conductivity at saturation, h^n_{sur} the water height and V^n_{inf} the infiltrated water volume. Thus we have the infiltration rate

$$I^n = \frac{\min(h^n_{sur}, \Delta t I^n_C)}{\Delta t} \tag{4}$$

and the infiltrated volume

$$V^{n+1}_{inf} = V^n_{inf} + \Delta t I^n \tag{5}$$

where Δt is the time step. In the case of a two-layer soil, we consider a modification of this model (see [7] and [5]).

2 The numerical method

The scheme will be presented in one dimension (SW1D)

$$\begin{cases} \partial_t h + \partial_x (hu) = R - I \\ \partial_t (hu) + \partial_x \left(hu^2 + gh^2/2\right) = gh\left(S_{0_x} - S_{f_x}\right) \end{cases}, \tag{6}$$

the extension to SW2D on structured grid is straightforward and is integrated in an object oriented code in C++ : FullSWOF_2D[1] (for details about the code see [4] and [5]). In what follows, we note the discharge $q = hu$ [m^2/s] and the vector of conservative variables $U = \binom{h}{hu}$.

2.1 Convective step

A finite volume discretization of SW1D writes

$$U_i^* = U_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2L}^n - F_{i-1/2R}^n - Fc_i^n\right], \tag{7}$$

with Δx the space step and

$$\begin{cases} F_{i+1/2L}^n = F_{i+1/2}^n + S_{i+1/2L}^n \\ F_{i-1/2R}^n = F_{i-1/2}^n + S_{i-1/2R}^n \end{cases} \tag{8}$$

left (respectively right) modification of the numerical flux for the homogeneous problem (see section 2.3)

$$F_{i+1/2}^n = \mathcal{F}(U_{i+1/2L}^n, U_{i+1/2R}^n) \tag{9}$$

The values $U_{i+1/2L}$ and $U_{i+1/2R}$ are obtained thanks to two consecutive reconstructions. Firstly a MUSCL reconstruction [2, 5] is performed on u, h and $h+z$ in order to get a second order scheme in space. This gives us the reconstructed values (U_-, z_-) and (U_+, z_+). Secondly we apply the hydrostatic reconstruction [1, 2] on the water height which allows us to get a positive preserving well-balanced scheme (in the sense it preserves steady state at rest)

$$\begin{cases} h_{i+1/2L} = \max(h_{i+1/2-} + z_{i+1/2-} - \max(z_{i+1/2-}, z_{i+1/2+}), 0) \\ U_{i+1/2L} = (h_{i+1/2L}, h_{i+1/2L}u_{i+1/2-}) \\ h_{i+1/2R} = \max(h_{i+1/2+} + z_{i+1/2+} - \max(z_{i+1/2-}, z_{i+1/2+}), 0) \\ U_{i+1/2R} = (h_{i+1/2R}, h_{i+1/2R}u_{i+1/2+}) \end{cases} \tag{10}$$

We introduce

$$S_{i+1/2L}^n = \begin{pmatrix} 0 \\ \frac{g}{2}(h_{i+1/2-}^2 - h_{i+1/2L}^2) \end{pmatrix}, S_{i-1/2R}^n = \begin{pmatrix} 0 \\ \frac{g}{2}(h_{i-1/2+}^2 - h_{i-1/2R}^2) \end{pmatrix} \tag{11}$$

and a centered source term is added to preserve consistency and well-balancing (see [1] and [2])

$$Sc_i = \begin{pmatrix} 0 \\ -g\frac{h_{i-1/2+} + h_{i+1/2-}}{2}(z_{i+1/2-} - z_{i-1/2+}) \end{pmatrix} \tag{12}$$

The rain and the infiltration are treated explicitly (for details see [5]).

[1] free software with CeCILL licence : http://www.cecill.info/licences/Licence_CeCILL_V2-fr.html

2.2 Friction treatment step

In this step, the friction term is taken into account with the following system

$$\partial_t U = \begin{pmatrix} 0 \\ -ghS_f \end{pmatrix} \tag{13}$$

This system is solved thanks to a semi-implicit method (as in [3])

$$\begin{cases} h^{n+1} = h^* \\ \dfrac{q^{n+1} - q^*}{\Delta t} = -\dfrac{f}{8}\dfrac{q^{n+1}|q^n|}{h^n h^{n+1}} \end{cases} \tag{14}$$

where h^*, q^* and u^* are the variables from the convection step. This method allows to preserve stability (under a classical CFL condition) and steady states at rest. Finally, these two steps are combined in a Heun method to get a second order scheme in time.

2.3 Numerical flux

We use the HLL flux which writes

$$\mathcal{F}(U_L, U_R) = \begin{cases} F(U_L) & if\ 0 < c_1 \\ \dfrac{c_1 F(U_L) - c_2 F(U_R)}{c_2 - c_1} + \dfrac{c_1 c_2}{c_2 - c_1}(U_R - U_L) & if\ c_1 < 0 < c_2 \\ F(U_R) & if\ c_2 < 0 \end{cases} \tag{15}$$

with two parameters $c_1 < c_2$ given by

$$c_1 = \inf_{U=U_G,U_D}(\inf_{j\in\{1,2\}} \lambda_j(U))\ and\ c_2 = \sup_{U=U_G,U_D}(\sup_{j\in\{1,2\}} \lambda_j(U)), \tag{16}$$

where $\lambda_1(U) = u - \sqrt{gh}$ and $\lambda_2(U) = u + \sqrt{gh}$ are the eigenvalues of SW1D.

2.4 MUSCL-reconstruction

We define the MUSCL reconstruction of a scalar function $s \in \mathbb{R}$ by

$$s_{i-1/2+} = s_i - \frac{\Delta x}{2} Ds_i \text{ and } s_{i+1/2-} = s_i + \frac{\Delta x}{2} Ds_i \tag{17}$$

with the operator

$$Ds_i = \text{minmod}\left(\frac{s_i - s_{i-1}}{\Delta x}, \frac{s_{i+1} - s_i}{\Delta x}\right) \tag{18}$$

and the minmod limiter

$$\text{minmod}(x, y) = \begin{cases} \min(x, y) & \text{if } x, y \geq 0 \\ \max(x, y) & \text{if } x, y \leq 0 \\ 0 & \text{else} \end{cases} \tag{19}$$

3 Validation

3.1 Experimental data

To validate FullSWOF_2D, it is applied to experimental results from studies conducted in Niger (West Africa) in 1994 by IRD[2] in the framework of the HAPEX[3]-Sahel experiment (see [7]). This data set is composed of the topography of two sites JAC and ERO, twelve typical rainfall events in [mm/h] (or hyetographs), the discharges measured downstream in [mm/h] (or experimental hydrographs) and the infiltrated water volume in the middle of each sites thanks to neutron probe measurements. JAC (20m $\times$ 5m) is a fallow site made of erosion crusts and sandy mounts thus with a friction coefficient f depending on the surface type, ERO (14.25m $\times$ 5m) is an erosion site made of erosion crusts. The application of FullSWOF_2D requires values of infiltration parameters and Darcy-Weisbach. This data set has been used to validate PSEM_2D[4] (see [7]), thus we already have all the parameters needed for the simulations (see Table 1) and numerical results of PSEM_2D as references.

Surface properties			Soil properties	
Surface type	Erosion crust	Sandy mounts	Soil type	Loamy sand
Site	JAC/ERO	JAC	Site	JAC/ERO
Z_c (m)	0.005	0.05	θ_s	0.296
h_f (m)	1.3795	0.18	h_f (m)	1.3795
K_s (m/s)	1.7E-8	1.9E-6	K_s (m/s)	2.15E-5
f	0.25	0.7		

Table 1: Parameters used for JAC and ERO sites.

3.2 Numerical results

As previously said, twelve rainfall-runoff events were simulated on two sites. In order to evaluate the numerical method, we estimate the Nash-Sutcliffe number (see [13]) as illustrated for ERO site in table 2

$$NS = 1 - \frac{\sum_{n=0}^{M} \left(Q_n - Q_n^s\right)^2}{\sum_{n=0}^{M} \left(Q_n - \overline{Q}_n\right)^2} \tag{20}$$

with Q_n is the observed runoff rate at time n, $\overline{Q}_n$ the mean observed runoff rate and Q_n^s the simulated runoff rate. We will focus on the ERO site, we got the same kind of results on JAC site. On the whole, the results obtained with FullSWOF_2D are in good agreement with both those observed and those simulated with PSEM_2D. As we can notice on figure 2b, the numerical results are very closed to the measures. This is confirmed with the Nash-Sutcliffe numbers (table 2). We can see that peaks of runoff are well catched (figures 2e and 2f). However, the time before runoff occurs is always underestimated both

[2]Institut de Recherche pour le Développement
[3]Hydrology and Atmosphere Pilot EXperiment
[4]Plot Soil Erosion Model 2D, a software based on MacCormack scheme

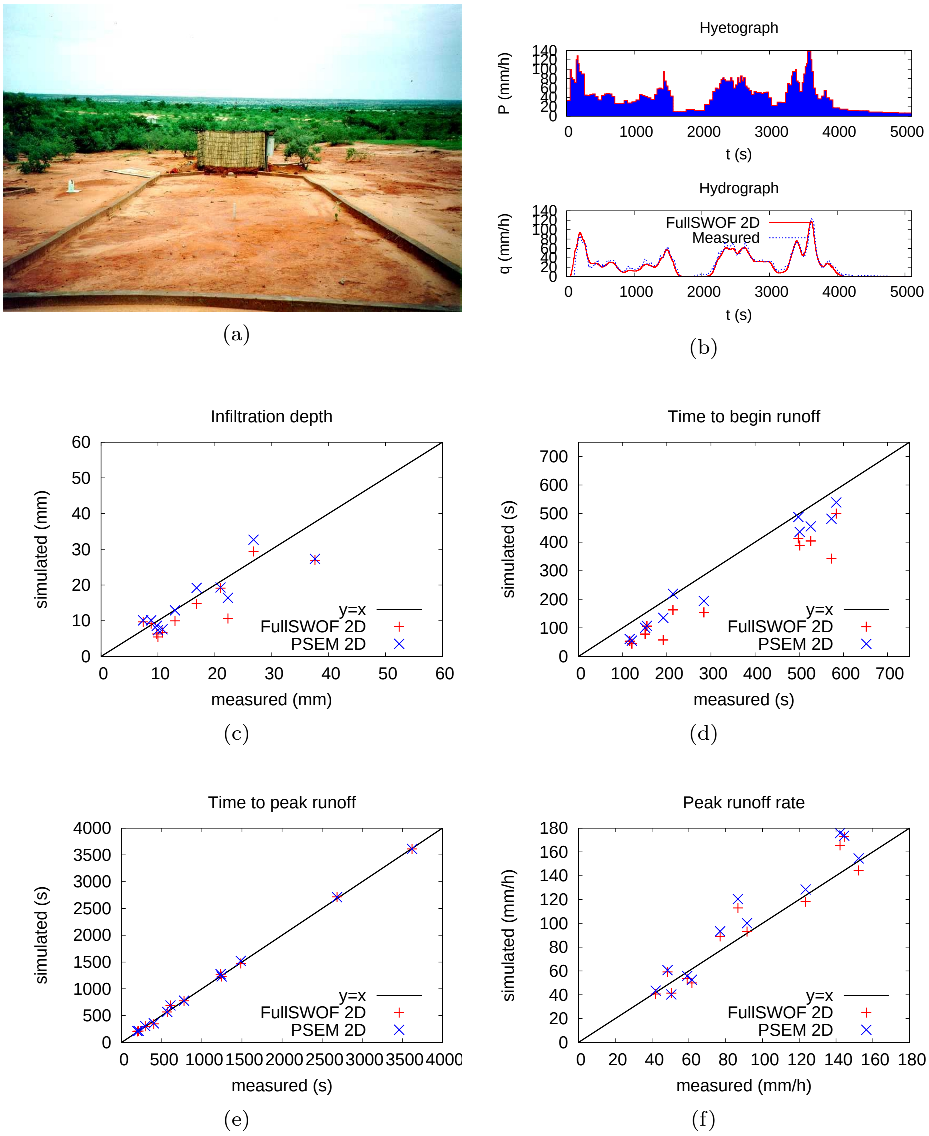

Figure 2: ERO : (a) the site, (b) comparison of the computed hydrograph with measures (event 9) and comparison of the measured vs simulated values of : (c) infiltration depth at the neutron probe, (d) time from the beginning of the event to the beginning of the runoff, (e) time from the beginning of the event to the main peak of runoff and (f) peak runoff rate.

with FullSWOF_2D and PSEM_2D (figure 2d). This might be an illustration of a lack in the model (SW2D) : the friction due to rain is not taken into account. Finally, figure 2c illustrates a comparison of the depth of infiltration as calculated by the codes and measured from the neutron probe. This results are quite good.

Event	Date	θ_i	NS FullSWOF_2D	NS PSEM_2D
1	20 July 1994	0.048	0.85	0.854
2	21 July 1994	0.103	0.748	0.661
3	07 August 1994	0.1	0.839	0.9
4	10 August 1994	0.106	0.873	0.896
5	20 August 1994	0.067	0.618	0.625
6	20 August 1994 2	0.095	0.618	0.896
7	24 August 1994	0.088	0.843	0.883
8	25 August 1994	0.085	0.922	0.885
9	04 September 1994	0.082	0.928	0.928
10	08 September 1994	0.082	0.605	0.460
11	12 September 1994	0.07	0.755	0.738
12	16 September 1994	0.061	0.661	0.662

Table 2: ERO : initial soil moisture and Nash-Sutcliffe numbers.

Conclusion

As a conclusion, we can consider that FullSWOF_2D has been validated on these experimental data. We notice that FullSWOF_2D and PSEM_2D gave very closed results but we have to be aware that MacCormack scheme is neither positive preserving nor well-balanced. So we will have to do further comparisons between the two methods. At last the systematic underestimation of the time before runoff occurs shows a lack in the model. As suggested in [15], we should take into account the friction effect due to rain.

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