

# A generalized 3D inverted pendulum to represent human normal walking



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## 1. Summary

This paper compares different inverted pendulum models to represent the stance phase of human normal walking. We have developed a model which takes into account the mechanism of the foot during the single support phase, by defining a pivot point under the ground level. Similarly to other models, the pivot point as well as the rod length remain constant during the complete single support phase. Lowering the position of the pivot point allows reducing the vertical amplitude of the center of mass (CoM) trajectory and therefore approaching the real CoM trajectory. We have measured the whole body kinematics of a representative healthy male subject and set a reference CoM trajectory based on multibody modeling of the human body (16 segments). Then, we have determined a common mathematical definition of two inverted pendulum models extended in the three dimensional space: the classical IP-3D and our GIP-3D model.

## 2. Objectives

Study the current natural models to make them compatible with the humanoid control based on inverted pendulum. A way to make them look more natural is to directly implement an inverted pendulum model of human walking. This study focuses on existing inverted pendulum models for human normal walking.

## 3. Protocol and post-treatment of the data

3D trajectories of 58 markers (100Hz) stucked to the skin of a healthy adult male subject walking at his comfortable speed on a 10-meter walk way. Reference trajectory: the human body is modeled by 16 rigid segments representing the feet, legs, thighs, hands, forearms, arms, pelvis, abdomen, thorax and head (including neck). Reference kinematics of the CoM:

$$\begin{bmatrix} \text{OC} \\ 1 \end{bmatrix}_{\mathcal{R}_0} = \frac{1}{M} \sum_{i=1}^{16} m_i T_0^i \begin{bmatrix} \text{O}_i \text{C}_i \\ 1 \end{bmatrix}_{\mathcal{R}_i} \quad (1)$$

OC: absolute position of the CoM;  $m_i$ : mass of the  $i^{\text{th}}$  body;  $M = \sum_{i=1}^{16} m_i$ : total mass of the subject;  $T_j^k$ : homogeneous transformation matrix from  $\mathcal{R}_j$  to  $\mathcal{R}_k$ ;  $\mathcal{R}_0$ : reference frame.

## 4. Pendulum models of human walking

- The gait is in steady state;
- The walking is symmetric;
- The system is conservative;
- The double support phase is an homothetic transformation of the single support phase which center is the position of the approached CoM ( $C'$ ) at the transition instants between a single support phase and the next double support phase (THS) and the homothety ratio (Fig. 1):

$$k_0 = -\frac{S_d}{S_s} \quad (2)$$

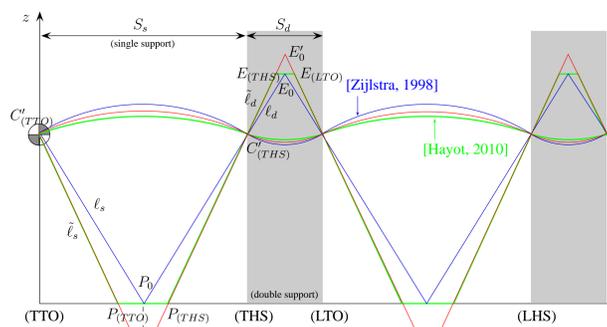


Fig. 1: Sagittal inverted pendulum representations of normal walking

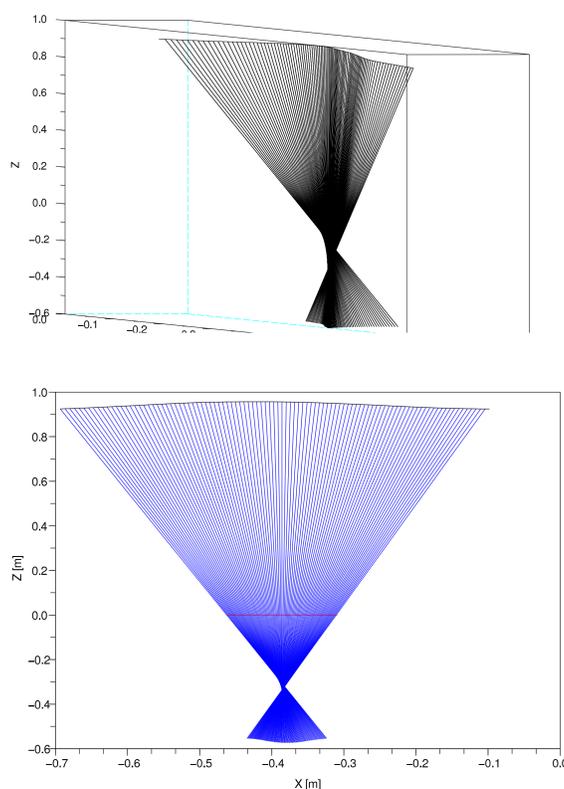


Fig. 2: 3D and sagittal views of the dynamics support lines of the single support phase. Black line: trajectory of the CoM; red line: trajectory of the CoP ( $z = 0$ ).

We observe a convergence area located 34 centimeters under the ground level of the instantaneous dynamics support lines (straight lines joining the CoM and the CoP at each time step of the single support phase). The pivot point is under the ground level!  $P'_0$ : the observed pivot point. How to determine the location of  $P'_0$ ?

$$\begin{bmatrix} \text{OC}' \\ 1 \end{bmatrix}_{\mathcal{R}_0} = \begin{cases} T_0^{P_k} \begin{bmatrix} \mathbf{P}_k \mathbf{C}' \\ 1 \end{bmatrix}_{\mathcal{R}_{P_k}} & ; t \in [\text{TTO}, \text{THS}] \\ T_0^{E_k} \begin{bmatrix} \mathbf{E}_k \mathbf{C}' \\ 1 \end{bmatrix}_{\mathcal{R}_{E_k}} & ; t \in [\text{THS}, \text{LTO}] \end{cases} \quad (3)$$

$$T_0^i = \begin{bmatrix} \cos \alpha_j & \sin \alpha_j \sin \beta_j & \sin \alpha_j \cos \beta_j & x_i \\ 0 & \cos \beta_j & -\sin \beta_j & y_i \\ -\sin \alpha_j & \cos \alpha_j \sin \beta_j & \cos \alpha_j \cos \beta_j & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Indexes  $s$  and  $d$  refer to the single and double support phases, respectively.  $\alpha(t)$  and  $\beta(t)$  are the sweep angles, around the lateral and the frontal axis, respectively.

$$\mathbf{P}_k \mathbf{C}' = \ell_s^k \mathbf{z}_{P_k} \quad \mathbf{E}_k \mathbf{C}' = \ell_d^k \mathbf{z}_{E_k} \quad (5)$$

$\ell_j^k$  ( $j = \{s, d\}$ ): constant rod lengths over their respective support phases:

1.  $\ell_s^k$  denotes the length between the pivot point and the CoM at  $t = \text{TTO}$ ;
2. at  $t = \text{THS}$ , the three points  $P_k$ ,  $C'$  and  $E_k$  are aligned and

$$\ell_d^k = |k_0| \ell_s^k.$$

## 5. Experimental results

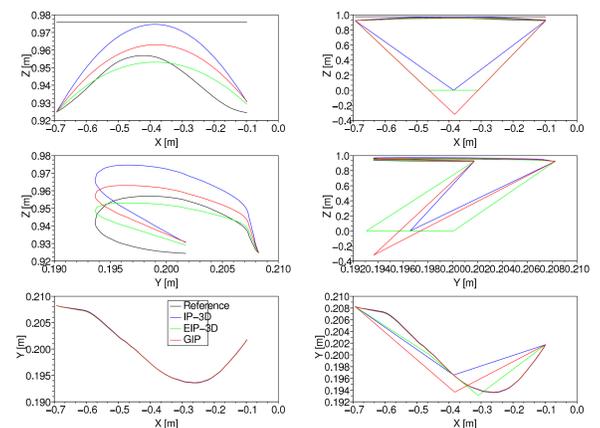


Fig. 4: Reference trajectory of the CoM and its approximations using IP-3D, EIP-3D and GIP-3D.

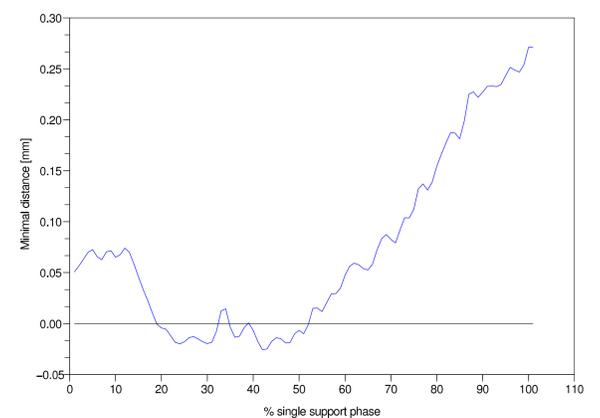


Fig. 5: The minimal distance between two adjacent instantaneous support lines does not exceed

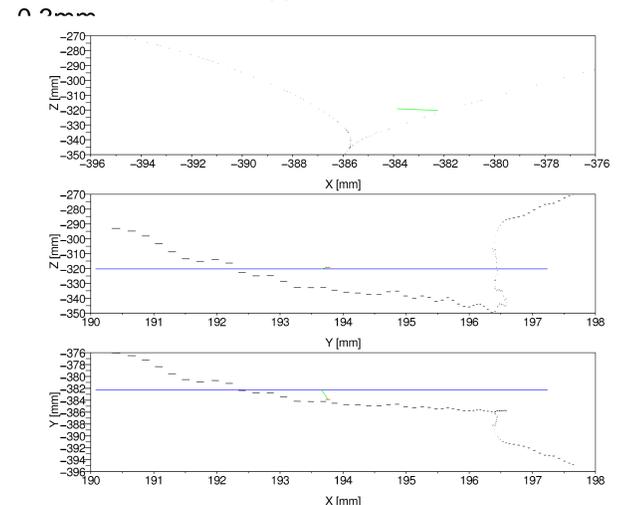


Fig. 6: Position of the pivot points with time.

## 6. Conclusions

- New pendulum model of human normal walking
- Pivot point is located 34 cm under the ground level
- easy approximated location of the pivot using the boundary support lines of a single support phase
- Approached CoM trajectory closer to the reference one compared to traditional pendulum model (IP3D)
- Behavior of the pivot point must be studied on a larger population to extract key characteristics of walking subjects.
- Extension possible to characterize human inspired robot walking control.