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# Lost work, extra work and entropy production for a system with complexity: the stepwise ideal-gas Carnot cycle

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## Abstract

This paper is an extension of a previous paper [F. di Liberto-Phil. Mag. **87**, 569 (2007)] devoted to lost work and entropy production; here we introduce also the extra work (i.e.  $W_{Extra} = W_{in} - W_{Rev}$ ) in an irreversible process and apply both concepts to the analysis of a system with complexity: the stepwise ideal-gas Carnot cycle. A stepwise Carnot cycle is performed by means of  $N$  small weights, (here called  $dw$ 's), which are first added and then removed from the piston of the vessel containing the gas. The work performed by the gas can be found as an increase in the potential energy of the  $dw$ 's. We identify each single  $dw$  and thus evaluate its raising i.e. its increase in potential energy. In such a way we find how the energy output of the cycle is distributed among the  $dw$ 's. The size of the  $dw$ 's affects the entropy production and therefore the lost and extra work. The raising distribution depends on the removing process we choose. Since these processes are  $N!$ , there are  $N!$  distributions of the raisings of the  $dw$ 's

## 1. Introduction

As pointed out in a previous paper [1], entropy production and its relation to the available energy are fascinating subjects which in last years have attracted much physics research [5-11].

It is well known [1-10] that for some elementary irreversible process, such as the irreversible isothermal expansion of a gas in contact with a heat source  $T$ , the work done by the gas  $W_{out}$  is related to the reversible isothermal work  $W_{Rev}$  (i.e. the work performed by the gas in the corresponding reversible process) by the relation

$$W_{out} = W_{Rev} - T \Delta S_U \quad (1)$$

where  $\Delta S_U$  is the total entropy change of the universe (system + environment). The degraded energy  $T \Delta S_U$  is usually called 'the lost work'  $W_{Lost}$

$$W_{Lost} = W_{Rev} - W_{out} \quad (2)$$

The latter can be interpreted as the missing work: i.e. the additional work that could have been done in the related reversible process (here the reversible isothermal expansion); it is also called 'energy unavailable to do work'.

On the other hand, in an irreversible isothermal compression,  $T \Delta S_U$  is called  $W_{Extra}$  i.e. the excess of work performed on the system in the irreversible process with respect to the reversible one.

$$W_{Extra} = W_{in} - W_{Rev} \quad (3)$$

Owing to the energy balance criterion, the same relation holds for the amount of heat given to the source  $T$ , i.e. we have

$$Q_{out} = Q_{Rev} + T \Delta S_U \quad (4)$$

Therefore  $T \Delta S_U$  is also called the 'excess of heat' ( $Q_{Extra}$ ), i.e. the additional heat that has been given to the source [8,9].

The total variation of entropy,  $\Delta S_U$ , is usually called 'entropy production'; we shall call  $\pi_U$ .

The second law of thermodynamics claims that  $\pi_U \geq 0$  and the entropy is an extensive quantity, which in transfers between systems can only increase or stay unchanged.

## 2. Entropy production, lost work and extra work in isothermal irreversible processes.

Let us first consider the isothermal irreversible expansion ( $A \rightarrow B$ ) of an ideal gas in contact with a heat source  $T$  where  $V_B = V_A + \Delta V$  and  $P_B = P_A - \Delta P$  with  $\Delta P > 0$ ,  $\Delta V > 0$ . In such a simple process, some heat  $Q_{in} \equiv W_{out} = P_B \Delta V = (P_A - \Delta P) \Delta V$ <sup>1</sup> goes from the heat source  $T$  to the ideal gas. There is an increase in the entropy of the ideal gas,  $\Delta S_{gas} = \int_A^B \frac{\delta Q_{Rev}}{T} = \frac{Q_{Rev}}{T}$  and a decrease in the entropy of the heat source ( $-\frac{Q_{in}}{T}$ ), where

$$Q_{Rev} = \int_A^B \delta Q_{Rev} = W_{Rev} = \int_A^B P dV = RT \ln \frac{V_B}{V_A} = RT \ln \left( 1 + \frac{\Delta V}{V_A} \right) = RT \left[ \frac{\Delta V}{V_A} - \frac{1}{2} \left( \frac{\Delta V}{V_A} \right)^2 + \dots \right].$$

Therefore the entropy production is  $\pi_U \equiv \Delta S_U = \frac{Q_{Rev}}{T} - \frac{Q_{in}}{T} > 0$  (5)

Since  $Q_{Rev} > Q_{in}$  we find in the ideal gas an amount of entropy greater than that taken from the heat source  $T$ .

If, for example  $\Delta V = 3V_A$  we have  $\pi_U = R \ln 4 - \frac{3}{4} R = 0,636 R$ .

On another hand for an isothermal irreversible compression of the ideal gas ( $B \rightarrow A$ ), some heat  $Q_{out}$  goes from the gas to the source  $T$ . We have a decrease of the gas entropy  $\Delta S_{gas} = \int_B^A \frac{\delta Q_{Rev}}{T} = -\frac{Q_{Rev}}{T}$  and an increase of the source entropy, ( $\frac{Q_{out}}{T}$ ) where  $Q_{out} = W_{in} = P_A (V_B - V_A) = P_A \Delta V$ .

Therefore in the compression the entropy production, is

$$\pi_U \equiv \Delta S_U = \frac{Q_{out}}{T} - \frac{Q_{Rev}}{T} > 0 \quad (\text{since } Q_{out} > Q_{Rev}) \quad (6)$$

Which, for  $\Delta V = 3V_A$  gives  $\pi_U = \frac{P_A \Delta V}{T} - R \ln \left( 1 + \frac{\Delta V}{V_A} \right) = 3R - R \ln 4 = 1,614 R$

In order to find how the previous entropy productions affect the dissipation of energy, we have to remark that the irreversibility of a generic process ( $A \rightarrow B$ ) is due, in general, to internal and external irreversibility. Therefore, as shown in [1,2,5], the related entropy production  $\pi_U$  can be expressed as a sum of two terms: the internal entropy production,  $\pi_{int} \geq 0$  and the external entropy production,  $\pi_{ext} \geq 0$  i. e.

$$\pi_U = \pi_{int} + \pi_{ext} \quad (7)$$

This result is not trivial since  $\pi_{int} \neq \Delta S_{sys}$ ; there are in fact many processes for which  $\Delta S_{sys} < 0$ , and  $\pi_{int} \geq 0$ . The system entropy production  $\pi_{int}$  is defined [1,2,5] by the relation

$$\Delta S_{syst} = S_{in} - S_{out} + \pi_{int} \quad (8)$$

where  $S_{in}$  e  $S_{out}$  are respectively the quantity of entropy which comes into and comes out of the system in the irreversible process;  $\Delta S_{syst} = \int_A^B \frac{\delta Q_{Rev}}{T}$  is the entropy variation of the system from A to B and does not depend

on the particular process. Similarly the external entropy production,  $\pi_{ext}$  is given by the relation

$$\Delta S_{ext} = S_{in}^{ext} - S_{out}^{ext} + \pi_{ext} \quad (9)$$

or by relation (7). It is easy to verify that for both previous irreversible isothermal processes,  $\pi_{ext} = 0$  and therefore that for both the expansion and the compression,  $\pi_{int} = \pi_U$ .

In the Appendix we give relations for the lost work and extra work for isothermal processes with internal and external irreversibility ( $\pi_{ext} \neq 0$ ). From relations (A4) and (A6) it follows that the lost work for an isothermal expansion at temperature  $T = T_0$  and without external irreversibility ( $\pi_{ext} = 0$ ) is

<sup>1</sup> The quantities  $Q_{in}$ ,  $Q_{out}$ ,  $Q_{Rev}$  and  $\pi_U$  are positive. <https://www.manuscriptcentral.com/pm-pml>

$$W_{Lost} = W_{Rev} - W_{out} = T_0 \pi_{int} \quad (10)$$

and that the extra work for an isothermal compression at  $T = T_B$  (with  $\pi_{ext} = 0$ ) is

$$W_{Extra} = W_{in} - W_{Rev} = T_B \pi_{int} \quad (11)$$

In the next section, by means of relations (10) and (11), we study the lost work and the extra work for a stepwise Carnot cycle.

### 3. The stepwise ideal-gas Carnot cycle and dissipated energy

In order to perform an ideal-gas stepwise cycle we need a heat source, a heat sink, a vessel with a free piston and a large number ( $N$ ) of small “driving weights” to increase or decrease slowly, step by step, the external pressure  $P$ . If the steps are infinitesimally small the cycle is “reversible”.

In order to evaluate the work performed by the ideal gas during the cycle, the displacements of the small driving weights ( $dw$ ) must be done carefully. We let them move on and off the piston only horizontally. To this end we assume that the handle of the piston is endowed with so many shelves that we can move each  $dw$  horizontally (and without friction) from (or to) the corresponding fixed shelf which belongs to the  $dw$ 's reservoir. (The  $dw$ 's reservoir is a vertical sequence of horizontal shelves on which the  $dw$ 's are initially located). Such an ideal device is shown schematically in Fig.1

#### FIGURE 1A, 1B

A cycle as the Carnot or Stirling cycles, in which there are no isobaric steps, can be performed through  $Z = 2N$  steps. In each of the first  $N$  steps one  $dw$  is *added* on the piston (and removed from the reservoir at its initial height  $h_0$ ); in each of the following  $N$  steps one  $dw$  is *removed* from the piston (and brought back to the reservoir at its final height, say  $h_f$ ). The  $k^{th}$   $dw$  is the  $dw$  which has been added on the piston at the  $k^{th}$  step in the compression.

Therefore at the end of the cycle the overall raising, on the  $dw$ 's reservoir, of the  $k^{th}$   $dw$  from its initial height ( $h_{k0}$ ) to the final one ( $h_{kf}$ ) is

$$h_k = h_{kf} - h_{k0} \quad (12)$$

Since a friction-less process is assumed, the vertical motion of the  $dw$ 's is only due to the gas and the total work ( $W$ ) performed by the ideal gas can be found as increase of potential energy of the  $dw$ 's on the reservoir, i.e.

$$W = \sum_{i=1}^Z P_i \Delta V_i = mg \sum_{k=1}^N h_k \quad (13)$$

where  $P_i$  is the external pressure at step  $i$  (after the addition or removal of the  $i^{th}$   $dw$ )  $\Delta V_i = V_i - V_{i-1}$ , is the volume variation from step  $(i-1)$  to step  $i$  and  $mg$  is the weight of the generic  $dw$ . Relation (13) has been proved elsewhere [3]. In the next section the raisings of the single  $dw$  on the reservoir are evaluated.

#### 3.1. The raisings of the $dw$ 's for a stepwise Carnot cycle

The cycle we consider is reported in Figure 2. The chosen values of  $P$  and  $V$  are easily available in ordinary conditions. In the first  $N$  steps the  $dw$ 's are *added* on the piston to perform first an isothermal compression ( $O \rightarrow A$ ) and then an adiabatic compression ( $A \rightarrow B$ ). In the remaining  $N$  steps the  $dw$ 's are *removed* from the piston in order to return to the initial state ( $B \rightarrow C \rightarrow O$ ). See figure 2.

#### FIGURE 2

The working fluid is the ideal gas and the free piston is massless. The vertical vessel's walls are heat insulating and the vessel's diathermal floor is made adiabatic when needed. The values in Fig. 2 are  $P_0 = 1$  at.,  $V_0 = 22.4$  l,  $T_0 = 273.15$  K,  $\gamma = 1.4$ ,  $P_A = 1.3969$  at.,  $V_A = 16.034$  l,  $P_C = 1.1385$  at.,  $V_C = 20.416$  l,  $T_C = 283.47$  K.

We have considered here  $N = 610$   $dw$ 's and therefore  $2N = 1220$  steps. The mass of each  $dw$  is  $m = 0.1$  kg. The surface of the piston is  $S = 100$  cm<sup>2</sup>, so that at each step in the compression the pressure increase is  $\Delta P = P_0/1033$ , i.e.

$$P_i = P_0 + i\Delta P \text{ for } i \in [1, N] \quad (14a)$$

And for each step in the expansion the pressure decreases by  $\Delta P$  i.e.

$$P_{N+l} = P_0 + (N-l)\Delta P \text{ for } l \in [1, N] \quad (14b)$$

Notice moreover that  $V_{2N} = V_0$  and  $V_N = V_B$  i.e. the volume at step  $2N$  is the initial volume and the volume at step  $N$  is the smallest volume in the cycle.

Keeping in mind how we perform the Carnot cycle, let us take a closer look at the last  $dw$ . It is clear that its raising in the reservoir is negative: when it leaves the reservoir and is added on the piston, it (together with the piston and the previous  $dw$ 's) moves downward; afterwards, at step  $N+1$ , during the expansion, it is removed and it goes to rest on the fixed shelf of the reservoir in front of it. It will stay on the piston for one step only! i.e.

$$h_N = (V_N - V_{N-1})/S.$$

Similarly the last but one  $dw$  moves downward for two steps and moves upward for one step. It performs two "negative" steps and one "positive" step

$$h_{N-1} = \frac{1}{S} (V_{N+1} - V_{N-2}) = \frac{1}{S} (V_{2N-(N-1)} - V_{(N-1)-1}) \quad (15)$$

By means of relations (15) one can calculate [4] the overall raising of each of the  $N$   $dw$ 's on the reservoir i.e. the  $h_k$ 's. The overall raisings are reported in Fig.3, where  $h_N$  is given for  $r=0$ ,  $h_{N-l}$  for  $r=l$  and so on.  $h_1$  for  $r=609$ . The inserts in the upper part of Fig. 3 show that the last  $dw$ 's and the first ones have negative raisings; it is clear that the negative raisings disappear only for the reversible cycle i.e. in the limit  $N \rightarrow \infty$  (and therefore  $\Delta P \rightarrow 0$ ,  $m \rightarrow 0$ ). Therefore for the  $k^{th}$   $dw$

$$h_k = \frac{1}{S} (V_{2N-k} - V_{k-1})$$

and  $h_1 = \frac{1}{S} (V_{2N-1} - V_0)$ : the first  $dw$  moves downward for  $N$  steps and moves upward for  $N-1$  steps.

### FIGURE 3

### 3.2. Lost work and extra work, step by step, and the total dissipated energy

One may observe that in the cycle there has been an entropy production: in fact in each isothermal step of the compression ( $O \rightarrow A$ ) at temperature  $T_0$ , from relation (6) we have

$$\pi_{i-int} = \frac{P_{i+1}\Delta V_i}{T_0} - R \ln\left(1 + \frac{\Delta V_i}{V_{i+1}}\right) \approx \frac{P_{i+1}\Delta V_i}{T_0} - R \left[ \frac{\Delta V_i}{V_{i+1}} - \frac{1}{2} \left( \frac{\Delta V_i}{V_{i+1}} \right)^2 \right] \approx \frac{R}{2} \left( \frac{\Delta V_i}{V_{i+1}} \right)^2 \quad (16a)$$

Therefore  $W_{i-Extra} = W_{i-irrev} - W_{i-Rev} = T_0 \pi_{i-int}$

$$\text{and in each isothermal step in the expansion } \pi_{i-int} = \frac{R}{2} \left( \frac{\Delta V}{V} \right)^2 = \frac{R}{2} \left( \frac{\Delta P}{P} \right)^2 \quad (16b)$$

and  $W_{i-Lost} = W_{i-Rev} - W_{i-irrev} = T_B \pi_{i-int}$  where  $T_B$  is the temperature of the hot heat source.

To evaluate the entropy production in the adiabatic steps we must recall that for such steps it has been shown [4]

$$\text{that } \frac{\Delta V_i}{V_i} = -\frac{1}{\gamma} \frac{\Delta P_i}{P_{i+1}} \text{ where } \gamma = \frac{C_P}{C_V} \quad V_{i+1} - V_i = \Delta V_i \quad \text{and} \quad P_{i+1} - P_i = \Delta P_i$$

Therefore the entropy production is

$$\pi_{i-int} = \Delta S_U = \int_i^{i+1} \frac{\delta Q}{T} = C_V \ln \frac{T_{i+1}}{T_i} + R \ln \frac{V_{i+1}}{V_i} \text{ which to first order gives}$$

$$\pi_{i-int} = C_V \frac{T_{i+1} - T_i}{T_i} + R \frac{V_{i+1} - V_i}{V_i} \text{ and since } C_V (T_{i+1} - T_i) = -P_{i+1} (V_{i+1} - V_i)$$

$$\pi_{i-int} = -\frac{P_{i+1}(V_{i+1} - V_i)}{T_i} + R \frac{V_{i+1} - V_i}{V_i} = -\frac{RP_{i+1}\Delta V_i}{P_i V_i} + \frac{R\Delta V_i}{V_i} = R \frac{\Delta V_i}{V_i} \left( 1 - \frac{P_{i+1}}{P_i} \right) = R \gamma \left( \frac{\Delta V_i}{V_i} \right)^2 = \frac{R}{\gamma} \left( \frac{\Delta P_i}{P_{i+1}} \right)^2 \quad (17)$$

Finally we can conclude that the dissipated energy i.e.  $W_{-}$  is

$$W_D = \sum_{i=1}^{2N} T_i \pi_{i-\text{int}} = W_{\text{Extra}} + W_{\text{Lost}} \text{ where}$$

$$W_{\text{Extra}} = \sum_{i=1}^N T_i \pi_{i-\text{int}} = \sum_{i=1}^{n_1} T_0 \pi_{i-\text{int}} + \sum_{i=n_1+1}^{n_1+n_2} T_i \pi_{i-\text{int}} \quad (18)$$

and

$$W_{\text{Lost}} = \sum_{i=N+1}^{2N} T_i \pi_{i-\text{int}} = \sum_{i=1}^{N+n_3} T_B \pi_{i-\text{int}} + \sum_{i=N+n_3+1}^{2N} T_i \pi_{i-\text{int}} \quad (19)$$

Here  $n_1, n_3$  are the numbers of isothermal steps and  $n_2, n_4$  are the numbers of adiabatic steps.

### 3.3. Complexity in the raisings pattern

The raising  $h_i$  depends on the removing process we choose (the number of such processes is  $N!$ ). In the previous sections we have chosen the simplest process (SP) (the removing process that starts with the last  $dw$  and ends with the first  $dw$ ). Let us see how the raisings of the single  $dw$ 's change for a complex process (CP), in which we start from the  $(N-L)^{\text{th}}$   $dw$ , ( $L < N$ ), go progressively to the last one, and then from the  $(N-L-1)^{\text{th}}$   $dw$ , go to the first. If the first removed  $dw$  is the  $(N-L)^{\text{th}}$  with  $1 < L < N$  the history of each  $dw$  will change. For the  $(N-L)^{\text{th}}$   $dw$  (which has been on the piston for the last  $L+1$  steps of the compression) we clearly have

$$h_{N-L} = \sum_{i=N-L}^N \Delta H_i = H_N - H_{N-L-1} = \frac{1}{S} (V_N - V_{N-L-1})$$

which is negative and remains negative in the  $N \rightarrow \infty$  limit i. e. also for "reversible cycles" [4].

For the  $(N-(L-1))^{\text{th}}$   $dw$ , which has been on the piston for the last  $L$  steps of the compression and the first step of the expansion (i.e. the step in which the  $(N-L)^{\text{th}}$   $dw$  has been removed) we clearly have

$$h_{N-(L-1)} = \sum_{i=N-(L-1)}^{N+1} \Delta H_i = H_{N+1} - H_{N-L} = \frac{1}{S} (V_{N+1} - V_{N-L})$$

Therefore if we denote by  $h_{N-r}^L$  the raisings in such CP for  $r \leq L$  we have

$$\begin{aligned} h_{N-r}^L &= \sum_{i=N-r}^N \Delta H_i + \sum_{i=N+1}^{N+L-r} \Delta H_i = H_{N+L-r} - H_{N-r-1} = \\ &= \frac{1}{S} (V_{N+L-r} - V_{N-r-1}). \end{aligned} \quad (20)$$

This last relation allows evaluation of the raisings of the  $dw$ 's directly from the  $P$ - $V$  diagram of the Carnot cycle. We have only to look at the  $\Delta V$  relative to a pressure variation  $(\Delta P)^* = (2r+1-L)\Delta P$ .

From the  $P$ - $V$  diagram we see that for  $(\Delta P)^* > 0$  we have  $\Delta V < 0$  and hence  $h_{N-r}^L < 0$  and for  $(\Delta P)^* < 0$  we have  $\Delta V > 0$  and hence  $h_{N-r}^L > 0$ .

For  $r > L$  the way in which the previous  $L$   $dw$ 's have been removed has no influence. Therefore for  $r > L$  the raisings are the same as in the SP i. e.

$$h_{N-r}^L = h_{N-r} = H_{N-r} - H_{N-r-1} = \frac{1}{S} (V_{N-r} - V_{N-r-1})$$

### FIGURE 4

Obviously we expect that

$$W \equiv \sum_{i=1}^Z P_i \Delta V_i = g \sum_{k=1}^N m_k h_k = g \sum_{k=1}^N m_k h_k^L \quad (21)$$

since for each of the  $N!$  possible processes the work performed by the gas in the cycle is the same.

In Figure 4 we report the raisings of the  $dw$ 's for a complex process with  $L = 50$ . Notice that the raisings of the last 50  $dw$ 's are here completely different from those in the SP.

The previous removing process is only one of the many complex processes that one can perform. The removing process are  $N!$  For each process we have a different distribution for the raisings  $h_k$ .

## 4. Summary



In this paper we have introduced the extra work which, together with the lost work, gives the dissipated energy in the irreversible processes. The analysis is confined mainly to irreversible isothermal process even if the known results [2,4] are used for the adiabatic processes.

The new and previous results are used to evaluate the dissipated energy for a stepwise ideal-gas Carnot cycle, a system with complexity. The complexity arises from the fact that there are  $N!$  different distribution for the raisings  $h_k$ , i.e the work output of the Carnot cycle can be distributed in  $N!$  ways among the  $N$   $dw$ 's.

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I greatly indebted to M. Zannetti and G. Monroy for useful comments.

### Appendix :Lost work and extra work for isothermal processes with external irreversibility

Here we evaluate the lost work for the expansion and the extra work for the compression when there is external irreversibility.

In sec.2.2 and in sec.3 of paper[1] we have shown that if the irreversible isothermal expansion is performed by means of a (shorter) contact with an heat source at  $T_{ext} > T$ , we have  $\pi_{ext} \neq 0$  i.e.

$$\pi_{ext} = \frac{Q_{in}}{T} - \frac{Q_{in}}{T_{ext}} \quad (A1)$$

and for the endo-reversible process, i.e. the process in which the gas performs the reversible isothermal expansion  $A \rightarrow B$

$$\pi_{ext}^{Endo} = \frac{Q_{Rev}}{T} - \frac{Q_{Rev}}{T_{ext}} \quad (A2)$$

Similarly if the irreversible isothermal compression is performed by means of a (shorter) contact with a heat source at  $T_{ext} < T$ , we have  $\pi_{ext} \neq 0$  i.e.

$$\pi_{ext} = \frac{Q_{out}}{T_{ext}} - \frac{Q_{out}}{T} \text{ and } \pi_{ext}^{Endo} = \frac{Q_{Rev}}{T_{ext}} - \frac{Q_{Rev}}{T} \quad (A3)$$

To evaluate the lost work for the expansion with  $T_{ext} > T$ , we calculate the work available in the related reversible process and subtract from it  $W_{out}$ , the effective work done in the irreversible process. This difference gives the lost work.

The reversible work is the reversible work of the gas + the work of an auxiliary reversible engine working between  $T_{ext}$  and  $T$ . For the gas  $W_{Rev}(gas) = Q_{Rev}$ ; the auxiliary reversible engine, which brings the heat

$Q_{Rev}$  to the system (the ideal gas at temperature  $T$ ) and takes from the heat source  $T_{ext}$  the heat  $Q_{Rev} \frac{T_{ext}}{T}$

performs the work  $W_{Rev}(engine) = Q_{Rev} \frac{T_{ext}}{T} (1 - \frac{T}{T_{ext}})$ . Therefore the total reversible work is

$$W_{RevTotal} = W_{Rev}(gas) + W_{Rev}(engine) = Q_{Rev} + Q_{Rev} \frac{T_{ext}}{T} (1 - \frac{T}{T_{ext}})$$

The work performed by the gas in the irreversible expansion is  $W_{out} = Q_{in}$ . Therefore

$$W_{Lost} = W_{RevTotal} - W_{out} = Q_{Rev} - Q_{in} + Q_{Rev} \frac{T_{ext}}{T} - Q_{Rev} = T\pi_{in} + T_{ext}\pi_{ext}^{Endo} \quad (A4)$$

On the other hand, for compression with an heat source at  $T_{ext} = T$

$$W_{Extra} = W_{in} - W_{Rev} = Q_{out} - Q_{Rev} = T\pi_{int} \quad (A5)$$



but if one uses a heat source at  $T_{ext} < T$ , from the reversible work necessary to perform the isothermal compression at temperature  $T$  we have to subtract the work of the reversible engine which subtracts  $Q_{out}$  from heat source  $T$  (the gas) and gives to the source  $T_{ext}$  the heat  $Q^{Min} = \frac{T_{ext}}{T} Q_{out}$ , i.e.

$$W_{Rev}^{ext} = Q_{out} - Q^{Min} = T_{ext} \left( \frac{Q_{out}}{T_{ext}} - \frac{Q_{out}}{T} \right) = T_{ext} \pi_{ext}$$

Therefore the extra work is

$$W_{Extra} = W_{in} - (W_{Rev} - W_{Rev}^{ext}) = Q_{out} - Q_{Rev} + Q_{out} - T_{ext} \frac{Q_{out}}{T} = T \pi_{int} + T_{ext} \pi_{ext} \quad (A6)$$

## References

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## Figure Captions

Figure 1. a) The adiabatic vessel with some  $dw$ 's on the piston. b) Cross-section view of the vessel showing two supports for the  $dw$ 's (the  $dw$ 's reservoir)

Figure 2. The stepwise Carnot cycle with very small steps.

Figure 3. Overall raising on the reservoir of each  $dw$ . In the enlargement of the extremity of the graph, one can see that the last  $dw$ 's and the first ones have negative raising

Figure 4. Overall raising for each  $dw$  for the removing process which starts from the  $(N - L)^{\text{th}}$   $dw$  i.e. the  $560 - dw$

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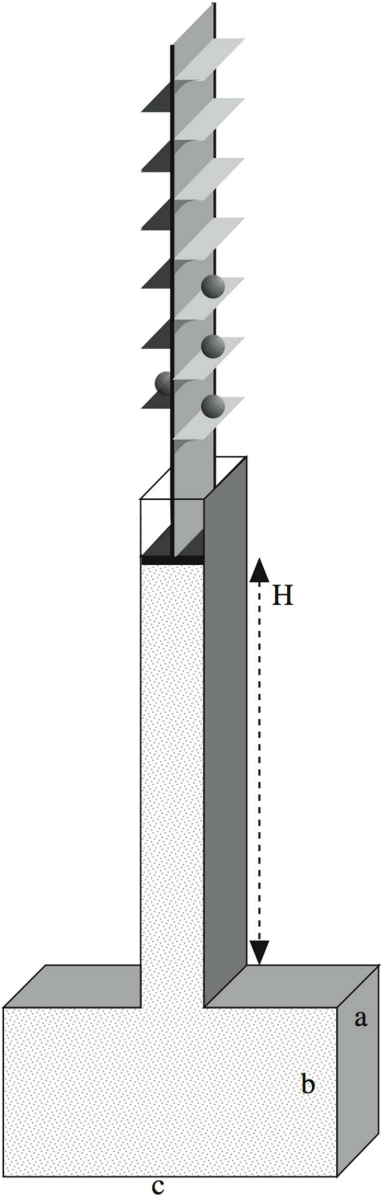


Figure 1a  
75x235mm (300 x 300 DPI)

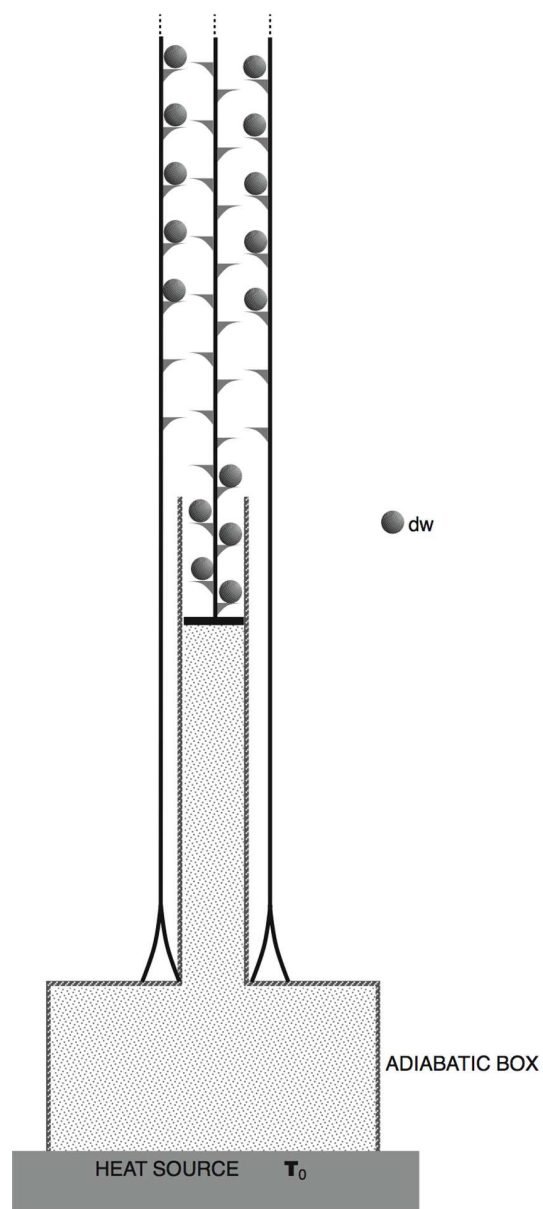


Figure 1b  
80x161mm (300 x 300 DPI)

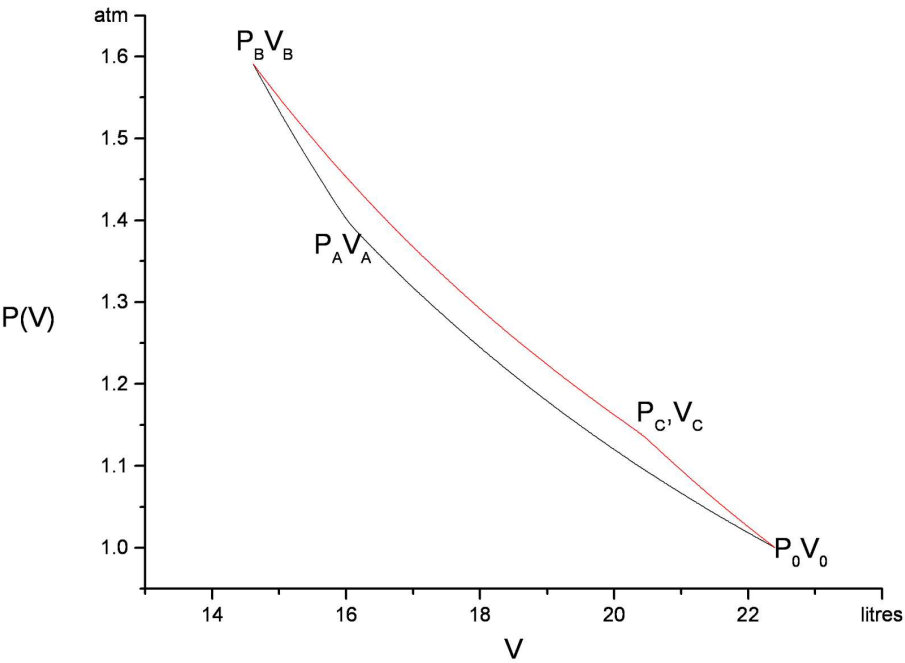


Figure 2  
150x114mm (300 x 300 DPI)

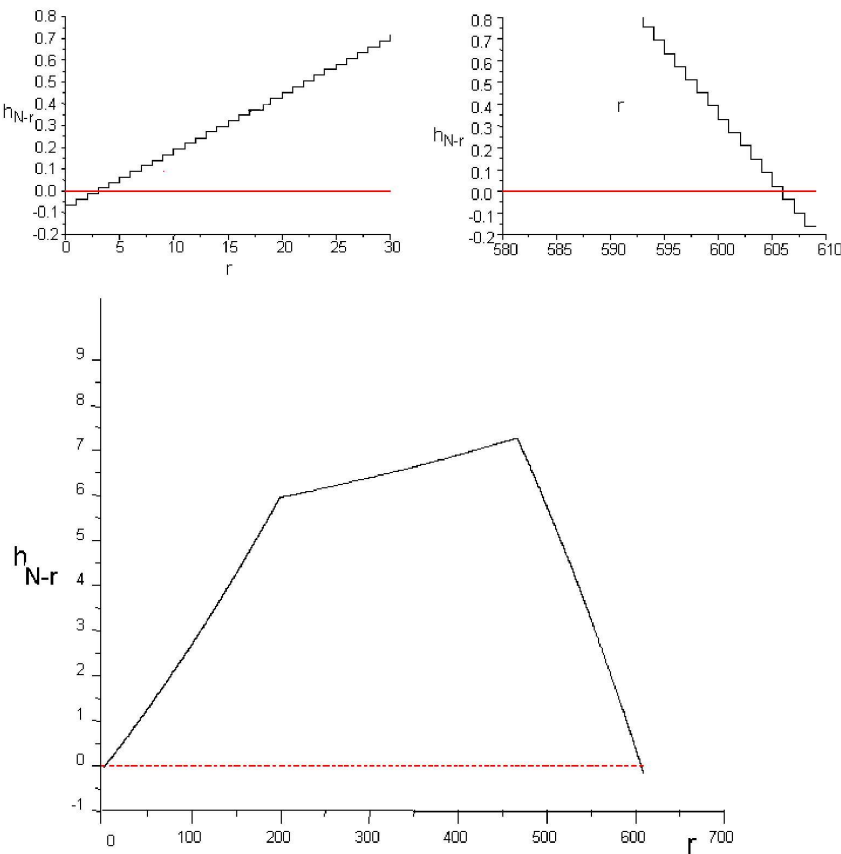


Figure 3  
140x186mm (300 x 300 DPI)

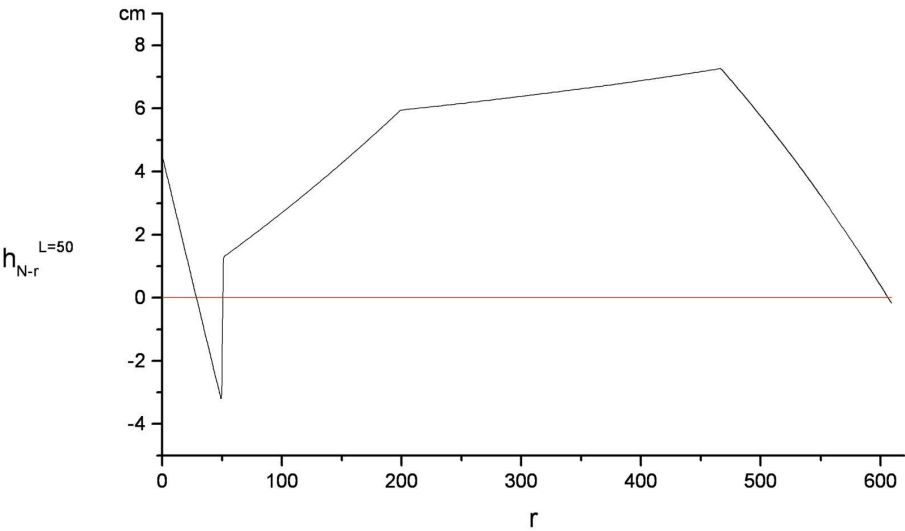


Figure 4  
150x95mm (300 x 300 DPI)