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► To cite this version:

David Rey, Christophe Rapine, Sophie Constans, Rémy Fondacci. A MIP for potential conflicts minimization by speed regulations. 24th European Conference on Operational Research, Jul 2010, Lisbonne, Portugal. hal-00520060

HAL Id: hal-00520060

<https://hal.science/hal-00520060>

Submitted on 22 Sep 2010

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A MIP for potential conflicts minimization by speed regulations

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July 14, 2010



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les transports et leur sécurité



Outline

Global Context

Resolution Framework

Mathematical Formulation

Conclusion

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Some facts about ATM (Air Traffic Management)

ATM background in 2010 :

- ▶ European air traffic yearly growth stands between 3% to 5%,
- ▶ In 15 to 20 years : the number of flights over Europe is doubled \implies Airspace capacity issue.
- ▶ ATM is currently mostly human-operated and the ATC (Air Traffic Control) system is slowly reaching its limits, \implies need for introducing automation in ATM.

Toward a New Airspace Configuration

Launched in 2007, the SESAR program aims at re-organizing ATM in Europe. It includes increasing airspace capacity while ensuring a high level of safety and reducing the environmental impact of flights.

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WP 4 : En-Route Operations

One of the objective of the SESAR Master Plan is to optimize 4D Business Trajectories.

The ERASMUS Project : 2006-09

- ▶ The ERASMUS project has investigated [3] the effects of subliminal speed control on air traffic controllers.
- ▶ The validation of the concept as a deconfliction method (*i.e prevent loss of separation*) has led researchers to develop mathematical models to provide insight on the concept [6], [1].

Research Focus

PhD Thesis - Started in Sept 2009

Air Conflict Risk Minimization through Speed Regulation

- ▶ The objective is to investigate the limits of the speed regulation deconfliction method through a mathematical framework.
- ▶ Tests and simulations have to be performed to validate the robustness of our model for an hypothetical online implementation.

I work as a PhD student at the French National Institute for Transport and Safety Research (INRETS) in the Transport and Traffic Engineering Laboratory (LICIT).

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Conflict Definition

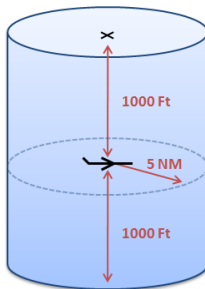


FIGURE: Separation Norms - (Off Scale)

When two cylinders intersect, there is a conflict between the two concerned aircraft. This model works well for path-crossing conflicts, trailing aircraft spacing is however subject to more complex rules (ICAO doc. 4444).

Resolution Process

This centralized mid-term CD&R method was first introduced in [2].

1. Most potential conflicts occur in the neighborhood of waypoints belonging to multiple flight trajectories. We focus on those intersection points for conflict detection.
2. Each conflict is attributed a cost, which value is proportional to the time required to maintain separation between the two concerned aircraft.
3. Adding all conflicts costs over a given region of airspace, we obtain a global cost for this region.
4. The speed regulation problem can then be modelled as an optimization problem. The computational time is bounded for it has to cope with ATM constraints (i.e less than 5 min).

Sliding Horizon Loop

At t_0 , the optimization problem is solved for the aircraft having crossing times between t_H and $t_H + H$.

p : Regulation Step

H : Optimization Horizon

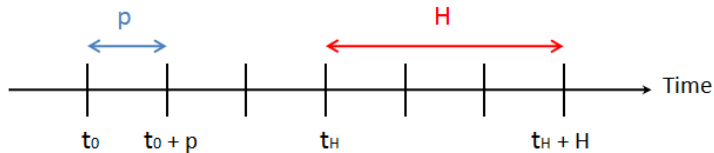


FIGURE: Sliding Horizon Loop

The optimization horizon, the regulation step and the relation between t_0 and t_H (look-ahead time) are to be determined together to fit ATC constraints.

Open Questions

Several questions still remain :

1. Is the time spent by two aircraft below the separation norm the right way to model a conflict cost ?
Can the severity of a conflict (from the controllers' workload point of view) be quantified in order to attribute higher costs to severe conflicts ?
2. How to handle border-related situations ? Assuming each FAB (Functional Air Block) has its own CD&R method, how to handle the border question ?
3. How to deal with speed uncertainty [4] ?

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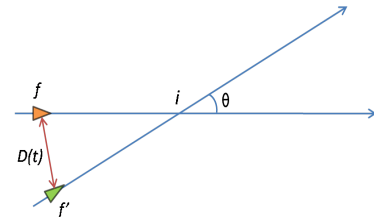
Conclusion

Conflict Cost

The right-hand side of both equations represents the time required for both aircraft to maintain space separation according to the separation norm D_s .

$(t_f^i, t_{f'}^i)$: Crossing time of flights f and f' at intersection point i .

$(v_f, v_{f'})$: En-route speed of flights f and f' .



$$|t_f^i - t_{f'}^i| < \sqrt{\frac{D_s^2(v_f^2 + v_{f'}^2 - 2\alpha v_f v_{f'})}{v_f^2 v_{f'}^2 (1 - \alpha^2)}} \quad (1)$$

with : $\alpha = \cos(\theta)$



$$|t_f^i - t_{f'}^i| < \frac{D_s}{v_f} \quad (2)$$

Denoting $\Gamma_{ff'}^i$ the right-hand side of either equation (1) or (2), we define the conflict cost as : $\max(\Gamma_{ff'}^i - |t_f^i - t_{f'}^i|, 0)$.

Speed Regulation Concept

Let $\{i_1, \dots, i_7\}$ be the path of flight f , each i_j being a waypoint. Denoting $\tau_{3,4}$ the travel time from i_3 to i_4 , we have :

$$\tau_{3,4} = \frac{d_{3,4}}{v_f}$$

As the distance is constant, bounding the travel time such as :

$$\tau_{3,4}^{min} \leq \tau_{3,4} \leq \tau_{3,4}^{max}$$

is equivalent to bounding the speed variation range. In order to cope with SESAR requirements, we set the speed regulation range from -6% to $+3\%$ of the nominal en-route speed of aircraft.

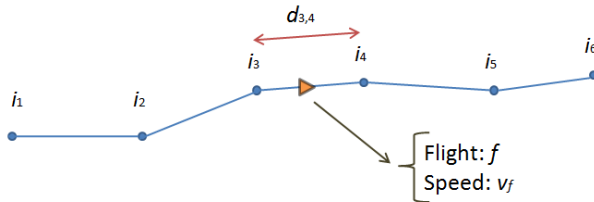


FIGURE: Speed control can be turned into travel time control

Decision Variables

The speed regulation problem is modelled as an optimization problem with decision variables t_f , the crossing times of flight $f \in F$ and F the set of all flights. The objective is to minimize the global conflict cost over a given region of airspace :

$$\begin{aligned} \min \sum_{i \in I} \left(\sum_{f, f' \in F_i} \max \left(\Gamma_{ff'i} - |t_f^i - t_{f'}^i|, 0 \right) \right) \\ \text{s.t. } \tau_f^{\min} \leq B t_f \leq \tau_f^{\max}, \quad \forall f \in F \end{aligned} \quad (3)$$

with B a square matrix such that $B t_f$ is the nominal travel time vector from two consecutive intersection points. $\tau_f^{\min}, \tau_f^{\max}$ the minimum and maximum travel time vectors of flight f . F_i the set of all flights having intersection point i in their path.

Meanwhile (3) is not a linear program and hence cannot be solved by an LP optimizer.

Linearization of the model

Introducing binary variables $y_{ff'}^i$ and $y_{f'f}^i$ to model the crossing order between two flights at a given intersection point i , we have :

$$y_{ff'}^i = \begin{cases} 1 & \text{if } t_f^i < t_{f'}^i \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

and :

$$y_{f'f}^i = \begin{cases} 1 & \text{if } t_{f'}^i < t_f^i \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

We include the priority order constraint as follows :

$$y_{ff'}^i + y_{f'f}^i = 1 \quad (6)$$

Optimization Problem

The speed regulation problem can now be defined as a Mixed Integer Linear Program :

$$\min \sum_{\substack{f, f' \in F_i \\ i \in I}} \omega_{ff'}^i \quad (7)$$

Such that :

$$\tau_f^{\min} \leq B t_f \leq \tau_f^{\max}, \quad \forall f \in F \quad (8a)$$

$$\omega_{ff'}^i \geq \Gamma_{ff'}^i - (t_f^i - t_{f'}^i) - (\Gamma_{ff'}^i + (\overline{t_{f'}^i} - \underline{t_f^i})) \cdot y_{ff'}^i \quad (8b)$$

$$\omega_{ff'}^i \geq \Gamma_{ff'}^i - (t_{f'}^i - t_f^i) - (\Gamma_{ff'}^i + (\overline{t_f^i} - \underline{t_{f'}^i})) \cdot y_{f'f}^i \quad (8c)$$

$$t_{f'}^i \leq t_f^i + (\overline{t_{f'}^i} - \underline{t_f^i}) \cdot y_{ff'}^i \quad (8d)$$

$$t_f^i \leq t_{f'}^i + (\overline{t_f^i} - \underline{t_{f'}^i}) \cdot y_{f'f}^i \quad (8e)$$

$$1 = y_{ff'}^i + y_{f'f}^i \quad (8f)$$

$$\omega_{ff'}^i, t_f^i, t_{f'}^i \in \mathbb{R}^+ \quad (8g)$$

$$y_{ff'}^i, y_{f'f}^i \in \{0, 1\}. \quad (8h)$$

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Future Works

- ▶ Investigate open questions,
- ▶ Include RTA (Required Time of Arrival) window in our model to limit global delay,
- ▶ Reduce optimization problem size to cope with ATM real time constraints.

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Thank You !

Optimization Problem Insight 1

Suppose flight f crosses intersection point i before flight f' , we have :

$$t_f^i < t_{f'}^i \quad \text{and} \quad y_{ff'}^i = 1 \quad \Rightarrow y_{f'f}^i = 0$$

the constraint set becomes :

$$\tau_f^{min} \leq Bt_f \leq \tau_f^{max}, \quad \forall f \in F \quad (9a)$$

$$\omega_{ff'}^i \geq (\underline{t}_f^i - \overline{t}_{f'}^i) - (t_f^i - t_{f'}^i) \quad (9b)$$

$$\omega_{ff'}^i \geq \Gamma_{ff'}^i - (t_{f'}^i - \underline{t}_f^i) \quad (9c)$$

$$t_{f'}^i \leq t_f^i + (\overline{t}_{f'}^i - \underline{t}_f^i) \quad (9d)$$

$$t_f^i \leq t_{f'}^i \quad (9e)$$

$$1 = 1 + 0 \quad (9f)$$

$$\omega_{ff'}^i, t_f^i, t_{f'}^i \in \mathbb{R}^+ \quad (9g)$$

$$y_{ff'}^i, y_{f'f}^i \in \{0, 1\}. \quad (9h)$$

Optimization Problem Insight 2

In constraint (9b) :

$$\omega_{ff'}^i \geq (t_{f'}^i - \overline{t_{f'}^i}) + (\underline{t_f^i} - t_f^i)$$

the right-hand side is negative since $\overline{t_{f'}^i}$ is the maximum crossing time of flight f' at intersection point i and $\underline{t_f^i}$ is the minimum crossing time of flight f at intersection point i . Thus the constraint vanishes since $\omega_{ff'}^i \in \mathbb{R}^+$.

Constraint (9d) is satisfied and necessary to prevent $t_{f'}^i$ from exploding. Constraint (9e) is satisfied by the hypothesis.

Optimization Problem Insight 3

Removing all satisfied and vanished constraints, we have :

$$\min \sum_{\substack{f, f' \in F_i \\ i \in I}} \omega_{ff'}^i \quad (10)$$

such that :

$$\tau_f^{\min} \leq B t_f \leq \tau_f^{\max}, \quad \forall f \in F \quad (11a)$$

$$\omega_{ff'}^i \geq \Gamma_{ff'}^i - (t_{f'}^i - t_f^i) \quad (11b)$$

$$\omega_{ff'}^i, t_f^i, t_{f'}^i \in \mathbb{R}^+ \quad (11c)$$

$$y_{ff'}^i, y_{f'f}^i \in \{0, 1\}. \quad (11d)$$

This optimization problem is equivalent to the non-linear formulation.