

High resolution infrared study of the 2 nu9 and nu4 bands of 10BF2OH and 11BF2OH:
Evidence of large amplitude effects for the OH - torsion bending modes in the $\mathbf{9 \wedge}^{\wedge} \mathbf{2}$ and $\mathbf{4 \wedge}^{\wedge} 1$ and excited states.

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| Complete List of Authors: | Perrin, Agnes; CNRS, Universite Paris 12 BERTSEVA, Elena; LISA <br> Flaud, J. M.; LISA, Uni. Paris12 <br> Collet, david; FBC Universitat <br> Buerger, Hans; FBC Universitat <br> masiello, tony; PNNL <br> Blake, Thomas; PNNL |
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High resolution infrared study of the $2 v_{9}$ and $v_{4}$ bands of ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ and ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$ : Evidence of large amplitude effects for the OH - torsion -bending modes in the $9^{2}$ and $4^{1}$ and excited states.

A.PERRIN ${ }^{\S}(1)$, E.BERTSEVA ${ }^{\mathbb{I}}(1)$, J.-M.FLAUD (1) D.COLLET(2), H. BÜRGER(2) T. MASIELLO(3), and T. A. BLAKE(3),

(1) Laboratoire Interuniversitaire des Systèmes Atmosphériques (LISA), CNRS/Universités Paris 12 et Paris 7, 61 av. du Général de Gaulle, 94010 Créteil Cedex, France
(2) Anorganische Chemie, FBC, Universität, D-42097 Wuppertal, Germany
(3) Pacific Northwest National Laboratory (PNNL), P. O. Box 999, Mail Stop K8-88, Richland, WA 99352, USA

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${ }^{8}$ Corresponding author: A.Perrin, Laboratoire Interuniversitaire des Systèmes Atmosphériques, (LISA), CNRS/Universités Paris 12 et Paris 7, 61 av. du Général de Gaulle, 94010 Créteil Cedex, France, Tel. 33(0)14517 6557, Fax. 33(0)145171564, Email: perrin@lisa.univ-paris12.fr ${ }^{\text {I }}$ Present address : E.Bertseva, Institut de Physique de la Matière Complexe, Ecole Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland


#### Abstract

High resolution $\left(2-3 \times 10^{-3} \mathrm{~cm}^{-1}\right)$ Fourier transform infrared spectra of gas phase ${ }^{10} \mathrm{~B}$ and ${ }^{11} \mathrm{~B}$ enriched and natural samples of $\mathrm{BF}_{2} \mathrm{OH}$ (difluoroboric acid) were recorded at Wuppertal and Richland. Starting from the results of previous studies [A.Perrin, M.Carvajal-Zaera, Z.Dutkiewicz, J.-M.Flaud, D.Collet, H.Bürger, J.Demaison, F.Willaert, H.Mäder, and N.W.Larsen, Mol. Phys. 102 , 1641 (2004); J. Breidung, J. Demaison, J.-F. D'Eu, L. Margulès, D. Collet, E.B. Mkadmi, A. Perrin and W. Thiel, J. Mol. Spectrosc. 228, 7, (2004)], it was possible to perform the first rovibrational analysis of the $2 \mathrm{v}_{9}$ (first overtone of $\mathrm{v}_{9}$, the OH torsion) and $\mathrm{v}_{4}$ (BOH bending) bands located at about 1043.9 and $961.7 \mathrm{~cm}^{-1}$ and 1042.9 and $961.5 \mathrm{~cm}^{-1}$ for the ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ and ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$ isotopic species respectively. Numerous "classical" perturbations were observed in the analysis of the $2 v_{9}$ and $v_{4}$ bands. The energy levels of the $9^{2}$ bright state are indeed involved in a B- type Coriolis resonance with those of the $6^{1} 9^{1}$ dark state. The $4^{1}$ levels are perturbed by a B-type Coriolis resonance and by an anharmonic resonance with the levels of the $7^{1} 9^{1}$ and the $6^{1} 7^{1}$ dark states respectively. In addition large amplitude effects were observed for the $2 \mathrm{v}_{9}$ and also, more surprisingly, the $v_{4}$ bands. This results in splittings of the energy levels of about 0.005 and $0.0035 \mathrm{~cm}^{-1}$ for the $9^{2}$ and $4^{1}$ states respectively which are easily observable in the P and R branches for both bands. The theoretical model used to reproduce the experimental levels accounts for the classical vibration -rotation resonances. Also the large amplitude torsional (or bending) effects are accounted for within the frame of the IAM (Internal Axis Method) -like approach. The Coriolis resonances between the two torsional (or bending) substates are taken into account by $\left\{J_{x}, J_{z}\right\}$ non orthorhombic terms in the $v$-diagonal blocks. This means that the $z-$ quantification axis deviates from the $a$ inertial axis by an axis switching effect of $\sim 35^{\circ}$ for the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ system and of $\sim 16.6^{\circ}$ for the $\left.\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}\right)$ system of interacting vibrational states. The calculation of the relative line intensities for the $2 v_{9}$ and $v_{4}$ bands accounts for these axis switching effects as well as for the intensity alternation which is due to the nuclear spin statistical weights since the OH large amplitude torsion and/or bending motion results indeed in an exchange of the two fluorine nuclei.


Keywords: large amplitude torsion, large amplitude bending mode, splitting, infrared, staggering

## 1. Introduction

$\mathrm{BF}_{2} \mathrm{OH}$, difluoroboric acid, is a reactive intermediate in the hydrolysis of $\mathrm{BF}_{3}$. It is a planar asymmetric rotor which is isovalent with $\mathrm{HONO}_{2}$ (nitric acid). The microwave spectrum was first measured by Takeo and Curl [1] and later on by Vormann and Dreizler [2]. These investigations were extended recently since at Lille new millimeterwave spectra of the ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ and ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$ isotopic species ${ }^{1}$ in their ground vibrational state were measured [3]. As a result, accurate ground state rotational and centrifugal distortion constants were determined for these isotopic species. Also, new and accurate $a b$ initio structure and anharmonic force fields have been calculated leading to the determination of an accurate equilibrium structure for $\mathrm{BF}_{2} \mathrm{OH}$ [3]. Figure 1 gives a picture of the $\mathrm{BF}_{2} \mathrm{OH}$ molecule together with some of the conventions which are used in this study.
As far as infrared data are concerned, Jacox et al. [4] observed the matrix infrared (IR) spectra of eight isotopologues. The assignments were supported by ab initio calculations of the structure and the harmonic force field. The first high resolution IR spectrum of $\mathrm{BF}_{2} \mathrm{OH}$ in the gas phase was observed by Collet et al. [5] using a Fourier transform (FT) spectrometer at Wuppertal. In this way the first analyses of the $v_{8}$ and $v_{9}$ fundamental bands of 11 B located at 684.160 and $522.870 \mathrm{~cm}^{-1}$, respectively were performed. Later on, the $v_{5}, v_{8}, v_{9}$ and $v_{8}+v_{9}$ bands of $10 B$, and $v_{7}, v_{5}$, and $v_{8}+v_{9}$ bands of 11 B were analysed for the first time up to very high rotational quantum numbers [6]. It was observed that the $v_{5}, v_{8}, v_{9}$ and $v_{8}+v_{9}$ bands of $10 B$ and the $v_{8}+v_{9}$ band of 11B are not significantly affected by perturbations. For the $v_{5}$ and $v_{7}$ bands of 11B C-type Coriolis interactions coupling the $5^{1}$ and $7^{1}$ energy levels with those of

[^0]the $7^{2}$ and $6^{1}$ dark states respectively had to be accounted for in the calculations. Table 1 lists the infrared bands of $\mathrm{BF}_{2} \mathrm{OH}$.

The present study presents the first analysis of the $2 v_{9}$ and $v_{4}$ bands both for 10B and 11B. These bands which correspond respectively to the first overtone of the OH torsional mode ( $v_{9}$ ) and the OH bending mode ( $v_{4}$ ) are analysed together in the present study because the levels of the $9^{2}$ and $4^{1}$ upper states are both split due to large amplitude motions of the OH bond. The line splittings, of about 0.005 and $0.0035 \mathrm{~cm}^{-1}$ in $2 v_{9}$ and $v_{4}$ respectively, are easily observed in the whole range of the IR spectra. It should be mentioned that on the contrary no such splittings were detected during the infrared analyses of the $v_{5}, v_{7}, v_{8}, v_{9}$ and $v_{8}+v_{9}$ bands [6].

It is interesting, for theoretical reasons, to compare difluoroboric acid, $\left(\underset{\mathrm{F}}{\mathrm{F}}>\mathrm{B}-\mathrm{O}{ }^{\mathrm{H}}\right)$
to one of its isoelectronic molecules, nitric acid $\left(\underset{\mathrm{O}}{\mathrm{O}}>\mathrm{N}-\mathrm{O}\right.$ ). For $\mathrm{HNO}_{3}$, the $v_{9}$ mode (torsion of the OH bond relative to the $\mathrm{NO}_{2}$ moiety) is a large amplitude motion. This induces a splitting of the energy levels which increases with increasing excitation of the $v_{9}$ vibrational mode [7]. The splitting of the energy levels is indeed about $0.002 \mathrm{~cm}^{-1}$ for the $9^{2}$ levels and about $0.06 \mathrm{~cm}^{-1}$ for the $9^{3}$ energy levels. For several other vibrational states of nitric acid involving the $v_{9}$ mode, large amplitude splittings were also evidenced by millimeter or submillimeter wave measurements (see [8] and references herein). Therefore, the occurence of large amplitude torsional splittings observed in the analysis of the $2 v_{9}$ band of $\mathrm{BF}_{2} \mathrm{OH}$ was not completely a surprise.

On the other hand the $v_{4}\left(\mathrm{OH}\right.$ bending) band of $\mathrm{HNO}_{3}$ does not reveal large amplitude splittings [9] contrary to what is observed for $\mathrm{BF}_{2} \mathrm{OH}$. It is interesting to investigate in detail this large amplitude splitting in the $v_{4}$ band of $\mathrm{BF}_{2} \mathrm{OH}$, since evidence for an interconversion pathway from the (lower energy, linear) HBO to the (higher energy, bend) BOH radicals was predicted by recent ab initio calculations [10, 11]. This may suggest that the $\mathrm{H} \hat{B} O$ bending mode may also be a large amplitude motion for $\mathrm{BF}_{2} \mathrm{OH}$.

## 2. Experimental details

## 2-A: Wuppertal (Germany)

Details of the synthesis of 10B (92.4 \%) and 11B (99 \%) and the recording of the high resolution FTIR spectra performed in Wuppertal were already given in Table 2 of Ref. [6]. Only the spectra denoted 1044 (10B) and 1043c (11B) were employed. In brief, $\mathrm{BF}_{2} \mathrm{OH}$ was synthesized free of contaminating $\mathrm{SiF}_{4}$ from $\mathrm{BF}_{3}$ and $\mathrm{H}_{2} \mathrm{O}$ in a glass-free reactor and the IR spectrum investigated in a stainless steel absorption tube measuring 1.5 m in length and outfitted with NaCl windows. The chosen total pressure was 250 Pa for 10B and 11B. A Bruker IFS 120HR interferometer was used, The resolution ( $1 /$ maximum optical path difference) was adjusted to $2.4 \times 10^{-3}$ $\mathrm{cm}^{-1}$.

## 2-B PNNL (Richland, USA)

At the Pacific Northwest National Laboratory (PNNL), the 10B and 11B spectra were recorded using a Bruker IFS 120 HR Fourier transform spectrometer. While recording rotationally-resolved combination, overtone and hot band spectra of isotopically enriched ${ }^{10} \mathrm{BF}_{3}$ and ${ }^{11} \mathrm{BF}_{3}$ samples (separately) in an adjustable pathlength White cell (Bruker A134), bands of 10 B and 11B were observed. It was assumed that the 10 B and 11 B were produced by the reaction of ${ }^{10} \mathrm{BF}_{3}$ and ${ }^{11} \mathrm{BF}_{3}$ with residual water on the surface of the White cell.

Optical, electronic and digital filters were used to limit the spectral window. A KBr beamsplitter, Globar source, optical filter with 900 to $1300 \mathrm{~cm}^{-1}$ bandpass, and MCT detector were employed for these experiments. Spectra were recorded with an aperture diameter of $1.5 \mathrm{~mm}, 40 \mathrm{kHz}$ scanner velocity and resolution of $0.0018 \mathrm{~cm}^{-1}$. Bruker defines the maximum optical path difference (MOPD) as 0.9/resolution, so in this case MOPD $=0.9 / 0.0018 \mathrm{~cm}^{-1}=500 \mathrm{~cm}$. For each isotopologue sample, 384 single-sided interferograms were coadded. For a background, a lower resolution ( $0.0288 \mathrm{~cm}^{-1}$ ) spectrum was recorded of the evacuated White cell; here 512 singlesided interferograms were coadded.

For the FT conversion, a Mertz phase correction of $1 \mathrm{~cm}^{-1}$ and boxcar apodization were applied to the averaged interferograms. For boxcar apodization, Bruker gives the instrument linewidth as $0.61 /$ OPD $=0.0012 \mathrm{~cm}^{-1}$ FWHM. The
averaged interferograms were post zerofilled to a factor of eight before calculating the absorbance spectra.

The enriched ${ }^{10} \mathrm{BF}_{3}$ and ${ }^{11} \mathrm{BF}_{3}$ samples were purchased from Voltaix, Inc., each with a 99.5 atom \% purity and a $99.9 \%$ chemical purity. The White cell pathlength for both isotopic samples was 6.4 meters. The cell windows were CsI. The pressure for the ${ }^{10} \mathrm{BF}_{3}$ sample was 261 Pa and 235 Pa for the ${ }^{11} \mathrm{BF}_{3}$ sample. Spectra were recorded at room temperature, $22.5^{\circ} \mathrm{C}$. The spectrometer was evacuated to a background pressure of less than 4 Pa . The spectra were wavenumber calibrated by first recording a spectrum of OCS using the same instrument settings as described above and then using the designated OCS calibration lines in the NIST wavenumber calibration tables [12] as a reference. The average uncertainty of the OCS lines used for calibration was about $0.000013 \mathrm{~cm}^{-1}$, and the estimated accuracy of the calibrated wavenumber is of $\sim 0.00015 \mathrm{~cm}^{-1}$.

## 3. Analyses of the $2 v_{9}$ and $v_{4}$ bands

The $2 v_{9}$ and $v_{4}$ bands of the 10B and 11B isotopic species of $\mathrm{BF}_{2} \mathrm{OH}$ are in principle hybrid bands with both A- and B-type transitions. For the $2 v_{9}$ band only Atype transitions were observed. On the other hand, for the $v_{4}$ band the A-type character is more pronounced than the B - type one. For A - type bands, the P and R branches are structured in stacks of lines separated by $\left(B_{z}+B_{x}\right) / 2$. These stacks consist of $\left[\mathrm{J}^{\prime}, \mathrm{d}^{\prime}, \mathrm{K}_{\mathrm{c}}^{\prime}\right] \leftarrow\left[\mathrm{J}^{\prime \prime}, \mathrm{d}^{\prime \prime}, \mathrm{K}^{\prime \prime}{ }_{\mathrm{c}}\right]$ transitions (d denotes the degenerate $\mathrm{K}_{\mathrm{a}}$ values with $K_{a}=J-K_{c}$ or $J-K_{c}+1$ ), for given values of ( $2 J-K_{c}$ ) with rather high $K_{c}$ values $\left(K_{a} \ll K_{c} \sim J\right)$. We observed that for $2 v_{9}$ and $v_{4}$ bands each $P$ or R transition was furthermore split into two components separated by about 0.005 and $0.0035 \mathrm{~cm}^{-1}$. Examples of such splittings are illustrated in Figures 2 and 3 which show portions of the $R$ branch of the $2 v_{9}$ band of $11 B$ and of the $P$ branch of the $v_{4}$ band of $10 B$ respectively. These splittings which decrease slightly with increasing values of $\left(\mathrm{J}-\mathrm{K}_{\mathrm{c}}\right)$ tend to vanish for high values of $K_{a}\left(K_{a} \sim J\right)$.
On the other hand the $Q$ branches are congested and therefore difficult to assign since they group together transitions with high $\mathrm{K}_{\mathrm{a}}$ values $\left(\mathrm{K}_{\mathrm{c}}<\mathrm{K}_{\mathrm{a}} \sim \mathrm{J}\right)$. Figures 4 and 5 display various portions of the $2 v_{9} Q$ branches for $10 B$ and 11B, respectively. The torsional (or bending) splittings which are almost negligible for high $K_{a}$ values are usually more difficult to identify in the Q branches. Nevertheless, Figures 4 and 5
show examples of these splittings in the low frequency range of the $2 v_{9} Q$ branches for 10B and 11B respectively. This complexity is even more dramatic for the $v_{4}$ bands since both an A-type and a B-type component exist in the low and high frequency ranges of the Q branch, respectively (see Figure 6 for example).

Finally, it is worth noticing that the analyses were difficult since both the $2 v_{9}$ and $v_{4}$ bands are perturbed by "classical" vibration - rotation resonances.
In a first step, the assignments were found by following the regular structure of doublets in the $P$ and $R$ branches and by checking the assignments using combination differences calculated employing the ground state parameters of Ref. [3] for both 10B and 11B. In order to establish the assignments, it was necessary to use the predictions of a theoretical model (see next section) which accounts for the observed "classical" vibration rotation resonances together with the perturbations linked to the large amplitude motions. In this way rather complete assignments could be performed for the $2 v_{9}$ and $v_{4}$ bands for both the ${ }^{10} \mathrm{~B}$ and ${ }^{11} \mathrm{~B}$ isotopic species of $\mathrm{BF}_{2} \mathrm{OH}$. Tables 2 and 3 report on the statistics of the assignments.

## 4. Hamiltonian model

## 4-A Symmetry properties and large amplitude motions

Before describing the Hamiltonian model which is used to calculate the energy levels of $\mathrm{BF}_{2} \mathrm{OH}$ it is necessary to go into some details about the symmetry properties of this molecule. In its equilibrium configuration $\mathrm{BF}_{2} \mathrm{OH}$ is a planar molecule. The corresponding point group $\mathrm{C}_{\mathrm{s}}=\left\{\mathrm{E}, \sigma_{x z}\right\}$ (where xz is the plane of the molecule) is isomorphic to the inversion group $G^{*}=\left\{E, E^{*}\right\}$, where $E^{*}$ is the laboratory-fixed inversion.

For $\mathrm{BF}_{2} \mathrm{OH}$, both $v_{9}$ (torsion of the OH bond relative to the $\mathrm{BF}_{2}$ moiety) and $v_{4}(\mathrm{OH}$ in plane bending) correspond to large amplitude motions which lead to the exchange of the fluorine nuclei:

$$
\begin{align*}
& F_{1}>B-O  \tag{1}\\
& F_{2}
\end{aligned} \quad \Leftrightarrow \begin{aligned}
& F_{2}>B-O_{H} \\
& F_{1}
\end{align*}
$$

It can then be assumed that during this large amplitude OH torsion and/or OH bending motion, the "average" conformation of the $\underset{\mathrm{F}}{\mathrm{F}}>\mathrm{B}-\mathrm{O}$ frame is of $\mathrm{C}_{2 v}$ symmetry. This $\mathrm{C}_{2 v}$ point group is isomorphic to the permutation-inversion group $G_{4}=\left\{E, E^{*},(12),(12)^{*}\right\} \quad$ where (12) is the permutation of the two fluorine nuclei $F_{1}$ and $F_{2}$ and $E^{*}$ is the inversion. Table 4 gives the character tables for $C_{2 v}$ and $G_{4}$ together with the symmetry species of the components of the angular momentum operator $\mathbf{J}$ and of the electric dipole moment $\mu$.

Let us now describe in detail the electronic, vibrational, OH - torsion, OH -bend, rotational, and ${ }^{19} \mathrm{~F}$ nuclear -spin contributions to the symmetry properties of the total wavefunctions. The ground electronic state is totally symmetric for $\mathrm{BF}_{2} \mathrm{OH}$, and we are dealing in this study with vibrational states involving only vibrational excitation in $v_{9}$ and/or $v_{4}$. Because of the existence of large amplitude motions, each OHtorsional $9^{v} / \mathrm{OH}$ - bending $4^{\vee}$ vibrational state is split into two sublevels which are referenced here as ${ }^{\text {Low }} 9^{\vee}$ and ${ }^{H i g h} g^{v} / L^{L o w} 4^{\vee}$ and ${ }^{H i g h} 4^{\vee}$ for the higher and lower energy subcomponent, respectively. Table 4 also gives the symmetry properties for the "High" and "Low" subcomponents of each torsional $9^{v}$ or bending $4^{v}$ vibrational state: these results are deduced from those described in Ref. [7]. The parity for the v vibrational quantum number is relevant only for the symmetry properties of $9^{v}$ and not for $4^{v}$ because $v_{9}$ is of $A^{\prime \prime}$ symmetry while $v_{4}$ is of $A^{\prime}$ symmetry. This is because $v_{9}$ corresponds to the "out -of -plane" OH torsion while $v_{4}(\mathrm{OH}$ bending) is an "in -plane" mode. In Table 4 the symmetry properties of the rotational wavefunctions [ $\mathrm{J}, \mathrm{K}_{\mathrm{a}}, \mathrm{K}_{\mathrm{c}}$ ] which depend on the $K_{a}$ and $K_{c}$ parity are quoted. Finally, the permutation (12) of the $F_{1}$ and $F_{2}$ fluorine nuclei leads to the permutation of the fluorine nuclear spins. For ${ }^{19} \mathrm{~F}$ the nuclear spin is $\mathrm{I}=1 / 2$ and Table 4 gives the symmetry properties of the ${ }^{19} \mathrm{~F}$ fluorine nuclear spin symmetrical (resp. antisymmetrical) wavefunctions for which the nuclear spin statistics give a weight of $g_{n}=3$ (resp. $g_{n}=1$ ).

Since we are dealing with $\mathrm{B}^{19} \mathrm{~F}_{2} \mathrm{OH}$ isotopic species involving the ${ }^{19} \mathrm{~F}$ Fermion nuclei (with a $1 / 2$ nuclear spin) the total wavefunctions (i.e. including the electronic, vibrational, OH - torsion, OH -bend, rotational, and ${ }^{19} \mathrm{~F}$ nuclear -spin contributions) must change sign under the permutation (12) of the two fluorine nuclei $F_{1}$ and $F_{2}$. Therefore, only the energy levels corresponding to an antisymmetrical ( $B_{1}$ or $B_{2}$ ) total wavefunction exist. As a consequence, the vibrational, OH -torsion, OH -bending, rotation (vib-tors-bend-rot) energy levels with $\Gamma\left(\varphi_{\text {vib-tors-bend-rot }}\right)=A_{1}$ or $A_{2}\left(\right.$ resp. $\Gamma\left(\varphi_{\text {vib }}\right.$
tors-bend-rot) $=\mathrm{B}_{1}$ or $\mathrm{B}_{2}$ ) exist only when associated with the antisymmetric (resp. symmetric) nuclear -spin wavefunctions with a $\mathrm{g}_{\mathrm{n}}=1$ (resp. $\mathrm{g}_{\mathrm{n}}=3$ ) nuclear spin statistical weight.

The Hamiltonian matrices used for the calculation of the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ and $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ interacting energy levels split into four sub-matrices. Table 5 gives the classification in four symmetry types ( $\mathrm{A}_{1}$ to $\mathrm{B}_{2}$ ) of the $9^{\vee}$ torsion- rotation and of the $4^{\vee}$ bending-rotation wavefunctions, according to the parity of $\mathrm{K}_{\mathrm{c}}$ and to the nuclear spin statistical weights ( $\mathrm{g}_{\mathrm{n}}=1$ or $\mathrm{g}_{\mathrm{n}}=3$ ).

As far as the analyses of the spectra are concerned, only very little information can be obtained concerning the energy levels with $g_{n}=1$. For example, in the $P$ and $R$ -branches of the $2 v_{9}$ band the assigned transitions involve $9^{2}$ energy levels with high $K_{c}$ values ( $K_{c} \sim J$ ) for which the $K_{a}$ degeneracy ( $K_{a}=d$ for $K_{a}=J-K_{c} \Leftrightarrow K_{a}=J+1-K_{c}$ ) occurs. In this case the energy levels $\left\{{ }^{\left({ }^{\text {ligh }} 9^{2} \quad\left[J, K_{a}=\text { even, } K_{c}\right], \mathrm{g}_{n}=3\right) \text { and }}\right.$ $\left.\left.{ }^{\text {High }} 9^{2}\left[J, K_{a}=o d d, K_{d}\right], g_{n}=1\right)\right\} \quad$ (resp. $\quad\left\{\left({ }^{L o w} g^{2} \quad\left[J, K_{a}=\right.\right.\right.$ even, $\left.\left.K_{d}\right], g_{n}=3\right) \quad$ and $\quad{ }^{\text {Low }} 9^{2}$ $\left.\left[J, \mathrm{~K}_{\mathrm{a}}=\mathrm{Odd}, \mathrm{K}_{\mathrm{c}}, \mathrm{g}_{\mathrm{n}}=1\right)\right\}$ ) coincide in energy. It is therefore not possible to observe separately the transitions with $\mathrm{g}_{\mathrm{n}}=1$ and 3 . In fact, the observed doublets (see Figure 2) concern the $\left\{{ }^{\text {High }}{ }^{2}\left[J, \mathrm{~K}_{\mathrm{a}}=\mathrm{d}, \mathrm{K}_{\mathrm{c}}\right]\right\}$ and $\left\{{ }^{\text {Low }}{ }^{2}\left[\mathrm{~J}, \mathrm{~K}_{\mathrm{a}}=\mathrm{d}, \mathrm{K}_{\mathrm{c}}\right]\right\}$ levels each of them having a total statistic weight of 4 (corresponding to $g_{n}=3+g_{n}=1$ ). The same situation occurs for the P and R -branches of the $\mathrm{v}_{4}$ band as can be seen in Figure 3.

This $\mathrm{K}_{\mathrm{a}}$ degeneracy does not occur for the transitions involved in the Q branches. One could therefore expect to distinguish between strong ( $\mathrm{g}_{\mathrm{n}}=3$ ) and weak $\left(g_{n}=1\right)$ transitions. Unfortunately, the torsional splittings are significantly weaker (less than $0.001 \mathrm{~cm}^{-1}$ for $\mathrm{K}_{\mathrm{a}} \sim \mathrm{J}$ and $\mathrm{J}>18$ ) and the identification of the weaker " $\mathrm{g}_{\mathrm{n}}=1$ " transitions is difficult since the $Q$ branches are congested. However, the $\left(g_{n}=3 / \mathrm{g}_{\mathrm{n}}=1\right)$ staggering effect could be observed in portions of the low frequency range of the $2 \mathrm{v}_{9}$ Q branches, as shown in Figures 4 and 5 for the 10B and 11B isotopic species respectively. In this case the ( $\mathrm{g}_{\mathrm{n}}=3 / \mathrm{g}_{\mathrm{n}}=1$ ) staggering of the lines is obvious.

Of course the same situation occurs for $v_{4}$ except that the bending splittings are smaller (only $\sim 0.0035 \mathrm{~cm}^{-1}$ for the $P$ and $R$ branches of $v_{4}$ instead of $\sim 0.005 \mathrm{~cm}^{-1}$ for $2 \mathrm{v}_{9}$ ) and the ( $\mathrm{g}_{\mathrm{n}}=3 / \mathrm{g}_{\mathrm{n}}=1$ ) staggering effect could be only occasionally observed in the $Q$ branches.

## 4-B Classical vibration - rotation interactions

The $9^{2}$ energy levels:
The $9^{2}$ energy levels are perturbed through B- type and A-type Coriolis resonances with the levels of the $6^{1} 9^{1}$ dark state. More precisely, for $10 B(11 B)$ these rather weak resonances reach their maximum at $J \sim 62$ (58) for the $9^{2}$ levels with $K_{c}=44$ (43) for the $9^{2}$ levels and $K_{c}=41$ (40) for $6^{1} 9^{1}$.

## The $4^{1}$ energy levels:

In the course of the analysis of the $v_{4}$ band for the 10B (11B) isotopic species, we noticed that the $4^{1}$ levels involving $K_{c}$ values close to $K_{c}=28(25)$ were perturbed. Using the spectroscopic parameters determined from the analyses of the $v_{7}$ and $v_{9}$ bands [6] and using symmetry considerations, we identified the perturbing dark state as $7^{1} 9^{1}$ ( $\mathrm{A}^{\prime \prime}$ symmetry) located around $973 \mathrm{~cm}^{-1}$ ( $971 \mathrm{~cm}^{-1}$ ) for 10B (11B). More explicitly, due to this B-type Coriolis resonance, levels of the $4^{1}$ state of $10 \mathrm{~B}(11 \mathrm{~B})$ with $\mathrm{K}_{\mathrm{c}}=28(25)$ are resonating with the levels of the $7^{1} 9^{1}$ dark state with $\mathrm{K}_{\mathrm{c}}{ }_{\mathrm{c}}=27$ (24); the crossing of the two series occuring around $\mathrm{J}=30$ (32). Since this resonance is strong and involves rather low rotational quantum numbers, some transitions for the $v_{7}+v_{9}$ dark band could be clearly identified in the spectrum near the crossing of the $4^{1}$ and $7^{1} 9^{1}$ resonating series. An example illustrating these perturbations is given in Figure 3, which shows a portion of the $P$ branch of the $v_{4}$ band. As was mentioned previously, this $v_{4}$ band exhibits a doublet structure which does not appear for the transitions of the resonating dark $v_{7}+v_{9}$ band. This shows that the $7^{1} 9^{1}$ dark state is not responsible for the large amplitude effects that affect the $4{ }^{1}$ energy levels.

Furthermore, an additional resonance was observed at higher energies for the $v_{4}$ transitions involving $K_{c}^{\prime}$ values close to $K_{c}^{\prime}=44$ (48) for $10 B$ (11B). We identified this resonance as an anharmonic perturbation by the $6^{1} 7^{1}$ dark state. This perturbation involves mainly the $\mathrm{K}_{\mathrm{c}}^{\prime}=42$ (46) levels of the $6^{1} 7^{1}$ state. As this rather weak resonance affects only weak $v_{4}$ transitions, no lines belonging to the $v_{6}+v_{7}$ dark bands of both isotopes could be identified in the spectra of any of the two isotopes.

## 4-C Preliminary calculations

The $\mathrm{BF}_{2} \mathrm{OH}$ spectrum exhibits some analogies to the $\mathrm{HNO}_{3}$ spectrum:

- the $2 v_{9}$ bands for both species are strong
- the $2 v_{9}$ lines of both species are split.

On the other hand, the observation of large amplitude splittings in the analysis of the $v_{4}$ band of $\mathrm{BF}_{2} \mathrm{OH}$ (associated to the OH bending motion) was rather unexpected as compared to $\mathrm{HNO}_{3}$.
For nitric acid, the $2 v_{9}$ overtone band (near $896 \mathrm{~cm}^{-1}$ ) is almost as strong as the $v_{5}$ band (near $\left.879 \mathrm{~cm}^{-1}\right)$. In addition, although $v_{5}\left(\mathrm{NO}_{2}\right.$ in-plane bend) is a low amplitude mode, torsional splittings were easily observed in the millimeter wave region for rotational transitions of both the $9^{2}$ the $5^{1}$ excited states of $\mathrm{HNO}_{3}$ [13-16]. In fact both the high intensity for the $2 v_{9}$ band and the existence of splittings for rotational transitions in the $5^{1}$ state could be explained satisfactorily through the existence of an overall mixing of the $9^{2}$ and $5^{1}$ wavefunctions due to a very strong Fermi resonance coupling the $9^{2}$ and $5^{1}$ energy levels [16].
At the start of the present study, it seemed reasonable to make the assumption that strong Fermi resonances couple the $9^{2}$ and $4^{1}$ states of $\mathrm{BF}_{2} \mathrm{OH}$. Such resonances might explain the strength of the $2 v_{9}$ overtone band and the existence of large amplitude splittings in $v_{4}$. Therefore, a preliminary calculation of the $9^{2}$ and $4^{1}$ energy levels was performed assuming such resonances and using a Hamiltonian matrix analogous to the one used for $\mathrm{HNO}_{3}$ [16].
Assuming that the $9^{2}$ and $4^{1}$ splittings have their common physical origin in the OH torsional mode ( $v_{9}$ vibrational mode), it was necessary to fix the Fermi constant term to a value of about $40.3 \mathrm{~cm}^{-1}$ in order to reproduce the $v_{4}$ line splittings. This proved to be unrealistic since it would lead for 11B to a value of $\sim 1008.4 \mathrm{~cm}^{-1}$ for the vibrational energy of the $9^{2}$ state, which differs from the value of $1045.7 \mathrm{~cm}^{-1}$ obtained assuming that $\mathrm{E}_{99}=2 \mathrm{E}_{9}$. As a consequence, the vibrational energy of the $9^{2}$ state should indeed be larger than $1045.7 \mathrm{~cm}^{-1}$ because $v_{4}$ is located below $2 v_{9}$. Also the values obtained for the rotational constants of the $9^{2}$ and $4^{1}$ states were unrealistic. In particular, contrary to $9^{2}$, for $4^{1}$ the value of $B_{z}$ was found to be smaller than that of $B_{x}$. Finally, the fit was not satisfactory since, for levels with $J \leq 30$, the standard deviation of the fit was already $\sim 0.007 \mathrm{~cm}^{-1}$. Clearly the assumption that the $v_{4}$ splittings originated from those of $2 v_{9}$ was physically inacceptable. Therefore, separate calculations had to be made for the $9^{2}$ and $4^{1}$ energy levels.

## 4-D Hamiltonian matrix

The form of the Hamiltonian matrices used to calculate the energy levels are given in Tables 6-A and 6-B for the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ and $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ polyads of interacting states, respectively.
As was pointed out previously, various "classical" vibration - rotation resonances had to be accounted for:

- A B-type Coriolis resonance between the $9^{2}$ and $6^{1} 9^{1}$ vibrational states.
- For the $4^{1}$ energy levels, a B-type Coriolis resonance with the $7^{1} 9^{1}$ levels together with an additional anharmonic interaction and a C-type Coriolis resonance with $67^{1}$ state.

In the v- diagonal blocks, the rotational operators for each of the $9^{2}, 6^{1} 9^{1}, 4^{1}, 7^{1} 9^{1}$, and $6^{1} 7^{1}$ vibrational states include $X Z_{v}$ non-orthorhombic operators:

$$
\begin{equation*}
X Z_{v}=h_{x z}^{v}\left\{J_{x}, J_{z}\right\} \tag{2}
\end{equation*}
$$

in addition to Watson's operators written in an $I^{r}$ representation with an A-type reduction [17]. These $X Z_{v}$ operators account for the rather strong Coriolis interactions linking the two $9^{2}$ torsional (or $4^{1}$ bending) sub-states that are due to the large amplitude tunneling effects. In order to have a common reference system of axes $\left(\mathrm{X}_{99}, \mathrm{Y}_{99}, \mathrm{Z}_{99} / \mathrm{x}_{4}, \mathrm{y}_{4}, \mathrm{z}_{4}\right.$ ) for the $\left\{9^{2}, 6^{1} 9^{1} / 4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ resonating upper states, the $\mathrm{X} \mathrm{Z}_{v}$ non -orthorhombic operators also had to be included in the rotational operators of the $6^{1} 9^{1} / 7^{1} 9^{1}, 6^{1} 7^{1}$ resonating dark states. The $h_{\mathrm{xz}}^{\mathrm{v}}$ non -orthorhombic parameters for the $6^{1} 9^{1} / 7^{1} 9^{1}, 6^{1} 7^{1}$ dark states were fixed at the value derived for the interacting $9^{2} / 4^{1}$ bright states.

Additionally, the tunneling splittings due to the large amplitude OH torsion (for the $2 v_{9}$ band) or OH bending (for the $v_{4}$ band) had to be accounted for by specific operators in the $9^{2}$ and $4^{1}$ vibrational blocks, respectively. For nitric acid, the observed torsional splittings could be successfully modeled using the Internal Axis Method (IAM) -like approach [16, 18-21]. In that approach, the general form of the IAM operator ${ }^{\text {IAM }} H$ involves matrix elements of the $D(\chi, \theta, \varphi)$ Wigner's operators [22], where $\quad \chi, \theta$, and $\varphi$ are are the Euler angles. Because of symmetry $\chi=\varphi+\pi$ for $\mathrm{C}_{\mathrm{s}}$ -
type planar molecules like $\mathrm{HNO}_{3}$ or $\mathrm{BF}_{2} \mathrm{OH}$. Therefore only two independent angles ${ }^{2}$ $\theta$ and $\varphi$ are actually to be considered. In the analysis of the $\left\{5^{1}, 9^{2}\right\}$ interacting states of $\mathrm{HNO}_{3}$ [16] both the $\theta$ and $\varphi$ angles could be determined together with the torsional splitting parameters since a large set of rotational transitions in the $5^{1}$ and $9^{2}$ excited states measured by millimeter wave techniques were included in the least squares fit together with $5^{1}$ and $9^{2}$ infrared energy levels.
On the contrary, only infrared data are presently available for the $9^{2}$ and $4^{1}$ excited states of $\mathrm{BF}_{2} \mathrm{OH}$. Therefore, it turned out that only the $\varphi$ angle could be determined from the least squares fits, and consequently $\theta$ was set fixed to zero during the calculations.
The exact form of the rotational, vibration- rotational, torsional or bending operators is given in Table 6.

## 5. Results

The calculations of the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ and $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ resonating energy levels of 10B and 11B were performed using the Hamiltonian models given in Table 6, sections $A$ and $B$ respectively. For 10B (11B) the Hamiltonian constants resulting from the least squares fits of the experimental data are given together with their estimated uncertainties in Table 7 (8) and Table 9 (10) for the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ and $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ resonating levels respectively.
For 10B (11B) Table 2 (3) gathers details of the energy level calculations in terms of standard deviations and statistical analyses. It is clear that the results are excellent for both the $\left\{9^{2}, 6^{1} 9^{1} / 4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ resonating states. The quality of the calculations is furthermore proved by the consistency of the parameters derived for the 10B and 11 B isotopic species.

[^1]
## 6. Discussion

In this section we discuss problems concerning the values of the centrifugal distrortion constants, the signs of some Hamiltonian constants and the definition of the angles involved in the torsional or bending operators.

## 6-A Values of the centrifugal distortion constants for the bright states

In Tables 7-10, some of the centrifugal distortion constants for the $9^{2}$ and $4^{1}$ bright states differ significantly from the ground state values: this is because insufficient information is available for the $6^{1} 9^{1}$ and $6^{1} 7^{1}$ dark states.

## 6-B Signs of the parameters

As it was discussed in detail by Perrin et al. [16], absolute signs can be determined from the least squares fit only for some of the parameters quoted in Tables 7-10. For example, the signs of the torsional $h^{\text {TORS }}$ or bending $h^{\text {BEND }}$ constants together with all the parameters appearing in the orthorhombic part of the rotational Hamiltonians (i.e. the Watson's A-type expansion) are obtained from the fit. On the other hand, as usual, the signs of the higher order constants occurring in the $6^{1} 9^{1} \Leftrightarrow 9^{2}$ B-type, $7^{1} 9^{1} \Leftrightarrow 4^{1}$ B-type, $67^{1} \Leftrightarrow 4^{1}$ C-type Coriolis or $6^{1} 7^{1} \Leftrightarrow 4^{1}$ anharmonic operators are obtained only relative to the lower order constants $\left({ }^{69,99} \mathrm{~B}_{\mathrm{x}},{ }^{79,4} \mathrm{~B}_{\mathrm{x}}\right.$, ${ }^{67,4} \mathrm{C}_{\mathrm{xz}}$ and ${ }^{67,4} \mathrm{Anh}_{\mathrm{xy}}$ in the present case (see Table 6)). As a consequence, any of the following changes of sign leave the energy of the levels unchanged:

- For the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ interacting states:

$$
\begin{align*}
& \text { TORS } \varphi \rightarrow-{ }^{\text {TORS }} \varphi \\
& \text { or }{ }^{69,99} \mathrm{~B}_{x} \rightarrow-{ }^{69,99} \mathrm{~B}_{\mathrm{x}} \\
& \text { or } \mathrm{h}_{\mathrm{xz}}^{99} \rightarrow-\mathrm{h}_{\mathrm{xz}}^{99} \quad\left(\mathrm{~h}_{\mathrm{xz}}^{69}=\mathrm{h}_{\mathrm{xz}}^{99}\right) \tag{3}
\end{align*}
$$

- For the $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ interacting states:

$$
\begin{gather*}
\text { BEND }_{\varphi \rightarrow-{ }^{\text {BEND }} \varphi} \\
\text { or }{ }^{79,4} \mathrm{~B}_{\mathrm{x}} \rightarrow-{ }^{-79,4} \mathrm{~B}_{\mathrm{x}} \\
\text { or }\left({ }^{67,4} \mathrm{C}_{\mathrm{xz}} \rightarrow-{ }^{67,4} \mathrm{C}_{\mathrm{xz}} \quad \underline{\text { and }}{ }^{67,4} \mathrm{Anh}_{\mathrm{xy}} \rightarrow-{ }^{-67,4} \mathrm{Anh}_{\mathrm{xy}}\right) \\
\text { or } \mathrm{h}_{\mathrm{xz}}^{4} \rightarrow-\mathrm{h}_{\mathrm{xz}}^{4} \quad\left(\mathrm{~h}_{\mathrm{xz}}^{67}=\mathrm{h}_{\mathrm{xz}}^{79}=\mathrm{h}_{\mathrm{xz}}^{4}\right) \tag{4}
\end{gather*}
$$

## 6-C Axis switching effects

The signs chosen for the $h_{x z}^{99}$ and $h_{x z}^{4}$ parameters that appear in the $X Z_{v}$ nonorthorhombic terms (see Eq. 2 and Table 6) lead to an anti-clockwise rotation of the ( $a, b, c$ ) initial system around the c-axis which results in the non-orthorhombic ( $\mathrm{x}_{99}, \mathrm{Y}_{99}, \mathrm{z}_{99} / \mathrm{x}_{4}, \mathrm{y}_{4}, \mathrm{Z}_{4}$ ) reference system of axes for the $\left\{6^{1} 9^{1}, 9^{2} / 4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ resonating states respectively. The rotation angle $\alpha_{v}$ (see Figure 1) is given by [16]:

$$
\begin{equation*}
\alpha_{v} \approx 1 / 2 \operatorname{atan}\left(\frac{-2 h_{x z}^{v}}{\left(B_{z}^{v}-B_{x}^{v}\right)}\right) \tag{5}
\end{equation*}
$$

with $v=99 / 4$ for $\left\{9^{2}, 6^{1} 9^{1} / 4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$.
According to the values of the $\mathrm{h}_{\mathrm{xz}}^{\mathrm{v}}, \mathrm{B}_{\mathrm{z}}^{\mathrm{V}}$ and $\mathrm{B}_{\mathrm{x}}^{\mathrm{V}}$ constants (Tables 7-10), these angles are:

$$
\begin{equation*}
\alpha_{99} \approx 35.5(10)^{\circ} \quad\left(\alpha_{99} \approx 35.4(10)^{\circ}\right) \tag{6}
\end{equation*}
$$

for the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ states of $10 B(11 B)$, and:

$$
\begin{equation*}
\alpha_{4} \approx 17.91(3)^{\circ} \quad\left(\alpha_{4} \approx 15.37(6)^{\circ}\right) \tag{7}
\end{equation*}
$$

for the $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ states of $10 B(11 B)$.

The large differences between the $\alpha_{v}$ values for the two polyads show that the large amplitude OH - torsion and the large amplitude OH -bending motion cannot be accounted for simultaneously. This is confirmed by the fact that the splittings for the $J=0$ levels are very different for the torsion ( ${ }^{99} \mathrm{Split}^{\text {TORS }} \cong 5.1 \times 10^{-3} \mathrm{~cm}^{-1}$ ) and for the bending ( ${ }^{4}$ Split ${ }^{\text {BEND }} \cong 3.7 \times 10^{-3} \mathrm{~cm}^{-1}$ ).
Finally, as we will see in the next Section, this rather strong axis switching effect has to be considered properly if reliable line intensities are to be calculated.

## 7. Simulation of the experimental spectra

To emphasize the quality of the calculations we have compared the observed and calculated spectra in various spectral regions. The line positions were generated using the ground state constants from [3] and the upper state constants given in Tables 7-10 of this paper.

## 7-A Line intensity calculations

It should be stressed that only relative intensities were computed since no attempt was made to derive absolute experimental intensities.

The intensity of a line $[23,24]$ is proportional to $R_{A}^{B}$ which is the square of the matrix element of the transformed transition moment operator $\mu_{\mathrm{Z}}^{\prime}$ :

$$
\begin{equation*}
\left.R_{A}^{B}=\left|\left\langle v^{\prime}, j K_{a}^{\prime} K_{c}^{\prime}\right| \mu_{Z}^{\prime}\right| 0, J^{\prime \prime} K_{a}^{\prime \prime} K_{c}^{\prime \prime}\right\rangle\left.\right|^{2} \tag{8}
\end{equation*}
$$

where $\mu_{\mathrm{Z}}$ is the transformed dipole moment operator [23, 24], which can be expanded as

$$
\begin{equation*}
\mu_{Z}^{\prime}=\sum_{v^{\prime} \in B^{\prime}}|0\rangle{ }^{v_{\mu_{Z}}^{\prime}}\left\langle v^{\prime}\right| \tag{9}
\end{equation*}
$$

where $v^{\prime}$ belongs to the upper $B^{\prime}=\left\{9^{2}, 6^{1} 9^{1}\right\}$ or $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ polyad of interacting states. For the $2 v_{9}$ and $v_{4}$ bright bands, both A-type and B-type transitions are allowed for symmetry reasons. Therefore, up to first order, the expansion of the $2 v_{9}$ and $v_{4}$ transition moment operators can be written as [23, 24]:

$$
\begin{equation*}
v_{\mu_{Z}}=\varphi_{z}{ }^{v} \mu_{z}^{1}+\varphi_{x} v_{\mu_{x}}^{1}+\ldots \tag{10}
\end{equation*}
$$

where $\varphi_{z}$ and $\varphi_{x}$ stand for the direction cosines $\phi_{Z z}$ and $\phi_{Z x}$ respectively. In the usual calculations the upper and ground state rovibrational wavefunctions are expanded as on the Wang's type sub-bases (see Table 6):

$$
\begin{align*}
& \left|\mathrm{v}^{\prime}, J^{\prime} \mathrm{K}_{\mathrm{a}}^{\prime} \mathrm{K}_{\mathrm{c}}^{\prime}\right\rangle=\sum_{\mathrm{v}^{\prime} \in \mathrm{B}^{\prime}} \sum_{\mathrm{K}^{\prime}, \gamma^{\prime}} \mathrm{C}_{\mathrm{v}^{\prime}}^{\gamma^{\prime}} \mathrm{K}^{\prime}\left|\mathrm{v}^{\prime}\right\rangle\left|\mathrm{J}^{\prime} \mathrm{K}^{\prime} \gamma^{\prime}\right\rangle  \tag{11}\\
& \left|\mathrm{v}=0, \mathrm{~J} \mathrm{~K}_{\mathrm{a}} \mathrm{~K}_{\mathrm{c}}\right\rangle=\sum_{\mathrm{K}^{\prime \prime}} \mathrm{C}_{0}^{\gamma^{\prime \prime}} \mathrm{K}^{\prime \prime}\left|\mathrm{J} \mathrm{~K}^{\prime \prime} \gamma^{\prime \prime}\right\rangle \tag{12}
\end{align*}
$$

In Eqs.(11) and (12), the $C_{\mathrm{v}^{\prime}}^{\gamma^{\prime} K^{\prime}}$ and $C_{0}^{\gamma^{\prime \prime} K^{\prime \prime}}$ coefficients are obtained from the diagonalization of the Hamiltonian matrices.

However, as for $\mathrm{HNO}_{3}$ [16], it is necessary to account properly for the axis switching effects. Indeed, for both 10 B and 11 B isotopologues of $\mathrm{BF}_{2} \mathrm{OH}$, the ( $\mathrm{x}_{99}, \mathrm{y}_{99}, \mathrm{Z}_{99} / \mathrm{x}_{4}, \mathrm{y}_{4}, \mathrm{z}_{4}$ ) reference axes for the upper $\left\{9^{2}, 6^{1} 9^{1} / 4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ vibrational excited states are tilted anticlockwise around the $c$ axis by about $\alpha_{99} \sim 35.4^{\circ}$ / $\alpha_{4} \sim 16.6^{\circ}$ from the ( $a, b, c$ ) inertial axes (see Fig.(1)). To the contrary, the ground vibrational state wave functions are calculated using a standard $I^{r}$ Watson A-type Hamiltonian and the reference axes are the ( $a, b, c$ ) inertial axes of the molecule [17]. To solve this problem it is necessary to re-write the ground state wavefunctions so that they are consistent with the upper state wavefunctions.

$$
\begin{align*}
\left|\mathrm{v}=0, \mathrm{~J} \mathrm{~K}_{\mathrm{a}} \mathrm{~K}_{\mathrm{c}}\right\rangle= & \sum_{\mathrm{K}^{\prime \prime} \gamma^{\prime \prime} \in\left\{\mathrm{E}^{+}, \mathrm{O}^{-}\right\}}{ }^{\alpha_{v} \mathrm{C}_{0}^{\gamma^{\prime \prime}} \mathrm{K}^{\prime \prime} \mid}\left|\alpha_{\mathrm{v}} \mathrm{~J}^{\prime} \mathrm{K}^{\prime \prime} \gamma^{\prime \prime}\right\rangle .  \tag{13}\\
& \text { or } \mathrm{K}^{\prime \prime} \gamma^{\prime \prime} \in\left\{\mathrm{E}^{-}, \mathrm{O}^{+}\right\}
\end{align*}
$$

This new expansion is performed simultaneously on both the $\mathrm{E}^{+}$and $\mathrm{O}^{-}$(resp. on both the $\mathrm{E}^{-}$and $\mathrm{O}^{+}$) sub-blocks of the Wang bases (see Table 6) whereas in Eq.(12) the summation on $K$ " is performed on only one of the four sub-bolcks of the Wang bases $\{\mid J, K, \gamma>\}=E^{+}$or $E^{-}$or $\mathrm{O}^{+}$or $\mathrm{O}^{-}$, with $\mathrm{E}^{ \pm}$for $\mathrm{K}=$ even and $\mathrm{O}^{ \pm}$for $\mathrm{K}=$ odd and $\gamma= \pm 1\}$.
The relationship between the two expansions of the ground state wavefunctions is obtained using the Wigner tensorial approach [22] and the $\mathrm{d}_{\mathrm{K}^{\prime}, \mathrm{K}^{\prime \prime}}^{(\mathrm{J})}\left(\alpha_{0}\right)$, matrix elements (see details in Ref. [16]):

$$
\begin{equation*}
\alpha_{v} C_{0}^{\gamma^{\prime} K^{\prime}}=\sum_{K^{\prime \prime}} C_{0}^{\gamma^{\prime \prime} K^{\prime \prime}}\left(d_{K^{\prime}, K^{\prime \prime}}^{(J)}\left(\alpha_{v}\right)+\gamma^{\prime} \quad(-1)^{K^{\prime}-K^{\prime \prime}} d_{K^{\prime},-K "}^{(J)}\left(\alpha_{v}\right)\right) \tag{14}
\end{equation*}
$$

(with $\alpha_{v}=\alpha_{99} / \alpha_{v}=\alpha_{4}$ for the $\left\{9^{2}, 6^{1} 9^{1} / 4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ polyad).

We have used this formalism to calculate the line intensities for the two studied polyads of interacting bands. As $2 v_{9}$ and $v_{4}$ are in principle $A$ and $B$ hybrid type bands, the transition moments operators involve both the ${ }^{v} \mu_{z}^{1}$ and ${ }^{{ }_{\mu}}{ }_{x}^{1}$ parameters defined in the switched axis system ( $x, y, z$ ) (Eq. (10)). These parameters are related to the transition moment operator parameters in the principal axis system ( $a, b, c$ ) through:

$$
\begin{align*}
& v_{\mu}{ }_{a}^{1}=\cos \alpha_{v}{ }^{v_{\mu}}{ }_{z}^{1}-\sin \alpha_{v}{ }^{v_{\mu}} \mu_{x}^{1} \\
& { }^{v} \mu_{b}^{1}=\sin \alpha_{v}{ }^{v} \mu_{z}^{1}+\cos \alpha_{v}{ }^{v} \mu_{x}^{1} \tag{15}
\end{align*}
$$

with $v=99$ or 4 .

For the dark bands $v_{6}+v_{9}, v_{7}+v_{9}$ and $v_{6}+v_{7}$ the transition moments were set to zero.
For the bright $2 v_{9}$ band, only A-type transitions were observed. Therefore, for $2 v_{9}$ the transition moment operator is parallel to the a- inertial axis (see Figure 1), and the $2 v_{9}$ line intensities could be satisfactorily reproduced using the following ratio for the $z$ - and $x$-component of the $2 v_{9}$ transition moment operator (using $\alpha_{99} \approx 35.4^{\circ}$ for 10B and 11B):

$$
\begin{equation*}
99 \mu_{\mathrm{x}}^{1} / 99_{\mathrm{z}}^{1}=-\tan \left(\alpha_{99}\right) \approx-0.711 \tag{16}
\end{equation*}
$$

For the bright $v_{4}$ band, the problem is more difficult since both A- and B- type transitions were observed. In this case, from examination of the spectra and fitting a few relative line intensities, we estimate the ratio of the $z$ - to $x$-components of the $v_{4}$ transition moment operator to be:

$$
\begin{equation*}
{ }^{4} \mu_{x}^{1} / 4^{4} \mu_{z}^{1} \approx+0.71 \pm 20 \% \tag{17}
\end{equation*}
$$

When using $\alpha_{4} \approx 16.6^{\circ}$ for $10 B$ and $11 B$, this leads to the ratio for the $b-$ to $a$ components of the $v_{4}$ transition moment operator:

$$
\begin{equation*}
4 \mu_{\mathrm{b}}^{1} / 4 \mu_{\mathrm{a}}^{1} \approx+1.28 \pm 20 \% \tag{18}
\end{equation*}
$$

which is in good agreement with the ab initio prediction $[3,25]$ :

$$
\begin{equation*}
\left({ }^{4} \mu_{\mathrm{b}}^{1} / 4_{\mathrm{a}}^{1}\right)^{\text {Ab initio }} \approx 1.05 \tag{19}
\end{equation*}
$$

## 7-B Comparison between the experimental and calculated spectra

In Figures 2-7 we compare the observed and calculated spectra in various spectral regions, where different type of resonances were observed.

Figures 2 and 3 show a portion of the $R$ branch of the $2 v_{9}$ band of $11 B$ and of the $P$ branch of the $v_{4}$ band of 10B illustrating examples of torsional and bending splittings. In Figure 3, the resonance involving the $4^{1}$ and $7^{1} 9^{1}$ energy levels for $K_{c}^{\prime} \sim 28$ is clearly seen, and lines from the dark $v_{7}+v_{9}$ resonating band appear in spite of the corresponding transition moment fixed to zero.
Figures 4 and 5 show portions of the $Q$ branch of the $2 v_{9}$ band for $10 B$ and $11 B$ respectively. In Figure 5, the "staggering" effect, which is due to the ( $g_{n}=3 / g_{n}=1$ ) nuclear spin statistical weights, is clearly observable. Figures 6 and 7 give portions of the $Q$ branch for the $v_{4}$ band of $10 B$ and $11 B$ respectively: the $A-$ and $B$ - type character of the $v_{4}$ band is obvious.
In all cases, the agreement between the observed and calculated s is very satisfactory proving the quality of the model both in terms of line positions and of line intensities.

## 8. Conclusion:

From high resolution Fourier transform spectra of ${ }^{10} \mathrm{~B}$ - and ${ }^{11} \mathrm{~B}$ enriched as well as of natural samples of $\mathrm{BF}_{2} \mathrm{OH}$ the first rovibrational analysis of the $2 v_{9}$ and $v_{4}$ bands for ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ and ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$ has been performed up to very high quantum
numbers. Numerous perturbations were observed in the analysis and were accounted for in the Hamiltonian model. First, the $2 v_{9}$ bands and $v_{4}$ bands are perturbed by classical vibration - rotation resonances. More explicitly B-type Coriolis interactions couple the $9^{2}$ and $4^{1}$ energy levels with those of the $6^{1} 9^{1}$ and $7^{1} 9^{1}$ dark states, respectively. Also, anharmonic and C-type Coriolis interactions link the $4^{1}$ energy levels with those of the dark $6^{1} 7^{1}$ state. Most prominently, large amplitude effects were observed for the $2 v_{9}$ and $v_{4}$ bands, leading to splittings of the energy levels of about 0.005 and $0.0035 \mathrm{~cm}^{-1}$ for the $9^{2}$ and $4^{1}$ states respectively. These splittings are easily observable in the P and R branches for both bands. The theoretical model used to reproduce the experimental energies accounts for the "classical" vibration -rotation resonances as well as for the large amplitude torsional (or bending) effects. The latter were interpreted using an IAM (Internal Axis Method) like approach. In this approach the Coriolis resonances between the two torsional (or bending) substates were taken into account by means of $\left\{\mathrm{J}_{\mathrm{x}}, \mathrm{J}_{z}\right\}$ non-orthorhombic terms in the Hamiltonian $v$-diagonal blocks. As a consequence, the $z$-quantification axis deviates from the $a$ inertial axis with an axis switching effect of $\sim 35^{\circ} / \sim 16.6^{\circ}$ for the $\left\{9^{2}, 6^{1} 9^{1} / 4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ polyad of interacting vibrational states.
From the observation of different splittings and axis switching effects one can then conclude that both the OH - torsion and the OH -bending modes have to be considered as large amplitude motions. Finally, the experimental spectra were simulated very satisfactorily by considering both the above mentioned effects and the intensity alternation due to the large amplitude motions which cause an exchange of the two fluorine nuclei. .

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## References

[1] H.Takeo and R.H. Curl, J. Chem. Phys. 56, 4314-4317 (1972).
[2] K. Vormann, and H. Dreizler, Z. Naturforsch. 44a, 1191-1195 (1989).
[3] J. Breidung, J. Demaison, J.-F. D'Eu, L. Margulès, D. Collet, E.B. Mkadmi, A. Perrin and W. Thiel, J. Mol. Spectrosc. 228, 7-22 (2004).
[4] M.E. Jacox, K.K. Irikura, and W.E. Thomson, J. Chem. Phys. 113, 5705-5715 (2000).
[5] D.Collet, A.Perrin, H.Bürger and J.-M.Flaud, J. Mol. Spectrosc. 212, 118-124 (2002).
[6] A.Perrin, M.Carvajal-Zaera, Z.Dutkiewicz, J.-M.Flaud, D.Collet, H.Bürger, J.Demaison, F.Willaert, H.Mäder, and N.W.Larsen, Mol. Phys. 102, 1641-1652 (2004).
[7]: "High resolution Infrared Spectroscopy and One Dimentional Large Amplitude Motion in Asymmetric Tops: $\mathrm{HNO}_{3}$ and $\mathrm{H}_{2} \mathrm{O}_{2}$ ", J.-M.Flaud and A.Perrin, Chap 7 in "Vibration-Rotational Spectroscopy \& Molecular Dynamics", 396-460, in "Advanced Series in Physical Chemistry", D.Papoušek editor, World Scientific Publishing Company, Singapore (1997).
[8] D.T.Petkie, P.Helminger, M.Behnke, I.R.Medvedev and F.C. De Lucia, J. Mol. Spectrosc., 233, 189-196 (2005).
[9] A.Perrin, O.Lado-Bordowski, and A. Valentin, Mol. Phys. 67, 249-270 (1989).
[10] T. K. Ha, J. Makarewicz, Chemical Physics Letters 299 637-42 (1999).
[11] Qian-Peng, Yubin-Wang, Bing-Suo, Qizhen-Shi, and Zhenyi-Wen, J. Chem.
Phys. 121, 778-782 (2004).
[12] A.G.Maki and J.S.Wells, (1998), Wavenumber Calibration Tables from
Heterodyne Frequency Measurements (version 1.3). [Online] Available:
http://physics.nist.gov/wavenum [2006, November 24]. National Institute of Standards and Technology, Gaithersburg, MD.
[13] C.D.Paulse, L.H.Coudert, T.M.Goyette, R.L.Crownover, P.Helminger and F.C.De Lucia, J. Mol. Spectrosc. 177, 9-18 (1996).
[14] T.M.Goyette, L. C.Oesterling, D.T.Petkie, R.A.Booker, P.Helminger and F.C.De Lucia, J. Mol. Spectrosc. 175, 395-410 (1996).
[15] D.T.Petkie, T.M.Goyette, P.Helminger, H.M.Pickett and F.C.De Lucia, J. Mol. Spectrosc. 208, 121-135 (2001).
[16] A.Perrin, J.Orphal, J.-M.Flaud, S.Klee, G.Mellau, H. Mäder, D.Walbrodt and M.Winnewisser, J. Mol. Spectrosc. 228, 375-391 (2004).
[17] "Aspects of quartic and sextic centrifugal effects on rotational energy levels " J.K.G.Watson, Chap 1, in "Vibrational spectra and structure", J.Durig editor, Elsevier, (1977).
[18] J.T.Hougen, J. Mol. Spectrosc. 114, 395-426 (1985).
[19] L.H.Coudert and J.T.Hougen, J. Mol. Spectrosc. 130, 86-119 (1988).
[20] L.H.Coudert and J.T.Hougen, J. Mol. Spectrosc. 139, 259-277 (1990).
[21] L.H.Coudert and A.Perrin, J. Mol. Spectrosc. 172, 352-368 (1995).
[22] A.R.Edmonds, "Angular Momentum in Quantum Mechanics", Princeton University Press, Princeton, NJ, (1960).
[23]"Water Vapour Line Parameters from Microwave to Medium Infrared", J.-M.Flaud, C.Camy-Peyret, and R.A.Toth, Pergamon press, Oxford (1981). [24]"Vibration-rotation dipole moment operator for asymmetric rotors", C.Camy-Peyret, and J.-M.Flaud; in "Molecular Spectroscopy Modern Research" (K.Narahari Rao Ed.), p. 69-110 Vol. 3, Academic Press, New York, (1985).
[25] J.Breidung, private communication (2006).

## Figure Captions:

Figure 1:
Structure of the $\mathrm{BF}_{2} \mathrm{OH}$ molecule and definition of the axes.
Note:
Because of the existence of non-orthorhombic terms in the v-diagonal blocks of the $\left\{9^{2}, 6^{1} 9^{1} / 4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ Hamiltonians, the ( $\mathrm{x}_{99}, \mathrm{Z}_{99} / \mathrm{x}_{4}, \mathrm{Z}_{4}$ ) systems of axes differ significantly from the ( $b, a$ ) inertial system. The out-of-plane $y_{4}$ and $y_{99}$ reference axes coincide with the $c$ inertia axis. Also the directions of the $2 v_{9} / v_{4}$ transition moment operators ${ }^{99} \mu /{ }^{4} \mu$ are indicated.

Figure 2:
Portion of the R branch of the $2 v_{9}$ band of ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$ (spectrum recorded at Wuppertal).
Note:
The triangles indicate the split lines which belong to the $\left(2 J^{\prime}-K_{c}{ }_{c}=43\right)$ stack.

Figure 3:
Portion of the $P$ branch of the $v_{4}$ band of ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ (spectrum recorded at PNNL).
Note:
The triangles indicate the split lines which belong to the ( $2 J^{\prime}-K_{c}{ }_{c}=28$ ) stack. The assignments indicate the $K_{c}^{\prime}$ values. A resonance of the $4^{1}$ energy levels with those of $7^{1} 9^{1}$ is clearly visible for $K_{c}^{\prime}$ values near $K_{c}{ }_{c}=28$. Due to this resonance, unsplit transitions belonging to the $v_{7}+v_{9}$ (dark) resonating band are observed.

## Figure 4:

Portion of the central part of the $Q$ branch of the $2 v_{9}$ band of ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ (spectrum recorded at Wuppertal).

Note:
Assignments are given for the ${ }^{Q} Q_{K_{a}^{\prime \prime}=J, K_{C}^{\prime \prime}=0}$ transitions (black dots). For the
${ }^{Q} Q_{K_{a}^{\prime \prime}=J-2, K_{c}^{\prime \prime}=2}$ branch, the transitions with nuclear spin statistical weigths $g_{n}=3$ and
$g_{n}=1$ are identified by open and black diamonds, respectively. The staggering effect is visible.

Figure 5:
Portion of the $Q$ branch of the $2 v_{9}$ band of ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$ (spectrum recorded at Wuppertal).

## Note:

Assignments are given for the ${ }^{\circ} Q_{K_{a}^{\prime \prime}=J-1, K_{C}^{\prime \prime}=2}$ branch (with $\Delta K_{a}=-2, \Delta K_{c}=+1$ ). The $g_{n}=3$ and $g_{n}=1$ transitions are identified by open and black diamonds, respectively. The staggering effect is visible.

## Figure 6:

Overview of the $v_{4} \mathrm{Q}$ branch for ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ (spectrum recorded at PNNL).

## Figure 7:

Central part of the $v_{4} \mathrm{Q}$ branch for ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$ (spectrum recorded at Wuppertal).

## Table Captions

## Table 1:

Infrared bands of $\mathrm{BF}_{2} \mathrm{OH}\left(\mathrm{cm}^{-1}\right)$
Note:
${ }^{a} A$ ' and $A$ " are the symmetry species in the $C_{s}$ point group.
${ }^{\mathrm{b}}$ Matrix band centers from Ref.[4].
${ }^{\circ}$ Gas phase band centers from Ref. [6].
${ }^{\mathrm{d}}$ This work.

## Table 2:

Range of quantum numbers for energy levels probed by infrared transitions and statistical analysis of the results of the energy level calculations for the $9^{2}, 4^{1}$, and $7^{1} 9^{1}$ vibrational States of ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$.

## Table 3:

Range of quantum numbers for energy levels probed by infrared transitions and statistical analysis of the results of the energy level calculations for the $9^{2}, 4^{1}$, and $7^{1}$ $9^{1}$ vibrational States of ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$.

## Table 4:

Character tables for the $\mathrm{C}_{2 \mathrm{v}}$ and $\mathrm{G}_{4}$ symmetry groups.
Note:
Meaning of the different columns:

- $J_{x}, J_{y}, J_{z}$ and $\mu_{x}, \mu_{y}, \mu_{z}$ : molecular fixed components of $\vec{J}$ and $\vec{\mu}$ respectively.
- $J_{Z}$ and $\mu_{z}$ : Z-laboratory fixed components of $\vec{J}$ and $\vec{\mu}$ respectively.
- Torsion and Bend: Symmetry properties for the ${ }^{\text {Low }} 9^{v}$ and ${ }^{\text {High }} 9^{v}$ (large amplitude torsion) and ${ }^{\text {Low }} 4^{\vee}$ and ${ }^{\text {High }} 4^{\vee}$ (large amplitude bend) vibrational substates.
- $v$ : parity of $v(e=e v e n, o=o d d)$.
- [J K $\mathrm{K}_{\mathrm{d}}$ ]: symmetry of a rotational energy level according to the $\mathrm{K}_{\mathrm{a}}$ and $\mathrm{K}_{\mathrm{c}}$ parity ( $\mathrm{e}=$ =even, $\mathrm{o}=\mathrm{odd}$ ).
- Nuclear spin: symmetry properties of the nuclear spin wavefunctions attached to the fluorine nuclei $(\mathrm{l}=1 / 2)$. The symmetrical (resp. antisymmetrical) wavefunctions have nuclear spin degeneracy of $g_{n}=3$ (resp. $g_{n}=1$ ).
- The symbols ${ }^{\text {Low }} 9^{\vee}$ and ${ }^{H i g h} g^{v} /{ }^{\text {Low }} 4^{\vee}$ and ${ }^{H i g h} 4^{v}$ designate the higher and lower energy subcomponent, respectively, of each OH - torsional $9^{\mathrm{v}} / \mathrm{OH}$ - bending $4^{\mathrm{v}}$ vibrational state


## Table 5

a) Symmetry properties of the $9^{\vee} \mathrm{OH}$ torsion -rotation wavefunctions ( $\Gamma_{\text {Tors-Rot }}$ )
b) Symmetry properties of the $4^{\vee} \mathrm{OH}$-bending -rotation wavefunctions ( $\Gamma_{\text {Bend-Rot }}$ )

## Note:

Meaning of the different columns:

- $g_{n}$ : nuclear spin statistical weights
- $K_{a}$ and $K_{c}$ : parity of $K_{a}$ and $K_{c}(e=e v e n, ~ o=o d d)$.
- Low $9^{\vee}$ and ${ }^{H i g h} 9^{\vee} / /^{\text {Low }} 4^{\vee}$ and ${ }^{\text {High }} 4^{\vee}$ : higher and lower energy subcomponent, respectively, of each OH - torsional $9^{v} / \mathrm{OH}$ - bending $4^{v}$ vibrational state


## Table 6

Hamiltonian matrices and operators.

## Table 7

Hamiltonian constants (in $\mathrm{cm}^{-1}$ ) for the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ polyad of ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$
Note:
The quoted errors are one standard deviation.
${ }^{\text {a }}$ From Ref. [3],
${ }^{\mathrm{b}}$ Fixed to the ground state value.

[^2]
## Note:

The quoted errors are one standard deviation.
${ }^{\text {a }}$ From Ref. [3],
${ }^{\mathrm{b}}$ Fixed to the ground state value.

## Table 9

Hamiltonian constants (in $\mathrm{cm}^{-1}$ ) for the $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ polyad of ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ Note:
The quoted errors are one standard deviation.
${ }^{\mathrm{b}}$ Fixed to the ground state value [3].

## Table 10

Hamiltonian constants (in $\mathrm{cm}{ }^{-1}$ ) for the $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ polyad of ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$
Note:
The quoted errors are one standard deviation.
${ }^{\mathrm{b}}$ Fixed to the ground state values [3].

Table 1
Infrared bands of $\mathrm{BF}_{2} \mathrm{OH}\left(\mathrm{cm}^{-1}\right)$

| Vibrational assignment ${ }^{\text {a }}$ | Description | ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ | ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$ |
| :---: | :---: | :---: | :---: |
| $v_{1}, A^{\prime}$ | $v$ (OH) | $3712.5^{\text {b }}$ | $3712.5^{\text {b }}$ |
| $v_{2}, A^{\prime}$ | $v_{\text {as }}(\mathrm{BF})$ | $1515.8{ }^{\text {b }}$ | $1464.3{ }^{\text {b }}$ |
| $v_{3}, A^{\prime}$ | $v_{s}(\mathrm{BF})$ | $1456.9{ }^{\text {b }}$ | $1414.9{ }^{\text {b }}$ |
| $v_{4}, A^{\prime}$ | $\delta(\mathrm{BOH})$ i.p. | $961.74{ }^{\text {d }}$ | $961.49^{\text {d }}$ |
| $v_{5}, A^{\prime}$ | $\delta\left(\mathrm{BF}_{2}\right)$ i.p. | $880.64{ }^{\text {c }}$ | $880.74^{\text {c }}$ |
| $v_{6}, A^{\prime}$ | $v\left(\mathrm{~F}_{2} \mathrm{BO}\right)$ i.p. |  | $479.17^{\text {c }}$ |
| $v_{7}, A^{\prime}$ | $\delta\left(\mathrm{F}_{2} \mathrm{BO}\right)$ i.p. |  | $446.54{ }^{\text {c }}$ |
| $\mathrm{v}_{8}, \mathrm{~A}{ }^{\prime}$ | $\delta\left(\mathrm{F}_{2} \mathrm{BO}\right)$ o.p. | $711.41^{\text {c }}$ | $684.16^{\text {c }}$ |
| $v_{9}, A^{\prime \prime}$ | $\delta(\mathrm{BOH})$ o.p. | $523.04{ }^{\text {c }}$ | $522.87^{\circ}$ |
| $2 v_{9}, A^{\prime}$ |  | $1043.89{ }^{\text {d }}$ | $1042.87{ }^{\text {d }}$ |

## Table 2

Range of quantum numbers for energy levels probed by infrared transitions and statistical analysis of the results of the energy level calculations for the $9^{2}, 4^{1}$, and $7^{1} 9^{1}$ vibrational States of ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$.

| Vibrational states: | $9^{2}$ | $4^{1}$ | $7^{1} 9^{1}$ |
| :--- | :---: | :---: | :---: |
| Number of lines | 3958 | 4259 | 116 |
| $J$ and $\mathrm{K}_{\mathrm{a}}$ ranges | $\mathrm{J} \leq 64, \mathrm{~K}_{\mathrm{a}} \leq 49$ | $\mathrm{~J} \leq 65, \mathrm{~K}_{\mathrm{a}} \leq 46$ | $\mathrm{~J} \leq 54, \mathrm{~K}_{\mathrm{a}} \leq 48$ |
| Number of levels | 1679 | 1571 | 55 |
|  |  |  |  |
| $0.0 \times 10^{-3} \leq \delta<0.5 \times 10^{-3} \mathrm{~cm}^{-1}$ | $86.8 \%$ | $71.9 \%$ |  |
| $0.5 \times 10^{-3} \leq \delta<1.0 \times 10^{-3} \mathrm{~cm}^{-1}$ | $9.8 \%$ | $21.2 \%$ | $6.9 \%$ |
| $1.0 \times 10^{-3} \leq \delta<3.0 \times 10^{-3} \mathrm{~cm}^{-1}$ | $3.4 \%$ | 0.57 |  |
| Standard deviation $\left(10^{-3} \mathrm{~cm}^{-1}\right)$ | 0.42 |  |  |

## Table 3

Range of quantum numbers for energy levels probed by infrared transitions and statistical analysis of the results of the energy level calculations for the $9^{2}, 4^{1}$, and $7^{1}$ $9{ }^{1}$ vibrational States of ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$.

| Vibrational states: | $9^{2}$ | $\mathbf{4}^{1}$ | $7^{1} 9^{1}$ |
| :--- | :---: | :---: | :---: |
| Number of lines | 5381 | 4665 | 258 |
| J and $\mathrm{K}_{\mathrm{a}}$ ranges | $\mathrm{J} \leq 64, \mathrm{~K}_{\mathrm{a}} \leq 49$ | $\mathrm{~J} \leq 69, \mathrm{~K}_{\mathrm{a}} \leq 35$ | $\mathrm{~J} \leq 38, \mathrm{~K}_{\mathrm{a}} \leq 15$ |
| Number of levels | 2044 | 1687 | 111 |
|  |  |  |  |
|  |  | $60.9 \%$ |  |
| $0.0 \times 10^{-3} \leq \delta<0.5 \times 10^{-3} \mathrm{~cm}^{-1}$ | $82.6 \%$ | $26.9 \%$ |  |
| $0.5 \times 10^{-3} \leq \delta<1.0 \times 10^{-3} \mathrm{~cm}^{-1}$ | $12.5 \%$ | $9.1 \%$ |  |
| $1.0 \times 10^{-3} \leq \delta<2.0 \times 10^{-3} \mathrm{~cm}^{-1}$ | $4.0 \%$ | $3.1 \%$ |  |
| $2.0 \times 10^{-3} \leq \delta<4.0 \times 10^{-3} \mathrm{~cm}^{-1}$ | $0.9 \%$ | 0.84 |  |
| Standard deviation $\left(10^{-3} \mathrm{~cm}^{-1}\right)$ | 0.50 |  |  |

Table 4

Character tables for the $\mathrm{C}_{2 \mathrm{v}}$ and $\mathrm{G}_{4}$ symmetry groups.

|  |  | Torsion | Bend | [ $\mathrm{J} \mathrm{K}_{\mathrm{a}} \mathrm{K}_{\mathrm{c}}$ ] | Nuclear Spin |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ${ }^{\text {Low }} \mathrm{g}^{\mathrm{v}}$, ${ }^{\text {High }} \mathrm{g}^{\text {v }}$ | ${ }^{\text {Low }} \mathbf{4}^{\text {v }}$, ${ }^{\text {High }} 4^{\text {v }}$ | $\mathrm{K}_{\mathrm{a}} \mathrm{~K}_{\mathrm{c}}$ <br> Parities |  |
| $\mathrm{G}_{4} \mathrm{E}$ (12) (12)* ${ }^{*}$ |  |  |  |  |  |
| $\begin{array}{lllllll}\mathrm{C}_{2 v} & \mathrm{E} & \mathrm{C}_{2 z} & \sigma_{y z} & \sigma_{\mathrm{xz}}\end{array}$ |  |  |  |  |  |
| $\begin{array}{llllll}\mathrm{A}_{1} & 1 & 1 & 1 & 1\end{array}$ | $J_{z} \quad \mu_{z}$ | $\mathrm{v}=\mathrm{e}$ Low | Low | e e | Sym: $\mathrm{gn}_{\mathrm{n}}=3$ |
| $\begin{array}{llllll}\mathrm{A}_{2} & 1 & 1 & -1 & -1\end{array}$ | $\mu_{z} \quad J_{z}$ | $v=0$ High |  | e o |  |
| $\begin{array}{lllll}\mathrm{B}_{1} & 1 & -1 & 1 & -1\end{array}$ | $\mu_{y}, J_{x}$ | $\mathrm{v}=0$ Low |  | 0 O | Antisym: $\mathrm{g}_{\mathrm{n}}=1$ |
| $\begin{array}{lllll}\mathrm{B}_{2} & 1 & -1 & -1 & 1\end{array}$ | $\mu_{x}, J_{y}$ | $v=e$ High | High | 0 e |  |

## Table 5

a) Symmetry properties of the $9^{v} \mathrm{OH}$ torsion -rotation wavefunctions

|  |  | OH -Torsion: $9^{v}$ $\mathrm{v}=$ even |  |  | OH -Torsion: $9^{v}$ v=odd |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma_{\text {Tors-Rot }}$ | $\mathrm{gn}_{\mathrm{n}}$ | $\mathrm{K}_{\mathrm{c}}$ | $\mathrm{K}_{\mathrm{a}}$ |  | $\mathrm{K}_{\mathrm{c}}$ | $\mathrm{K}_{\mathrm{a}}$ |  |
|  |  |  | ${ }^{\text {Low }}{ }^{\text {V }}$ | $\mathrm{High}^{\text {V }}$ |  | ${ }^{\text {Low }}{ }^{\text {v }}$ | ${ }^{\text {High }}{ }^{\text {V }}$ |
| $\mathrm{B}_{2}$ | 3 | e | 0 | e | 0 | e | 0 |
| $\mathrm{B}_{1}$ | 3 | 0 | 0 | e | e | e | 0 |
| $\mathrm{A}_{1}$ | 1 | e | e | 0 | 0 | 0 | e |
| $\mathrm{A}_{2}$ | 1 | 0 | e | 0 | e | 0 | e |

b) Symmetry properties of the 4 vOH -bending -rotation wavefunctions:

|  |  |  | $4^{\mathrm{v}}$ OH-bending |  |
| :---: | :---: | :---: | :---: | :---: |
| $\Gamma_{\text {Bend-Rot }}$ | $\mathrm{g}_{\mathrm{n}}$ | $\mathrm{K}_{\mathrm{c}}$ | $\mathrm{K}_{\mathrm{a}}$ |  |
|  |  |  | Low $^{\mathrm{v}}$ | High $^{\mathrm{v}}$ |
| $\mathrm{B}_{2}$ | 3 | e | 0 | e |
| $\mathrm{B}_{1}$ | 3 | 0 | 0 | e |
| $\mathrm{A}_{1}$ | 1 | e | e | 0 |
| $\mathrm{~A}_{2}$ | 1 | 0 | e | 0 |

## Table 6

A: Hamiltonian matrix for the resonating $\left\{9^{2}, 6^{1} 9^{1}\right\}$ energy levels

|  | $9^{2}$ | $6^{1} 9^{1}$ |
| :---: | :---: | :---: |
| $9^{2}$ | $\mathrm{E}_{99}+\mathrm{H}_{99}^{\mathrm{ROT}}+\mathrm{H}_{99}^{\mathrm{TORS}}$ | c.c. |
| $6^{1} 9^{1}$ | $69,99 \mathrm{~B}$ | $\mathrm{E}_{69}+\mathrm{H}_{69}^{\mathrm{ROT}}$ |

B: Hamiltonian matrix for the resonating $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ energy levels

|  | $4^{1}$ | $7^{1} 9^{1}$ | $6^{1} 7^{1}$ |
| :---: | :---: | :---: | :---: |
| $4^{1}$ | $\mathrm{E}_{4}+\mathrm{H}_{4}^{\mathrm{ROT}}+\mathrm{H}_{\mathrm{V}}^{\mathrm{BEND}}$ | c.c. | c.c. |
| $7^{1} 9^{1}$ | ${ }^{79,4} \mathrm{~B}$ | $\mathrm{E}_{79}+\mathrm{H}_{79}^{\mathrm{ROT}}$ |  |
| $6^{1} 7^{1}$ | ${ }^{67,4} \mathrm{Anh}+{ }^{67,4} \mathrm{C}$ |  | $\mathrm{E}_{67}+\mathrm{H}_{67}^{\mathrm{ROT}}$ |

## v-diagonal operators

## $\mathrm{E}_{\mathrm{v}}$ =vibrational energy

$$
\begin{aligned}
& \text { The } H_{v}^{R O T} \text { rotational operator } \\
& \qquad H_{v}^{R O T}=W_{v}+X Z_{v}
\end{aligned}
$$

$\mathrm{W}_{\mathrm{v}}=$ Watson's A-type operators (Ir representation)

$$
\begin{gathered}
W_{v}=B_{z}^{v} J_{z}^{2}+B_{x}^{v} J_{x}^{2}+B_{y}^{v} C_{v} J_{y}^{2} \\
-\Delta_{K}^{v} J_{z}^{4}-\Delta_{J K}^{v} J^{2} J_{z}^{2}-\Delta_{J}^{v} \mathbf{J}^{4}-2 \delta_{J}^{v} J^{2} J_{x y}^{2}-\delta_{K}^{v}\left\{J_{z}^{2}, J_{x y}^{2}\right\}+\ldots
\end{gathered}
$$

$X Z_{v}=$ non orthorhombic terms

$$
X Z_{v}=h_{x z}^{v}\left\{J_{x}, J_{z}\right\}+\ldots
$$

## Torsion (in 6-A) or bending (in 6-B)

$$
\left\langle\left. J K^{\prime} \gamma^{\prime}\right|^{\mathrm{IAM}} \mathrm{H} \mid J \mathrm{~K}^{\prime \prime} \gamma^{\prime \prime}\right\rangle=\varepsilon(-1)^{\mathrm{K}^{\prime}}\left(\cos \left(\left(\mathrm{K}^{\prime}+\mathrm{K}^{\prime \prime}\right) \varphi\right) \mathrm{d}_{\mathrm{K}^{\prime}, \mathrm{K}^{\prime \prime}}^{(\mathrm{J})}(\theta)+\gamma^{\prime \prime} \cos \left(\left(\mathrm{K}^{\prime}-\mathrm{K}^{\prime \prime}\right) \varphi\right) \mathrm{d}_{\mathrm{K}^{\prime},-\mathrm{K}^{\prime \prime}}^{(\mathrm{J}}(\theta)\right)
$$

For $\theta=0$ (see text) this expression leads to:
In A: $\left\langle J K^{\prime} \gamma^{\prime}\right| H^{\text {TORS }}\left|J K^{\prime \prime}=K^{\prime} \gamma^{\prime \prime}=\gamma^{\prime}\right\rangle=\varepsilon h^{\text {TORS }}(-1)^{K^{\prime}} \cos \left(2 \mathrm{~K}^{\prime} \varphi^{\text {TORS }}\right)$
In B: $\left\langle J K^{\prime} \gamma^{\prime}\right| H^{B E N D}\left|J K^{\prime \prime}=K^{\prime} \gamma^{\prime \prime}=\gamma^{\prime}\right\rangle=\varepsilon h^{\mathrm{BEND}}(-1)^{\mathrm{K}^{\prime}} \cos \left(2 \mathrm{~K}^{\prime} \varphi^{\mathrm{BEND}}\right)$
with $\varepsilon=+1$ (resp. $\varepsilon=-1)$ for $\Gamma\left(\varphi_{\text {vib-tors-bend-rot }}\right)=\mathrm{B}_{1}$ or $\mathrm{B}_{2}\left(\right.$ resp. $\Gamma\left(\varphi_{\text {vib-tors-bend-rot }}\right)=\mathrm{A}_{1}$ or $\left.\mathrm{A}_{2}\right)$

$$
\begin{aligned}
& \quad\left\langle J K^{\prime} \gamma^{\prime}\right| H^{\mathrm{BEND}}\left|\mathrm{~J} \mathrm{~K}^{\prime \prime} \gamma^{\prime \prime}\right\rangle=0 ; \quad\left\langle J K^{\prime} \gamma^{\prime}\right| H^{\mathrm{TORS}}\left|\mathrm{~J} \mathrm{~K}^{\prime \prime} \gamma^{\prime \prime}\right\rangle=0 \\
& \text { if } \mathrm{K}^{\prime} \neq \mathrm{K}^{\prime \prime} \text { or } \gamma^{\prime} \neq \gamma^{\prime \prime} \text {. }
\end{aligned}
$$

## $v$ - off- diagonal operators

B: B-type Coriolis:
$v^{v^{\prime}, v} B=v^{v^{\prime}, v} B_{x} J_{x}+v^{v^{\prime}, v} B_{y z}\left\{I_{y}, J_{z}\right\} \psi^{v^{\prime}, v} B_{x J} J_{x} J^{2}+{ }^{v^{\prime}, v} B_{x z z}\left\{J_{z}^{2}, J_{x}\right\}+v^{v^{\prime}, v} B_{x 3}\left(J_{+}^{3}+J_{-}^{3}\right)$
C: C-type Coriolis: $\quad{ }^{v^{\prime}, v} C^{v}{ }^{v^{\prime}, v} C_{y} J_{y}+{ }^{v^{\prime}, v} C_{x z}\left\{J_{x}, J_{z}\right\}$
Anh: anharmonic operator $v^{v^{\prime}, v} A n h={ }^{v^{\prime}, v} A n h_{x y} J_{x y}^{2}+v^{v^{\prime}, v} A n h_{x y z z}\left\{J_{z}^{2}, J_{x y}^{2}\right\}$ with:
c.c. complex conjugate;
$\{A, B\}=A B+B A, J_{x y}^{2}=J_{x}^{2}-J_{y}^{2}, \quad J_{ \pm}=J_{x} \mp i J_{y}$

## Wang type functions

$|\mathrm{JK} \gamma\rangle=1 / \sqrt{2}(|\mathrm{JK}\rangle+\gamma|\mathrm{J}-\mathrm{K}\rangle)$ (for $\mathrm{K} \neq 0, \gamma= \pm 1)$ and $|\mathrm{JK}=0 \gamma=+1\rangle=|\mathrm{J} 0\rangle \quad$ (for $\mathrm{K}=0$ ).

## Table 7

Hamiltonian constants (in $\mathrm{cm}^{-1}$ ) for the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ polyad of ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$

|  | G.S. ${ }^{\text {a }}$ | $9^{2}$ | $6^{1} 9^{1}$ |
| :---: | :---: | :---: | :---: |
| $\varphi^{\text {TORS }}$ (in ${ }^{\circ}$ ) |  | 1.0933(580) |  |
| $\mathrm{h}^{\text {TORS }} \times 10^{3}$ |  | 2.5321 (500) |  |
| $E_{V}$ |  | 1043.88608(8) | 1000.5546(650) |
| $\mathrm{h}_{\mathrm{xz}}^{\mathrm{V}} \mathrm{x} 10^{3}$ | 0. | -3.78530(600) | -3.78530 |
| $\mathrm{B}_{\mathrm{z}}$ | 0.3442138510 | $0.34169714(1900)$ | 0.3397651 (2800) |
| $\mathrm{B}_{\mathrm{x}}$ | 0.3368401260 | $0.33909619(1900)$ | 0.3371721 (2600) |
| $\mathrm{B}_{\mathrm{y}}$ | 0.1699352790 | $0.170013475(180)$ | $0.16960468(5400)$ |
| $\Delta_{\mathrm{K}} \times 10^{6}$ | 0.361050 | - 0.26634(330) | b |
| $\Delta_{\mathrm{JK}} \times 10^{6}$ | -0.123810 | $0.06767(300)$ | b |
| $\Delta_{\mathrm{J}} \times 10^{6}$ | 0.208660 | 0.168630 (140) | -0.2047(170) |
| $\delta_{K} \times 10^{6}$ | 0.1730980 | $0.13020(140)$ | b |
| $\delta_{J} \times 10^{7}$ | 0.918320 | 0.718751 (780) | b |
| $\mathrm{H}_{\mathrm{K}} \times 10^{11}$ | 0.2545 | b | b |
| $\mathrm{H}_{\mathrm{KJ}} \times 10^{11}$ | -0.10374 | b | b |
| $\mathrm{H}_{\mathrm{JK}} \times 10^{12}$ | -0.6990 | b | b |
| $\mathrm{H}_{\mathrm{J}} \times 10^{12}$ | 0.5337 | b | b |
| $h_{K} \times 10^{11}$ | 0.25600 | b | b |
| $h_{\text {JK }} \times 10^{12}$ | 0.2902 | b | b |
| $h_{J} \times 10^{12}$ | 0.2795 | b | b |

(b) Interaction constant (B- type Coriolis) : ${ }^{69,99} \mathrm{~B}_{\mathrm{x}}=1.7857(140) \times 10^{-3}$

## Table 8

Hamiltonian constants (in $\mathrm{cm}^{-1}$ ) for the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ polyad of ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$
(a) Vibrational energies, rotational, large amplitude torsional parameters

|  | G.S. ${ }^{\text {a }}$ | $9^{2}$ | $6^{1} 9^{1}$ |
| :---: | :---: | :---: | :---: |
| $\varphi^{\text {TORS }}$ ( in $^{\circ}$ ) |  | 1.2136(390) |  |
| $\mathrm{h}^{\text {TORS }} \times 10^{3}$ |  | $2.54516(1000)$ |  |
| $E_{V}$ |  | 1042.87549(1) | 1001.6994(90) |
| $\mathrm{h}_{\mathrm{xz}}^{\mathrm{V}} \times 10^{3}$ | 0. | -3.81573(550) | -3.81573 |
| $\mathrm{B}_{\mathrm{z}}$ | 0.3442527674 | $0.34177105(1700)$ | $0.34048063(4300)$ |
| $\mathrm{B}_{x}$ | 0.3368801619 | $0.33912576(1700)$ | 0.33753430 (6900) |
| $\mathrm{B}_{\mathrm{y}}$ | 0.1699552162 | $0.170036435(160)$ | $0.16971814(1900)$ |
| $\Delta_{K} \times 10^{6}$ | 0.360006 | $0.279582(990)$ | b |
| $\Delta_{\mathrm{JK}} \times 10^{6}$ | -0.123285 | $0.05201(100)$ | b |
| $\Delta_{J} \times 10^{6}$ | 0.208611 | $0.170484(400)$ | -0.08018(740) |
| $\delta_{\mathrm{K}} \times 10^{6}$ | 0.1732762 | $0.138954(960)$ | b |
| $\delta_{J} \times 10^{7}$ | 0.918552 | $0.72798(190)$ | b |
| $\mathrm{H}_{\mathrm{K}} \times 10^{11}$ | 0.2595 | b | b |
| $\mathrm{H}_{\mathrm{KJ}} \times 10^{11}$ | -0.1121 | b | b |
| $\mathrm{H}_{\mathrm{JK}} \times 10^{12}$ | -0.6838 | b | b |
| $\mathrm{H}_{\mathrm{J}} \times 10^{12}$ | 0.5003 | b | b |
| $h_{K} \times 10^{11}$ | 0.24951 | b | b |
| $\mathrm{h}_{\mathrm{JK}} \times 10^{12}$ | 0.3035 | b | b |
| $h_{J} \times 10^{12}$ | 0.2782 | b | b |

(b) Interaction constant (B-type Coriolis) : ${ }^{69,99} \mathrm{~B}_{\mathrm{x}}=1.60934(770) \times 10^{-3}$

## Table 9

Hamiltonian constants (in $\mathrm{cm}^{-1}$ ) for the $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1},\right\}$ polyad of ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$
(a) Vibrational energies, rotational, large amplitude bending parameters

|  | $4^{1}$ | $7{ }^{1} 9^{1}$ | $6^{17} 7^{1}$ |
| :---: | :---: | :---: | :---: |
| $\varphi^{\text {BEND }}$ (in ${ }^{\text {a }}$ ) | 1.07425(190) |  |  |
| $h^{\text {BEND }} \times 10^{3}$ | 1.78523(100) |  |  |
| $E_{v}$ | 961.73359(1) | 972.80547(4) | 932.0071(1) |
| $\mathrm{h}_{\mathrm{xz}}^{\mathrm{V}} \mathrm{x} 10^{3}$ | -1.81396(130) | -1.81396 | -1.81396 |
| $B_{z}$ | 0.342819186 (670) | $0.346918002(730)$ | 0.34421697(920) |
| $B_{\text {x }}$ | 0.337792467 (620) | $0.334863884(740)$ | $0.33832555(100)$ |
| $\mathrm{B}_{\mathrm{y}}$ | 0.1698203486 (63) | $0.1680236075(840)$ | $0.1702683793(900)$ |
| $\Delta_{\mathrm{K}} \times 10^{6}$ | $0.1211745(100)$ | -0.81833(180) | b |
| $\Delta_{\mathrm{JK}} \times 10^{6}$ | 0.0345027(900) | 0.67876(160) | b |
| $\Delta_{\mathrm{J}} \times 10^{6}$ | 0.22767587(660) | $0.115517(190)$ | b |
| $\delta_{K} \times 10^{6}$ | 0.3007202(450) | 0.429436(530) | b |
| $\delta_{J} \times 10^{7}$ | 0.9897934(900) | 0.796839 (920) | b |
| $\mathrm{H}_{\mathrm{K}} \times 10^{11}$ | b | b | b |
| $\mathrm{H}_{\mathrm{KJ}} \times 10^{11}$ | b | b | b |
| $\mathrm{H}_{\mathrm{JK}} \times 10^{12}$ | b | b | b |
| $\mathrm{H}_{\mathrm{J}} \times 10^{12}$ | -0.013268(760) | b | b |
| $h_{K} \times 10^{11}$ | b | b | b |
| $h_{\text {JK }} \times 10^{12}$ | b | b | b |
| $h_{J} \times 10^{12}$ | b | b | b |

(b) Interaction constants

| $7^{1} 9^{1} \Leftrightarrow 4^{1}$ | B-Coriolis | ${ }^{79,4} \mathrm{~B}_{\mathrm{x}}$ | $-1.4277277(770) \times 10^{-1}$ |
| :--- | :--- | :--- | :--- |
|  |  | ${ }^{79,4} \mathrm{~B}_{\times J}$ | $6.02342(160) \times 10^{-6}$ |
|  |  | ${ }^{79,4} \mathrm{~B}_{\times 3}$ | $4.83401(140) \times 10^{-6}$ |
| $6^{1} 7^{1} \Leftrightarrow 4^{1}$ | C- Coriolis | ${ }^{67,4} \mathrm{C}_{x z}$ | $-2.94749(380) \times 10^{-4}$ |
|  | Anharmonic | ${ }^{67,4} \mathrm{An}_{\mathrm{xy}}$ | $5.20874(520) \times 10^{-4}$ |
|  |  | ${ }^{6,4} \mathrm{An}_{\mathrm{xyzz}}$ | $9.72892(150) \times 10^{-8}$ |

Hamiltonian constants (in cm ${ }^{-1}$ ) for the $\left\{4^{1}, 7^{1} 9^{1}, 6^{1} 7^{1}\right\}$ polyad of ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$

|  | $4{ }^{1}$ | $7{ }^{1} 9^{1}$ | $6^{17} 7^{1}$ |
| :---: | :---: | :---: | :---: |
| $\varphi^{\text {BEND }}$ (in ${ }^{\text {a }}$ ) | 1.20244(250) |  |  |
| $h^{\text {BEND }} \times 10^{3}$ | 1.89810(160) |  |  |
| $E_{v}$ | 961.49330(1) | 970.99570(1) | 929.2077(6) |
| $h_{\mathrm{xz}}^{\mathrm{V}} \times 10^{3}$ | -1.44363(170) | -1.44363 | -1.44363 |
| $\mathrm{B}_{\mathbf{z}}$ | $0.342982214(660)$ | 0.343420051 (900) | 0.37616993 (430) |
| $B_{x}$ | 0.338127986 (660) | $0.337759784(900)$ | 0.32158867 (190) |
| $\mathrm{B}_{\mathrm{y}}$ | $0.1698268850(70)$ | 0.1680406191 (440) | $0.172324425(360)$ |
| $\Delta_{\mathrm{K}} \times 10^{6}$ | $0.462518(110)$ | $0.284814(370)$ | -3.01463(270) |
| $\Delta_{\text {JK }} \times 10^{6}$ | -0.3377696(860) | -0.104067(380) | 4.41391 (200) |
| $\Delta_{\mathrm{J}} \times 10^{6}$ | $0.25790756(26)$ | $0.1859195(990)$ | 0.238714(530) |
| $\delta_{\mathrm{K}} \times 10^{6}$ | $0.1719722(840)$ | b | b |
| $\delta_{J} \times 10^{7}$ | 0.919588(160) | 1.362515(510) | -4.85529(260) |
| $\mathrm{H}_{\mathrm{K}} \times 10^{11}$ | b | b | b |
| $\mathrm{H}_{\mathrm{KJ}} \times 10^{11}$ | b | b | b |
| $\mathrm{H}_{\mathrm{JK}} \times 10^{12}$ | b | b | b |
| $\mathrm{H}_{\mathrm{J}} \times 10^{12}$ | b | b | b |
| $\mathrm{h}_{\mathrm{K}} \times 10^{11}$ | b | b | b |
| $\mathbf{h}_{\text {JK }} \times 10^{12}$ | b | b | b |
| $h_{J} \times 10^{12}$ | b | b | b |

(b) Interaction constants

| $7^{1} 9^{1} \Leftrightarrow 4^{1}$ | B- Coriolis | ${ }^{79,4} \mathrm{~B}_{\times}$ | $-1.5583049(210) \times 10^{-1}$ |
| :--- | :--- | :--- | :--- |
|  |  | ${ }^{79,4} \mathrm{~B}_{\mathrm{xJ}}$ | $-2.82945(380) \times 10^{-6}$ |
|  |  | ${ }^{79,4} \mathrm{~B}_{\mathrm{xzz}}$ | $4.77886(610) \times 10^{-6}$ |
|  |  | ${ }^{79,4} \mathrm{~B}_{\times 3}$ | $3.59167(280) \times 10^{-7}$ |
| $6^{1} 7^{1} \Leftrightarrow 4^{1}$ | Anharmonic | ${ }^{67,4} \mathrm{An}_{x y}$ | $3.76857(220) \times 10^{-4}$ |
|  |  | ${ }^{6,4} \mathrm{An}_{\mathrm{xyzz}}$ | $-1.03756(130) \times 10^{-7}$ |



$279 \times 215 \mathrm{~mm}(150 \times 150$ DPI)

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$279 \times 215 \mathrm{~mm}(150 \times 150$ DPI)


[^0]:    ${ }^{1}$ For the ${ }^{10} \mathrm{BF}_{2} \mathrm{OH}$ and ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$ isotopic species we will use henceforth in the text the abbreviated notation 10B and 11B respectively. Properties and values, X , given both for 10 B and 11 B are quoted ${ }^{10} \mathrm{X}\left({ }^{11} \mathrm{X}\right)$, with values for 11 B given in parentheses. For conciseness, properties $Y$ referring both to the $9^{2}$ and $4^{1}$ polyads of interacting systems are labelled $\mathrm{Y} 9^{2} / \mathrm{Y} 4^{1}$ using a slash throughout the present work.

[^1]:    ${ }^{2}$ By convention, the $\mathrm{BF}_{2}$ moiety is defined as the rotor and the OH bond as the frame and the $\varphi$ angles are related to the rotor and frame moments of inertia by $2 \varphi=180^{\circ} \frac{I_{\text {Rotor }}}{I_{\text {Rotor }}+I_{\text {Frame }}}$.

[^2]:    Table 8
    Hamiltonian constants (in cm ${ }^{-1}$ ) for the $\left\{9^{2}, 6^{1} 9^{1}\right\}$ polyad of ${ }^{11} \mathrm{BF}_{2} \mathrm{OH}$

