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A Mixed Integer Linear Model for Potential Conflict Minimization by Speed Modulations

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Abstract—As air traffic volume is growing around 5% each year in Europe it has become a priority to improve air traffic control in order to deal with tomorrow's air traffic configuration. In 2007 the SESAR (Single European Sky ATM Research) project was created under European Community law as an initiative to design the future of air traffic management over Europe. One of the objective of SESAR is to increase air traffic density and optimize flight route plans. This can be achieved through en-route deconfliction. Reducing the global number of conflicts through speed regulation has been tested in the ERASMUS project (En-Route Air traffic Soft Management Ultimate System) and efficiency of the concept has been analysed through tests and simulations [2]. It provided many insights related to en-route control using speed variations hence becoming a solid reference. In this paper we develop a mixed integer linear model for a speed regulation problem that suits SESAR requirements. We focus on flights crossing times at intersection points rather than distance to ensure separation along our resolution. Speed regulation is thus converted into travel time control. Finally we propose an integer linear program aiming at minimizing the global potential conflict quantity by speed modulations.

Index Terms—ATM, Conflicts, Mixed Integer Programming, Speed Control.

I. INTRODUCTION

ATM (Air Traffic Management) has become a challenge in Europe since the beginning of the 90's when it started to increase over 5% yearly. Airspace capacity is extended year after year in order to deal with growing demand, slowly reaching its limits. Europe's core area has one of the highest air traffic density in the world and generates many potential air conflicts every day. An air conflict occurs when two aircraft are too close to each other resulting in a loss of separation, which is a dangerous situation. Managing aircraft separation requires a high cognitive activity and directly affects ATCos' (Air Traffic Controllers) capacity to deal with large numbers of flights, therefore reducing the global airspace capacity. Hence today's ATM research and development is looking forward to reduce ATCos' workload by developing new tools to help controllers managing greater quantities of flights simultaneously while ensuring a high safety level. The ERASMUS project was funded to develop efficient ways to integrate advanced automation concepts in the SESAR framework. One of the objective of the SESAR Master Plan is to optimize flight route plans and as suggested in [2], automatic speed control could

be a way to improve air traffic deconfliction.

Many conflict detection and resolution methods have been tested and reviewed in [5]. Among the conflict resolution methods, lateral and vertical maneuvers are described in [3] and in [4]. We focus on minimizing the global conflict quantity through speed regulation. We chose a *global* multiple aircraft conflict resolution method as described in [5] thus considering the whole traffic simultaneously, using a mixed integer linear model. The conflict detection part is done using flight intentions and existing routes. Merging all this data, it is possible to come up with a map containing all intersections of all flight trajectories and thus all potential conflicts. Conflict resolution through speed regulation is currently not used in practice by ATCos. Indeed, ATCos rarely suggest speed variation to solve a potential conflict as it is generally not appreciated by pilots and airliners for fuel consumption and delay matters. Similarly, it is very difficult for an ATCo to perceive speed regulation on a radar screen making the automation of speed control a natural solution. In the next section we describe the framework of our resolution, then we introduce our model for the speed regulation problem and conclude with some perspectives.

II. RESOLUTION FRAMEWORK

The method developed here was first introduced in [1], we recall part of the original setup for it sets the basics of our model.

To reach their destination, aircraft follow trajectories that frequently intersect and overlap each other, especially in dense areas. Most conflicts happen around such intersection points thus we choose to focus on all those geographical references. The first step in our resolution method is to locate all intersection points of all routes on a two-dimensional map. Above each of these intersection points we identify the altitudes of all flights passing near the current intersection point in order to obtain a three-dimensional map of all potential conflict zones. If two aircraft are below the vertical separation norm in the neighborhood of an intersection point, they are considered in a potential conflict situation. We now define the notion of *conflict cost*. This notion is to be understood from the ATCo's point of view. Indeed, the longer two aircraft are predicted to be in conflict, the greater the monitoring and conflict solving workload is. Therefore,

the more time a controller spends to monitor and solve a conflict, the more expensive it should be. Defining a conflict cost in accordance with ATCos' workload seems to be the right manner to characterize the severity of a conflict. From a mathematical point of view, we define the cost of a conflict as a function of the time two aircraft spend under the separation norm. Adding up the costs of all potential conflicts over a given region we obtain a global conflict indicator for this region. Our objective is to minimize this indicator using speed modulations on concerned aircraft.

Flight trajectories, intersection points and aircraft performances are used as input data in our model. After calculating the theoretical arrival times of all flights on all intersection points and all the travel times from an intersection point to the next one, the next step is to optimize those arrival times through speed modulations in order to minimize the global conflict indicator. Recalling the method used in [1], we modify travel times to express speed variations. Indeed, bounding the speed variation range is equivalent to bounding the travel time from an intersection point to another for a given aircraft. Due to aircraft engine characteristics, the optimal speed variation range for an Airbus A320 is -6% to $+3\%$ of the nominal en-route speed according to [2]. Although this is the recommended speed variation range, it should be considered as one of the optimization problem parameter as well as the separation norm. As described in [1], we suppose that a sliding horizon loop with a fixed step is used to continuously regulate the traffic flow. The regulation time step has to be chosen carefully since the optimization problem is solved during each iteration. The optimization horizon (*i.e.* the period of time affected by the optimization) is to be determined together with the regulation step to limit the overall optimization problem size as it may reach extremely long computation times. Simulations are thus required to achieve better tuning of these parameters. The output data of our optimization problem are the optimized arrival times of all flights at intersection points in the optimization horizon. In the next section, we introduce our mathematical formulation for the speed regulation problem.

III. MATHEMATICAL FORMULATION

As mentioned in the previous section, the cost of a conflict depends on the configuration of the intersection as well as the aircraft speeds. In the en-route airspace the conflict area is a cylinder of 5 NM radius and 1000 ft height centered on the aircraft. Any other aircraft entering this area is considered "in conflict" with the current aircraft. In order to be more compatible with existing and future technology in ATM, we define the notion of conflict using a time interval. Focusing on the difference of the crossing times at a given intersection point of two aircraft, we can ensure separation. Quoting [1], we say that flights f and f' are in conflict at intersection point i if the instants t_{fi} and $t_{f'i}$ when flights f and f' cross

intersection point i , are such that:

$$|t_{fi} - t_{f'i}| < \sqrt{\frac{D_s^2(u^2 + u'^2 + 2\alpha uu')}{u^2 u'^2 (1 - \alpha)^2}} \quad (1)$$

where u and u' are the flights speeds, α is the cosinus of the angle between their trajectories and D_s the separation norm (*i.e.* 5 NM) parameter. The case where both aircraft share a common segment (*i.e.* $\alpha = 1$) is not described here. Information on this case can be found in [1].

Let $\Gamma_{ff'i}$ be the right hand side of (1). We define the cost of a potential conflict between flights f and f' at intersection point i such as:

$$\max(\Gamma_{ff'i} - |t_{fi} - t_{f'i}|, 0) \quad (2)$$

This local cost vanishes as soon as $|t_{fi} - t_{f'i}|$ is large enough to ensure separation between flights f and f' , and the closer the crossing times are, the higher the cost is. Defining the global cost as the sum of all the local ones, for all the flights and all the intersection points, our objective is to minimize this cost. In order to achieve en-route speed regulation on aircraft flying on intersecting routes, we focus on travel times from one intersection point to the next one. Let τ_{fi} be the travel time of flight f from its $(i-1)^{th}$ to i^{th} intersection point, we get the relations (3) and (4):

$$t_{fi} = t_{f_{i-1}} + \tau_{fi}, \quad \forall i \neq 1, \quad i \in I_f \quad (3)$$

$$t_{f1} = T_{fREF} + \tau_{f1} \quad (4)$$

Where I_f is the set of all intersection points of flight f . T_{fREF} is equal to T_{REF} , the time when the picture of the traffic is taken (*i.e.* the reference instant), if the aircraft is already airborne, and to the flight take-off time of flight f otherwise. Equations (3) and (4) can be rewritten in a matricial form:

$$Bt_f = \tau_f + T_{fR} \quad (5)$$

where T_{fR} is a vector of zeros, except for its first component, equal to T_{fREF} , and where B is a square matrix such that $b_{ij} = 1$ if $i = j$, $b_{ij} = -1$ if $i - 1 = j$ ($i > 1$), and $b_{ij} = 0$ otherwise.

Since we want to control the aircraft speeds along their trajectories, we have to bound the aircraft speed variation range. This is done by bounding their travel times. Let τ_f^m and τ_f^M be two vectors to limit the range of these modifications for flight f as lower and upper bounds on τ_f . We are now able to reproduce the optimization problem developed in [1]:

$$\begin{aligned} \min \sum_{i \in I} \left(\sum_{\substack{f, f' \in F_i \\ f < f'}} \max \left(\Gamma_{ff'i} - |t_{fi} - t_{f'i}|, 0 \right) \right) \\ \text{s.t. } \tau_f^m + T_{fR} \leq Bt_f \leq \tau_f^M + T_{fR}, \quad \forall f \in F \end{aligned} \quad (6)$$

where $t_{fi}, t_{f'i} \in \mathbb{R}^+$ are the decision variables of the problem, I is the set of the intersection points, F_i the set of the flights crossing intersection point i and F the set of all flights.

This formulation is not linear because of the absolute value in the objective function. In order to linearize this expression we introduce new parameters. Let t_{fi} and \bar{t}_{fi} be the minimal and maximal instants possible for flight f to cross intersection point i according to the speed variation range. We define two binary variables $y_{ff'i}$ and $y_{f'fi}$ to characterize the crossing order between flights f and f' at intersection point i . Variable $y_{ff'i}$ is such as:

$$y_{ff'i} \equiv \begin{cases} 1 & \text{if } t_{fi} \leq t_{f'i} \\ 0 & \text{otherwise} \end{cases}$$

and reciprocally for $y_{f'fi}$.

Consider the following optimization problem with decision variables $\omega_{ff'i}$, t_{fi} , $t_{f'i}$, $y_{ff'i}$ and $y_{f'fi}$:

$$\min \sum_{\substack{f, f' \in F_i \\ i \in I \\ f < f'}} \omega_{ff'i} \quad (7)$$

subject to:

$$\tau_f^m + T_{fR} \leq Bt_f \leq \tau_f^M + T_{fR}, \quad \forall f \in F \quad (8a)$$

$$\omega_{ff'i} \geq \Gamma_{ff'i} - (t_{fi} - t_{f'i}) - (\Gamma_{ff'i} + (\bar{t}_{f'i} - t_{fi})) \cdot y_{ff'i} \quad (8b)$$

$$\omega_{ff'i} \geq \Gamma_{ff'i} - (t_{f'i} - t_{fi}) - (\Gamma_{ff'i} + (\bar{t}_{fi} - t_{f'i})) \cdot y_{f'fi} \quad (8c)$$

$$t_{f'i} \leq t_{fi} + (\bar{t}_{f'i} - t_{fi}) \cdot y_{ff'i} \quad (8d)$$

$$t_{fi} \leq t_{f'i} + (\bar{t}_{fi} - t_{f'i}) \cdot y_{f'fi} \quad (8e)$$

$$1 = y_{ff'i} + y_{f'fi} \quad (8f)$$

$$\omega_{ff'i}, t_{fi}, t_{f'i} \in \mathbb{R}^+ \quad (8g)$$

$$y_{ff'i}, y_{f'fi} \in \{0, 1\}. \quad (8h)$$

We show that this mixed integer program is a correct formulation of optimization problem (6). Consider for instance the case where flight f crosses intersection point i before flight f' : we have $t_{fi} \leq t_{f'i}$ thus $y_{ff'i} = 1$. Constraint (8b) becomes:

$$\omega_{ff'i} \geq (t_{f'i} - \bar{t}_{f'i}) + (t_{fi} - t_{f'i})$$

which right-hand side is negative and thus vanishes because of (8g) (by redundancy). Constraint (8c) becomes:

$$\omega_{ff'i} \geq \Gamma_{ff'i} - (t_{f'i} - t_{fi})$$

Since $\omega_{ff'i}$ only appears in constraints (8b), (8c) and (8g), its value at the optimum is:

$$\omega_{ff'i}^* = \max(\Gamma_{ff'i} - (t_{f'i} - t_{fi}), 0).$$

This last expression of $\omega_{ff'i}$ is equivalent to the main part of the objective function in optimization problem (6). Similarly we can obtain the transposed formulation if $t_{f'i} \leq t_{fi}$. Therefore, optimization problem (7) subject to the set of constraints (8a) to (8h) is a linear formulation of optimization problem (6). This mixed integer program can now be implemented on a linear programming solver such as CPLEX.

IV. CONCLUSION

We developed an air conflict resolution method using a mixed integer linear model designed to solve the speed regulation problem in air traffic control. Experimentations are now required to set the limits of our approach, in particular to determine the maximum flight density we can deal with. As many mixed integer problems can have exponential computation times, we should watch carefully the program total running time when implemented on real-size scenarios, in order to cope with SESAR requirements. Tuning of the optimization problem parameters is essential to better achieve deconfliction through speed modulations, hence tests and simulations are to be performed in a near future. Results from these experimentations should guide us in our future work and could be compared to other results achieved in the ERASMUS project.

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