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Theory of the Kondo lattice : competition between Kondo effect and magnetic order

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Abstract

We present here a review of some theoretical features of the Kondo lattice problem, which can describe anomalous experimental properties of strongly correlated electron systems with mainly Cerium, Ytterbium or Uranium. In such Kondo systems, there exist both a Kondo effect on each site and strong intersite magnetic interactions. We present firstly the general case of the Kondo lattice and the mean-field approximation. We discuss then the case of the "undercreened" Kondo lattice with a strong coexistence between the Kondo effect and a ferromagnetic order, as for example observed in Uranium compounds such as UTe. We also discuss the Kondo-Spin Glass competition with eventually an additional magnetically ordered phase (ferromagnetic or antiferromagnetic one) and we can account for phase diagrams of some Cerium and Uranium disordered alloys such as $CeNi_{1-x}Cu_x$. Finally, the introduction of a transverse magnetic field in the Kondo-Spin Glass problem yields a Quantum Critical Point and allows a better description of the phase diagrams of disordered alloys.

1. Introduction

It is well established that an anomalous behaviour is observed in many cerium, ytterbium, uranium or other anomalous rare-earth (Europium, Samarium, Praseodymium and Thulium) and actinide (Neptunium and Plutonium) systems. There are different anomalous behaviours observed in such systems and a very important case concerns the "intermediate valence" systems, where the number of 4f electrons is generally not an integer and varies with the external pressure or the matrix concentration, like in the famous phase diagram of Cerium metal [1, 2, 3, 4, 5, 6] or in $Eu(Pd_{1-x}Au_x)_2Si_2$ alloys [7] or $YbInCu_4$ [8]. Another interesting and well studied case concerns the so-called "Kondo systems", which have a valence or a number of f-electrons close to an integer, but present however an anomalous behaviour due to a strong d-f hybridization. The Kondo systems have been extensively studied experimentally and theoretically after the first explanation of the resistivity minimum of magnetic dilute alloys by Kondo [9] and the exact solution obtained within the renormalisation group technique for the single Kondo impurity [10].

In the case of a lattice, there exists a strong competition between the Kondo effect and the magnetic order in many compounds or concentrated alloys containing rare-earths or actinides. This strong interplay between the Kondo effect on each site and the magnetic ordering arising from the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between rare-earth atoms at different lattice sites is well described by the Doniach diagram [11], which gives the variation of the Néel temperature and of the Kondo temperature with increasing antiferromagnetic intrasite exchange interaction J_K between

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localized spins and conduction electron spins. It is well known that the ordering temperature T_N is increasing initially with increasing ρJ_K , then passes through a maximum and tends to zero at a critical value ρJ_K^c corresponding to a "Quantum Critical Point" (QCP) in the Doniach diagram. Such a behavior of T_N has been experimentally observed with increasing pressure in many cerium compounds, such as $CeAl_2$ [12] or $CeRh_2Si_2$ [13]. A similar effect has been observed in $YbCu_2Si_2$ [14] or in related ytterbium compounds, where the Néel temperature starts from zero at a given pressure and increases rapidly with pressure, because pressure has an opposite effect in ytterbium compounds than in cerium compounds.

The Doniach diagram presents also the great advantage of separating cerium compounds in two categories : the first one includes cerium compounds which have a mixed Kondo-magnetic character corresponding to pressures below the critical pressure of the QCP; for example, $CeAl_2$ has a Néel temperature of 3.8 K and also a relatively large electronic constant γ of the specific heat equal to 135 mJ/moleK^2 . The second category corresponds to cerium compounds with only a huge Kondo or heavy fermion character and for example $CeAl_3$, which does not order magnetically, has an enormous value of $\gamma = 1600 \text{ mJ/moleK}^2$. In the magnetically ordered compounds, there is coexistence between the magnetic order (generally antiferromagnetic like in $CeAl_2$) and the heavy fermion character, although this effect is much smaller in magnetically ordered compounds than in non magnetic ones. On the other hand, the magnetic order can be antiferromagnetic, ferromagnetic in a few cases, a classical spin glass order or a more disordered one.

The purpose of the present paper is to review recent theoretical models

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8 which can account for the difficult problem of competition or coexistence
9 between the magnetic order and the Kondo effect which gives the heavy
10 fermion character. In the next section, we will firstly discuss the mean field
11 approximation and we will apply it to both the case of the Kondo effect in
12 presence of short-range magnetic correlations and to the case of the compe-
13 tition between the Kondo effect and the antiferromagnetic order. Then, we
14 will develop the case of the so-called "underscreened" Kondo model where
15 a coexistence between the Kondo effect and the magnetic order is obtained,
16 which can account for the magnetic properties of some uranium compounds.
17 Finally, we will discuss the occurrence of spin glass phases in Cerium or Ura-
18 nium systems and we will describe the competition between the spin glass
19 order, the Kondo effect and eventually a magnetically (ferromagnetic or an-
20 tiferromagnetic) ordered phase.
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34 **2. The Kondo lattice model**

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37 We have firstly developed the Kondo-lattice model which is well appropriate
38 to describe cerium (ytterbium) compounds corresponding to a configuration
39 close to $4f^1$ (or $4f^{13}$) with one 4f electron (or one hole). The Kondo-lattice
40 model is often studied with only the intrasite Kondo interaction. In principle,
41 both the RKKY magnetic interaction and the Kondo effect can be obtained
42 from the intrasite exchange term [15]. When dealing with approximation
43 schemes, it is hard to obtain both effects starting with the intrasite inter-
44 action alone and for this reason, an explicit intersite exchange interaction is
45 usually included in the model Hamiltonian. We will give here a very brief
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summary of the work presented in ref.[16, 17] and obviously details can be found in these papers.

The proposed Hamiltonian of the system is, therefore :

$$H = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} n_{\mathbf{k}\sigma}^c + J_K \sum_i \mathbf{s}_i \cdot \mathbf{S}_i + J_H \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j, \quad (1)$$

where $\varepsilon_{\mathbf{k}}$ is the energy of the conduction band, $J_K (> 0)$ is the Kondo coupling between the localized spin \mathbf{S}_i and the spin \mathbf{s}_i of a conduction electron at the same site, and J_H is the interaction between nearest-neighboring localized spins. Assuming spin-1/2 localized moments, we represented them by a zero-width f band with one electron per site, while the conduction band has width $2D$ and a constant density of states. We consider here the case of choosing J_H to be positive, implying that intersite interactions are antiferromagnetic, as it is the case of most cerium compounds.

We now write the spin operators in fermionic representation and we impose the constraint of single-occupancy of the f level at all sites, $n_i^f = 1$.

In order to discuss the Kondo effect and magnetic correlations, we define the operators :

$$\lambda_{i\sigma} \equiv c_{i\sigma}^\dagger f_{i\sigma}, \quad \Gamma_{ij\sigma} \equiv f_{i\sigma}^\dagger f_{j\sigma}, \quad (2)$$

where $\lambda_{i\sigma}$ describes the intrasite Kondo correlation and $\Gamma_{ij\sigma}$ represents an intersite correlation between two neighboring atoms. This is so because the on-site spin-spin correlation $\langle \mathbf{s}_i \cdot \mathbf{S}_i \rangle$ can be written in terms of the average $\langle \lambda_{i\sigma} \lambda_{i,-\sigma} \rangle$, and similarly for the intersite correlation using $\Gamma_{ij\sigma}$. Then, with this notation we perform an extended mean-field approximation [15, 18] which is presented in full detail in Ref. [16, 17]. Considering translational invariance and taking into account that there are no magnetic states, we can

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8 write $\lambda = \langle \lambda_{i\sigma} \rangle$ for all sites and $\Gamma = \langle \Gamma_{ij\sigma} \rangle$ for nearest-neighboring sites and
9 zero otherwise. It is important to note that, in all the present mean-field
10 descriptions of different Kondo lattice problems, we always use the same
11 mean-field approximation, i.e. $\lambda = \langle \lambda_{i\sigma} \rangle$ in the non magnetic case or a
12 spin-dependent $\lambda_\sigma = \langle \lambda_{i\sigma} \rangle$ in the magnetic case, for describing the Kondo
13 problem, while we use different mean-field terms to describe the different
14 parameters, like here the short-range magnetic correlations or then different
15 descriptions of the magnetic moments to study different cases of magnetic
16 orderings. In fact, the mean-field approximation used here for the Kondo
17 effect has the great advantage to describe simply and correctly the main fea-
18 tures of the Kondo effect, but obviously it has the disadvantage of giving a
19 too brutal mean-field-like transition rather than a continuous one.

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21 After performing this approximation, one deals with a one-electron Hamil-
22 tonian representing two hybridized bands : the conduction band of width $2D$
23 and the f band of effective band width $2BD$, with B given by :

$$24 \quad B = -zJ_H\Gamma/D, \quad (3)$$

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26 z being the number of nearest neighbors of a site.

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28 The magnitude of the hybridization gap is directly related to λ^2 and this
29 quantity is also a measure of the Kondo effect, as the Kondo correlation
30 function $\langle \mathbf{s}_i \cdot \mathbf{S}_i \rangle$ is proportional to λ^2 .

31
32 The mean-field parameters λ and Γ are obtained by minimizing the total
33 internal energy of the system at zero temperature, or the total free energy
34 at finite temperatures [17, 19]. The reference energy E_0 of the f band and
35 the chemical potential μ have to be determined self-consistently in order to
36 keep the average numbers of f and conduction electrons respectively equal
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to 1 and n .

We have previously treated the half-filled case, corresponding to a number of conduction electrons $n = 1$ [16] and then the general case of a non-integer number of conduction electrons [17]. In this previous paper, we have determined both the temperature T_{cor} for the occurrence of short-range magnetic correlations and the Kondo temperature T_K for the lattice, which is different from the one-impurity Kondo temperature T_{K0} obtained for $J_H = 0$.

In the limit $J_H = 0$ and for $J_K/D \ll 1$, we obtain :

$$T_K = C_0 D \sqrt{n(2-n)} \exp\left(-\frac{1}{\rho J_K}\right), \quad (4)$$

where C_0 is a numerical constant: $C_0 = 1.1337$. Eq. (4) gives T_K depending on the band filling as $\sqrt{n(2-n)}$, which is consistent with a \sqrt{n} dependence when $n \rightarrow 0$, as found in Ref. [20].

We have also extensively studied the variations of the Kondo temperature T_K with J_H and the conduction-band filling n [17]. We have found that T_K first increases and then decreases with J_H for fixed n , dropping abruptly to zero at some critical value of J_H . On the other hand, for a given J_H , T_K decreases rapidly as n departs from half filling. Thus, we have obtained a drastic suppression of the Kondo effect with increasing antiferromagnetic intersite coupling and reducing the band-filling factor. On the other hand, the correlation temperature T_{cor} is linear with J_H independently of the value of n and the short-range magnetic correlations occur above the Kondo temperature, in good agreement with neutron diffraction experiments in Cerium systems, such as CeCu_6 and CeRu_2Si_2 [21, 22].

We have established that the dependence with the coupling constant J_K of the Kondo temperature for the lattice can be significantly different from

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8 the single-impurity one [16, 17]. It is rather difficult to deduce experimen-
9 tally the Kondo temperature especially in Kondo systems which are close
10 to be magnetic and, therefore, to compare the preceding theoretical results
11 with experimental data. However, we can deduce from experimental results in
12 CeRh₂Si₂ [13], CeRu₂Ge₂ [23, 24] or Ce₂Rh₃Ge₅ [25] that the pressure depen-
13 dence of T_K observed in these systems is different from the single-impurity
14 case, in good agreement with our model.
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21 But, on the other hand, the lattice Kondo temperature is closely following
22 the single-impurity one in the other cerium and all ytterbium compounds. If
23 we would like to compare cerium and ytterbium compounds, the 4f electrons
24 are more localized in the ytterbium case than in the cerium one and it results
25 that the intersite interaction J_H must be smaller in ytterbium systems than
26 in cerium ones. Thus, in our present model, the pressure dependence of
27 the lattice Kondo temperature must be closer to the single-impurity one
28 in ytterbium compounds than in cerium ones, as experimentally observed.
29 Further experiments are needed to better understand the conditions which
30 can yield a Kondo temperature for the lattice much smaller than the single-
31 impurity one.
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41 In this section, we have introduced the Kondo lattice problem, but in
42 fact we have not accounted for the experimental curves of T_N versus pressure
43 which present a Quantum Critical Point (QCP). Thus, in order to better
44 describe the behaviour of many Cerium compounds, we have studied the
45 competition between an antiferromagnetic (AF) order and the Kondo effect
46 within the preceding model [26]. We use again the mean-field approximation
47 by taking two order parameters, first $\lambda = \langle \lambda_{i\sigma} \rangle$ for describing the Kondo
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8 effect and then the average magnetization for each sub-lattice of the antifer-
9 romagnetic system. We have obtained a phase diagram showing a first-order
10 transition from an antiferromagnetic order for small J_K values to a Kondo
11 phase for larger J_K values. The transition at low temperatures between the
12 AF phase to the Kondo one with increasing J_K (or equivalently increasing
13 pressure in Ce compounds) is a first order one without any QCP and we
14 have, therefore, obtained a competition between these two phases and not a
15 coexistence between them, as observed in many cerium compounds such as
16 $CeAl_2$. This mean-field model does not give any "mixed" AF-Kondo phase,
17 but is able to give successively an antiferromagnetic order and a Kondo phase
18 with increasing $|J_K|$.
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28 Recently, the competition between the ferromagnetic order and the Kondo
29 effect has been studied within the same Kondo lattice problem using the
30 same mean-field approximation [27, 28]. The ferromagnetic order exists even
31 for $J_H = 0$ and the magnetic moment is maximum for small J_K values.
32 The Kondo order parameter λ increases and the magnetic moment decreases
33 with increasing J_K , but however it remains an interval for J_K where there
34 exists a coexistence between the ferromagnetic order and the Kondo effect.
35 This effect is relatively small with respect to the large coexistence which
36 is obtained for the underscreened Kondo lattice as we will see in the next
37 section.
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3. The underscreened Kondo lattice model

As we have seen in the previous section, there is a strong competition between the Kondo effect and the magnetic order at low temperature in cerium and ytterbium compounds [17]. On the other hand, the situation is different in uranium systems. Uranium compounds, such as UBe_{13} [29], URu_2Si_2 [30], UNi_2Al_3 and UPd_2Al_3 [31, 32] present a heavy fermion behavior characterized by large values of the electronic specific heat constant, then order antiferromagnetically, and undergo finally a transition to a superconducting state, as temperature decreases. The situation of UPt_3 is even more complex, because probably short-range magnetic correlations rather than a normal antiferromagnetic order exist [33, 34]. Finally, UTe is a dense Kondo system undergoing a ferromagnetic ordering at the large Curie temperature of $T_c = 102\text{K}$ [35]. We can conclude that there is a strong coexistence between the heavy fermion character and the magnetic order in U-compounds rather than the competition generally invoked in Ce- or Yb-compounds.

Cerium and ytterbium Kondo compounds are well described by the regular Kondo effect corresponding to the $4f^1$ and $4f^{13}$ configurations, while uranium ions in such compounds have a configuration $5f^n$, with $n = 2$ or 3 . The dual nature of the $5f$ electrons and the possible coexistence of localized and itinerant $5f$ states has been developed experimentally [36] and theoretically [37] to explain the behavior of U-compounds.

We present here a theoretical model to describe the "underscreened Kondo-lattice" treated in the mean-field approximation [38]. This model describes

a lattice of ions in the $5f^2$ configuration with a $5f$ -spin $S = 1$, in Kondo-like interaction with classical conduction electrons with $s = 1/2$ corresponding, therefore, to an underscreened situation for the f -electrons. The underscreened Kondo effect has been solved exactly for the single-impurity case [39] and studied for the case of two $S = 1$ impurities [40].

To describe the mean-field approximation for the underscreened Kondo model, the $S = 1$ localized spins are decomposed into two $1/2$ spins $\tau_{i\sigma}^{fn}$. We impose the condition that these two spins are always ferromagnetically aligned and we take only the two z-components $+1$ and -1 of each spin $S = 1$ in order to have a fermionic description of the problem with only four fermionic operators [40].

Thus, the lattice Hamiltonian is given here by:

$$\begin{aligned}
 H = & \sum_{k,\sigma} \epsilon_k n_{k\sigma}^c + \sum_{i\sigma} E_0 (n_{i\sigma}^{f1} + n_{i\sigma}^{f2}) + \\
 & + J_K \sum_{i\sigma} \vec{s}_i^c (\vec{\tau}_{i\sigma}^{f1} + \vec{\tau}_{i\sigma}^{f2}) + \\
 & + J_H \sum_{i,j} (\vec{\tau}_{i\sigma}^{f1} + \vec{\tau}_{i\sigma}^{f2}) (\vec{\tau}_{j\sigma}^{f1} + \vec{\tau}_{j\sigma}^{f2})
 \end{aligned} \tag{5}$$

where the first term describes the conduction band, the second term the two-fold degenerate f -band without width, the third term the intra-site Kondo interaction (with $J_K > 0$) and the fourth term the inter-site interaction between f -magnetic moments.

In fact, a more complete representation will be to take a representation of the $S = 1$ spins with the three components $+1$, -1 and 0 . We are presently studying the problem with this full quantum description [41].

Then here, we use the mean-field (MF) approximation and expand the preceding Hamiltonian according to the four following MF values:

$$\begin{aligned}
 M &= \langle S_i^z \rangle = \langle \tau_{iz}^{f1} + \tau_{iz}^{f2} \rangle \\
 m &= \langle s_i^{cz} \rangle \\
 \lambda_\sigma &= \sum_n \langle f_{i\sigma n}^+ c_{i\sigma} \rangle
 \end{aligned} \tag{6}$$

λ_σ is taken independent of the site i and of $n = 1, 2$, but is spin-dependent.

The resulting bands are one not hybridized f -band at energy $E_0 + \sigma J_K m / 2 + \sigma J_H M z / 4$ and two $k - f$ hybridized bands with spin-dependent energy:

$$\begin{aligned}
 E_\pm^\sigma(k) &= \frac{1}{2} (\epsilon_k + E_0 + \sigma J_K (M + m) / 2 + \sigma J_H M z / 4) \pm \\
 &\pm \frac{1}{2} \sqrt{(\epsilon_k - E_0 + \sigma J_K (M - m) / 2 - \sigma J_H M z / 4)^2 + 2 J_K^2 \lambda_{-\sigma}^2}
 \end{aligned} \tag{7}$$

where $\sigma = \pm 1$ for the two spins \uparrow and \downarrow and z is the number of neighboring atoms. The bands for \uparrow and \downarrow spins are shifted from each other. The resulting hybridization is double here than in the $S = 1/2$ f -spin case.

Then, the equations giving M , m , λ_\uparrow and λ_\downarrow are obtained by taking the number of f -electrons $N_f = \sum_\sigma (n_\sigma^{f1} + n_\sigma^{f2}) = 2$ and a fixed number of conduction electrons $n = \sum_\sigma \langle n_\sigma^c \rangle$. The magnetic moment M arises from f -electrons and m from conduction electrons. E_0 is also expressed as a function of the four variables. The solutions for M , m , λ_\uparrow and λ_\downarrow are obtained for the Kondo (which corresponds to $J_K > 0$) and ferromagnetic (which corresponds to $J_H < 0$) case at $T = 0$ by solving the self-consistent equations.

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8 In conclusion, we emphasize the two following points : first, we obtain
9 solutions with both a magnetization ($M \neq 0$, $m \neq 0$) coming mostly from
10 the not hybridized band and a Kondo character (λ_{\uparrow} and $\lambda_{\downarrow} \neq 0$); this result
11 can account for the coexistence between magnetic order and heavy fermion
12 character observed in uranium compounds such as UTe. Second, we study
13 the influence of the different parameters, J_H which tends to give the ferro-
14 magnetic order and J_K which favours the Kondo effect. A full description of
15 the results will be published elsewhere [42].
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25 **4. The Kondo lattice - spin glass competition**

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28 Above the QCP, there exists a very strong heavy fermion character, but
29 several possible behaviours, i.e. the classical Fermi liquid one with eventually
30 a reduced Kondo temperature [13, 17] or different Non-Fermi-Liquid (NFL)
31 ones, have been observed in cerium or ytterbium compounds [43, 44, 45].
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35 On the other hand, the disorder can yield a Spin Glass (SG) phase in
36 addition to the Kondo (mainly NFL) behaviour at low temperatures around
37 the QCP in disordered cerium or uranium alloys. This is the case of the
38 magnetic phase diagram of $CeNi_{1-x}Cu_x$ alloys that has been extensively
39 studied [46, 47], while the phase diagram of $Ce_2Au_{1-x}Co_xSi_3$ alloys presents
40 the sequence of SG-AF-Kondo phases at low temperatures with increasing
41 Cobalt concentration [48]. The three phases, AF, SG and NFL have been
42 obtained at low temperatures for different concentrations in $UCu_{5-x}Pd_x$ [49]
43 or in $U_{1-x}La_xPd_2Al_3$ [50] disordered alloys. Thus, a SG-Kondo transition
44 has been observed experimentally with increasing concentration in disordered
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8 alloys around the QCP. Recent experiments performed in $CeNi_{1-x}Cu_x$ alloys
9 have shown that the phase diagram of these alloys is more complicate [51, 52].
10 In fact, when temperature decreases, for alloys with $0.7 > x > 0.1$, there
11 appears a "cluster Spin Glass" phase presenting magnetic inhomogeneities,
12 rather than a classical Spin Glass phase. At lower temperatures, there is a
13 percolative process which increases the size of the clusters, to finally yield
14 a ferromagnetic phase at very low temperatures. On the other hand, the
15 transition to the Kondo phase when x decreases below roughly 0.15 is very
16 smooth without any QCP and it seems that some magnetic clusters remain
17 at very low temperatures with x approaching zero.

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27 The transition between a SG phase and a Kondo one, with in addition
28 the presence of a magnetically ordered (ferromagnetic or antiferromagnetic),
29 has been recently studied in detail in the case of Kondo lattices. The SG-
30 Kondo transition was firstly studied theoretically [53], then in the presence
31 of ferromagnetic ordering [54] or antiferromagnetic ordering [55] and finally
32 the model has been improved in order to obtain a good description of the
33 QCP [56], by introducing a transverse magnetic field.

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40 In the first paper on spin glass-Kondo phase transitions, we take a Hamil-
41 tonian that couples the localized spins of a Kondo lattice with an additional
42 long range random interaction, like in the Sherrington- Kirkpatrick spin glass
43 model [57]. The localized spins of the Kondo lattice, with localized spins \vec{S}_i ,
44 interact with conduction electrons and the Kondo effect is studied in the
45 previously described mean field decoupling scheme [15, 17]. The spin glass
46 interaction is represented by the quantum Ising Hamiltonian where only in-
47 teract the z-components of the localized spins [58, 59, 60].
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The Hamiltonian of the model is:

$$\mathcal{H} = \sum_{k,\sigma} \epsilon_k n_{k\sigma} + \epsilon_0 \sum_{i,\sigma} n_{i\sigma}^f + J_K \sum_i [S_{fi}^+ s_{ci}^- + S_{fi}^- s_{ci}^+] - \sum_{i,j} J_{ij} S_{fi}^z S_{fj}^z \quad (8)$$

where, as usual, $J_K > 0$.

The localized and conduction electron spins are then written in terms of fermion operators $f_{i\sigma}$ or $c_{i\sigma}$. The energies $\epsilon_0(\epsilon_k)$ are referred to the chemical potentials $\mu_f(\mu_c)$, respectively.

To describe the nature of the spin glass, the coupling J_{ij} is taken here as an independent random variable with the gaussian distribution :

$$\mathcal{P}(J_{ij}) = \frac{1}{J} \sqrt{\frac{N}{64\pi}} \exp \left\{ -\frac{(J_{ij} + 2J_0/N)^2}{64J^2} N \right\}. \quad (9)$$

The preceding expression represents the random distribution of the exchange integrals J_{ij} . The average value J_0 has been taken equal to zero to describe the SG-Kondo transition [53], while the case $J_0 < 0$ produces a complex phase diagram with spin glass, ferromagnetic, mixed (a spin glass with spontaneous magnetization) and Kondo phases [54]. Finally, the case $J_0 > 0$ corresponds to an antiferromagnetic phase in competition with Spin Glass and Kondo phases [55].

The Kondo and spin glass transitions are both described within the mean field approximation. Using functional integrals techniques and a static, replica symmetric Ansatz for the Kondo and spin glass order parameters, a mean field expression for the free energy and the saddle point equations for the order parameters have been derived. Numerical solutions of the saddle-

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8 point equations allow us to determine the Kondo order parameter λ and the
9 spin glass (SG) one q .

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11 In the first studied case [53] with $J_0 = 0$, we obtain three phases : At high
12 temperatures, the "normal" phase is paramagnetic with $\lambda = q = 0$. When
13 temperature is lowered, for not too large values of the ratio J_K/J , a SG phase,
14 with $q > 0$ and $\lambda = 0$, is obtained and, for large values of the ratio J_K/J , we
15 recover the "Kondo" phase with a non-zero λ value and $q = 0$. The transition
16 line from the SG phase to the Kondo phase is a first-order one and ends at
17 J_K^c at $T = 0$. When the temperature is lowered, the transition temperature
18 does not vary very much with the value of J_K/J ; the separation between
19 the spin-glass and the Kondo phases departs completely from the behaviour
20 of the single-impurity Kondo temperature T_K and looks like the separation
21 between the antiferromagnetic and Kondo phases when these two phases are
22 considered [26]. We can also remark that we get here only "pure" Kondo or
23 SG phases and never a mixed SG-Kondo phase with the two order parameters
24 different from zero; this result is probably connected to the approximations
25 used here to treat the starting Hamiltonian.

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The diagram obtained theoretically for the SG-Kondo transition can give
a rough account for the magnetic phase diagram observed above the Curie
temperature for the $CeNi_{1-x}Cu_x$ [46] for small x values when there is a
transition from a SG state to a Kondo state and then to the intermediate
valence compound CeNi. However, the theoretical model gives an average
description, while recent experiments have shown the existence of magnetic
clusters and our model cannot be checked from that point of view [51, 52].
There is also probably a SG-Kondo transition in $CeCoGe_{3-x}Si_x$ alloys, but

there the experimental situation is also very complicate [61].

Indeed the experimental situation is generally more complex in disordered cerium or uranium alloys and there generally exists an antiferromagnetically (AF) ordered phase in addition to the SG phase and to the Kondo one, which moreover has a Non Fermi Liquid (NFL) close to the QCP. For example, the three phases, AF, SG, and NFL have been obtained at low temperatures for different concentrations in $UCu_{5-x}Pd_x$ [49] or in $U_{1-x}La_xPd_2Al_3$ [50] disordered alloys. The sequence AF-NFL-SG occurs with increasing x in $UCu_{5-x}Pd_x$ [49] and the opposite sequence AF-SG-NFL with increasing x in $U_{1-x}La_xPd_2Al_3$ [50].

The two theoretical cases SG-Ferromagnetic-Kondo [54] and SG-AF-Kondo [55] have been studied within the same type of mean-field approximation, but here there are at least three order parameters, the spin glass order parameter q , the magnetization M (or the magnetization of each sub-lattice in the AF case) and finally the Kondo parameter λ which is generally taken as spin independent. The calculation is even more difficult in the AF case, because we are obliged to consider two sub-lattices with opposite magnetizations. We present here the results obtained for the SG-AF-Kondo phase sequence with a positive value of J_0 . There are three parameters J_K , J_0 and J , but actually, J_K cannot be considered as independent of J_0 and J , since an intrasite exchange term would yield both a Kondo effect and a RKKY interaction. This problem has been already discussed and we have taken the two possible choices : either we take the values of the three parameters independent from each other; or we adopt some conjecture relating J_K , J_0 and J . In our theoretical approach, the variance J is kept constant and we take the following

relationship between J_0 and J_K :

$$\frac{J_0}{J} = \alpha \left(\frac{J_K}{J} \right)^2. \quad (10)$$

The quadratic choice in Eq. (10) is taken here in order to have a rough simulation of the fact that the RKKY interaction behaves as J_K^2 , as already assumed in Ref.[16]. A phase diagram obtained for a typical value $\alpha = 0.0051$ is shown in Figure 1.

The numerical solutions of the order parameters yield the sequence of phases SG-AF-Kondo shown in Figure 1. The Kondo temperature T_k is decreasing with decreasing values of J_K/J down to $J_K/J \approx 15$. From this point onwards, there is a phase transition leading to an AF order with the Néel temperature T_N monotonically decreasing with J_K/J . For still smaller values of J_K/J (and therefore smaller values of J_0/J), the spin glass behaviour becomes dominant.

Finally, Figure 1 can roughly account for the phase diagram of $Ce_2Au_{1-x}Co_xSi_3$ alloys which presents the sequence of SG-AF-Kondo phases at low temperatures with increasing Cobalt concentration [48]. However, in contrast to our theoretical model, the experimental AF phase ends at a critical point before reaching the Kondo phase when the Cobalt concentration x is still increasing.

Thus, in order to improve the theoretical description of the SG-Kondo or SG-AF-Kondo transitions and to obtain a good description of the QCP, a new model has been developed by adding a transverse field to the previous Hamiltonian [56]. In fact, in previous publications, the resultant RKKY interaction was introduced by means of random, infinite range couplings among the S^z components of the localized spins as in the Sherrington-Kirkpatrick

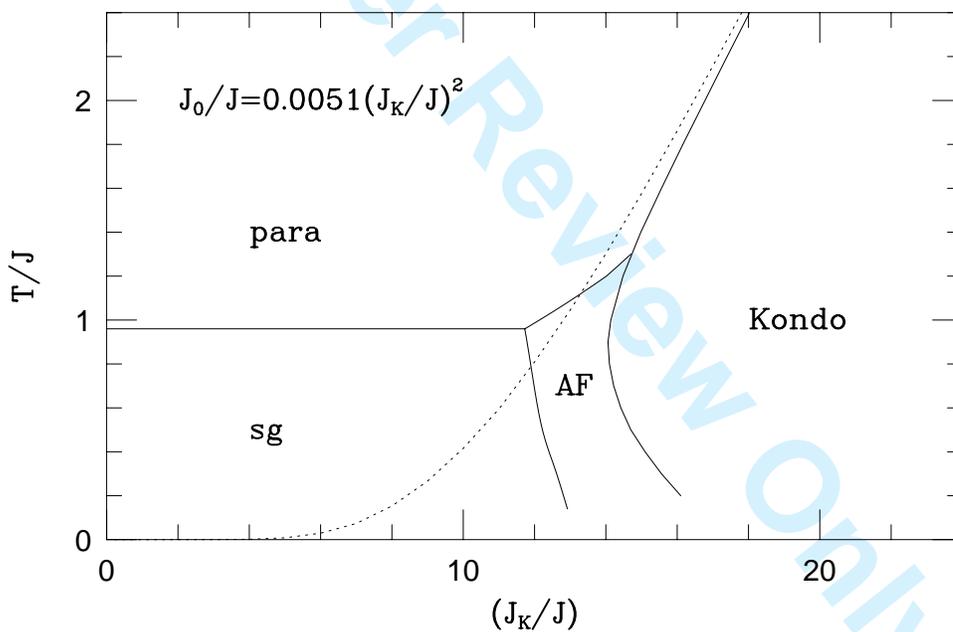


Figure 1: The phase diagram T/J versus J_K/J showing the sequence of sg (Spin Glass), AF (Antiferromagnetism) and Kondo phases. The variance J is kept constant. The dotted line means the "pure" (i.e. single-impurity) Kondo temperature.

model [57] and by neglecting the spin flip coupling of the transverse components. The previous models failed to describe a second order QCP at $T = 0$, because, by disregarding the spin flipping part of the Heisenberg Hamiltonian, we suppressed the tunneling mechanism and magnetic ordering occurs only along the z-axis. In order to introduce a spin flipping mechanism and to avoid the intricacies of the random Heisenberg model, the Heisenberg-like coupling among the three spin components induced by the RKKY interaction is mimicked by a quantum Ising spin glass in a transverse field. Thus, the Hamiltonian of the model is taken here as the sum of the Hamiltonian given by the eq.(8) and of an additional term given by :

$$\mathcal{H}_x = -2\Gamma \sum_i S_{fi}^x \quad (11)$$

The coupling J_{ij} in eq.(8) is described by a gaussian distribution of zero mean J_0 as previously used for the SG-Kondo transition [53].

Numerical solutions of the saddle-point equations allow us to draw in figure 2 the magnetic phase diagram in the J_K versus T plane, using the relation $\Gamma = \alpha J_K^2$ between the transverse field and J_K , fixed values of J and D , with $D/J = 12$ and a value of the parameter αJ : $\alpha J = 0.01348$ [56]. Figure 2 shows three different phases : At high temperatures, the "normal" phase is paramagnetic with vanishing Kondo and spin glass order parameters, i.e. $\lambda = q = 0$. When temperature is lowered, for not too large values of the ratio J_K/J , a second-order transition line is found at $T = T_1$ to a spin glass phase with $q > 0$ and $\lambda = 0$. The critical line $T_1(J_K)$ ends at a quantum critical point J_{K1}^c . Then, for larger values of the ratio J_K/J , there is a second order transition line starting at J_{K2}^c to a Kondo state with $\lambda \geq 0$.

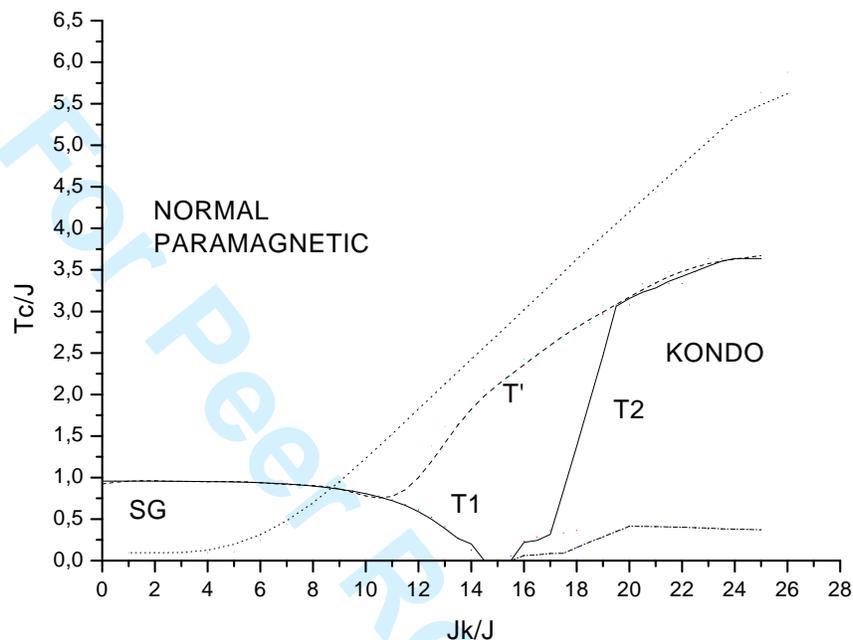


Figure 2: Phase diagram in the $T - J_K$ plane as a function of T/J and J_K/J for the transverse field $\Gamma = \alpha J_K^2$, for fixed values of J and D , with $D/J = 12$, and for two values $\alpha J = 0.01348$ (solid lines) and $\alpha J = 0.01344$ (dashed line). The critical (solid) second order line T_1 for low values of J_K separates the paramagnetic phase ($q = \lambda = 0$) for high temperatures from the spin-glass phase ($q \geq 0, \lambda = 0$) at low temperatures and ends at a QCP. The critical second order solid line T_2 for large values of J_K separates the paramagnetic phase from the Kondo phase ($\lambda \geq 0, q = 0$) and ends at a second QCP. The dotted line represents the “pure” (i.e. single impurity) Kondo temperature T_K and the dashed-dotted line represents the residual magnetic moment $\bar{\chi}_c(J_K)$ on the line $T = T_2$. The critical line (dashed) labelled by T' corresponds to $\alpha J = 0.01344$ and has no QCP.

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8 These results are very sensitive to the value of D/J and αJ . For $D/J = 12$
9 and for example $\alpha J = 0.01344$, the two QCP disappear, as shown in figure
10 2, where the new transition is given by the dashed curve labelled T'. Thus,
11 the occurrence of two QCP is extremely sensitive to the precise of value of
12 αJ . However, the phase diagram shown in figure 2 with two very close QCP
13 improves considerably the theoretical description of the SG-Kondo transition
14 [56]. We can also remark that we get here only "pure" Kondo or SG phases
15 and never a mixed SG-Kondo phase with the two order parameters different
16 from zero, as already noticed in our mean field calculations [26].
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19 A QCP has been observed experimentally in several cerium and uranium
20 disordered alloys [48, 49, 50] and the phase diagram shown in Figure 2 im-
21 proves considerably the description of the spin glass-Kondo transition with
22 respect to Ref. [53]. However, the experimental situation appears to be re-
23 ally complicate, as shown for example by recent experiments on $CeNi_{1-x}Cu_x$
24 alloys [51, 52]. Thus, further experimental work is certainly necessary, but
25 however the models presented here can account roughly for the behavior of
26 heavy fermion disordered alloys, especially in the vicinity of the quantum
27 critical point.
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44 5. Conclusion

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46 We have presented here a brief review on recent works on the Kondo lattice
47 problem. We have always used here the same mean field approximation for
48 the Kondo effect, which is simple and gives correct results, but yields however
49 a too brutal (typically mean-field-like) transition rather than the expected
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8 continuous one around the Kondo temperature. Here, we have firstly consid-
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10 ered the case of cerium (ytterbium) compounds corresponding to a configu-
11 ration close to $4f^1$ (or $4f^{13}$) with one 4f electron (or one hole) and then the
12 case of uranium described here by a configuration close to $5f^2$. The "nor-
13 mal" Kondo effect describes the first case, while the "underscreened" Kondo
14 effect is more appropriate for the second one. Finally, we have discussed the
15 interaction between the Kondo effect and a disordered phase that we describe
16 as a spin glass phase, but in fact the spin glass-Kondo problem is more com-
17 plicate, as shown for example by recent experiments on $CeNi_{1-x}Cu_x$ alloys
18 which have yielded a new "cluster spin glass". Finally, further experimen-
19 tal results and a more sophisticated theoretical approach, eventually beyond
20 our mean-field approximation, are necessary to improve the description of
21 anomalous rare-earth and actinide systems.
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