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# Resistance measurements and weak localization in long SWNTs

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In many I-V measurements an evidence has emerged pointing towards the existence of Luttinger liquid behavior in metallic SWNT, as expected for strongly interacting electrons in 1-d, such behavior was observed via the power law temperature and bias-voltage dependence of the current through tunneling contacts attached to the nanotubes. In particular recent advances in the growth of extremely long nanotube ( $>1$  mm) have allowed for experimental measurements on the scaling behavior of resistance in individual, millimeter long SWNT for the temperature range of 1.6-300 K. From the linear scaling of resistance, the temperature dependent electron mean free path has been calculated for each temperature and, beyond the linear scaling regime, it has been observed that the resistance increases exponentially with length, indicating localization behavior. In this work we analyse the results of the resistance measurements of different lengths SWNT indicating the weak localization behavior.

**Keywords:** quantum transport, , weak localization, carbon nanotube, phase coherence

## 1 INTRODUCTION

The rapidly advancing technology of nanometric devices has led to the production of smaller and smaller systems: among the main items in the design of these electronic devices there are the measurement and understanding of the current-voltage (I-V) response of electronic circuits in which carbon nanotubes act as conducting elements. These devices, often called mesoscopic systems, are large on the atomic scale but sufficiently small that the electron wavefunction is coherent over the entire sample: the condition for coherence is that the electron traverse the wire without undergoing any inelastic collision. Indeed in perfect single-walled carbon nanotube (SWNT) electrons propagate ballistically if the inelastic scattering can be neglected.

On the other hand it is also well known that at low-temperature charge transport in any disordered conductor is governed by the interplay between inelastic scattering and elastic scattering off static disorder (impurities and defects) of electrons, therefore, in low dimensions systems like SWNTs, an arbitrarily weak disorder localizes [1,2] all single-electron states and there would be no transport without inelastic processes (Anderson localization<sup>1</sup> of electronic states leads to metal-insulator transition at zero temperature [3]: note that, for electrons in a given conduction band, strong enough disorder can localize the whole band; the metal-insulator transition induced by disorder is called Anderson transition.).

Coherent quantum transport in low dimensional systems can be investigated with either the Kubo or the Landauer-

Buttiker formalism [4]. The first approach, which derives from the fluctuation-dissipation theorem, allows one to evaluate the intrinsic conduction regime within the linear response and gives direct access to the fundamental transport length scales<sup>2</sup> such as the elastic mean free path  $l_0$  and the localization length  $\xi$ . While  $l_0$  results from elastic backscattering by static disorder,  $\xi$  denotes the scale beyond which quantum conductance decays exponentially with the system length  $L$  driving the system from weak to strong localization. In other words the localization length  $\xi$  gives the scale beyond which weak localization are fully suppressed, since the quantum interference effects are usually reduced by inelastic scattering as decoherence owing to mechanisms such as electron-phonon (e-ph) or electron-electron (e-e) coupling. Within the framework of weak localization theory, it has been possible to derive perturbatively the relation between the measured conductance becomes more appropriate, since it rigorously treats  $G$ , its quantum correction  $\delta G$  and the coherence length  $l_\phi$  that fixes the scale beyond which quantum interference effect are destroyed. The estimation of the coherence lengths is a central issue in mesoscopic physics, and weak localization provides an elegant framework to extract the behaviour of  $l_\phi$ , that mainly depends on the dimensionality of charge transport. When  $l_0$  becomes longer than the length of the nanotube between the leads, the carriers propagate ballistically and contact effects prevail. In such a situation, the Landauer-Buttger

<sup>1</sup>The localization is a property of the states in random QM systems and can be interpreted by total back-reflection of particles from potential barriers so that they become localized in a single potential well.

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<sup>2</sup> Important length scales: the coherence length  $l_\phi$ , the energy relaxation length  $l$ , the elastic mean free length  $l_0$ , the Fermi wave length  $\lambda_F$  of the electron, the sample size  $L$ : in mesoscopic systems it will be  $\lambda_F \ll l_0 < L < l_\phi \ll l$ . Note that  $k_F l \geq 1$  is called the Ioffe-Regel limit and the atomic Bohr radius  $a_0 \ll \lambda_F$ .

formalism transmission properties for open systems and arbitrary interface geometries.

## 2 QUANTUM TRANSPORT IN SWNT

### 2.1 Conductivity and Transport

The more general formula for actual local current measured by experimentalists (generalized Ohm's law) for conductivity in infinite length system is

$$J_{\sigma}(\mathbf{r}, t) = \int d\mathbf{r}' \int_{\sigma} \chi_{\sigma}(\mathbf{r}, \mathbf{r}', \sigma) E_{\sigma}(\mathbf{r}', t) \quad (1)$$

where the (non-local) conductivity  $\chi_{\sigma}$  is response to actual (external + induced) electric field and is given by the Kubo formula [5] (where periodic boundary conditions and coupling of only the charge degrees of freedom, none spin degree of freedom is considered, to external  $\mathbf{E}$ -field are assumed)

$$\chi_{\sigma}(\mathbf{r}, \mathbf{r}', \sigma) = \frac{e^2}{\hbar \sigma} \int_0^{\infty} dt e^{i\omega t} \langle \sigma | [j_{\sigma}^+(\mathbf{r}, t), j_{\sigma}(\mathbf{r}', 0)] | \sigma \rangle + \frac{ne^2}{m\sigma} i\chi_{\sigma}(\mathbf{r} \parallel \mathbf{r}') \quad (2)$$

The wave function  $|\sigma\rangle$  is the ground state of the many-body Hamiltonian which contains all possible interactions in the solid (except the interaction between the total electric field and the particles of the system) and the first term is called retarded current-current correlation function. Kubo first derived the equations for electrical conductivity in the solid and Kubo formulas are the name applied to the correlation function which describes the linear response.

But this procedure is physically incorrect as a way of defining the conductivity in a finite system in which electrons enter from an external electrode at one end and are removed at the other end collected by another external electrode. In a finite (ideal) sample if the chemical potential is higher at one lead, electrons of the large reservoir with constant  $\mu_1$ , than at the other lead, with constant  $\mu_2$ , the current is the response to the gradient of chemical potential for electrons not to the electric field; in other words, if  $\mu_1 > \mu_2$  and  $e$  is the absolute value of the electronic charge, the voltage difference  $\Delta V$  between the two baths due to flow across the sample of the current  $I$  is

$$\Delta V = \frac{I}{G_c} = \frac{\mu_1 - \mu_2}{e} \quad (3)$$

where

$$G_c = \frac{e^2}{\hbar} \chi(E_f) \quad (4)$$

is the irreducible conductance measured between the two outside reservoirs being  $\Gamma$  the transmission probability for channel (to go from electrode 1 to electrode 2). The inelastic processes (which break the time-reversal invariance and the phase coherence of the states at the two extremities, dissipate energy and restore equilibrium) in this case are assumed to exist only in the two electrons baths, so that the randomized phase of the injected and absorbed electrons through these processes results in no phase relation between particles.

At low temperature, in presence of only elastic scattering for the electrons at the Fermi surface with a linear series of random scatterers connecting the two reservoirs, the true conductance  $G$  due to barrier (including spin degeneracy) is correctly given by the transmission and reflection coefficients of the sample by the Landauer-Buttiker, not the Kubo, formula [4,5]

$$G = \frac{e^2}{\hbar} \frac{\Gamma(E_f)}{1 + \Gamma(E_f)} \quad (5)$$

Electronic transport measurements [6] on individual SWNT demonstrate that, in the absence of scattering (then the transmission probability is  $\Gamma = 1$ ), the momentum relaxation length and the localization length  $\lambda$  are much larger than the wire length and the transport in these systems is ballistic: the wavefunction of the electron is extended over the total length of the nanotube and there are only two channels which contribute to the electronic transport giving  $G = 2G_c$ . However, as already outlined, in presence of some mechanism of scattering the conductance is described by the Landauer formula (4) and the conductance is no longer exactly quantized.

Because the electron can lose energy and equilibrate with heat bath only via inelastic collisions it is necessary to reexamine the conventional concept of energy dissipation in a quasi 1-d resistor systems. In the theory of the 1-d electron systems, called Luttinger-liquid (LL)[7,8] (see AppendixA), the correlated electron state is characterized by a parameter  $g$  that measures the strength of the interaction between the electrons ( $g=1$  for non-interacting electrons gas). The most important feature of the LL, in contrast to Landau Fermi-Liquid theory<sup>3</sup> (FL), is the absence of the fermion quasiparticle branch at low energy: excited states of the system must be described by the bosonic fluctuations of the charge and spin densities dispersing with different velocities, which correspond to many-body electron state with a huge number of the electron-hole pairs. This have a pronounced effect on the tunneling into a LL conductor: the I-V curve of a tunnel

<sup>3</sup> Landau Fermi-Liquid theory is concerned with the properties of many-fermion system at low temperature (much lower than Fermi energy) in the normal state, i.e. in the absence or at least at temperatures above any symmetry breaking phase transition (superconduction, etc).

1  
2 junction between a normal FL and a LL conductor is  
3 expected to be non-ohmic and described by a power law  
4 with an exponent depending on interaction strength.  
5 Not only, in [9] Bockrath et al. observe a Luttinger-liquid  
6 behavior in (rope of) SWNT's in measurements of the  
7 electrical transport as a function of temperature resulting in a  
8 power laws for linear response conductance

$$G(T) \propto T^\alpha \quad (6)$$

9  
10  
11 where  $\alpha = f(g)$  and  $g = \frac{2U}{\Delta} + \frac{2U}{\Delta}^{1/2}$  ( $U$  is the charging  
12 energy of the tube and  $\Delta$  [10] is the single-particle level  
13 spacing).

## 14 2.2 Weak localization

15  
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21 When the temperature is so high that the conductivity can  
22 be treated as local quantity, like in Anderson localization  
23 which deals with wave function of the single electron in  
24 presence of impurities, the conductance is obtained by  
25 combination of smaller parts of materials. But when the  
26 temperature is low or the sample is small the dephasing  
27 length  $l_\phi$  is greater than the sample linear dimensions  $L$  so  
28 that the quantum corrections to the conductivity are non-  
29 local and the conductance can no longer be treated as a self-  
30 averaging quantity. Since the lack of self-averaging of the  
31 conductance is a feature of mesoscopic conductors, in  
32 SWNT it is necessary to analyze the effects of weak disorder  
33 and the inelastic scatterings on charges transport.

34 Due to the existence of the impurities the transport is more  
35 diffusive than ballistic (the existence of scattering is possible  
36 in both regimes but in diffusive one we have that  
37  $l = v_F \tau \ll L$  so that the material is characterized by a  
38 relatively low mobility) though the elastic scattering of the  
39 electrons, if these impurities are equivalent to static defects,  
40 can modify the interference terms but does not cause  
41 decoherence [4]. At low temperature the conduction take  
42 place mainly with electrons at Fermi energy and, due to  
43 some gate potential, the Fermi point upon which the  
44 electrons travel can be shifted slightly, therefore it is  
45 possible that an electron that moved on one side (path) of an  
46 impurity begins to move on the other side (path) after the  
47 shift. This process (analogous to Aharonov-Bohm effect in  
48 the presence of some magnetic field) induced a quantum  
49 fluctuation of the conductance of the order of  $2e^2/h$  and  
50 depends on the exact configuration of scattering centers  
51 within the sample: these two paths are time-reversed with  
52 respect to one another and since the electron return to its  
53 original position it can interfere with itself creating an  
54 additional resistance called weak localization [11,12]. Then  
55 the weak localization is caused by the quantum interference  
56 effect on the diffusive motion of a single electron.

57 From semiclassical point of view it is possible to  
58 calculate this additional resistance considering that the

conductivity is related to the current-current correlation  
function, as in eq. (2), and being  $D = v_F^2 \tau$  the diffusion  
coefficient and  $l_\phi = \sqrt{D \tau_\phi}$  for 1D we get

$$\sigma_{wl} = \sigma \left[ \frac{e^2}{h} l_\phi \left( \frac{1}{L} \right) + \frac{l_\phi}{L} \left( \frac{1}{L} \right)^{1/2} \right] \quad (7)$$

Even though in [13] it has been suggested that zero-point  
fluctuations cause the dephasing in one dimensional  
quantum wire at low temperature (ascribed to finite  
broadening of Fermi surface) and presenting a zero-point-  
limited dephasing time  $\tau_\phi$  in good agreement with the  
measured saturation values  $l_\phi$  found in many  
experiments, in [14] this hypothesis is rejected using  
purely physical arguments.

A more sophisticated theory [15,16] tells to us that in  
presence of a vector potential  $\mathbf{A}$ , for  $|\mathbf{A}| \ll 1$ , the  
probability of return path in a disordered SWNT can be  
conveniently obtained by calculating the Cooperon  
 $C_0(\mathbf{r}, \mathbf{r}')$ , which is a retarded classical electron-electron  
propagator satisfying a modified diffusion equation in the  
frequency domain [16]

$$\left[ D \nabla^2 - i\omega + \frac{2ie}{\hbar} \mathbf{A} \cdot \nabla \right] C_0(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \quad (8)$$

The WL correction to the conductivity is due to enhanced  
probability to return, so that

$$\sigma_{wl}(\omega) = \sigma \frac{2e^2 D}{\hbar} C_0(\mathbf{r}, \mathbf{r}') \quad (9)$$

## 3 RESISTANCE IN LONG SWNT

The conductance of metallic SWNT has been shown to  
depend strongly on the nature of the contacts between the  
nanotube and the leads. In a typical experimental setup  
[10] a bias voltage is applied across a nanotube connected  
to metallic leads, while a gate voltage applied to a third  
electrode acts as a chemical potential and modulates the  
charge on the nanotube (see [10] fig.1,3 and 4). At room  
temperature the main origin of the resistivity at low bias  
in high-quality metallic SWNT is believed to be inelastic  
scattering by acoustic phonon: the scattering is weak  
resulting in long mean-free path in a range from few  
hundred nanometers to several micrometers both in the  
measurements and in the calculations. Then at low bias  
regime we get ballistic transport. When sufficiently large  
bias are applied to drive the electric current, higher energy  
vibrational modes are activated and e-ph coupling limits  
ballistic transport: electrons gain enough energy to emit  
optical or zone-boundary phonons leading to a saturation



of the current, in [17] indicated at  $\sim 20 \mu\text{A}$ . The effect of electron-(optical) phonon coupling was found to strongly affect electronic conductance and to induce some energy dependence of the coherence length scale, completely similar to the experimental data obtained in the weak localization regime [18,19].

## 4 CONCLUSION

We study the suppression of the quasi-ballistic conduction in long (many  $l_D$ ) SWNT noting that some environmental conditions also at low temperature introduce some dynamic disorder which involves, by means inelastic scattering [18-20], a weak localization correction to the conductance.

### Appendix A LUTTINGER LIQUID: VERY BRIEF REVIEW

A Luttinger liquid (LL) is a one-dimensional (Fermi liquid) correlated electron state characterized by a parameter  $g$  that measures the strength of the interaction between electrons: strong repulsive interactions have  $g \ll 1$ , whereas  $g=1$  for the non-interacting electron gas (remembering that weakly interacting electrons in normal metal are described by quasiparticles of the Fermi liquid). The LL's are very special in that they retain a Fermi surface enclosing the same  $k$ -space volume as that of free fermions, but there are no fermionic quasi-particles (like in normal Fermi liquids), their elementary excitations are bosonic collective charge and spin fluctuations dispersing with different velocities. An incoming electron decays into such charge and spin excitations which then spatially separate with time (charge-spin separation): the correlations between these excitations are anomalous and show up as interaction-dependent non-universal power laws in many physical properties where those of ordinary metals are characterized by universal (interaction-independent) powers. A list of such properties includes: 1) a continuous momentum distribution function  $n(k)$ , varying with as  $|k - k_c|^\nu$  with an interaction-dependent exponent  $\nu$ , and a pseudogap in the single-particle density of states  $|\rho|^\nu$ , consequences of the non-existence of fermionic quasi-particles; 2) similar power-law behavior in all correlation functions (in those for charge or spin density wave fluctuations) with universal scaling relations between the different non-universal exponents,

which depend only on one effective coupling constant per degree of freedom; 3) finite spin and charge response at small wave vectors and finite Drude weight in the conductivity; 4) spin-charge separation; persistent currents quantized in units of  $2k_F$ .

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