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Stefan Staicu. Dynamics analysis of the Minuteman cover drive. European Journal of Mechanics - A/Solids, 2009, 29 (1), pp.91. 10.1016/j.euromechsol.2009.07.001 . hal-00516877

HAL Id: hal-00516877

<https://hal.science/hal-00516877>

Submitted on 13 Sep 2010

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Accepted Manuscript

Title: Dynamics analysis of the Minuteman cover drive

Authors: Stefan Staicu

PII: S0997-7538(09)00096-5

DOI: [10.1016/j.euromechsol.2009.07.001](https://doi.org/10.1016/j.euromechsol.2009.07.001)

Reference: EJMSOL 2540

To appear in: *European Journal of Mechanics / A Solids*

Received Date: 13 November 2008

Accepted Date: 17 July 2009

Please cite this article as: Staicu, S. Dynamics analysis of the Minuteman cover drive, *European Journal of Mechanics / A Solids* (2009), doi: 10.1016/j.euromechsol.2009.07.001

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Dynamics analysis of the Minuteman cover drive

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ABSTRACT

Recursive matrix relations in kinematics and dynamics analysis of a 1-DOF compound planetary gear train are established in the paper. The mechanism of the Minuteman cover drive is a system with four moving links and three gear pairs controlled by one electric motor. Knowing the rotation motion of the output link, the inverse dynamic problem is solved using an approach based on the principle of virtual work, but it has been verified the results in the framework of the Lagrange equations. Finally, some simulation graphs for the input and output angles of rotation, the torque and the power of the actuator are obtained.

Keywords: Dynamics modelling; Gear mechanism; Kinematics; Lagrange equations; Virtual work

1. Introduction

The epicyclical gear trains are incorporated in the structure of industrial robots and have one, two or three output rotations. Including in their architecture conical and cylindrical teathed elements, the input axes of these mechanisms are parallel while the output axes can be nonparallel.

The industrial robots with orienting gear trains can perform several operations such as: welding, flame cutting, spray painting, milling or assembling. Being comparatively simple and compact in size, the bevel-gear wrist mechanisms can be sealed in a metallic box that keeps the device of contamination. Furthermore, using bevel gear trains for power transmission, the actuators can be mounted remotely on the forearm, thereby reducing the weight and inertia of a robot manipulator.

Numerous methods for kinematics analysis of epicyclical gear trains have been proposed by several researchers. Planetary gear trains with three degrees of freedom are adopted as the design concept for robotic wrist (Hsieh and Sheu,

1995; Paul and Stevenson, 1983; Willis, 1982; Ma and Gupta, 1989; White, 1988). The gear drives are commonly used for speed reduction and torque amplification in mechanical systems.

2. Inverse kinematics model

Recursive relations for kinematics and dynamics of a 1-DOF orienting gear train are developed in this paper. The mechanism topology of the Minuteman cover drive consists of four moving links, four turning pairs and three gear pairs (Fig. 1).

First, we wish to find the overall speed reduction ratio of this mechanism. A matrix methodology for the kinematics analysis based on the concept of fundamental circuit of an open-loop chain is presented. This method involves the identification of all open-loop chains and the derivation of the geometric relationships between the orientation of the output link and the joint angles of the chains, including the input actuator displacements (Tsai, 1999, 1988, 1987).

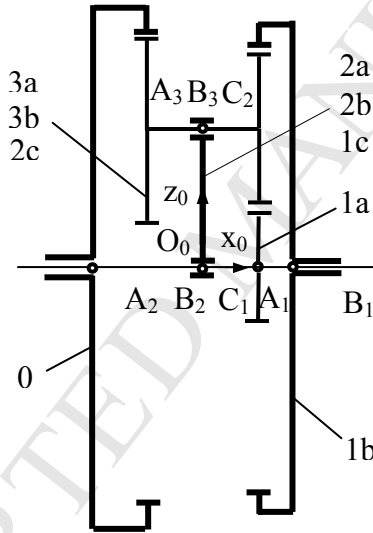


Fig. 1 The Minuteman cover drive

Let $O_0x_0y_0z_0(T_0)$ be a fixed Cartesian orthogonal frame, about which the mechanism moves. In the Minuteman cover drive, the first ring gear 0 is fixed to the ground, the sun gear $1a$ of radius r_1 , mass m_1 and tensor of inertia \hat{J}_1 is the input link connected to link 0 , while the moving ring gear $1b$ of mass m_4 and tensor of inertia \hat{J}_4 serves as output member. The compound planet gear $3a = 3b = 2c$ meshes with sun gear $1a$ as well as the two ring gears and is supported with a revolute joint by the carrier $2a = 2b = 1c$ as a connected coupling shaft of h in height, mass m_2 and tensor of inertia \hat{J}_2 . The central body $3a$, of mass m_3 and tensor of inertia \hat{J}_3 is adjacent to carrier $2a$ and consists of two cylindrical gears of

radius r_2, r_3 , respectively. Otherwise, the reduced bi-coupled transmission becomes a simply one-DOF compounding planetary gear train.

In the followings, we apply the method of successive displacements to geometric analysis of closed-loop chains and we note that a joint variable is the displacement required to move a link from the initial location to the actual position. If every link is connected to least two other links, the chain forms one or more independent closed-loops. The variable angles $\varphi_{k,k-1}$ of rotation about the joint axes z_k are the parameters needed to bring the next link from a reference configuration to the next configuration. We call the matrix $a_{k,k-1}^\varphi$, for example, the orthogonal transformation 3×3 matrix of relative rotation with the angle $\varphi_{k,k-1}^A$ of link T_k^A around z_k^A axis.

In the study of the kinematics of constrained systems, we are interested in deriving a matrix equation relating the location of an arbitrary T_k body to the joint variables. When the change of coordinates is successively considered, the corresponding matrices are multiplied.

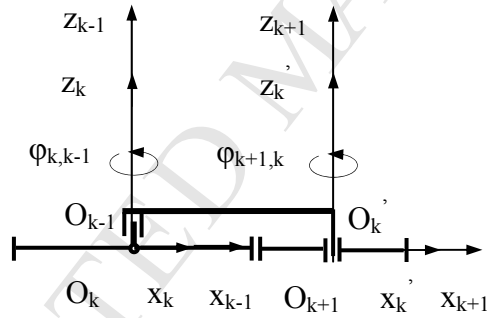


Fig. 2 Gear fundamental circuit

In what follows, we introduce a matrix approach which utilizes the *theory of fundamental circuits* developed by Tsai (1999). There exists a *real* or *fictitious* carrier for every gear pair in a planetary gear train and consequently a fundamental matrix equation for each loop can be written:

$$q_{k+1,k-1} = q_{k+1,k}^\varphi q_{k,k-1}^\varphi, \quad \varphi_{k,k-1} = n_{k+1,k-1} \varphi_{k+1,k}, \quad (1)$$

where $\varphi_{k,k-1}$ and $\varphi_{k+1,k}$ denote two successive relative angles of rotation of the carrier T_k and the planet gear T_{k+1} , respectively. The gear ratios of a gear pair is defined as

$$n_{k+1,k-1} = r_{k+1} / r_{k-1} = z_{k+1} / z_{k-1} \quad (2)$$

where r_{k-1} , r_{k+1} and z_{k-1} , z_{k+1} are the radius and the number of teeth of two gears, respectively (Fig. 2).

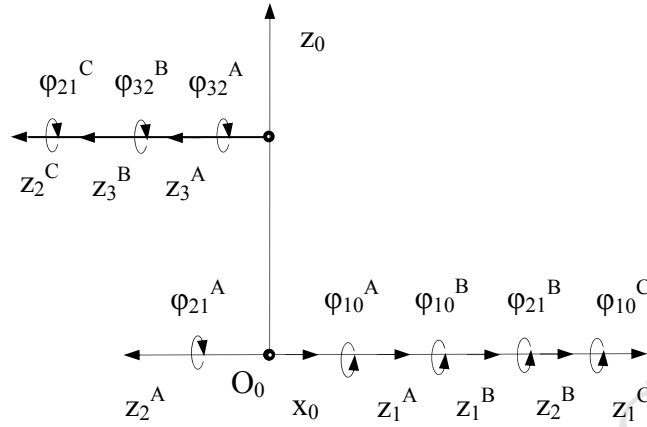


Fig. 3 Kinematical scheme of the mechanism

We consider the rotation angles φ_{10}^A of the actuator A_1 as single variable giving the instantaneous position of the mechanism (Fig. 3). Pursuing three kinematical chains $0-1a-2a-3a$, $0-1b-2b-3b$, $0-1c-2c$, we obtain following successive matrices of transformation (Staicu, Liu and Wang, 2007; Staicu and Zhang, 2008):

$$a_{10} = a_{10}^\varphi \theta_1, a_{21} = a_{21}^\varphi \theta_2, a_{32} = a_{32}^\varphi, b_{10} = b_{10}^\varphi \theta_1, b_{21} = b_{21}^\varphi, b_{32} = b_{32}^\varphi \theta_2, c_{10} = c_{10}^\varphi \theta_1, c_{21} = c_{21}^\varphi \theta_2. \quad (3)$$

where one denoted

$$\theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \theta_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, q_{k,k-1}^\varphi = \begin{bmatrix} \cos \varphi_{k,k-1}^i & \sin \varphi_{k,k-1}^i & 0 \\ -\sin \varphi_{k,k-1}^i & \cos \varphi_{k,k-1}^i & 0 \\ 0 & 0 & 1 \end{bmatrix}, (k = 1, 2, 3) \quad (4)$$

$$q_{k0} = \prod_{s=1}^k q_{k-s+1,k-j}^\varphi, (q = a, b, c), (i = A, B, C).$$

Let us suppose that the absolute motion of the end-effector attached at the output link $1b$ is a rotation of angle expressed by the analytical function

$$\varphi_{10}^B = \phi = \phi_0 \left[1 - \cos\left(\frac{\pi}{6} t\right) \right]. \quad (5)$$

The value $2\phi_0$ is a parameter characterizing the final position of the end-effector.

The rotation conditions for the central planet gear $3a = 3b = 2c$ are given by the following identities

$$a_{30} = b_{30} = c_{20}. \quad (6)$$

On the other hand, some constraint geometric conditions established along above three kinematical chains will be expressed by matrix equations

$$a_{20}^T \vec{r}_{32}^A = b_{30}^T \vec{r}_{32}^B = c_{20}^T \vec{r}_{21}^C, \quad (7)$$

where, for example, one denoted

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \vec{r}_{32}^A = -\vec{r}_{32}^B = -\vec{r}_{21}^C = h \vec{u}_1, \quad h = r_1 + r_2. \quad (8)$$

From the equations (6), (7), we obtain easily the real-time evolution of all characteristic joint angles, as follows

$$\varphi_{10}^B = \phi, \varphi_{32}^A = \varphi_{32}^B = \varphi_{21}^C = \frac{\phi}{n_3 - n_2}, \varphi_{10}^A = (n_1 + n_3)\varphi_{21}^C, \varphi_{21}^A = n_1\varphi_{21}^C, \varphi_{21}^B = n_2\varphi_{21}^C, \varphi_{10}^C = n_3\varphi_{21}^C \quad (9)$$

$$n_1 = \frac{r_2}{r_1}, n_2 = \frac{r_2}{r_1 + 2r_2}, n_3 = \frac{r_3}{r_1 + r_2 + r_3}.$$

In the design of power transmission mechanisms such as speed reducers or automotive transmissions, it is necessary to analyze the speed ratios between their input and output members and, sometimes, angular velocities or angular accelerations of all intermediate members.

The analysis of the kinematics of bevel-gear wrist mechanisms of gyroscopic structure, for example, is very complex due to the fact that the carriers and planet gears may possess simultaneous angular velocities about nonparallel axes. The conventional tabular or analytical method, which concentrates on planar epicyclical gear trains, is no longer applicable. To overcome this difficulty, Freudenstein, Longman and Chen (1984) applied the dual relative velocity and dual matrix of transformation for the analysis of epicyclical bevel-gear trains. The most straightforward approaches make use of the theory of fundamental circuits introduced by Freudenstein and Yang (1972), Tsai, Chen and Lin (1998), Chang and Tsai (1989) and Hedman (1993) and show that the kinematical analysis of geared robotic mechanisms can be accomplished by applying this systematic method.

Since a kinematical chain is an assemblage of links and joints, these can be symbolized in a more abstract form known as equivalent graph representation (Fig. 4). So, we use the *associated graph* to represent the topology of the mechanism. From this equivalent graph the fundamental circuits can be easily identified.

In the kinematical graph representation we denote the links by vertices and the joints by edges (Yan and Hsieh, 1991, 1994). Two small concentric circles label the vertex denoting the fixed link 0. To distinguish the difference between the pairs connections, the gear pairs $1a-3a, 1b-3b, 0-2c$ are designed by thick edges and the revolute joints $0-1a, 1a-$

$2a, 2a-3a, 1a-1b$ by thin edges. Four edged paths, which start from the base link 0 and end at the central link $3a = 3b = 2c$ consist of following vertices $1a, 2a, 3a, 1b, 2b, 3b$ and $1c, 2c$. There are three independent loops, three fundamental circuits $(1a, 2a, 3a), (1b, 2b, 3b), (0, 1c, 2c)$ and we identify one real and two *fictitious* carriers.

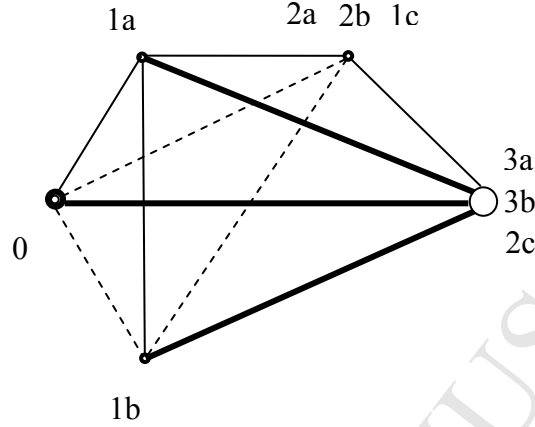


Fig. 4 Associated graph of the mechanism

The kinematics of an element for each circuit is characterized by skew-symmetric matrices given by the recursive relations (Staicu, 2009):

$$\tilde{\omega}_{k0}^i = q_{k,k-1} \tilde{\omega}_{k-1,0}^i q_{k,k-1}^T + \tilde{\omega}_{k,k-1}^i, \tilde{\omega}_{k,k-1}^i = \dot{\phi}_{k,k-1}^i \tilde{u}_3, \quad (10)$$

where \tilde{u}_3 is a skew-symmetric matrix associated with the unit vector \bar{u}_3 . These matrices are *associated* to the angular velocities

$$\bar{\omega}_{k0}^i = q_{k,k-1} \bar{\omega}_{k-1,0}^i + \bar{\omega}_{k,k-1}^i, \bar{\omega}_{k,k-1}^i = \dot{\phi}_{k,k-1}^i \bar{u}_3. \quad (11)$$

Knowing the rotation motion of the output link $1b$ by the relations (5), one develops the inverse kinematical problem and determines the velocities $\bar{v}_{k0}^i, \bar{\omega}_{k0}^i$ and the accelerations $\bar{\gamma}_{k0}^i, \bar{\varepsilon}_{k0}^i$ of each of the moving links. Based on the important remark

$$\omega_{k,k-1} = n_{k+1,k-1} \omega_{k+1,k}, \quad (12)$$

the derivatives with respect to time of the relations (9) lead to the relative angular velocities of all links as function of the angular velocity $\dot{\phi}_{10}^B = \dot{\phi}$ of the output gear:

$$\omega_{10}^B = \dot{\phi}, \omega_{32}^A = \omega_{32}^B = \omega_{21}^C = \frac{\dot{\phi}}{n_3 - n_2}, \omega_{10}^A = (n_1 + n_3) \omega_{21}^C, \omega_{21}^A = n_1 \omega_{21}^C, \omega_{21}^B = n_2 \omega_{21}^C, \omega_{10}^C = n_3 \omega_{21}^C. \quad (13)$$

These kinematical expressions will be further required in the computation of virtual velocity distribution of the elements of the mechanism. Starting from these results, a complete expression of the Jacobian of the mechanism is easily written in an invariant form. This square invertible matrix is an essential element for the analysis of singularity loci into mechanism workspace.

Let us assume now that the mechanism has a *virtual motion*. Characteristic virtual velocities expressed as function of robot's position are given by the above relations (13):

$$\omega_{10v}^A = 1, \omega_{32v}^A = \omega_{32v}^B = \omega_{21v}^C = \frac{1}{n_1 + n_3}, \omega_{21v}^A = n_1 \omega_{21v}^C, \omega_{10v}^B = (n_3 - n_2) \omega_{21v}^C, \omega_{21v}^B = n_2 \omega_{21v}^C, \omega_{10v}^C = n_3 \omega_{21v}^C. \quad (14)$$

These virtual velocities are required into the computation of the virtual work of the forces applied to the component elements of the gear train.

Concerning the relative angular accelerations of the compounding elements of the mechanism, these are immediately given by deriving the relations on the above velocities: $\mathcal{E}_{k,k-1}^i = \dot{\omega}_{k,k-1}^i$.

The angular accelerations $\tilde{\mathcal{E}}_{k0}^i$ and the useful square matrices $\tilde{\omega}_{k0}^i \tilde{\omega}_{k0}^i + \tilde{\mathcal{E}}_{k0}^i$ are calculated with the following recursive formulae (Staicu, 2008):

$$\tilde{\mathcal{E}}_{k0}^i = q_{k,k-1} \tilde{\mathcal{E}}_{k-1,0}^i + \mathcal{E}_{k,k-1}^i \tilde{u}_3 + \omega_{k,k-1}^i q_{k,k-1} \tilde{\omega}_{k-1,0}^i q_{k,k-1}^T \tilde{u}_3 \quad (15)$$

$$\tilde{\omega}_{k0}^i \tilde{\omega}_{k0}^i + \tilde{\mathcal{E}}_{k0}^i = q_{k,k-1} (\tilde{\omega}_{k-1,0}^i \tilde{\omega}_{k-1,0}^i + \tilde{\mathcal{E}}_{k-1,0}^i) q_{k,k-1}^T + \omega_{k,k-1}^i \omega_{k,k-1}^i \tilde{u}_3 \tilde{u}_3 + \mathcal{E}_{k,k-1}^i \tilde{u}_3 + 2\omega_{k,k-1}^i q_{k,k-1} \tilde{\omega}_{k-1,0}^i q_{k,k-1}^T \tilde{u}_3.$$

Knowing the rotation motion of the output link *lb* by the relations (5), one determines also the absolute velocity \vec{v}_{k0}^i and the absolute acceleration $\vec{\gamma}_{k0}^i$ of each of the moving links:

$$\vec{v}_{k0}^i = q_{k,k-1} \vec{v}_{k-1,0}^i + q_{k,k-1} \tilde{\omega}_{k-1,0}^i \vec{r}_{k,k-1}^i + v_{k,k-1}^i \tilde{u}_3, \quad v_{k,k-1}^i = 0, \quad (16)$$

$$\vec{\gamma}_{k0}^i = q_{k,k-1} \vec{\gamma}_{k-1,0}^i + q_{k,k-1} \{ \tilde{\omega}_{k-1,0}^i \tilde{\omega}_{k-1,0}^i + \tilde{\mathcal{E}}_{k-1,0}^i \} \vec{r}_{k,k-1}^i + 2v_{k,k-1}^i q_{k,k-1} \tilde{\omega}_{k-1,0}^i q_{k,k-1}^T \tilde{u}_3 + \gamma_{k,k-1}^i \tilde{u}_3, \quad \gamma_{k,k-1}^i = 0.$$

The velocity \vec{v}_k^{Ci} and the acceleration $\vec{\gamma}_k^{Ci}$ of mass centre of T_k^i rigid body are calculated from two basic matrix relations

$$\vec{v}_k^{Ci} = \vec{v}_{k0}^i + \tilde{\omega}_{k0}^i \vec{r}_k^{Ci}, \quad \vec{\gamma}_k^{Ci} = \vec{\gamma}_{k0}^i + \{ \tilde{\omega}_{k0}^i \tilde{\omega}_{k0}^i + \tilde{\mathcal{E}}_{k0}^i \} \vec{r}_k^{Ci}. \quad (17)$$

The matrix relations (10), (11), (15), (16) and (17) will be further used for the computation of the wrench of the inertia forces for every rigid component of the mechanism.

The dynamic model would only be established in regard with the complete geometrical analysis and kinematics of the mechanical system.

3. Equations of motion

In the context of the real-time control, neglecting the friction forces and considering the gravitational effects, the relevant objective of the dynamics is to determine the input torques and powers, which must be exerted by the actuators in order to produce a given trajectory of the end-effector.

3.1. Principle of virtual work

The torque of moment $\vec{m}_{10}^A = m_{10}^A \vec{u}_3$ can control by intermediate of an electric motor the motion of the geared mechanism. The derivation of a dynamic model has a very important effect in the determination of the actuator torque (Tsai, 2001; Muller, Mannhardt and Glover, 1982; Castillo, 2002).

In the inverse dynamic problem, in the present paper one applies first the principle of virtual work in order to establish recursive matrix relations for the torque and the power of the active system. The mechanism of the gear train can artificially be transformed in a set of three open serial chains C_j ($j = A, B, C$) subject to the constraints. This is possible by cutting successively several joints for the central link $3a$ and taking their effects into account by introducing the corresponding constraint conditions.

Considering that the motion of the output link $1b$ is given, the position, angular velocity, angular acceleration as well as the velocity and acceleration of the centre of mass are known of each element. The force of inertia of an arbitrary rigid body T_k^A , for example

$$\vec{f}_{k0}^{inA} = -m_k^A [\vec{\gamma}_{k0}^A + (\tilde{\omega}_{k0}^A \tilde{\omega}_{k0}^A + \tilde{\varepsilon}_{k0}^A) \vec{r}_k^{CA}] \quad (18)$$

and the resulting moment of the forces of inertia

$$\vec{m}_{k0}^{inA} = -[m_k^A \tilde{r}_k^{CA} \vec{\gamma}_{k0}^A + \hat{J}_k^A \tilde{\varepsilon}_{k0}^A + \tilde{\omega}_{k0}^A \hat{J}_k^A \tilde{\omega}_{k0}^A] \quad (19)$$

are determined with respect to the centre of the joint A_k . The wrench of two vectors \vec{f}_k^{*A} and \vec{m}_k^{*A} evaluates the influence of the action of the external and internal forces applied to the same element T_k^A or of its weight $m_k^A \vec{g}$, for example:

$$\vec{f}_k^{*A} = 9.81 m_k^A a_{k0} \vec{u}_3, \quad \vec{m}_k^{*A} = 9.81 m_k^A \tilde{r}_k^{CA} a_{k0} \vec{u}_3 \quad (k = 1, 2, 3). \quad (20)$$

Finally, two significant recursive relations generate the vectors

$$\vec{F}_k^A = \vec{F}_{k0}^A + a_{k+1,k}^T \vec{F}_{k+1}^A, \quad \vec{M}_k^A = \vec{M}_{k0}^A + a_{k+1,k}^T \vec{M}_{k+1}^A + \tilde{r}_{k+1,k}^A a_{k+1,k}^T \vec{F}_{k+1}^A, \quad (21)$$

where one denote

$$\vec{F}_{k0}^A = -\vec{f}_{k0}^{inA} - \vec{f}_k^{*A}, \quad \vec{M}_{k0}^A = -\vec{m}_{k0}^{inA} - \vec{m}_k^{*A}. \quad (22)$$

The fundamental principle states that a mechanism is under dynamic equilibrium if and only if the virtual work developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism. Applying *the fundamental equations of parallel robots dynamics* obtained in a compact form by Stefan Staicu (2005, 2009), the following little matrix relation results

$$m_{10}^A = \vec{u}_3^T \{ \omega_{10v}^A \vec{M}_1^A + \omega_{10v}^B \vec{M}_1^B + \omega_{10v}^C \vec{M}_1^C + \omega_{21v}^C \vec{M}_2^C \}. \quad (23)$$

for the torque of the actuator.

The relations (21) and (23) represent the *inverse dynamic model* of the Minuteman cover drive. The procedure leads to very good estimates of the actuator torque for given displacement of end-effector, provided that the inertial properties of the gears are known with sufficient accuracy and that friction is not significant. This new dynamic approach developed here can be extended to any gyroscopic bevel-gear train with revolute actuators.

In that follows we can apply the Newton-Euler procedure to establish the set of analytical equations for each compounding rigid body of a prototype gear train in a real application. These equations give all connecting forces in the external and internal joints. Several relations from the general system of equations could eventually constitute verification for the input forces or active torques obtained by the method based on the principle of virtual work. Now, we can calculate the friction forces and the friction torques in the joints, based on the friction coefficients and the maximum of the connecting forces in the joints. We apply again the explicit equations (21) and (23), where the contribution of the virtual work of friction forces in joints it is added. The new active torque is compared to the value obtained in the first calculus.

3.2. Equations of Lagrange

A solution of the dynamics problem of the Minuteman cover drive can be developed based on the Lagrange equations of second kind. The single generalized coordinate of the robot are represented by the rotation angle of the actuator: $\varphi_{10}^A = \alpha$.

The Lagrange's equation will be expressed by one differential relation

$$\frac{d}{dt} \left\{ \frac{\partial E}{\partial \dot{\alpha}} \right\} - \frac{\partial E}{\partial \alpha} = Q_\alpha, \quad (24)$$

which contain following generalized force

$$Q_\alpha = m_{10}^A - \frac{n_3 - n_2}{n_1 + n_3} M_r + \left(\frac{1}{2} m_2 + m_3 \right) gh \frac{n_3}{n_1 + n_3} \sin\left(\frac{n_3}{n_1 + n_3} \alpha \right). \quad (25)$$

The components of the general expression of the kinetic energy $E = \sum_{\nu=1}^3 E_\nu^A + E_1^B$ are expressed as analytical function of

first derivative with respect to time of the generalized coordinate:

$$E_1^A = \frac{1}{2} \bar{\omega}_{10}^{AT} \hat{J}_1 \bar{\omega}_{10}^A, E_2^A = \frac{1}{2} \bar{\omega}_{20}^{AT} \hat{J}_2 \bar{\omega}_{20}^A, E_3^A = \frac{1}{2} m_3 \bar{v}_{30}^{AT} \bar{v}_{30}^A + \frac{1}{2} \bar{\omega}_{30}^{AT} \hat{J}_3 \bar{\omega}_{30}^A, E_1^B = \frac{1}{2} \bar{\omega}_{10}^{BT} \hat{J}_4 \bar{\omega}_{10}^B, \quad (26)$$

where the absolute angular velocities have the expressions:

$$\bar{\omega}_{10}^A = \dot{\alpha} \bar{u}_3, \bar{\omega}_{20}^A = \dot{\alpha} \frac{n_3}{n_1 + n_3} \bar{u}_3, \bar{\omega}_{30}^A = \dot{\alpha} \frac{n_3 - 1}{n_1 + n_3} \bar{u}_3, \bar{v}_{30}^A = -\dot{\alpha} h \frac{n_3}{n_1 + n_3} \bar{u}_2, \bar{\omega}_{10}^B = \dot{\alpha} \frac{n_3 - n_2}{n_1 + n_3} \bar{u}_3. \quad (27)$$

In the inverse dynamics problem, a calculus of the derivatives with respect to time $\frac{d}{dt} \left\{ \frac{\partial E}{\partial \dot{\alpha}} \right\}$ of all above functions leads

quickly to same expression (23) for the input torque m_{10}^A required by the actuator, now given as analytical solution:

$$m_{10}^A = \left\{ J_{1z} + \frac{n_3^2}{(n_1 + n_3)^2} (J_{2z} + m_3 h^2) + \frac{(n_3 - 1)^2}{(n_1 + n_3)^2} J_{3z} + \frac{(n_3 - n_1)^2}{(n_1 + n_3)^2} J_{4z} \right\} \ddot{\alpha} + \frac{n_3 - n_2}{n_1 + n_3} M_r - \left(\frac{1}{2} m_2 + m_3 \right) g h \frac{n_3}{n_1 + n_3} \sin\left(\frac{n_3}{n_1 + n_3} \alpha\right). \quad (28)$$

4. Simulation procedure

The procedure for solving the inverse dynamics of the Minuteman cover drive by using the principle of virtual work can be summarised in several basic steps.

1°. For a period of $\Delta t = 6$ second, it is assumed that the time-history of the moving output link is specified from equation

(5). The relations (9) give the evolution of joint variables $\varphi_{10}^i, \varphi_{21}^i, \varphi_{32}^i$ ($i = A, B, C$).

2°. Using the relations (3), (4) and (6) we compute the transformation matrices of three legs A, B, C : $q_{10}^i, q_{21}^i, q_{32}^i$,

$$q_{20}^i = q_{21}^i q_{10}^i, q_{30}^i = q_{32}^i q_{20}^i.$$

3°. Determine the velocities and accelerations of all links by performing the inverse kinematics analysis in terms of prescribed angular velocity $\dot{\phi}$ and angular acceleration $\ddot{\phi}$ of the output link $1b$. Specifically, for each kinematical chain, from

the relations (13), we compute the relative angular velocities $\omega_{k,k-1}^i$ and the relative angular accelerations $\varepsilon_{k,k-1}^i = \dot{\omega}_{k,k-1}^i$.

4°. Starting from same equations (13), where we introduce $\omega_{10v}^A = 1$, we compute the virtual characteristic velocities (14) of all moving elements.

5°. We compute the velocity and the acceleration of the centre of mass as well as the angular velocity and the angular acceleration of each link using the equations (11), (15), (16) and (17).

6°. Decompose artificially the gear mechanism in three independent open-loop kinematical chains $1a, 1b$ and $1c - 2c$.

7°. We determine the inertia force (18), the moment of inertia forces (19), the resulting force and the resulting moment (excluding the actuator torque) exerted to the rigid body T_k , from recursive equations (20).

8°. Finally, we find the input torque m_{10}^A and the power $p_{10}^A = \omega_{10}^A m_{10}^A$ from the equations (23) or (28).

For simulation purposes let us consider a geared mechanism which has the following characteristics

$$r_1 = 0.03 \text{ m}, r_2 = 0.02 \text{ m}, r_3 = 0.04 \text{ m}, m_1 = 0.3 \text{ kg}, m_2 = 0.25 \text{ kg}, m_3 = 0.6 \text{ kg}, m_4 = 0.85 \text{ kg}$$

$$h = d = 0.05 \text{ m}, M_r = -1.75 \text{ Nm}, \phi_0 = \pi, \Delta t = 6 \text{ s}.$$

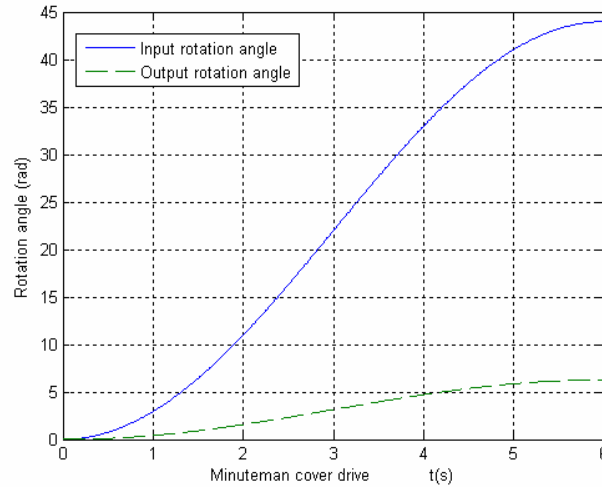


Fig. 5 Input and output rotation angles ϕ_{10}^A, ϕ_{10}^B

A program which implements the suggested algorithm is developed in MATLAB to solve first the inverse kinematics of the gear train. For illustration, it is assumed that the end-effector starts at rest from its initial position and is moving in a known rotation motion. A numerical study of the kinematics is carried out by computation of the input and output angles of rotation ϕ_{10}^A, ϕ_{10}^B , for example (Fig. 5).

Based on the algorithm derived from the above recursive relations, a computer program solve the inverse dynamics modelling of the mechanism, using the MATLAB software. Assuming that a *resistant torque* of constant moment $M_r = -1.75 \text{ Nm}$ applied at the end-effector and the weights $m_k^A \bar{g}$ of compounding rigid bodies constitute the external forces acting on the mechanism during its evolution, a numerical computation in the dynamics is developed, based on the determination of the input torque m_{10}^A (Fig. 6) and its active power p_{10}^A (Fig.7). The time-history evolution of the torque and power required by the active system is shown for a period of six second of motion.

5. Conclusions

Within the inverse kinematics analysis, some exact matrix relations giving the position, velocity and acceleration of each link for a 1-DOF epicyclic gear train have been established.

Knowing the rotation motion of the output link, the inverse dynamic problem is solved using an approach based on the principle of virtual work, but it has been verified the results in the framework of the Lagrange equations. The new approach described above is very efficient and establishes a direct recursive determination of the variation in real-time of torque and power of the actuator. The matrix relations, given by this dynamic simulation, can be transformed in a model for automatic command of the gear mechanism.

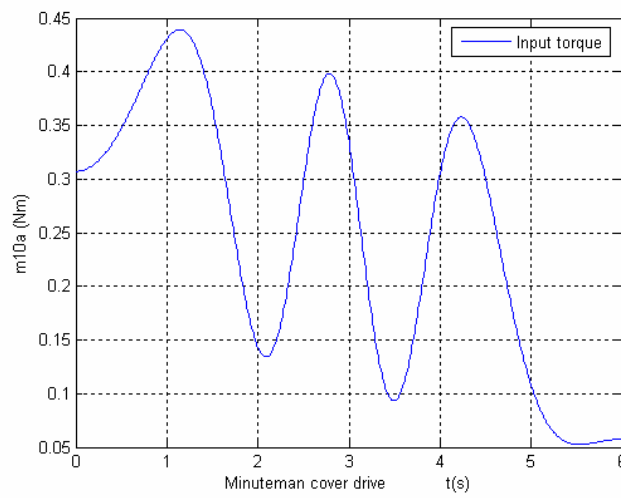


Fig. 6 Input torque m_{10}^A of the actuator

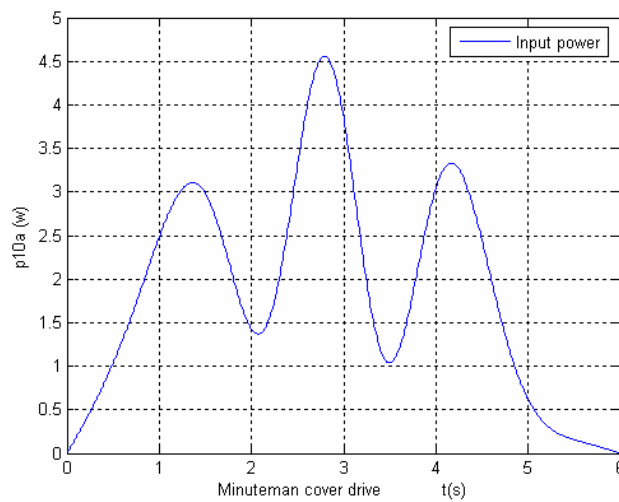


Fig. 7 Input power p_{10}^A of the actuator

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