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EXTENSION OF THE TARGET SCATTERING VECTOR MODEL TO THE BISTATIC CASE

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ABSTRACT

The polarimetric information has been widely used to interpret the Synthetic Aperture Radar (SAR) scene. Hence, many decompositions have been introduced to extract polarimetric parameters with a physical meaning. Nevertheless, for most of them, the reciprocity assumption is assumed. For a bistatic PolSAR sensor, the cross-polarization terms of the scattering matrix are not equal. This paper presents a generalization of the Target Scattering Vector Model (TSVM) to the bistatic case.

Index Terms— Bistatic Polarimetry, Polarimetric Synthetic Aperture Radar, Roll-invariant decomposition, Target Scattering Vector Model.

1. INTRODUCTION

In the context of Polarimetric Synthetic Aperture Radar (PolSAR) imagery, the extraction of roll-invariant parameters is one of the major point of interest for segmentation, classification and detection. In 2007, for the monostatic case, Ridha Touzi has proposed a new Target Scattering Vector Model (TSVM) [1]. Based on the Kennaugh-Huynen decomposition, this model allows to extract four roll-invariant parameters.

For the bistatic case, the reciprocity assumption is in general no more valid. This paper presents a extension of the TSVM when the cross-polarization terms are not equal. First, a presentation of bistatic polarimetry is exposed by means of the Kennaugh-Huynen decomposition [2]. Then, the TSVM is introduced as a projection of the scattering matrix in the Pauli basis to extract roll-invariant parameters [1] and a comparison with the monostatic case is carried out. Next, a presentation of the computation of the TSVM parameters is exposed. Finally, some comparisons with the classical α/β model are shown.

2. THE KENNAUGH-HUYNEN CON-DIAGONALIZATION

Coherent targets are fully described by their scattering matrix \mathbf{S} . In the context bistatic polarimetry, \mathbf{S} is a complex 2×2 matrix, $\mathbf{S} = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$ where the cross-polarization elements S_{HV} and S_{VH} are not equal in general.

Kennaugh and Huynen have proposed to apply the characteristic decomposition on the scattering matrix to retrieve physical parameters [2] [3] [4]. The Kennaugh-Huynen decomposition is parametrized by means of 8 independent parameters: $\theta_R, \tau_R, \theta_E, \tau_E, \nu, \mu, \kappa$ and γ by [2] [5] [6]:

$$\mathbf{S} = e^{-j\theta_R\sigma_3} e^{-j\tau_R\sigma_2} e^{-j\nu\sigma_1} \mathbf{S}_0 e^{j\nu\sigma_1} e^{-j\tau_E\sigma_2} e^{j\theta_E\sigma_3} \quad (1)$$

where:

$$\mathbf{S}_0 = \mu e^{j\kappa} \begin{bmatrix} 1 & 0 \\ 0 & \tan^2 \gamma \end{bmatrix} \text{ and } e^{j\alpha\sigma_k} = \sigma_0 \cos \alpha + j\sigma_k \sin \alpha. \quad (2)$$

σ_i are the spin Pauli matrices defined by:

$$\begin{aligned} \sigma_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \\ \sigma_3 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_4 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}. \end{aligned} \quad (3)$$

θ_R and θ_E are the tilt angles. τ_R and τ_E are the helicity. The subscript R and E stand respectively for reception and emission. μ is the maximum amplitude return. γ and ν are respectively referred as the characteristic and skip angles. κ is the absolute phase of the target, this term is generally ignored except for interferometric applications.

Moreover, it can be shown that:

$$\begin{aligned} e^{-j\nu\sigma_1} \mathbf{S}_0 e^{j\nu\sigma_1} &= \begin{bmatrix} \mu e^{2j(\nu+\kappa/2)} & 0 \\ 0 & \mu \tan^2 \gamma e^{-2j(\nu-\kappa/2)} \end{bmatrix} \\ &= \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \end{aligned} \quad (4)$$

where λ_1 and λ_2 are the two complex con-eigenvalues of \mathbf{S} .

3. THE TARGET SCATTERING VECTOR MODEL

3.1. Definition

The TSVM consists in the projection in the Pauli basis of the scattering matrix con-diagonalized by the Takagi method. It yields that $\mathbf{k}_P = 1/\sqrt{2} [S_{HH} + S_{VV}, S_{HH} - S_{VV}, S_{HV} + S_{VH}, j(S_{HV} - S_{VH})]^T$. After some mathematical manipulations, one can express the target vector \mathbf{k}_P by means of Huynen's parameters (See (5) at the top of the next page).

By following the same procedure as proposed by Touzi in [1], one can introduce the symmetric scattering type magnitude and phase parameters, denoted respectively α_s and Φ_{α_s} by:

$$\tan(\alpha_s) e^{j\Phi_{\alpha_s}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}. \quad (6)$$

According to (5), one can decompose \mathbf{k}_P as the product of three terms (see (7) at the top of the next page). Φ_s corresponds to the phase of $\lambda_1 + \lambda_2$. It can be noticed that the first and second terms are "rotation" matrices which depend only on the tilt angles θ_R and θ_E .

3.2. Roll-invariant target vector

As a consequence, for the bistatic case, the expression of the roll-invariant target vector $\mathbf{k}_P^{\text{roll-inv}}$ is given by:

$$\mathbf{k}_P^{\text{roll-inv}} = \mu \begin{bmatrix} \cos \alpha_s \cos(\tau_1) \\ \sin \alpha_s e^{j\Phi_{\alpha_s}} \cos(\tau_2) \\ -j \cos \alpha_s \sin(\tau_1) \\ -j \sin \alpha_s e^{j\Phi_{\alpha_s}} \sin(\tau_2) \end{bmatrix}, \quad (8)$$

where $\tau_1 = \tau_R + \tau_E$ and $\tau_2 = \tau_R - \tau_E$. In the context of bistatic polarimetry, five parameters (namely μ , τ_R , τ_E , α_s and Φ_{α_s}) are necessary for an unambiguous description of a coherent target.

3.3. Link with the monostatic case

The monostatic case can be retrieved from the bistatic case by assuming $\theta = \theta_R = \theta_E$ and $\tau_m = \tau_R = \tau_E$. Consequently, when the reciprocity assumption holds, the roll-invariant target vector, introduced by Touzi, is obtained:

$$\mathbf{k}_P^{\text{roll-inv}} = \mu \begin{bmatrix} \cos \alpha_s \cos(2\tau_m) \\ \sin \alpha_s e^{j\Phi_{\alpha_s}} \\ -j \cos \alpha_s \sin(2\tau_m) \\ 0 \end{bmatrix}. \quad (9)$$

4. TSVM PARAMETERS COMPUTATION

4.1. The Kennaugh matrix

The Kennaugh matrix \mathbf{K} is another representation of the scattering matrix \mathbf{S} , its expression is given by $\mathbf{K} = 2\mathbf{A}^* \mathbf{W} \mathbf{A}^{-1}$ with $\mathbf{W} = \mathbf{S} \otimes \mathbf{S}$. \otimes is the Kronecker product, and:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix}. \quad (10)$$

4.2. The Kennaugh matrices of orders 0 to 2

Let \mathbf{O}_1 , \mathbf{O}_2 and \mathbf{O}_3 be the three "rotation matrices" defined by [5]:

$$\mathbf{O}_1(2\nu) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\nu) & -\sin(2\nu) \\ 0 & 0 & \sin(2\nu) & \cos(2\nu) \end{bmatrix} \quad (11)$$

$$\mathbf{O}_2(2\tau) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & \sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & -\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \quad (12)$$

$$\mathbf{O}_3(2\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (13)$$

The Kennaugh matrices of orders 0 to 2, denoted $\mathbf{K}^{(i)}$, are defined by:

$$\begin{cases} \mathbf{K}^{(2)} = \mathbf{O}_3(-2\theta_R) \mathbf{K} \mathbf{O}_3(2\theta_E) \\ \mathbf{K}^{(1)} = \mathbf{O}_2(2\tau_R) \mathbf{K}^{(2)} \mathbf{O}_2(-2\tau_E) \\ \mathbf{K}^{(0)} = \mathbf{O}_1(-2\nu) \mathbf{K}^{(1)} \mathbf{O}_1(2\nu) \end{cases} \quad (14)$$

4.3. TSVM parameters computation

4.3.1. Tilt angles

In practice, thanks to the scattering matrix \mathbf{S} , the Kennaugh matrix \mathbf{K} is first computed. The tilt angles θ_E and θ_R are then directly deduced from the Kennaugh matrix \mathbf{K} by [7]:

$$\tan(2\theta_E) = \frac{\mathbf{K}_{02}}{\mathbf{K}_{01}} \quad \text{and} \quad \tan(2\theta_R) = \frac{\mathbf{K}_{20}}{\mathbf{K}_{10}}. \quad (15)$$

In (15), \mathbf{K}_{ij} corresponds to the element of \mathbf{K} at position $(i + 1, j + 1)$. Once θ_E and θ_R are found, the Kennaugh matrix of order 2, namely $\mathbf{K}^{(2)}$, is computed according to (14). As this matrix does not depend on the tilt angles, it can be viewed as the roll-invariant Kennaugh matrix.

$$\mathbf{k}_P = \frac{1}{\sqrt{2}} \begin{bmatrix} (\lambda_1 + \lambda_2) \cos(\tau_R + \tau_E) \cos(\theta_R - \theta_E) + j(\lambda_1 - \lambda_2) \sin(\tau_E - \tau_R) \sin(\theta_E - \theta_R) \\ (\lambda_1 - \lambda_2) \cos(\tau_R - \tau_E) \cos(\theta_R + \theta_E) + j(\lambda_1 + \lambda_2) \sin(\tau_R + \tau_E) \sin(\theta_R + \theta_E) \\ (\lambda_1 - \lambda_2) \cos(\tau_R - \tau_E) \sin(\theta_R + \theta_E) - j(\lambda_1 + \lambda_2) \sin(\tau_R + \tau_E) \cos(\theta_R + \theta_E) \\ (\lambda_1 - \lambda_2) \sin(\tau_E - \tau_R) \cos(\theta_R - \theta_E) + j(\lambda_1 + \lambda_2) \cos(\tau_r + \tau_E) \sin(\theta_E - \theta_R) \end{bmatrix}. \quad (5)$$

$$\mathbf{k}_P = \mu e^{j\Phi_s} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_R + \theta_E) & -\sin(\theta_R + \theta_E) & 0 \\ 0 & \sin(\theta_R + \theta_E) & \cos(\theta_R + \theta_E) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_R - \theta_E) & 0 & 0 & -\sin(\theta_R - \theta_E) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -j \sin(\theta_R - \theta_E) & 0 & 0 & -j \cos(\theta_R - \theta_E) \end{bmatrix} \begin{bmatrix} \cos \alpha_s \cos(\tau_R + \tau_E) \\ \sin \alpha_s e^{j\Phi_{\alpha_s}} \cos(\tau_R - \tau_E) \\ -j \cos \alpha_s \sin(\tau_R + \tau_E) \\ j \sin \alpha_s e^{j\Phi_{\alpha_s}} \sin(\tau_E - \tau_R) \end{bmatrix}. \quad (7)$$

4.3.2. Helicity angles

Similarly, the helicity angles τ_R and τ_E are issued from the Kennaugh matrix of order 2 by [7]:

$$\tan(2\tau_R) = \frac{\mathbf{K}_{30}^{(2)}}{\mathbf{K}_{10}^{(2)}} \quad \text{and} \quad \tan(2\tau_E) = \frac{\mathbf{K}_{03}^{(2)}}{\mathbf{K}_{01}^{(2)}}. \quad (16)$$

4.3.3. Characteristic and skip angles

Next, the skip and characteristic angles (ν and γ) are deduced from the Kennaugh matrices of order 1 and 0 by:

$$\tan(4\nu) = \frac{\mathbf{K}_{32}^{(1)}}{\mathbf{K}_{33}^{(1)}} \quad \text{and} \quad \cos(2\gamma) = A \pm \sqrt{A^2 - 1} \quad (17)$$

with $A = \frac{\mathbf{K}_{11}^{(0)}}{\mathbf{K}_{01}^{(0)}}$. The solution adopted is the $A \pm \sqrt{A^2 - 1}$ ranging in the interval $[-1, 1]$.

4.3.4. Symmetric scattering type magnitude and phase

Finally, the symmetric scattering type magnitude and phase, α_s and Φ_{α_s} , are directly deduced from parameters ν and γ by:

$$\tan(\alpha_s) e^{j\Phi_{\alpha_s}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \frac{e^{2j\nu} - e^{-2j\nu} \tan^2 \gamma}{e^{2j\nu} + e^{-2j\nu} \tan^2 \gamma} = B. \quad (18)$$

It yields:

$$\tan \alpha_s = |B| \quad \text{and} \quad \Phi_{\alpha_s} = \arg(B). \quad (19)$$

4.4. Con-eigenvalue phase ambiguity

Due to the con-eigenvalue phase ambiguity, Huynen's parameters need to be reevaluated. To overcome this problem, Touzi has proposed to restrict the tilt angles $\theta_1 = \theta_R + \theta_E$ and $\theta_2 = \theta_R - \theta_E$ domain definition to the interval $[-\pi/2, \pi/2]$ [1]. If the tilts angles (θ_1, θ_2) are solution of (7), then $(\theta_1 \pm$

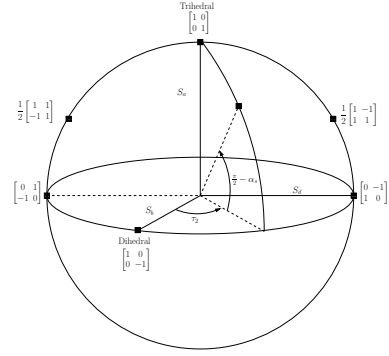


Fig. 1. Symmetric scattering type magnitude α_s and helicity τ_2 Poincaré sphere ($\tau_2 = 0$ and $\Phi_{\alpha_s} = 0$).

$\pi, \theta_2 \pm \pi$), $(\theta_1 \pm \pi, \theta_2)$ and $(\theta_1, \theta_2 \pm \pi)$ are also solutions of (7). It yields the following three relations:

$$\begin{aligned} \mathbf{k}_P &= \mathbf{k}_P(\Phi_s \pm \pi, \theta_1 \pm \pi, \theta_2 \pm \pi, \tau_1, \tau_2, \mu, \alpha_s, \Phi_{\alpha_s}) \\ &= \mathbf{k}_P(\Phi_s, \theta_1 \pm \pi, \theta_2, -\tau_1, -\tau_2, \mu, \alpha_s, \Phi_{\alpha_s} \pm \pi) \\ &= \mathbf{k}_P(\Phi_s \pm \pi, \theta_1, \theta_2 \pm \pi, -\tau_1, -\tau_2, \mu, \alpha_s, \Phi_{\alpha_s} \pm \pi). \end{aligned}$$

Those equations are implemented to solve the con-eigenvalue phase ambiguity problem. After this step θ_1 and θ_2 belong to the interval $[-\pi/2, \pi/2]$.

5. INTERPRETATION

5.1. Poincaré Sphere

To understand the influence of the 4 roll-invariant parameters α_s , Φ_{α_s} , τ_1 and τ_2 , the Poincaré sphere representation can be used. Here, only the symmetric scattering type magnitude α_s and helicity τ_2 Poincaré sphere is shown (Fig. 1). The other spheres can be found in [1]. A symmetric scatterer ($\tau_1 = 0$) with a null symmetric scattering type phase ($\Phi_{\alpha_s} = 0$) is uniquely mapped by a point located at a longitude τ_2 and a latitude $\pi/2 - \alpha_s$ at the surface of this Poincaré sphere.

The symmetric target scattering type phase Φ_{α_s} is the trihedral-dihedral channel phase difference. This roll-invariant parameter can be exploited only under coherence conditions.

The degree of coherence of Φ_{α_s} (denoted $p_{\Phi_{\alpha_s}}$) is therefore introduced. Its expression is given by:

$$p_{\Phi_{\alpha_s}} = \frac{\sqrt{\left(\langle |a|^2 - |b|^2 \rangle\right)^2 + 4|\langle a \cdot b^* \rangle|^2}}{\langle |a|^2 + |b|^2 \rangle}, \quad (20)$$

where $a = \cos \alpha_s \cos \tau_1$ and $b = \sin \alpha_s e^{j\Phi_{\alpha_s}} \cos \tau_2$ for a bistatic polarimetric radar. Therefore, a partially coherent scatterer is represented as a point inside the Poincaré sphere at a distance $p_{\Phi_{\alpha_s}}$ from the sphere center.

5.2. Comparison with the α/β bistatic model

In 2005, S.R. Cloude has proposed to extend the well-known α/β model to the bistatic case [8]. The target vector \mathbf{k}_P is defined by means of 8 parameters:

$$\mathbf{k}_P = \mu e^{j\Phi_s} \begin{bmatrix} \cos \alpha \\ \sin \alpha \cos \beta e^{j\delta} \\ \sin \alpha \sin \beta \cos \chi e^{j\gamma} \\ \sin \alpha \sin \beta \sin \chi e^{j\epsilon} \end{bmatrix}. \quad (21)$$

For the monostatic case ($\chi = 0$ or $\theta_R = \theta_E$ and $\tau_R = \tau_E$), the α angle has been widely used to characterize the backscattered mechanism. Indeed, Touzi has proved in [1] that the symmetric scattering type magnitude α_s is equal to α for a symmetrical target ($\tau_m = 0$) which corresponds to a wide class of targets including dihedral, trihedral, dipole, ... Fig. 2 shows a comparison between parameters α and α_s issued respectively from the α/β model and the bistatic TSVM. This plot shows their evolution as a function of the tilt angle $\theta_2 = \theta_R - \theta_E$ for different set of target helicity $\tau_2 = \tau_R - \tau_E$. Fig. 2(a) and 2(b) are respectively done for $\tau_1 = 0$ and $\tau_1 \neq 0$. First, it can be seen that α depends on the tilt angle θ_2 . It yields that, for the bistatic case, α is not a roll-invariant parameter.

In the monostatic case, α and α_s are equal for a symmetrical target. This phenomenon is observed in Fig. 2(a). Indeed, a symmetrical target has a null target helicity (*i.e.* $\tau_m = 0 = \tau_1/2$) and the monostatic case is retrieved for $\theta_R = \theta_E$ (*i.e.* $\theta_2 = 0$).

It yields that the α/β model cannot be directly transposed to the bistatic case to extract a roll-invariant quantity. The bistatic TSVM should be used instead to provide an unique and roll-invariant target decomposition by means of five independent parameters α_s , Φ_{α_s} , τ_1 , τ_2 and μ . As for the monostatic case, those parameters are necessary for an unambiguous description of the backscattering mechanism.

6. CONCLUSION

In this paper, a generalization of the Target Scattering Vector Model to the bistatic case has been proposed. Based on the Kennaugh-Huynen decomposition, five parameters, namely

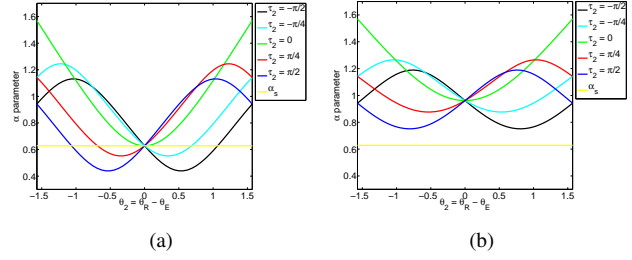


Fig. 2. Comparison between α and α_s as a function of θ_2 and τ_2 with $\theta_1 = \pi/3$ for: (a) $\tau_1 = 0$ and (b) $\tau_1 \neq 0$

α_s , Φ_{α_s} , τ_1 , τ_2 and μ , are necessary for an unambiguous description of a coherent target. The "monostatic" TSVM has been retrieved as a particular case of the proposed bistatic decomposition. Some comparisons with the so-called α/β model parameters have been done. It yields that α is not roll-invariant for the general case of bistatic polarimetry.

Further works will deal with the development of a bistatic incoherent target decomposition in terms of roll-invariant parameters.

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7. REFERENCES

- [1] R. Touzi, "Target Scattering Decomposition in Terms of Roll-Invariant Target Parameters," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 45, no. 1, pp. 73–84, January 2007.
- [2] J.R. Huynen, *Phenomenological Theory of Radar Targets*, Academic Press, 1978.
- [3] K. Kennaugh, "Effects of Type of Polarization on Echo Characteristics," *Ohio State Univ., Research Foundation Columbus Antenna Lab, Tech. Rep. 389-4, 381-9*, 1951.
- [4] J.R. Huynen, "Measurement of the Target Scattering Matrix," *Proceedings of the IEEE*, vol. 53, no. 8, pp. 936–946, August 1965.
- [5] A.-L. Germond, *Théorie de la Polarimétrie Radar en Bistatique*, Ph.D. thesis, Université de Nantes, Nantes, France, 1999.
- [6] Z.H. Czyz, "Fundamentals of Bistatic Radar Polarimetry Using the Poincare Sphere Transformations," Technical report, Telecommunications Research Institute, <http://airex.tksc.jaxa.jp/pl/dr/20010100106/en>, 2001.
- [7] C. Titin-Schnaider, "Polarimetric Characterization of Bistatic Coherent Mechanisms," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 46, no. 5, pp. 1535–1546, May 2008.
- [8] S.R. Cloude, "On the Status of Bistatic Polarimetry Theory," in *Geoscience and Remote Sensing, IGARSS'05, Seoul, Korea*, 2005, pp. 2003–2006.