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The Nabarro equation for thermally activated plastic glide

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ABSTRACT

A special feature of thermally activated plastic glide is that the activation energy is not a constant but depends strongly on the applied stress, all the way down to zero at the low-temperature yield stress. The Nabarro equation gives the relation for this dependence and is particularly applicable when the obstacles to glide dislocations are localized, as in forest work hardening. Some characteristic features of plastic flow and creep, which result from this, are briefly discussed.

1. THE EQUATION

It is a pleasure to help, here, celebrate Frank Nabarro's 90th birthday and to applaud his long – over 60 years – continuous record of distinguished research, almost all on the theory of dislocations.

Of his many contributions I will here consider just one, his equation for the stress-dependent thermal activation of a glide dislocation past a localised obstacle. This equation first appeared in a landmark paper that he wrote with Nevill Mott [1] on precipitation hardening in solid-solution alloys. It was subsequently developed and applied more widely by Nabarro [2] and is

$$G = G_0 \left(\frac{\sigma_g}{\sigma_0} \right)^{3/2} \quad (1)$$

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3 where G is the stress-dependent activation energy to overcome the
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6 obstacle, with G_0 as its limiting value under zero stress; σ_g is the ‘stress
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9 gap’, i.e. the shortfall, $\sigma_0 - \sigma_a$, between the applied driving stress σ_a and the
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12 ‘obstacle stress’ σ_0 at which the obstacle can be overcome without thermal
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15 help.
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22 A much earlier theory of thermally activated glide was given by Becker
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24 [3] before the days of dislocation theory. He argued that, when there is a
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26 stress gap, the thermal vibration of the system could occasionally provide
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28 a stress fluctuation in a limited volume of the material, sufficient to span
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30 the stress gap and so allow an increment of plastic glide to occur. This led
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32 to an equation apparently different from eqn. 1, with G varying as the
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34 square of the stress gap; but it has now been shown that, when this volume
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36 is optimized, instead of being assumed constant as in Becker’s work, then
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38 his equation leads to the same $3/2$ dependence as the Nabarro equation [4].
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55 Dislocation processes such as the thermal vibration of a short segment of a
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57 free glide dislocation line against a localized obstacle occur with a
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3 frequency of order 10^{11} s^{-1} . It follows that, if such obstacles are to be
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6 overcome with reasonable frequency, say 1 s^{-1} , then G cannot be much
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9 larger than about 25 kT, ie about 0.62 eV at room temperature. Thus,
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12 unless G_0 is also small, not more than about 2.5 eV, Nabarro's equation
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15 requires that σ_g must be very small, i.e. the applied stress is always close
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18 to its zero temperature value, σ_0 . There is then almost no temperature
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21 dependence of the yield stress. Mott and Nabarro recognized that typical
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24 precipitates in alloys would be too large to give G_0 this required small
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27 value and thus that the strength would hardly vary with temperature.
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30 Having given this result, it seemed that Nabarro's equation would have
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33 little further useful to say. The position changed greatly, however, when
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36 the equation was later applied to work-hardened states.
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45 2. FOREST HARDENING

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48 Work-hardened metals such as aluminium and copper show a temperature
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51 dependence, both in their flow stress for a given dislocated state and in the
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54 type of dislocated state they develop. A major role is thus played in the
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57 hardening by obstacles with small G_0 values, i.e. obstacles of atomic order
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3 of size. This points clearly to forest hardening, in which the glide
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6 dislocations are obstructed by the narrow cores of forest dislocation lines
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8
9 which pass through the glide planes. Then G_o is given by the core energy,
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11
12 about $\frac{1}{2}\mu b^3$ (μ = shear modulus, b = Burgers vector length), typically
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17 about 2.5 eV for such metals. Moreover, as Nabarro [2] has shown, when
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20 the finite size ($\sim b$) and strength ($\sim \frac{1}{2}\mu b^2$) of forest obstacles is taken into
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23 account, forest hardening is able to explain the observed constancy of the
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26 temperature dependence of the flow stress at various levels of work
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31 hardening.
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37 Nabarro's equation enables us to check the value of G_o , which, on the
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40 forest theory, is about $\frac{1}{2}\mu b^3$, i.e. 1.9 eV for aluminium. The observed flow
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43 stress of an aluminium crystal in a given work-hardened state, after
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46 correction for the temperature dependence of μ , falls by about 0.21 in the
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49 first 100 K above zero. Thus, taking $\sigma_g/\sigma_o = 0.21$ in eqn.1 and
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$$G = kT \ln(\nu t) \quad (2)$$

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4 with $\nu t \simeq 10^{11}$, we deduce that $G = 0.21$ eV at 100 K and $G_0 = 2.2$ eV. At
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6
7 higher temperatures the flow stress falls off more gradually, which may be
8
9 attributed partly to an increased apparent size which the forest obstacles
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11 then present to the glide dislocations when σ_g/σ_0 is no longer small [5];
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14 and partly to the fact that Nabarro's equation is not then strictly valid.
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23 3. CELL STRUCTURES

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26 The thermo-mechanical activation represented by eqn.1 not only
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28 determines the above temperature dependence of flow stress for a given
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30 work-hardened state; it also determines the form of such states, so leading
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32 to work-softening (dynamical recovery) effects. The contribution of
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34 electron microscopy in this field is to show the existence of cell structures
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36 in work-hardened systems. The forest dislocations are mainly localized, as
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38 dense irregular mats in which the glide dislocations are entangled, in *walls*
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40 that surround almost dislocation-free *cells*, typically about 10^{-6} across.
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51 About one-fifth of the total volume is occupied by the walls.
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3 Evidently, the glide dislocations are created from within the cells and
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6 glide, fairly freely, until they run into and are then obstructed by the forest
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9 dislocations of the walls. In the walls they are not only pushed forwards by
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12 the applied stress and other glide dislocations coming along behind them;
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15 they are also pulled into the walls by elastic attraction to their opposite
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18 numbers similarly coming into the walls from cells on the other side.
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21 Nevertheless, they can usually make progress through the forest tangles
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24 (which Nabarro and I, years ago, once described as *birds' nests*) only with
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27 the help of thermal activation, through the Nabarro equation. If they
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30 penetrate far enough they can then meet their opposite numbers and so
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33 either mutually annihilate or form close dipoles.
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41 An important aspect of this has been recognised by Mughrabi (e.g. [6]);
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44 internal stresses are developed between the cells and walls, due to plastic
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47 deformation initiated in the cells which is not transmitted equally through
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50 the walls, since the glide dislocations from the cells are obstructed at the
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53 walls. These *Mughrabi stresses* form a back stress in the cells, σ_i , i.e.
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60 opposing the applied stress, and a forward stress, approximately $4\sigma_i$, in the

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walls. Suppose that the sources in the cells operate when the stress acting on them is σ_s . Then

$$\sigma_a - \sigma_i = \sigma_s. \quad (3)$$

If $\sigma_a - \sigma_i$ were greater than this the cells would then make more dislocations to pile up against the walls, so increasing σ_i and restoring the balance in eqn. 3.

Mughrabi stresses are involved with eqn. 1 through the work-hardening process. Consider the extreme case where the walls are completely impenetrable and deform only elastically (with the same shear modulus). Then, since the total strain is approximately the same everywhere, a plastic strain γ in the cells produces an elastic strain and hence a stress $\mu\gamma$ in the walls, requiring an increase of σ_i in the cells of about $0.25 \mu\gamma$. To continue plastic flow the applied stress would thus have to be increased by this same amount, implying an apparent work-hardening coefficient of 0.25μ , about 7000 MPa in aluminium, instead of the observed value which is a maximum of only about 0.003μ .

4. WORK SOFTENING

So, in practice, the Mughrabi stresses do not accumulate because the walls are penetrable by forest cutting processes represented by the Nabarro equation. Through thermo-mechanical activation the obstructed glide dislocations are able to pass into the walls to the centre, where they can 'disappear', as above. Mughrabi proved the existence of his stresses experimentally, but they rose to only about 10 MPa in the cells and 80 MPa in the walls, and so were evidently only the residue of survivors of the annihilation and dipole forming processes. Work softening (dynamical recovery) is usually considered to make its appearance when the stress-strain curve evolves from stage II, where the work-hardening coefficient is about 0.003μ , to stage III, where it falls below this value. But we see that it is also active in stage II where it prevents σ_i and hence the applied stress σ_a from rising rapidly.

The Mughrabi stresses provide a transmission mechanism, with approximately 4 to 1 gearing, which communicates the work hardening of the forest, in the walls, out to the applied stress via eqn. 3. When the

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4 sample is being strained at a constant rate, $\dot{\gamma}$, as in conventional stress-
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7 strain experiments, G has then to remain constant. Thus, σ_g has to be
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10 increased in proportion to σ_o ; and so also $4\sigma_i$ in the walls; σ_i in the cells;
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13 and finally σ_a so as to maintain the equality of eqn. 3. The work softening
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16 in the walls, which removes most of the glide dislocations otherwise
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19 accumulating there, reduces $4\sigma_i$ there at almost the same rate as $4\sigma_i$ builds
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22 up from the applied strain rate, $\dot{\gamma}$, the difference being due to the relatively
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25 small increase of $4\sigma_i$ required to hold σ_g/σ_o constant in the Nabarro
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28 equation.
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36 Most of the Mughrabi effect is thus hidden in conventional stress–strain
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39 experiments. But it can be revealed in experiments which make abrupt
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42 changes to the penetrability of the wall forests; particularly by finite step
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45 changes in temperature. which, through eqn. 2, drastically alter the value
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48 of G (e.g. [7]). Steps up lead to fast forest cutting, from the Nabarro
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51 equation, which produces spectacular work softening yield drops [8]. Of
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54 greater interest here are step down changes which render the previously
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57 penetrable walls resistant to the now enfeebled thermal activation. The
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3 glide dislocations which are then produced accumulate at the walls, with
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6 virtually no softening processes, and so generate Mughrabi stresses
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9 undiluted by dynamical recovery, which reveal themselves in a very rapid
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12 increase in the applied stress, giving an apparent rate of work hardening
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15 that is barely distinguishable from purely elastic loading.
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22 The transition from stage II to the lower rate of work hardening in stage III
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24 is usually attributed to dynamical recovery, as indicated by its temperature
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26 dependence. But it cannot be the same process as that above, i.e. involving
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28 the removal of Mughrabi stresses through forest cutting by obstructed
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31 glide dislocations, since, during steady running at constant strain rate, this
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34 process merely acts as an intermediary which communicates the work
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37 hardening of the forest out to the applied stress. Electron microscopy has
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40 shown that the softening in stage III is related in fact to a clarification of
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43 the structure of the forests. This may be due to the forest dislocations
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46 themselves cutting through 'their' forests, these being made up partly by
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49 the glide dislocations; and also to cross-slip processes involving screw
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52 dislocations, not here considered.
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5. ANDRADE CREEP

Andrade creep, in which the plastic strain of a soft metal, under constant applied stress, increases fairly rapidly, varying with time as $t^{1/3}$, has proved a difficult challenge to theory. This is because all theories which attribute the slowing down of the creep flow to a standard rate equation, i.e.

$$\dot{\gamma} = Ae^{\frac{-G}{kT}}; \quad (4)$$

and have G increasing with γ by work hardening (of which the Nabarro equation could of course provide an example), lead almost unavoidably to a creep which increases logarithmically with time, not to Andrade creep.

As an alternative, it has been proposed that the declining rate in Andrade creep is due to the system moving gradually away from the singularity of a critical point, where different governing conditions apply. Evidence for such criticality has been provided by Wyatt [9], who showed, with aluminium, that very small increases of applied stress, after Andrade creep had been occurring for some time, e.g. 1000 s, under constant stress, were able fully to restore the original fast flow rate.

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3 The criticality shows itself through the formation of *avalanches*. When a
4 glide dislocation segment in the forest overcomes its obstacle and breaks
5 free, the consequential internal stress change can trigger neighbouring
6 segments, which are also near the critical state for such release. Thus, an
7 avalanche of triggered releases may ensue, giving a large contribution to
8 the plastic deformation. Very close to the critical point avalanches of
9 almost unlimited size can form, so giving a fast flow rate. But as the
10 developing forest thereby becomes less penetrable the avalanche sizes
11 diminish and the creep rate drops. The avalanche size is so extremely
12 sensitive to obstacle strength, near the critical point, that its variation can
13 dominate the creep rate under such conditions.
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41 A recent version of this theory [10] expresses the criticality through a
42 *reproductive factor*, k , which measures the population ratio of successive
43 generations of triggerings. During the “instantaneous” flow, which occurs
44 on first loading, $k > 1$ so that each triggered segment then, on average,
45 triggers more than one successor and the wall forests hence become
46 swamped with numerous uncontrolled avalanches. The critical point is
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3 then reached when $k = 1$, at which singularity each generation can trigger a
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7 successor of statistically equal size. Beyond this point is the creep range,
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10 where $k < 1$, in which sequences die out at finite avalanche sizes unless
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13 reactivated by thermal fluctuations which support the driving stress at
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16 segments held up by blocking cages of forest obstacles. The analysis then
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19 leads to

$$\dot{\gamma} \propto (1-k)^{-2} \propto \left(\frac{\eta\gamma}{\sigma_a} \right)^{-2}, \quad (5)$$

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27 with η as the coefficient of hardening; and thus to Andrade creep.
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34 This scheme depends on two things: 1) that the waiting time of a segment
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37 at an obstacle before it is released does not increase (through the effect of
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40 the rising $\eta\gamma$) sufficiently to alter the creep rate significantly; 2) that the k
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43 = 1 point is arrived at in a timescale typical of creep experiments. It will
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47 be seen that these are related.

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52 The first follows from the low rate of work hardening in Andrade creep, as
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56 observed in soft metals at temperatures where thermal softening is
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3 appreciable. Additionally, Wyatt's experiments, as above, showed that
4 such creep strains in aluminium never raised the stress for rapid flow by
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7 such creep strains in aluminium never raised the stress for rapid flow by
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9
10 much . From his results we take $\sigma_a = 30$ Mpa, $\eta = 40$ MPa, $\gamma = 0.01$ and
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12 hence $\eta\gamma = 0.4$ MPa, so that, with $\sigma_g/\sigma_o = 0.4/30$ and $G_o = 2$ eV in
13
14 Nabarro's equation, $G = 0.003$ eV. According to eqn. 4 this would give a
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17 reduction in $\dot{\gamma}$ of only 12%, which is negligible compared with the 200-
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20 fold reduction over the typical 3000 s lifetime of an Andrade creep
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27 experiment.

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32 There is an additional reason for the low hardening rate in such creep, one
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35 which links 1) and 2) above. The creep follows, more slowly, the
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38 extremely rapid flow of the "instantaneous" phase and such a large drop in
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41 strain rate can produce exceptional rates of work softening, similar to
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44 those produced by increasing the temperature finitely, as noted in § 4
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48 above.

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54 Because of inertia the strain rate in the "instantaneous" phase is not more
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57 than very high. As a result, the applied load rises to its intended final value
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4 over a short time interval. For example, to minimise dynamic load effects,
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7 Wyatt [9] arranged for the interval to be about 2 s. In practice then, the
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10 equilibrium, eqn. 3, (e.g. typically in Wyatt's experiments on aluminium,
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13 $\sigma_a = 30$ MPa, $\sigma_i = 20$ Mpa and σ_s , which we take to equal the initial yield
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15
16 stress, = 10 MPa), is established in a time of order 1s. This sets the
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19 timescale for the beginning of creep.
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26 The back-stress σ_i is a result of the build-up of forest density and of
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29 obstructed glide dislocations in the walls. Its rise during the
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32 "instantaneous" phase brought the system to the equilibrium point, eqn. 3.
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35 The rise then attempts to continue beyond this point, but this is countered
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38 by a strong reduction in generation rate at the dislocation sources in the
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41 cells. The effect is that the strain rate drops severely, after this point is
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44 reached, so marking the end of the "instantaneous" phase.
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52 During the "instantaneous" phase the glide dislocations in the walls come
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55 under increasing pressure, both from the rising applied stress directly, and
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58 from other glide dislocations following up behind. As a result, it is
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3 expected that for these dislocations k does not fall to $k = 1$ (and then to $k <$
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7 1) until the equilibrium point, eqn. 3, is reached at the final, steady, value
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10 of the applied stress and a consequential drop in strain rate ensues. It is
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13 thus expected that the critical point, $k = 1$, is reached soon after the other
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16 critical point, eqn. 3, has been reached. On this basis, the system begins its
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19 creep phase, at $k = 1$, after an “instantaneous” phase of duration about 1 s.
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26
27
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29
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32
33
34 course of this work.
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