



HAL
open science

Shallow water acoustic communications : how far are we from the channel capacity ?

Jean-Michel Passerieux, François-Xavier Socheleau, Christophe Laot

► To cite this version:

Jean-Michel Passerieux, François-Xavier Socheleau, Christophe Laot. Shallow water acoustic communications : how far are we from the channel capacity ?. European Conference on Underwater Acoustics, Jul 2010, Istanbul, Turkey. hal-00515385

HAL Id: hal-00515385

<https://hal.science/hal-00515385>

Submitted on 10 Jun 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Shallow Water Acoustic Communications: How Far are we from the Channel Capacity ?

Jean-Michel Passerieux¹, Francois-Xavier Socheleau^{1,2} and Christophe Laot²
jean-michel.passerieux@fr.thalesgroup.com, {fx.socheleau, christophe.laot}@fr.telecom-bretagne.eu

¹Thales Underwater Systems SAS, 525 route des Dolines, BP 157, 06903 Sophia Antipolis Cedex, France

²Telecom Bretagne; Technopole Brest-Iroise-CS 83818, 29238 Brest Cedex, France

This paper focuses on the capacity of the underwater acoustic communications (UAC) channel under realistic assumptions: time-varying multi-paths channel, modelled as a stochastic doubly-spread frequency-selective channel, unknown actual realization of the channel at both transmitter and receiver, constraints on both transmitted power (rms and/or peak) and available frequency bandwidth. The exact channel capacity under such assumptions is still unknown. Therefore, several bounds for this capacity are given and then numerically assessed for a few typical shallow water UAC channels. Main conclusion is that, even with the above comprehensive set of demanding assumptions, as far as the theoretical channel capacity is considered, transmission with higher than often now spectral efficiency (e.g. 2 to 4 bits/sec/Hz) appears as a reasonable objective in typical time-varying underwater acoustic communication channels, providing SNR about 15 to 20 dB.

Introduction

After pioneering works in the 70's and 80's [1] and first assessments of high bitrate phase coherent communications in the 90's [2], a few attractive techniques [3] are now emerging in underwater acoustic communications (UAC), especially for the difficult shallow water UAC channel. Among these techniques, the most promising¹ are likely the multi-carriers (MC) modulations (with Orthogonal Frequency Division Multiplex (OFDM) [4, 5] as a special case), efficient channel coding techniques such as turbo or LDPC codes, and iterative reception algorithms such as turbo-equalization [6, 7].

The results obtained with these techniques are generally quite promising. A few recently reported experimental results, such as the successful transmission, with a 32-QAM modulation and turbo-equalization, in the Baltic Sea at 50 km and 10 kbits/sec reported in [8] even appear quite amazing. Hence, an assessment of the ultimate performance of shallow water UAC, by the means of computation of the Shannon capacity [9] appears critical to determine whether these new techniques could actually yield a significant breakthrough in UAC, or whether the reported results are only the consequence of exceptionnally favorable environmental conditions. Meanwhile, this interest is also renewed by the outcomes of many recent works on the capacity of fading channel, [10–13], mainly motivated by the wireless [14] or ultra-wideband (UWB) channels, but which are applicable to the UAC channel as well.

Unlike the capacity of other channels, the capacity of the shallow water UAC channel has been seldomly addressed. Apart from recent publications [15] related to MIMO sys-

tems, we are only aware of a very few works in the UAC context. Hayward et al. [16] apply a Gaussian-beam propagation code to get the amplitudes and phases of the multipaths in a flat bottom shallow water environment (100 meters depth, range up to 20 km) with an almost unlimited available frequency band (from 100 to 10⁶ Hz). Then, assuming a time-invariant frequency-selective channel, they allocate the transmitted power across the available frequency band in order to maximize the channel capacity (by using the water-filling algorithm [14]). Assuming a 193 dB rePa@1 m source level, they eventually get very high theoretical bitrates, about 1 Mbits/sec at 1 km or 100 kbits/sec at 10 km. More recently, Stojanovic et al. [17, 18] consider a simpler propagation model (a single path whose signal-to-noise ratio (SNR) at reception varies with frequency and range according to simple models). Then, they also apply the water-filling algorithm to maximize the channel capacity and derive approximate simple relations between range, transmitted power and channel capacity. Unlike [16], they do not provide numerical values of the channel capacity, however some values of the spectral efficiency plotted in [17] also appear optimistic.

Main point is that all the channel capacities above, even if interesting are not fully realistic since they neglect some critical characteristics of the UAC channel, its random and highly time-varying impulse response, and of the transmitting devices, the often encountered peak-power and bandwidth limitations. To be more explicit, on the basis of the shallow water UAC characteristics, we believe that a model that aims at being realistic should take the following aspects into account:

- The channel is selective in both time and frequency.
- Each channel tap can be modelled as a Rice or Rayleigh-fading process, with memory, that satisfies the wide-

¹Note that multiple-inputs multiple-outputs (MIMO) techniques, even if they also appear attractive in UAC, are not addressed here where the focus is laid upon single-input single-output (SISO) systems.

sense stationary uncorrelated scattered assumptions (WSSUS)².

- Neither the transmitter (TX) or the receiver (RX) knows the current realization of the channel. However, thanks to the WSSUS assumption, we consider that both TX and RX know the channel statistics.
- The available frequency bandwidth is limited.
- The peak power of the transmitted symbols is limited.

These capacity computations aim at determining the gap between theoretical capacity in a AWGN channel and the same capacity with the realistic above assumptions. Meanwhile, they also provide some reliable pieces of information about how far the current experimental results are from the ultimate theoretical performance of the shallow water UAC.

This paper is organized as follows. Section 1 is devoted to a formal presentation of the relevant assumptions and notations. Previous results related to the addressed capacity of constrained peak-power frequency-selective channels are recalled in Section 2. Then, upper and lower bounds for the capacity of a doubly spread frequency-selective Ricean channel are derived in Section 3. These bounds are finally discussed and applied to typical shallow water UAC channels in Section 4.

1 Main assumptions and system model

Underwater acoustic channel and WSSUS modelling

The properties of the underwater acoustic channel have been widely investigated for several decades. With respect to the shallow water UAC [20–22], its main characteristics are the low propagation speed (about 1500 m/s), the increase of the propagation losses with frequency (which results in a small available bandwidth at medium or large ranges, typically a few kHz at 10 km), the spreading of the channel impulse response (CIR), both in time and frequency, the lack of direct path at medium or large range (due to the refraction of the sound paths), the random fluctuations of the propagation delays and angles of arrival due to multiple reflexions on rough or moving interfaces (sea bottom and surface) and, finally, a long diffuse reverberation.

A general statistical model to deal with such channels, dispersive both in time and frequency, is the wide sense stationary uncorrelated scattered (WSSUS) model [23], where the channel is modelled as a continuous-time random linear time-varying (LTV) filter

$$y(t) = \int h(t, \tau)x(t - \tau) + w(t) \quad (1)$$

²To be more accurate, in [19] it is shown that the shallow water channel taps are rather trend-stationary than wide-sense stationary processes. However, only the local mean of these processes is time-varying, besides with slow fluctuations.

and where $x(t)$, $y(t)$ and $w(t)$ are the channel input, its output and the additive noise, respectively, and $h(t, \tau)$ the continuous-time channel impulse response, assumed to be a two-dimensional stochastic process, characterized by its mean $\bar{h}(\tau) \triangleq E[h(t, \tau)]$, its correlation function $C_H(\Delta t, \tau)$ and its scattering function $S_H(\tau, \nu)$ defined as

$$E[[h(t, \tau) - \bar{h}(\tau)] \cdot [h(t', \tau') - \bar{h}(\tau')]] \quad (2)$$

$$\triangleq C_H(t - t', \tau) \cdot \delta(\tau - \tau')$$

$$S_H(\tau, \nu) \triangleq \int C_H(\Delta t, \tau) e^{-2j\pi \Delta t \cdot \nu} \cdot d \Delta t. \quad (3)$$

Note that, as in [23], the WSSUS channel is generally assumed to be zero-mean, which corresponds to channel taps distributed according to a Rayleigh distribution. However, the generalization to a non-zero mean channel, which corresponds to Ricean channel taps, is straightforward. Then, apart from the scattering function above, another critical parameter is the Rice factor, which now depends upon the propagation delay τ , and is defined as $\kappa(\tau) \triangleq \frac{|\bar{h}(\tau)|^2}{C_H(0, \tau)}$.

Relation between channel input and output

In the sequels, the transmitted and received signals, $x(t)$ and $y(t)$, as well as the CIR $h(t, \tau)$ and the noise $w(t)$, are assumed to be sampled at the symbol rate B , where B is also equal to the available frequency band. It is also assumed that the symbol rate B is such that $B \gg \nu_{max}$, where ν_{max} is the Doppler spread of the channel. Therefore, the Nyquist condition is met when sampling the channel response both along time t and delay τ .

The transmitted symbols $X = (x_0, \dots, x_{N-1})^t$ are assumed identically independent distributed (i.i.d.) with a peak-power constraint expressed as

$$|x_n|^2 \leq \Omega_x^2. \quad (4)$$

Note that here the average power $E[|x_n|^2] \triangleq \sigma_x^2$ is not explicitly constrained. However $\sigma_x^2 \leq \Omega_x^2$ is always true, with equality when the input symbols are distributed on the circle with radius Ω_x .

The discrete time channel output $Y = (y_0, \dots, y_{N-1})^t$ is given by

$$Y = \mathcal{T}_H \cdot X + W \quad (5)$$

where W is the vector of the N noise samples, assumed i.i.d. gaussian and zero-mean with variance σ_w^2 , and \mathcal{T}_H the $N \times N$ random channel matrix defined as

$$\mathcal{T}_H \triangleq \begin{pmatrix} h_{0,0} & 0 & \dots & \dots & \dots & 0 \\ \vdots & h_{1,0} & \ddots & & & \vdots \\ h_{L-1,L-1} & \vdots & \ddots & \ddots & & \vdots \\ 0 & h_{L,L-1} & & h_{L,0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \ddots & 0 \\ 0 & \dots & 0 & h_{N-1,L-1} & \dots & h_{N-1,0} \end{pmatrix} \quad (6)$$

and $h_{n,k}$ is the gain at time n of the channel tap k , for $n \in [0, N - 1]$ and $k \in [0, L - 1]$, and L the length of CIR. Depending upon the context, it can also be convenient to rewrite (5) as

$$Y = \sum_{k=1}^{L-1} H_k \odot X_{\downarrow k} + W \quad (7)$$

where H_k (for $k \in [0, L - 1]$) is the $N \times 1$ vector of the gains of the k th channel tap (the k th low subdiagonal of the channel matrix \mathcal{T}_H), $X_{\downarrow k}$ the vector obtained from X by shifting its $N - k$ first elements k times downward, and then padding the k upper elements with zeros. Finally the symbol \odot stands for the Hadamard (element-wise) product. From the WSSUS assumption, it follows

$$E[H_k] = \bar{h}_k \cdot 1_N \triangleq \bar{H}_k \quad (8)$$

$$E\{[H_k - \bar{H}_k] \cdot [H_l - \bar{H}_l]^+\} \triangleq C_H(k) \cdot \delta_{k,l} \quad (9)$$

where \bar{h}_k and $C_H(k)$ are the mean and the covariance matrix of the k th channel tap, respectively.

For commodity³, we denote now by $\sigma_h^2(k)$ the elements of the main diagonal of the matrices $C_H(k)$ and

$$\Xi_H^2 \triangleq \sum_k \sigma_h^2(k) \quad (10)$$

$$\Psi_H^2 \triangleq \sum_k |\bar{h}_k|^2 = \int_{-1/2}^{1/2} |\psi_H(\theta)|^2 d\theta \quad (11)$$

where θ is the normalized frequency ($\theta \triangleq (f - f_{carrier})/B$), and $\psi_H(\theta)$ the discrete Fourier transform of the mean of the CIR, $\psi_H(\theta) = \sum_k \bar{h}_k e^{-2j\pi k \theta}$.

Note that the two above parameters Ψ_H^2 and Ξ_H^2 can be viewed as the energies of the mean and of the random (zero-mean) parts of the CIR of an equivalent time-varying flat fading channel obtained by summing the contributions of the different multipaths or channel taps.

Finally, we also denote by \mathcal{C}_H the sum over the channel taps of the correlation matrices $C_H(k)$, $\mathcal{C}_H \triangleq \sum_k C_H(k)$, and, by $\mathcal{S}_H(\theta)$ and $\tilde{\mathcal{S}}_H(\theta)$, the raw and the normalized spectral density of this equivalent time-varying flat-fading channel

$$\mathcal{S}_H(\theta) = \sum_n [\mathcal{C}_H]_{n,1} e^{-2j\pi n\theta/N} \quad (12)$$

$$\tilde{\mathcal{S}}_H(\theta) = \mathcal{S}_H(\theta) / \Xi_H^2. \quad (13)$$

Assumptions for capacity assessment

As expressed in [14, 24], the definition of the capacity for a random linear time-varying communication channel is not as simple as for the AWGN channel. Indeed, one has to consider how information on the channel state (CSI) is available.

Here, the most general case is considered, where the actual realization of the channel is unknown to TX and RX, but the statistical properties of the channel are assumed to be known to TX and RX. Then, with the above notations, the channel capacity is given by [9]

$$C_p = \lim_{N \rightarrow \infty} \frac{1}{N} \left[\sup_{p_X \in \mathcal{P}_X} I(X; Y) \right] \quad (14)$$

where $I(Y; X) = h_E(Y) - h_E(Y|X)$ is the mutual information between X and Y , $h_E(X)$ the differential entropy of X , and the sup is taken for p_X in the set \mathcal{P}_X of the input symbol distributions which meet the constraints (4).

2 Recall of previous works on peak-limited doubly-spread fading channel

When dealing with frequency-selective channels, a common practice [14] is to notice that these channels can be made approximately equivalent to a bank of parallel narrow-band flat-fading channels. It allows to apply in this context most of the available results for flat-fading channels [11, 12, 25].

A possible approach consists in expanding the signals on a set of Gabor functions $g_{k,n}(t)$ ⁴. Denote by $x_{k,n}$, $h_{k,n}$, $y_{k,n}$ and $w_{k,n}$ the coefficients of this expansion for $x(t)$, $h(t)$, $y(t)$ and $w(t)$, respectively. In [13], it is argued that the $g_{k,n}(t)$ are approximate eigen-functions of the channel, with the $h_{k,n}$ as corresponding eigen values (i.e. $\int h(t, \tau) g_{k,n}(t - \tau) d\tau = h_{k,n} \cdot g_{k,n}(t)$). Hence $y_{k,n} = h_{k,n} \cdot x_{k,n} + w_{k,n}$, for all k and n , and the relation between the channel input and output reads

$$\mathcal{Y} = \mathcal{H} \odot \mathcal{X} + \mathcal{W} \quad (15)$$

where the $KN \times 1$ vectors \mathcal{X} , \mathcal{H} and \mathcal{W} are defined by stacking the $x_{k,N}$, $y_{k,N}$, $h_{k,N}$ and $w_{k,N}$.

Therefore, thanks to (15), the WSSUS channel has been transformed into a set of NK flat-fading channels, with inputs and outputs in the time/frequency domain, to which it is possible to apply all the results applicable for a flat fading channels. After a few manipulations, this yields to the upper and lower capacity bounds given in [13] for the Rayleigh frequency-fading channel, and extended to the Ricean case in [26], which are applicable in the case where the channel is peak-limited in the time and frequency domain. Here we just recall the upper bound.

Theorem 1: Assume that, when represented on the above Gabor expansion, the input symbols to a time-varying Ricean frequency-selective fading channel are i.i.d. and bounded as $|x_{k,n}|^2 \leq \Omega_x^2$ (constraint in the time/frequency domain). Then, the capacity of this channel is upper bounded as

$$C_{upper}^{FT} = \max_{0 \leq \alpha \leq 1} \left[\int \log \left(1 + \alpha \frac{\Omega_x^2 (\Xi_H^2 + \Psi_H^2(\theta))}{\sigma_w^2} \right) d\theta - \alpha \int \int_{-\infty}^{\infty} \log \left(1 + \frac{\Omega_x^2 \mathcal{S}_H(\nu, \tau)}{\sigma_w^2 B} \right) d\nu d\tau \right] \quad (16)$$

³Note that, thanks to the WSSUS assumption, the matrices $C_H(k)$ are Toeplitz.

⁴ $g_{k,n}(t)$ defined by $u(t - n\Delta T_e) \exp(-2j\pi k \Delta F_e t)$, where the generating function $u(t)$ is taken well localized both in time and frequency

where $S_H(\nu, \tau)$ is the scattering function of the channel as defined in Section 1.

3 Doubly-spread fading channel with peak-limited power in time

Even if interesting, the approach summarized in Section 2 applies only when the peak-power constraint is applied on a time-frequency representation of the input signal (e.g. on the symbols of an OFDM modulation). In our opinion, this is correct for the applications where the peak-power constraint results from spectrum allocation regulations (eg. UWB or wireless communication systems [14]). However, in the UAC context, the peak-power constraint results from the transducer technology and has to be applied in the time domain. This is why two new bounds are given below, whose proofs are outlined in Appendices A and B.

Theorem 2: The channel capacity of a discrete-time Ricean WSSUS channel with i.i.d. input symbols and peak-power constraint in the time domain is upper bounded by

$$C_{upper}^{FSEL} = \max_{0 \leq \alpha \leq 1} \int_{-1/2}^{1/2} \log \left(1 + \frac{\alpha \Omega_x^2}{\sigma_w^2} (\Xi_H^2 + |\psi_H|^2(\theta)) \right) d\theta - \alpha \int_{-1/2}^{1/2} \log \left(1 + \frac{\Omega_x^2 \Xi_H^2}{\sigma_w^2} \tilde{S}_H(\theta) \right) d\theta \quad (17)$$

where all quantities are as defined in Section 1.

Concerning this first bound, it is worth noticing that, given the parameter α which is determined numerically, it is the difference of two terms:

- the first term is the capacity of a AWGN channel, with SNR obtained by summing the energy incoming from all multipaths. It takes into account for the whole CIR, i.e. its mean and its random zero-mean parts.
- the second term, which is a penalty term, corresponds to the capacity loss due to the fact that the CIR is time-varying and unknown. This term takes into account only for the random zero-mean part of the channel response (for all multipaths).

Theorem 3: The capacity of a discrete-time Ricean WSSUS channel with i.i.d. input symbols and peak-power constraint in the time domain is lower bounded as

$$C_{lower}^{FSEL} = \lim_{N \rightarrow \infty} \frac{1}{N} E_H \left[\log \det \left(I_N + \frac{1}{e} \frac{\Omega_x^2}{\sigma_w^2} \mathcal{T}_H \mathcal{T}_H^+ \right) \right] - \int_{-1/2}^{1/2} \log \left[1 + \frac{\sigma_x^2 \Xi_H^2}{\sigma_w^2} \tilde{S}_H(\theta) \right] d\theta \quad (18)$$

where all quantities are as defined in Section 1.

Note that, unlike the upper bound, this lower bound is not fully analytic. In our best knowledge, the expectation of the first term is still unknown. Hence, it has been numerically assessed via a Monte-Carlo technique.

As the upper bound, this lower bound is also the difference of two terms. The first term is now the coherent capacity of the channel (i.e. the capacity when the CIR is available to RX, but not to TX). Hence, the penalty term appears as the capacity loss due to the fact that the CIR is unknown. The second term, a penalty term, is similar to the second term in Theorem 2, with average SNR of the zero-mean random time-varying part of the CIR, while in Theorem 2 it depends upon the peak SNR of this time-varying part of the CIR.

As a first application, the capacity bounds obtained with Theorems 1-3 for four simple WSSUS channels are plotted⁵ on Figure 1 (in bits/sec/Hz vs peak SNR in dB). The parameters corresponding to these channels are listed in the table below (number of path of the CIR, power profile, Rice factors).

	paths	power (dB)	Rice factor	T_3
I	3	0 -5 -5	30 10 10	50
II	3	0 -5 -5	10 5 5	50
III	3	0 -5 -5	5 0 0	50
IV	1	0	5	50

Table 1: Main parameters of channels considered on Figure 1.

The correlation of the channel taps has been assumed exponentially decreasing with time, i.e. such that the elements of the p th subdiagonal of the channel covariance matrices $C_H(k)$ are given by $[C_H(k)]_{l,l+p} = \sigma_h^2 \cdot \exp(-\frac{p \log(2)}{T_3})$. The parameter T_3 is given in the last column of table.

Main conclusion of these first plots is that, even for very demanding UCA channels (channels II and III), the theoretical capacity remains far better than the bitrate of existing high data rate modems (for instance, 2 to 3 bits/sec/Hz at 15 dB, vs typically at most 1 bit/sec/Hz for modems).

4 Application to at sea measured UAC channels

Figure 2 shows some examples of real channel impulse responses recorded at sea as well as their respective capacity bounds. The first two impulse responses were recorded in the Mediterranean sea at 6 kHz, with a 60 to 120 m depth, and TX and RX separated by 2500 and 9000 m respectively. The third CIR was recorded in the Atlantic ocean at 17.5 kHz with a 10 to 30 m depth. The transmission range was approximately 2000 m.

The analysis of the figure leads to the main conclusion that, in the operating SNR range of existing high data rate modems (approx. 15 dB), these channels should allow us to communicate at 2 to 3 bits/sec/Hz. This means that there is still a 200 to 300% potential capacity gain compared to the performance of existing SISO modems since they usually work with a capacity of approximately 1 bit/sec/Hz. Similarly,

⁵For comparison with known results, the capacity of a AWGN channel has also been plotted, even if it is not rigorously correct since abscissa is here the peak SNR and not the averaged SNR.

this also means that there is a 5 to 10 dB gap between what is implemented today and the ultimate theoretical limits.

Note that despite the difference between the three CIR's, the bound C_{upper}^{FSEL} is almost the same for all channels. This can be explained by the strong Ricean nature of these channels. In the examples, most of the energy of the channels is conveyed by a few paths with a very high Rice factor (between 40 to 700). Consequently, the penalty term of C_{upper}^{FSEL} does not impact much the overall bound.

In addition, as expected the bound C_{upper}^{TF} is lower than C_{upper}^{FSEL} since both dimensions (time and frequency) are constrained. Moreover, this bound varies quite significantly from a channel to another. This comes from its penalty term that considers both the time and frequency selectivity of the random components of the channel whereas C_{upper}^{FSEL} only considers the time selectivity of these components and is therefore looser.

Conclusions

Upper and lower bounds for the capacity of a peak limited doubly-spread frequency selective communication channel have been given. This channel, even if still a theoretical model of the true is likely a far better model than the AWGN channel usually taken for assessment of the UAC channel capacity. Conclusion is that, even with this model, there is still a possible bitrate improvement with respect to the existing high data rate modems.

Appendix A: Proof of Theorem 2

The proof is derived from the proof given in [11] with more general constraint on the transmitted power, but only for the time-varying Rayleigh flat-fading channel. It is detailed in [26]. Here, we just outline it.

A first step is to notice that, in (14), the term $h_E(Y)$ can be upper bounded by the differential entropy of a complex gaussian vector \tilde{Y} with same covariance matrix as vector Y (see [9]). Hence, $h_E(Y) \leq \log \pi e + \log \det \Gamma_Y$, where Γ_Y is the covariance matrix of Y . In other respects, conditionally to the input symbols X , the channel output Y is (exactly) complex gaussian, with covariance $\Gamma_{Y|X}$. Hence $h_E(Y|X) = \log \pi e + \log \det \Gamma_{Y|X}$. Finally, it turns out that

$$C \leq \sup_{P_X \in \mathcal{P}_X} \lim_{N \rightarrow \infty} \frac{1}{N} \left[\log \det \Gamma_Y - E_X \log \det \Gamma_{Y|X} \right]. \quad (19)$$

The second step for the proof is to compute the two covariance matrices Γ_Y and $\Gamma_{Y|X}$ and then to substitute their log det into (19). The corresponding lengthy computations, detailed in [26], yield to

$$\lim_{N \rightarrow \infty} \frac{1}{N} \log \det \Gamma_Y = \log \sigma_w^2$$

$$+ \int_{-1/2}^{1/2} \log \left[1 + \frac{\sigma_x^2}{\sigma_w^2} (\Xi_H^2 + \|\psi_H(\theta)\|^2) \right] d\theta \quad (20)$$

and

$$\lim_{N \rightarrow \infty} \frac{1}{N} \log \det \Gamma_{Y|X} = \lim_{N \rightarrow \infty} \log \sigma_w^2 + \frac{1}{N} \log \left[I_N + \frac{\Omega_x^2}{\sigma_w^2} \mathcal{C}_H \odot X X^+ \right]. \quad (21)$$

where all quantities are as defined in Section 1. Hence

$$C \leq \int_{-1/2}^{1/2} \log \left[1 + \frac{\sigma_x^2}{\sigma_w^2} (\Xi_H^2 + \|\psi_H(\theta)\|^2) \right] d\theta - \lim_{N \rightarrow \infty} \frac{1}{N} E_X \left[\log \left[I_N + \frac{1}{\sigma_w^2} \mathcal{C}_H \odot X X^+ \right] \right]. \quad (22)$$

Finally the third step consists in lower bounding the second term of (22) as

$$\begin{aligned} & \lim_{N \rightarrow \infty} \frac{1}{N} E_X \left[\log \det \left[I_N + \frac{1}{\sigma_w^2} \mathcal{C}_H \odot X X^+ \right] \right] \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \int_X \log \det \left[I_N + \frac{1}{\sigma_w^2} \mathcal{C}_H \odot X X^+ \right] p_X(X) dX \\ &\geq \lim_{N \rightarrow \infty} \frac{1}{N} \left[\inf_{P_X \in \mathcal{P}_X} \frac{\log \det \left[I_N + \frac{1}{\sigma_w^2} \mathcal{C}_H \odot X X^+ \right]}{\|X\|^2} \right] E\|X\|^2 \\ &= \sigma_x^2 \lim_{N \rightarrow \infty} \left[\inf_{P_X \in \mathcal{P}_X} \frac{\log \det \left[I_N + \frac{1}{\sigma_w^2} \mathcal{C}_H \odot X X^+ \right]}{\|X\|^2} \right] \\ &\geq \frac{\sigma_x^2}{\Omega_x^2} \int_{-1/2}^{1/2} \log \left(1 + \frac{\Omega_x^2}{\sigma_w^2} S_H(\theta) \right) d\theta \end{aligned} \quad (23)$$

where the last inequality follows from [13, lemma 11, pp. 383] and, as in Section 1, $S_H(\theta)$ is the spectral density of the equivalent flat-fading channel.

Then, Theorem 2 finally follows from substitution of (23) into (22) and a few manipulations.

Appendix B: Proof of Theorem 3

The starting point for the proof of Theorem 3 is a well known [11, 25] information theoretic inequality:

$$I(Y; X) \geq I(Y; X | \mathcal{T}_H) - I(Y; \mathcal{T}_H | X). \quad (24)$$

Let's now notice that the computation of the second term of (24), $I(Y; \mathcal{T}_H | X) = h_E(Y | \mathcal{T}_H) - h_E(Y | \mathcal{T}_H, X)$, is straightforward since

- conditionally to X , Y is distributed according to a complex gaussian distribution with covariance $\sigma_w^2 I_N + \sigma_x^2 \mathcal{C}_H$,
- conditionally to X and \mathcal{T}_H , Y is complex gaussian with covariance matrix $\sigma_w^2 I_N$.

It yields $I(Y; H|X) = \log \det(I_N + \frac{\sigma_x^2}{\sigma_w^2} C_H)$ and, therefore, the channel capacity can be lower bounded as

$$C_p \geq \frac{1}{N} I(Y; X|\mathcal{T}_H) - \lim_{N \rightarrow \infty} \frac{1}{N} \log \det \left[I_N + \frac{\sigma_x^2}{\sigma_w^2} C_H \right]. \quad (25)$$

The computation of the first term is not as easy. However, this term can be lower bounded thanks to a generalization of the entropy power inequality [9].

Lemma (Zamir [27]): For any $N \times 1$ random vector \mathcal{U} with independent components $(u_0 \cdots u_{N-1})$ and for any deterministic $M \times N$ matrix A , we have $h_E(A \cdot \mathcal{U}) \geq h_E(A \cdot \tilde{\mathcal{U}})$, where $\tilde{\mathcal{U}}$ a $N \times 1$ is a random vector with independent gaussian components, $(\tilde{u}_0 \cdots \tilde{u}_{N-1})$, such that $h_E(\tilde{u}_k) = h_E(u_k)$.

Note that this lemma applies only for real matrix A and vector U . Hence, before applying it to (5), we start by splitting the channel matrix \mathcal{T}_H , and the vectors X and W , into their real and imaginary parts $[\mathcal{T}_H]_R$ $[\mathcal{T}_H]_I$, X_R , X_I , etc. Then, we set

$$\mathcal{A} = \begin{pmatrix} [\mathcal{T}_H^0]_R & -[\mathcal{T}_H^0]_I & I & 0 \\ [\mathcal{T}_H^0]_I & [\mathcal{T}_H^0]_R & 0 & I \end{pmatrix} \quad (26)$$

$$\mathcal{Y} = \begin{pmatrix} Y_R \\ Y_I \end{pmatrix} \text{ and } \mathcal{U} = \begin{pmatrix} X_R \\ X_I \\ W_R \\ W_I \end{pmatrix} \quad (27)$$

This allows us to rewrite the channel input-output Equation (5) as

$$\mathcal{Y} = \mathcal{A} \cdot \mathcal{U} \quad (28)$$

Assume now that the input symbols x_n are taken mutually independent, with real and imaginary parts both uniform distributed on $[-\Omega_x/\sqrt{2}, \Omega_x/\sqrt{2}]^6$. The differential entropy of these real and imaginary parts is $\log(\Omega_x \sqrt{2})$. This is also the differential entropy of a real gaussian distribution with variance $\Omega_x/\sqrt{\pi e}$. Let's now denote by $\hat{\mathcal{U}}$ a complex gaussian vector whose components are mutually independent, with variances $\Omega_x \sqrt{2}$ for its N first components, and σ_w^2 for its last component. The differential entropy of $\hat{\mathcal{U}}$ is equal to the differential entropy of \mathcal{U} and the lemma applies. Then, using the fact that the entropy of the complex gaussian vector, is the sum of the entropies of its real and imaginary parts, it turns out

$$\begin{aligned} (Y; X|\mathcal{T}_H) &= h_E(Y|\mathcal{T}_H) - h_E(Y|X, \mathcal{T}_H) \\ &= h_E(A \cdot \mathcal{U}|\mathcal{T}_H) - N \log(\pi e \sigma_w^2) \\ &\geq h_E(A \cdot \hat{\mathcal{U}}|\mathcal{T}_H) - N \log(\pi e \sigma_w^2) \\ &\geq E_H \left[A E_{X,W} (\hat{\mathcal{U}} \cdot \hat{\mathcal{U}}^+) \cdot A^+ \right] \\ &= E_H \left[\log \det \left(I_N + \frac{2 \Omega_x^2}{\pi e \sigma_w^2} \mathcal{T}_H \mathcal{T}_H^+ \right) \right] \end{aligned} \quad (29)$$

Theorem 3 follows by dividing (29) by N , substituting the result into (24), taking the limit $N \rightarrow \infty$ and, finally, subtracting the penalty term given by (25).

⁶Note that this is required since $x_n \leq \Omega_x$.

References

- [1] A.B. Baggeroer. Acoustic telemetry – An overview. *IEEE Journal of Oceanic Engineering*, 9(4), Oct. 1984.
- [2] M. Stojanovic, J.A. Catipovic, and J.G. Proakis. Phase coherent digital communications for underwater acoustic channel. *IEEE Journal of Oceanic Engineering*, 19(1), January 1994.
- [3] M. Stojanovic, S. Shahabudeen, and M. Chitre. Underwater acoustic communications and networking: Recent advances and future challenges. *Marine Technology Society Journal*, 42(1):103–116, Spring 2008.
- [4] F. Frassati, C. Lafon, P.A. Laurent, and J.M. Passerieux. Experimental assessment of OFDM and DSSS modulations for use in littoral waters underwater acoustic communications. In *IEEE Oceans-Europe 2005*, Brest (FR), June 2005.
- [5] M. Chitre, S.H. Ong, and J. Potter. Performance of coded OFDM in very shallow water channel and snapping schrimp noise. In *Proc. OCEANS 2005, MTS/IEEE*, Brest (FR), June 2005.
- [6] C. Laot, A. Glavieux, and J. Labat. Turbo equalization: Adaptive equalization and channel decoding jointly optimized. *IEEE Journal on Selected Areas of Commun.*, 19:1744–1751, Sep. 2001.
- [7] R. Otnes and T.H. Eggen. Underwater acoustic communications: Long-term test of turbo equalization in shallow water. *IEEE Journal of Oceanic Engineering*, 33(3):321–334, July 2008.
- [8] E. Sangfelt, T. Öberg, B. Nilsson, and M. Lundberg-Nordenvaad. Underwater acoustic communications in the Baltic sea. In *Proc. UDT Asia 2008*, Sydney, UK, November 2008.
- [9] T.M. Cover and J.A. Thomas. *Elements of Information Theory (2nd ed.)*. Wiley, 2006.
- [10] A. Lapidoth and S.M. Moser. Capacity bounds via duality with applications to multiple-antenna systems on flat-fading channels. *IEEE Transactions on Information Theory*, 49(10), October 2003.
- [11] V. Sethuraman and B. Hajek. Capacity per unit energy of fading channels with a peak constraint. *IEEE Transactions on Information Theory*, 51(9), September 2005.
- [12] M. Gursoy, H.V. Poor, and S. Verdú. The noncoherent Rician fading channel Part I: Structure of the capacity-achieving input. *IEEE Trans. Inf. Theory*, 4(5), September 2005.
- [13] G. Durisi, U.G. Schuster, H. Bölcskei, and S. Shamai (Shitz). Noncoherent capacity of underspread fading channels. *IEEE Transactions on Information Theory*, 56(1), January 2010.

[14] A. Goldsmith. *Wireless Communications*. Cambridge University Press, 2005.

[15] A. Radošević, J.G. Proakis, and M. Stojanović. Statistical characterization and capacity of shallow water acoustic channels. In *IEEE Oceans 09 Conference*, Bremen (GE), May 2009.

[16] T. Hayward and T.C. Yang. Underwater acoustic communication channel capacity: A simulation study. In *Proceeding of AIP Conference*, pages 114–121, November 2004.

[17] M. Stojanović. On the relationship between capacity and distance in an underwater acoustic communication channel. *SIGMOBILE Mob. Comput. Commun. Rev.*, 11(4):34–43, 2007.

[18] D.Lucani, M.Medard, and M.Stojanović. On the relationship between transmission power and capacity of an underwater acoustic communication channel. In *Proc. IEEE Oceans 08 Conference*, Kobe, Japan, April 2008.

[19] F.X. Socheleau, C. Laot, and J.M. Passerieux. Characterization of the time-varying underwater acoustic communications channel with application to channel capacity. In *Underwater Acoustics Measurements*, Nafplion (GR), June 2009.

[20] D.B. Kilfoyle and A.B. Baggeroer. The state of the art in underwater acoustic telemetry. *IEEE Journal of Oceanic Engineering*, 25(1), Jan. 2000.

[21] X. Cristol. NARCISSUS-2005: A global model of fading channel for application to acoustic communication in marine environment. In *Proc. Oceans Europe 2005*, Brest (FR), June 2005.

[22] M. Stojanović and J. Preisig. Underwater acoustic communication channels: Propagation models and statistical characterization. *IEEE Communications Magazine*, January 2009.

[23] J.G. Proakis. *Digital Communications 4th ed.* Mc Graw Hill, New-York, 2001.

[24] E. Biglieri, J.G. Proakis, and S. Shamai (Shitz). Fading channels: Information-theoretic and communication aspects. *IEEE Transactions on Information Theory*, 44:2619–2692, 1998.

[25] X. Deng, A. Haimovich, and H. Bölcskei. Information rates of time varying rayleigh fading channels. In *Proc. IEEE Int. Conf. on Communications (ICC)*, Paris (FR), June 2004.

[26] J.M. Passerieux, F.X. Socheleau, and C. Laot. *On the Non-Coherent Capacity of Peak-Limited Rice-Fading Channel with Memory*. in preparation, July 2010.

[27] R. Zamir and M. Feder. A generalization of the entropy power inequality with applications. *IEEE Trans. Inf. Theory*, 39(5), September 1993.

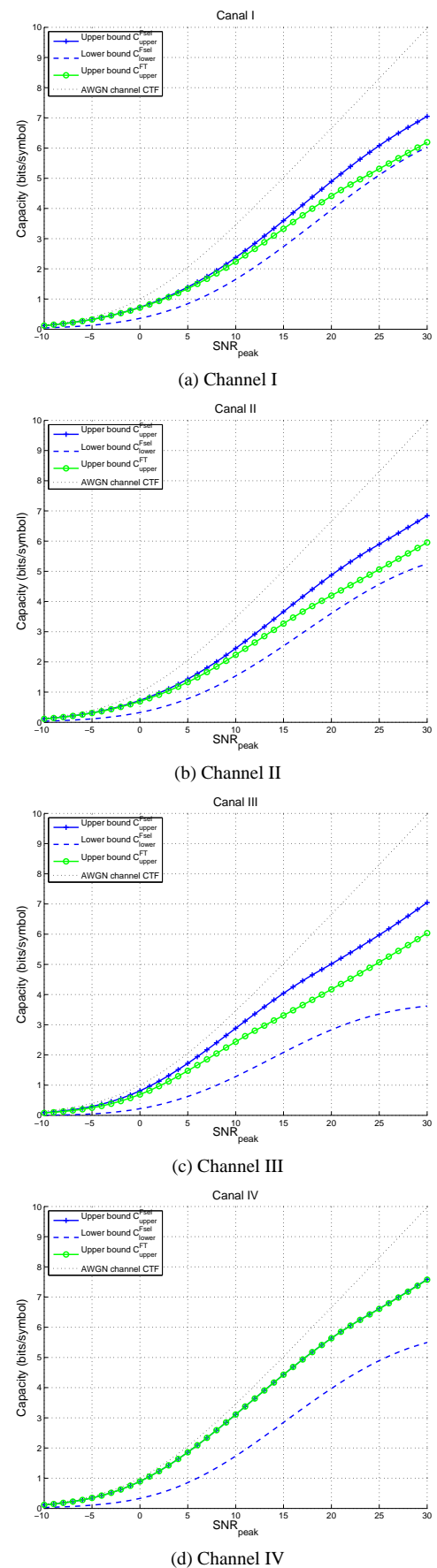
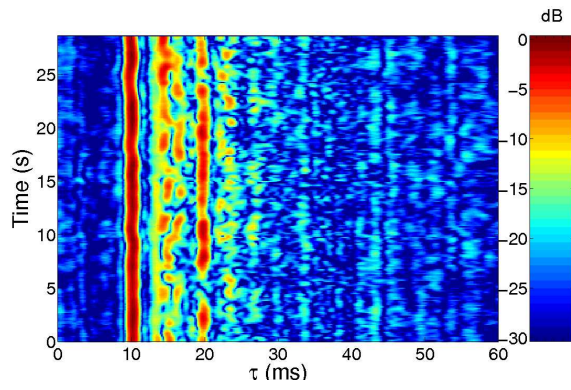
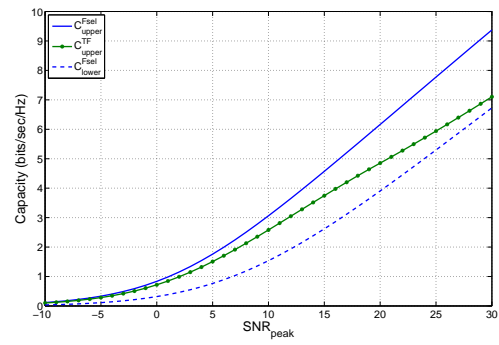


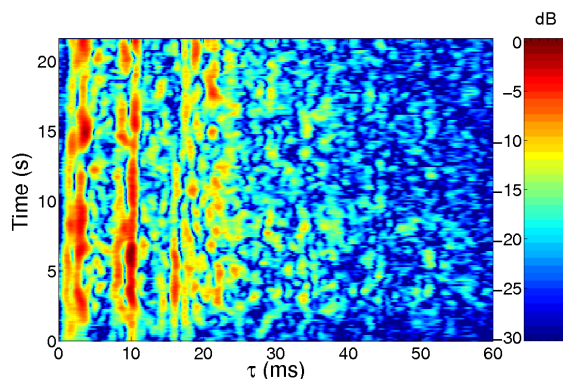
Figure 1: Capacity bounds for simple ideal channels



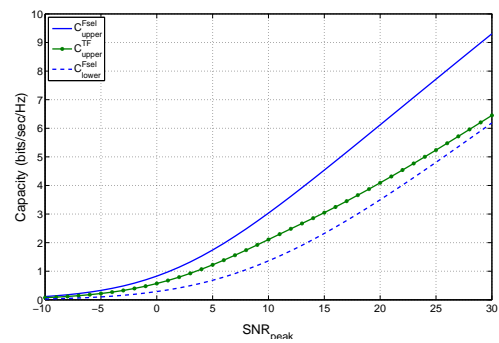
(a) Mediterranean channel (range 2500 m)



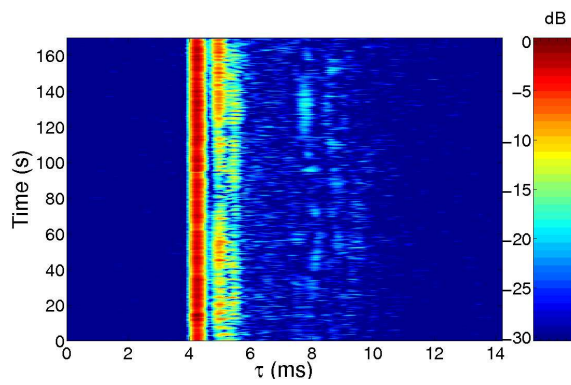
(b) Mediterranean channel (range 2500 m)



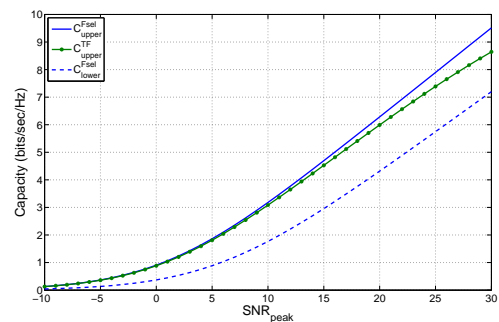
(c) Mediterranean channel (range 9000 m)



(d) Mediterranean channel (range 9000 m)



(e) Atlantic channel



(f) Atlantic channel

Figure 2: Capacity of at-sea measured channels