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## Comment on “Phase Transitions in Systems of Self-Propelled Agents and Related Network Models”

In a recent letter [1], Aldana *et al.* study order-disorder phase transitions in random network models and show that the nature of these transitions may change with the way noise is implemented in the dynamics. Arguing that these networks are limiting cases of simple models of interacting self-propelled agents of the type of the Vicsek model (VM) [2], they claim that the conclusions reached for the networks may carry over to the transitions to collective motion of the VM-like systems. They suggest in particular that in the case of “angular” noise (i.e. as in the original VM [5], or in their Eq. (1)) the transition to collective motion is *continuous*, in contradiction with some of the conclusions of [3]. While we agree with the analysis of the network models, we argue here that it has no bearing on VM-like systems. We show in particular that the transition to collective motion, for angular noise, remains *discontinuous* for any finite microscopic velocity  $v$  and finite density  $\rho$ , however large, confirming [3].

In [3], it was shown that the transition in the original VM appears continuous only when the linear system size  $L$  is smaller than some crossover size  $L^*(\rho, v)$  and thus is discontinuous in the thermodynamic limit (see Fig. 2a there). These results were obtained for values of  $\rho$  and  $v$  of order unity. The same scenario occurs in the large- $v$  or large  $\rho$  limit of interest here. Figure 1a here displays results akin to Fig. 1 of [1] except that the VM order parameter curves were obtained in systems such that  $L > L^*(\rho, v)$ , whereas in [1]  $L \simeq 100 < L^*(\rho, v)$  (see below for our estimates of  $L^*$ ). Clearly, the transition is discontinuous, as demonstrated by the minimum shown by the Binder cumulant (Fig. 1b) and by the bimodality of the order parameter distribution in the transition region (not shown).

The key difference between the network models and interacting self-propelled agents is indeed that the latter move, inducing a local coupling between order and density, which is well-known to be crucial for understanding collective properties of active particles [4]. In the VM, this coupling gives rise to strong density and order variations on lengthscales of the order of  $L^*$ . While the network models in [1] capture the long-range interactions due to large velocities, they obviously cannot account for any coupling between density and order. The network models only represent VM-like models of size  $L \lesssim L^*$ .

The crossover scale  $L^*$  is difficult to estimate with high accuracy, but our data indicate that  $L^*$  increases roughly linearly with  $v$  (for  $\rho = 2$ ,  $L^* = 150 \pm 25, 175 \pm 25, 250 \pm 50, 550 \pm 50$  for  $v = 5, 10, 20, 40$  respectively). Thus, we expect  $L^*$  to be finite at any finite  $v$ . As a consequence, the transition is always discontinuous in the thermodynamic limit, although its asymptotic behavior is harder to observe as  $v$  is taken larger and larger. When  $v$  is

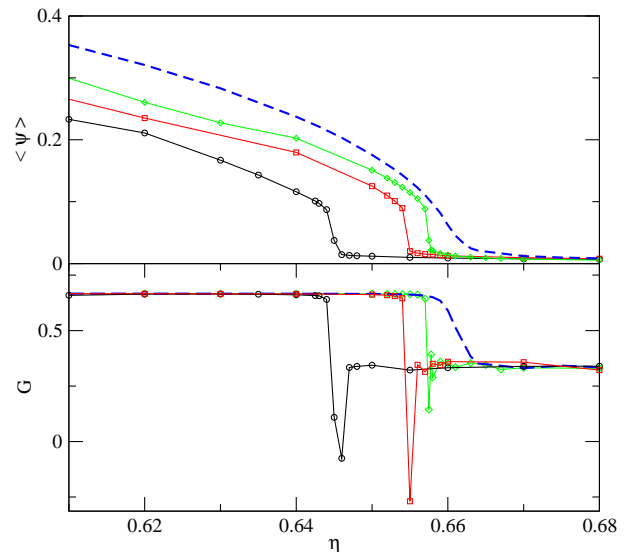


FIG. 1: (Color online) Order-disorder transition for the Vicsek model at different  $v$  values (solid lines) and the network of fixed vectors of [1] (dashed line), both with angular noise. (a) Time-averaged order parameter  $\langle \psi \rangle$  vs noise strength  $\eta$ . (b) same as (a) for the Binder cumulant  $G = 1 - \langle \psi^2 \rangle^2 / \langle \psi^4 \rangle$ . For the VM,  $\rho = 2$ , and  $v = 5$  (circles),  $v = 10$  (squares),  $v = 20$  (triangles) with  $L = 200, 250$  and  $300$  respectively. For the network, the number of nodes is  $N = 4 \times 10^5$  and the average connectivity is set to  $K = 2\pi$  (i.e. each node interacts with 6 or 7 neighbors, with suitable probabilities), which corresponds to  $\rho = 2$  in the VM.

taken to infinity first, the transition is continuous for all finite “size”, but then the notion of distance in physical space is abolished.

Summary: the transition to collective motion in VM-like systems with angular noise remains discontinuous for large  $v$  values. Thus, the networks studied in [1] at best constitute a *singular*  $v \rightarrow \infty$  limit of these systems [6].

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  - [5] Note, however, that Vicsek *et al* use a slightly different updating rule than in [1, 3].

[6] In the large  $\rho$  limit, we obtain similar results: for  $L > L^*$  the transition reveals its discontinuous character and  $L^*$  increases with  $\rho$  (results obtained at  $v = 0.5$ , not shown).