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Economic Lot Scheduling Problems Incorporating a Cost of Using the Production Facility

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Abstract

This paper considers scheduling the production of several different items on a single machine with constrained capacity, commonly known as the Economic Lot Scheduling Problem (ELSP). Most traditional approaches for the ELSP consider the sum of the setup cost and inventory holding cost and provide cyclic schedules that minimize this sum. In practice, there are not only costs for setups and inventory holding, but also costs for operating the production facility due to e.g. electricity, service, maintenance, tools, operators etc, which depend on the number of hours the facility is operating per working day. In this paper, we modify the traditional cost function to include not only setup and inventory holding cost but also a time variable cost for operating the production facility. The paper shows it is possible to adapt a previous heuristic procedure to this complemented cost. The model can help to determine cyclic schedules and the number of production hours per working day.

Keywords: ELSP, Multi products, Single machine, Capacity constrained; Production facility cost

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1 Introduction

The economic lot scheduling problem, ELSP, is a common problem within the research literature. The problem is found in many practical applications: e.g. milling of gear houses, painting of metal rolls, paper production etc, both in process industries with more or less continuous flow and in work shops. The problem is concerned with scheduling the production of several different items consuming capacity on a single machine. The capacity is constrained but assumed to be sufficient to satisfy total demand. The solution to the ELSP is order quantities and a realizable schedule without backorders and at the same time minimizing total costs.

For this problem, the items are often scheduled in cycles and three categories of solutions have been found useful (i.e. common cycle approach, basic period approach, and extended basic period approach). The common cycle approach assumes that all products have the same cycle time, i.e. each item is produced once in every cycle. Imbalances in demand rates, product cost, or setup cost motivate the basic period approach, where the products are allowed to have different cycle times, which are integer multiples of a basic period. High volume products are produced in every cycle or basic period and lower volume products are produced less frequently (every other cycle, every fourth cycle etc.). This means that in some basic period all products may be produced, which implies that the sum of the setup times and operations times of all items must be less than or equal to the basic period. In the extended basic period approach, this constraint is relaxed but the basic period must be long enough to cover the average setup times and processing times for all products. Power-of-two (PoT) policies, where each production frequency is a multiplier of two, have become popular for this problem since they provide reasonably tight worst case bounds and simplify the construction

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3 of cyclic schedules (cf. Yao and Elmaghraby (2001)). Lopez and Kingsman (1991) suggest
4 that the power-of-two is a requirement for achieving schedule feasibility in practice. There are
5 many algorithms for determination of cyclic schedules (e.g. Bomberger (1966), Doll and
6 Whybark (1973), Goyal (1975), Haessler and Hogue (1976), Haessler (1979), Axsäter (1987),
7 Zipkin (1991), Gallego and Roundy (1992), Segerstedt (1999)). Elmaghraby (1978) presents
8 an overview of early research upon ELSP and he also presents own contribution. Lopez and
9 Kingsman (1991) make a review and compare different solution methods in one common
10 machine. Silver (1990) presents a model where the production rates can vary and they are
11 decision variables together with a common basic period. Among more recent research that
12 covers ELSP and special problems of ELSP Khoury et al (2001), Soman et al (2004), Cooke
13 et al (2004), Yao (2005) can be mentioned. Segerstedt (2004) shows that the heuristic from
14 Segerstedt (1999) can be modified to treat capacity-constrained multilevel production also in
15 more than one machine.

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36 Traditionally, in ELSP the setup/order costs together with inventory holding costs are
37 minimised under given capacity and demand restrictions; an exception is Silver (1990) that
38 introduces a cost per unit of operating time that can be avoided by idling the machine. In a
39 practical application it is often difficult to state the real setup/order cost since the setup cost is
40 mostly a result of the situation and the solution (cf. Karmarkar (1987)); e.g. low demand
41 means no lost-sales from many possible setups. A company has different expenditures and
42 costs if its production facility is producing 14 hours per working day instead of 7 hours per
43 day. Many costs are there and almost constant per hour regardless of if the facility is idle,
44 under setup, or under production, as long as the production facility is not closed down for the
45 evening. The cost per time unit of both production and setup in the production facility arises
46 due to the cost for having the facility available. To operate the facility demands mostly a lot

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3 of people present creating different types of costs: direct labour costs, indirect manufacturing
4 costs, consumption of electricity and energy, costs for tools, maintenance, necessities etc.;
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8 additions of costs just for the extra hour per day of production. If employees could leave
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10 earlier and the production could be stopped earlier every working day costs would be reduced.
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12 The facility cost affects the time (hours, shifts) the production facility should be operating
13 from an economical point of view and the facility cost will thus affect the scheduling and lot
14 sizing. The scheduling and lot sizing in their turn affect how many hours per working day the
15 facility must produce to satisfy current demand.
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24 The main purpose with this paper is to treat the ELSP-problem with an untraditional cost
25 model: a cost per time unit of both production and setup in the production facility; an out-of
26 pocket cost for the setup independent of the setup time and traditional inventory holding
27 costs. (All demand is assumed to be satisfied therefore the setup cost does not include any lost
28 sales, opportunity costs, the setup cost includes only expenditures created by the setup.) A
29 heuristic procedure, that is a modification of the heuristic in Nilsson and Segerstedt (2007), is
30 developed to determine cyclic schedules. (The heuristic easily finds the best known solution
31 32.07 to the Bomberger (1966) problem (cf. Segerstedt (1999), Nilsson and Segerstedt
32 (2008)).) The main idea behind this heuristic is to find as equal balance as possible between
33 the replenishment and the inventory holding costs for all items; this idea originates from the
34 simple economic order quantity (inventory holding cost = replenishment cost; cf. Harris
35 (1913)) and it has shown high-quality results for multiple items when it concerns the Joint
36 Replenishment Problem (cf. Nilsson et al (2007)) and the one warehouse N-retailer
37 distribution problem (cf. Abdul-Jalbar et al (2007)).
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3 From a set number of production hours for the facility the heuristic searches the lowest cost
4 for different and realisable cycles, in the sense that demand is satisfied and there is enough
5 time to produce and schedule the demanded quantities. Due to labour contracts and legislation
6 it may not be possible to vary the working hours below a normal 8-hour shift and it may also
7 for the same reasons be constrained how several shifts during twenty-four hours can be
8 designed. This in its turn constrains how many possible working hours can be made available
9 per working day. The number of production hours per working day can therefore not be seen
10 as a continuous variable; and the cost per production hour may also vary depending on how
11 many production hours, and shifts, the facility has per day. Investment costs; depreciations,
12 capital costs, are sunk costs (c.f. Horngren et al (2003)) and not variable costs with the
13 number of working hours. Costs for the sales department should not either be included in the
14 production facility costs per working hour, only costs that vary with the number of working
15 hours should be included.
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36 For different number of production hours per day the facility cost and its cost per hour can be
37 settled. The facility cost per hour for 7-8 hours per day may differ from the facility costs of
38 15-16 hours per day; the latter may be lower. For different possible number of production
39 hours and their facility costs per hour, our model can identify the number of hours resulting in
40 the lowest cost and the result can be used as an aid to decide how to operate the facility. From
41 the model the number of working hours per day the production facility should be opened can
42 be decided and economical cyclical patterns for the different produced items to follow.
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55 The remainder of the paper is organized as follows. In the next section we present the
56 assumptions and notations that are used. Section three presents the model in detail. In order to
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test the model, it is applied to numerical examples in section four. Conclusions and extensions are given in the final section.

2 Notation and Assumptions

We produce multiple items, one at a time, on a single facility. The capacity is constrained, but sufficient to meet the total demand. No backorders are allowed. There is a setup time and setup cost between the production of each item and these are independent of the production order. Product demand rates as well as the production rate for each item are deterministic and constant over time. The inventory holding costs are determined from the price of the item together with an interest rate and are assumed to be proportional to the number of units in the inventory. The facility cost is assumed to be linear (within certain intervals, i.e. different shifts may change the cost stepwise) to the time the facility is operating. A power-of-two policy is used for the scheduling of items since it simplifies the determination of a cyclic schedule.

We consider N items, which are produced on a capacity constrained machine and introduce the following notations. (As we have to deal with costs and money we chose to use SEK (1 SEK \approx 1/9 EUR \approx 1/7 USD)):

- A_i Set-up cost per production lot for item i (SEK)
- $C_i(f_i)$ Average total cost for item i (SEK/day)
- d_i Demand rate for item i (units/day)

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f_i	Number of times (frequency) that item i is produced in a specified time interval
FC	Production facility cost (SEK/hour)
h_i	Inventory holding cost of item i (SEK/unit and day)
o_i	Operation time of item i (hours/unit)
p_i	Production rate for item i (units/day)
q_i	Order quantity of item i
s_i	Set-up time for item i (days)
t_i	Set-up time for item i (hours)
T	Common cycle time (days)
T_i	Cycle time of item i (days)
V	Number of hours the facility is available for production (set-ups and operations) per day
$\rho = \sum_i \frac{d_i}{p_i}$	Utilization of the facility
Θ_K	A set of K items from the total numbers of items $\{1, 2, \dots, N\}$

3 Model formulation

In the traditional ELSP the items are produced in cycles. We consider an extended basic period approach where the production cycle of each item is a multiple of a basic period. We introduce f_i as the number of times ($\dots, 1/8, 1/4, 1/2, 1, 2, 4, 8, \dots$ etc in powers-of-two) that item i is produced during a specified time interval, T . This implies that item i has a cycle time equal to $T_i = T/f_i$. The traditional way to formulate the average total cost per day for a

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possible specified time interval and the chosen frequencies is (first the setup cost and then the inventory holding cost, cf. Segerstedt (1999), Nilsson and Segerstedt (2008)):

$$C = C(\mathbf{f}) = \sum_{i=1}^N \left(\frac{f_i A_i}{T} + h_i \frac{d_i T}{2 f_i} \left(1 - \frac{d_i}{p_i} \right) \right) \quad (1)$$

This can be differentiated with respect to T , which results in the cycle time that presents the lowest cost as:

$$T_{opt} = \sqrt{\frac{\sum_i f_i A_i}{\sum_i h_i d_i \left(1 - \frac{d_i}{p_i} \right) / (2 f_i)}} \quad (2)$$

Additionally, the shortest time interval all items can be produced with the chosen frequencies without shortages is (cf. Segerstedt (1999) and in line with the feasibility conditions for ELSP presented in Eilon (1962) and Haessler and Houge (1976)):

$$T_{inf}(\mathbf{f}) = \frac{\sum_i f_i s_i}{1 - \sum_i \frac{d_i}{p_i}} \quad (3)$$

Therefore the shortest time, T , that presents the lowest cost but also possible solution without shortages is:

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$$T^* = \max \left\{ \sqrt{\frac{\sum_i f_i A_i}{\sum_i h_i d_i \left(1 - \frac{d_i}{p_i}\right) / (2f_i)}}, \frac{\sum_i f_i s_i}{1 - \sum_i \frac{d_i}{p_i}} \right\} \quad (4)$$

The operation time, o_i , (hours per unit (or minutes per unit)), for each item is mostly limited to the inherent production speed in the facility or machine. It is an important parameter mostly registered in companies' computer systems for Enterprise Resource Planning (ERP), used for calculations of work load etc. However, what can be produced per working day is also decided by how many production hours there are per day. The production rate (units/working day), p_i , can be determined as:

$$p_i = \frac{V}{o_i} \quad (5)$$

where V is the number of hours per day the facility is operating. t_i is the setup time in hours (or minutes), it is an important parameter also mostly registered in the companies ERP-systems, used for calculations of work load etc. Changing the number of hours the facility is operating will affect s_i , which is the setup time in days:

$$s_i = \frac{t_i}{V} \quad (6)$$

We assume that there is a cost for the production facility regardless of if the facility is operating, under setup, or idle. The setup cost will then only represent the cost for waste of material and other possible expenditures due to the setup. We introduce the facility cost per

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hour as FC , here a constant, and the cost for the facility would be $FC \cdot V$. We introduce this; eq. (5) and eq. (6) to the cost function in eq. (1) and get:

$$C = \sum_{i=1}^N \left(\frac{f_i A_i}{T} + h_i \frac{d_i T}{2 f_i} \left(1 - \frac{o_i d_i}{V} \right) \right) + FC \cdot V \quad (7)$$

Eq. (5) and eq. (6) introduced to eq. (4) presents:

$$T^* = \max \left\{ \sqrt{\frac{\sum_i f_i A_i}{\sum_i h_i d_i \left(1 - \frac{o_i d_i}{V} \right) / (2 f_i)}}, \frac{\sum_i f_i \frac{t_i}{V}}{1 - \sum_i \frac{o_i d_i}{V}} \right\} \quad (8)$$

For the search for and determination of frequencies, we formulate the following heuristic procedure (analogous to Nilsson and Segerstedt (2008)):

STEP 0 (An initial solution with a common frequency (or cycle) is created)

$$f_i = 1 \quad \forall i \quad (9)$$

Determine the shortest time interval due to the capacity all items can be produced with the chosen frequencies:

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$$T_{\text{inf}}(\mathbf{f}) = \frac{\sum_i f_i \frac{t_i}{V}}{1 - \sum_i \frac{o_i d_i}{V}} \quad (10)$$

If $T_{\text{inf}}(\mathbf{f}) < 0$ Go to STOP. (i.e. there is no possible solution; the capacity does not cover the required demand. It may be possible to increase the number of production hours per day and thereby add more available time per production day)

Compute the lowest cost according to a possible specified time interval and the chosen frequencies:

$$C_0 = C(\mathbf{f}) = \sum_{i=1}^N C_i(f_i) = \sum_{i=1}^N \left(\frac{f_i A_i}{T^*} + \frac{h_i d_i T^*}{2 f_i} \left(1 - \frac{o_i d_i}{V} \right) \right) + FC \cdot V \quad (11)$$

where

$$T^* = \max \left\{ \sqrt{\frac{\sum_i f_i A_i}{\sum_i h_i d_i \left(1 - \frac{o_i d_i}{V} \right) / (2 f_i)}}, \frac{\sum_i f_i \frac{t_i}{V}}{1 - \sum_i \frac{o_i d_i}{V}} \right\} \quad (12)$$

Compute the ratio between the set-up cost and inventory holding cost for all items:

$$R_i(\mathbf{f}) = \frac{f_i A_i / T^*}{h_i \left(1 - \frac{o_i d_i}{V} \right) d_i T^* / (2 f_i)} \quad \forall i \quad (13)$$

Let:

$$K \leftarrow N \quad (14)$$

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STEP 1

Find the item with the maximum ratio or maximum inverted ratio:

$$k \leftarrow \arg \max_{i \in \Theta_K} (R_i, 1/R_i) \quad (15)$$

If $R_k > 1$ then $f'_k \leftarrow f_k / 2$ else $f'_k \leftarrow 2 \cdot f_k$

($R_k > 1$ means that the setup cost is large compared to the holding cost then a decrease in the frequency may create a lower total cost; $R_k < 1$ implies that the holding cost is large compared to the setup cost then an increase in the frequency may create a lower total cost.)

If $T_{\text{inf}}(\mathbf{f}') < 0$ then ($\mathbf{f}' \leftarrow \mathbf{f}$, $\Theta_{K-1} \leftarrow \Theta_K \setminus \{k\}$, $K \leftarrow K - 1$, If $K \geq 1$ Go to STEP 1 else Go to STEP 2).

If $C(\mathbf{f}') < C_0$ then ($C_0 = C(\mathbf{f}')$, $\mathbf{f} \leftarrow \mathbf{f}'$, $K \leftarrow N$, Compute $R_i(\mathbf{f}) \forall i$)

else ($\Theta_{K-1} \leftarrow \Theta_K \setminus \{k\}$, $K \leftarrow K - 1$)

(If a change of the frequency for the “best” item ends up with a shortage or not a lower total cost this item is eliminated from the lot of items tested, and then the “next best” item is tested etc until the lot of items is empty, i. e. all items are investigated. Observe that if a lower cost is found; new ratios are calculated and the test starts again with all items.)

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If $K \geq 1$ Go to Step 1.

STEP 2 (The recursion procedure does not find any better solution)

The frequencies are then scaled so that the lowest f_i is set to one (cf. Nilsson and Segerstedt (2008)). T^* should then be recalculated for these frequencies from eq. (12).

$$q_i = d_i T^* / f_i \quad \forall i$$

STOP.

From the frequencies, a cyclic schedule can be created. During T^* days, all days working V hours per day, the demanded volume $d_i \cdot T^*$ is produced (for all items i), for some item maybe once during the T^* days in one order quantity if $f_i = 1$, but for others f_j times each time with an order quantity $d_j \cdot T^* / f_j$. The total working hours $T^* \cdot V$ is enough for all necessary set-up times and operation times for all items, and after T^* days the ending inventories of all items will be of the same size as the initial inventories and during the next T^* days the same production cycles can be repeated. But sequences in the production, in which order the different items are produced, will influence the average inventories and therefore the inventory holding costs. The replenishment of one item may have to start before its inventory comes down to zero, to avoid shortages of other items; and there are a lot of possible sequences, so it is hard to find the minimum cost (cf. Nilsson & Segerstedt (2008)).

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4 Numerical example

In order to show and test the heuristic presented in section 3, it is applied to two numerical examples considering the production of five items. (We only treat five items in our examples and therefore we exaggerate the differences between the items. We want to show that the model can manage a variety in values. In practical situations, according to our opinion, the differences for price/value (and therefore h_i) and operation times may be less separated. In a machine or facility product “families” may also be manufactured, where the differences between families are more diverse than inside the “family”. Our examples can therefore be seen as the different items representing different families.)

Example 1

The characteristics of the items in our first example are shown in table 1.

(Table 1)

To produce this demand without shortages the possible lowest integer number of production hours per day is 5 hours. 4 hours per day is not possible because then demand can not be satisfied ($V = 4 \rightarrow \rho = 1.20$ and $T_{\text{inf}} = -19.3$). The outcome for a number of production hours, V , between 5 and 16 is investigated and showed in table 3. First in table 2 is shown how the start solution could be seen in a spreadsheet model ($V = 8, FC = 0$). The next step is then to increase the frequency of item D to 2. The search algorithm eventually ends up with the frequencies (1, 2, 2, 2, 1) shown in table 3. With the spreadsheet model it is easy to see what the extra cost would be if we instead of $T = 8.4$ days used $T = 9$ days to repeat our schedule, much more useful from a practical point of view; the cost then increases from 3690

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to 3914. ($T = 8$ days, $V = 8$ hours/day and frequencies (1, 2, 2, 2, 1) are not possible, because $T_{\text{inf}} = 8.4$, shortages will then happen.) The increase of costs in this case is not negligible, but with longer cycle times the increase to the nearest integer is often small.

(Table 2)

Table 3 shows the end solutions with a zero facility cost per hour, it shows the daily total setup cost, TSC ; daily total holding cost, THC ; and the daily facility cost, TFC . C is the total daily cost. The most “economical” frequencies depend on the number of hours since the setup times (in days) change when V is changed. (The calculation for 9 hours, and 15 hours, results in a frequency of 1/2 for item A. Therefore, the frequencies are scaled so that the smallest frequency is one and the cycle time is recalculated.) 15 hours per day are the lowest number of hours per day where for the common cycle solution ($f_i = 1 \quad \forall i$) T_{inf} is smaller than T^* ($1.47 < 1.51$).

(Table 3)

Table 3 shows that the most economical plan is to have the facility operating 16 hours and even more ($V = 24$, $C = 1804$)! This makes no sense, when most of the time the facility would be idle and the demand can easily be satisfied within 8 or less hours.

Therefore we assume a facility cost greater than 0. We test three different facility costs 600 SEK/hour, 1800 SEK/hour and 6000 SEK/hour. The results are shown in table 4, 5 and 6.

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(Table 4)**(Table 5)****(Table 6)**

As we can see from tables 3-6, the facility cost, FC , will highly affect the number of production hours that are profitable. As the facility cost increases, it is profitable to decrease the number of production hours and increase the utilisation of the facility. The “economical” frequencies will not change with the facility cost, only the daily number of production hours will affect the frequencies and hence the schedule. However, the schedule in its turn influences how many production hours that are necessary to cover the demand. One shift in a machine or production facility may mostly mean 8 hours of working time, but dependent of how t_i and o_i are defined then there would be no time for cleaning and maintenance, therefore $V = 7$ could be the proper time for a one shift solution. In example 1 the set-up costs are small compared to the holding costs, therefore T_{inf} constrains the order quantities presenting the lowest cost even though the utilisation factor, ρ , is small. In the next example, example 2, setup costs are larger compared to inventory holding costs; then T_{inf} does not constrain the solutions so much.

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Example 2

The items in our second example have the characteristics in table 7.

(Table 7)

Table 7 presents the solutions with $FC = 0$ SEK/hour.

(Table 8)

The number of production hours cannot be decreased below 6, since the utilization exceeds one in that case and demand can not be satisfied. We now assume a facility cost greater than 0 SEK/hour. First, it is set to moderately to 100 SEK/hour and then increased a bit to 500.

(Table 9)

(Table 10)

As can be seen from tables 8-10, when the inventory holding cost exceeds the setup cost for T_{inf} , the cycle time is prolonged by idle time so that these costs are equal. Then, the setup cost and inventory holding cost are balanced (i.e. equal). A solution like in table 10 in a practical application, where the production cycle is repeated first after 173 days, with 40% of the items produced only once during these 173 days (a little less than twice a year), is a dangerous strategy if something happens with the demand. We believe an order quantity larger than half of the yearly demand would be allowed only in special cases, and a sensible restriction in practical applications.

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5 Discussion, Conclusions and Extensions

In this paper we present a complementary cost analysis to the economic lot scheduling problem; we consider not only setup costs and inventory holding cost, but also a facility cost in form of a cost per hour. A heuristic (from Nilsson and Segerstedt (2008)) we modify to make it handle this additional cost parameter. The new heuristic and model can be used to investigate the effect of the number of hours the production facility should operate per working day and from it can be determined cyclic production schedules. Profitability is not created by financial goals; it is created by how efficiently the company can manage their processes. It is created by low investments in inventory and work-in-process, high utilisation of equipment, buildings and personnel, high delivery performance and service to customers. The model presented in this paper can hopefully act as a tool for managers/planners to make useful decisions for how to manage the facility.

In the introduction of the paper we mentioned costs to include in the production facility cost, such as labour costs, indirect manufacturing costs, electricity, tools, operators, maintenance, materials etc. From an operational point of view time variable costs are interesting; investment costs and depreciations are sunk costs and should not influence the decision of how many hours per day the facility should be in work. Our examples show that neglecting the time variable costs of a machine or production facility can lead to most peculiar and illogical solutions (cf. table 3).

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3 In our test we use the same facility cost for different numbers of production hours, but in a
4 practical situation our opinion is that the facility cost is not totally invariant to the number of
5 production hours. 16 hours per day are expected to have a somewhat lower cost per hour than
6 8 hours per day, because the next 8 hours may require a little less assisting crew (less indirect
7 manufacturing costs). However, our small examples show that ($FC =$) 1000 SEK/hour or
8 1200 SEK/hour do not change the solution so much; the solution is more dependent on the
9 available hours ($= V$).

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22 In a practical application where several items are produced in the same machine or facility it
23 is necessary to have decided order sizes and/or a cyclic schedule follow. But if we always
24 start production of the next item immediately after another item; the inventories will increase.
25 That happens because the production rate of the machine and the set-up times are enough for
26 the demand, which should it be otherwise there will be shortages, and mostly there is also a
27 (small) surplus of capacity (i. e. $\sum_i (f_i s_i + d_i T/p_i) < T$). This possible surplus then without
28 sometimes stopping or idling the machine will cause inventories to rise. When in practical
29 applications there also are variations in demand as well as not deterministic operation times it
30 is important in the short run to avoid shortages of some items and in the long run to avoid
31 large and increasing inventories. Brander et al (2005) show that it possible to use
32 deterministic ELSP models when demand is stochastic, chosen order size is important but
33 very important is a decision rule, when to produce and when not to produce. Levén and
34 Segerstedt (2007) present a heuristic method that is based on an extension of Leachman and
35 Gascon (1988), Leachman et al (1991). This heuristic contains decision rules that try to avoid
36 shortages in the short run by decreasing the order sizes and in the long run to avoid excess
37 inventories by complementary decision rules when to idle the machine.

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3 As seen in our examples a production facility time variable cost, even a small one, in most
4 cases, makes a high usage, ρ , of the facility economically beneficial. Nilsson and Segerstedt
5 (2008) show that even for ELSP problems with high utilisation it is possible to find a feasible
6 solution (fulfilling feasibility conditions from Eilon (1962), Goyal (1975), Segerstedt (1999)
7 when $T \geq T_{\text{inf}}(\mathbf{f})$), but the real inventory holding cost is mostly higher than the common used
8 approximation. This is because the production of an item has to start before the inventory of
9 the same item reaches zero, to avoid a future shortage of the same item. The common cycle
10 solution always presents the “right” inventory holding cost; but diversifications of the
11 frequencies may not accomplish what it promises. Therefore, a most important extension to
12 the ELSP in general is to find a solution method that can evaluate different schedules due to
13 different frequencies, and find the right inventory holding cost without the common
14 approximation and combine it with a facility time variable cost as in this paper. An idea to
15 start from could be Cooke et al (2004). They have formulated a mixed integer programming
16 (MIP) problem that from a given basic period and integer number of basic periods for each
17 item minimises costs and avoids shortages. Such a model could be applied as an extension to
18 our problem, but an important disadvantage of MIP is long calculation times not suitable in an
19 application especially when in our case to find the best solution several MIP calculations have
20 to be done after each other. Then proper order sizes and production cycles from correct
21 economical analysis can be created, but in practical applications they must still be
22 complemented by decision rules when to produce and when not to produce, i.e. when possible
23 idle times should be implemented.

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56 Another extension to consider is sequence dependent set-up times (cf. Brander (2006)).
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58 However, if in a machine or facility the same items are produced with regularity it may not be
59 as complicated in practice as it is in theory. A schedule with the frequencies attained from the
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3 model in this paper could be established in such a way that most of the sequence from
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5 minimised set-up times also could be achieved. Another extension would be to maximise cash
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7 flow, the contribution or the production per time unit instead of minimising costs. By
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9 computing the present value of cash flow a more correct estimation of costs is also provided
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11 (cf. Grubbström (1980)). Such an extension may also present a more reliable model that
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13 increases the value for the possible user.
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Tables:

Table 1. Item characteristics.

Item	A	B	C	D	E
d_i (units/day)	400	400	800	1600	80
o_i (hours/unit)	0.0027	0.0010	0.0008	0.0016	0.0013
h_i (SEK/unit & day)	0.125	1.25	0.3125	0.625	0.9167
A_i (SEK)	800	200	300	100	500
t_i (hours)	1	6	2	4	2

Table 2. Start solution, the common cycle solution, in a spreadsheet model; $V = 8$, $FC = 0$

$T^* = 4.66$	i = A	B	C	D	E	Σ
f_i	1	1	1	1	1	
$\frac{f_i A_i}{T^*}$	171.52	42.88	64.32	21.44	107.20	407
$\frac{h_i d_i T^*}{2 f_i} \left(1 - \frac{o_i d_i}{V} \right)$	100.86	1107.74	536.38	1585.82	168.80	3500
$V \cdot FC$	-	-	-	-	-	0
C						3907
R_i	1.701	0.039	0.120	0.014	0.635	
$q_i = d_i T^* / f_i$	1866	1866	3731	7463	373	$T_{inf} = 4.66$

Table 3. End solutions with $FC = 0$.

V	ρ	f_A	f_B	f_C	f_D	f_E	T^*	T_{inf}	TSC	THC	TFC	C
5	0.96	1	1	1	1	1	69.4	69.4	27	44341	0	44368
6	0.80	1	2	2	2	1	22.2	22.2	113	8268	0	8381
7	0.68	1	2	2	2	1	12.2	12.2	205	4758	0	4963
8	0.60	1	2	2	2	1	8.40	8.40	298	3392	0	3690
9	0.53	1	2	2	4	2	8.77	8.77	366	2695	0	3059
15	0.32	1	4	4	8	2	6.75	6.75	681	1243	0	1924
16	0.30	1	4	4	8	2	6.15	6.15	748	1138	0	1886

Table 4. End solutions with $FC = 600$ SEK/hour.

V	ρ	f_A	f_B	f_C	f_D	f_E	T^*	T_{inf}	TSC	THC	TFC	C
5	0.96	1	1	1	1	1	69.4	69.4	27	44341	3000	47368
6	0.80	1	2	2	2	1	22.2	22.2	113	8268	3600	11981
7	0.69	1	2	2	2	1	12.2	12.2	205	4758	4200	9163
8	0.60	1	2	2	2	1	8.40	8.40	298	3392	4800	8490
9	0.53	1	2	2	4	2	8.78	8.78	365	2695	5400	8459
15	0.32	1	4	4	8	2	6.75	6.75	681	1243	9000	10924
16	0.30	1	4	4	8	2	6.15	6.15	748	1138	9600	11486

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Table 5. End solutions with $FC = 1800$ SEK/hour.

V	ρ	f_A	f_B	f_C	f_D	f_E	T^*	T_{inf}	TSC	THC	TFC	C
5	0.96	1	1	1	1	1	69.4	69.4	27	44341	9000	53368
6	0.80	1	2	2	2	1	22.6	22.6	113	8268	10800	19181
7	0.69	1	2	2	2	1	12.2	12.2	205	4758	12600	17563
8	0.60	1	2	2	2	1	8.40	8.40	298	3392	14400	18161
9	0.53	1	2	2	4	2	8.78	8.78	365	2695	16200	19260
15	0.32	1	4	4	8	2	6.75	6.75	681	1243	27000	28924
16	0.30	1	4	4	8	2	6.15	6.15	748	1138	28800	30686

Table 6. End solutions with $FC = 6000$ SEK/hour.

V	ρ	f_A	f_B	f_C	f_D	f_E	T^*	T_{inf}	TSC	THC	TFC	C
5	0.96	1	1	1	1	1	69.4	69.4	27	44341	30000	74408
6	0.80	1	2	2	2	1	22.6	22.6	113	8268	36000	44381
7	0.69	1	2	2	2	1	12.2	12.2	205	4758	42000	46963
8	0.60	1	2	2	2	1	8.40	8.40	298	3392	48000	51690
9	0.53	1	2	2	4	2	8.78	8.78	365	2695	54000	57060
15	0.32	1	4	4	8	2	6.75	6.75	681	1243	90000	91924
16	0.30	1	4	4	8	2	6.15	6.15	748	1138	96000	97886

Table 7. Item characteristics.

Item	A	B	C	D	E
d_i (units/day)	450	300	50	700	125
o_i (hours/unit)	0.0027	0.0030	0.0267	0.0020	0.0080
h_i (SEK/unit & day)	0.0042	0.0104	0.0313	0.0625	0.0208
A_i (SEK)	1500	450	300	700	500
t_i (hours)	4	3	4	1	2

Table 8. End solutions with $FC = 0$ SEK/hour.

V	ρ	f_A	f_B	f_C	f_D	f_E	T^*	T_{inf}	TSC	THC	TFC	C
6	0.97	1	2	1	8	2	173	173	54	808	0	862
7	0.83	1	4	4	4	4	39.6	38.3	235	235	0	470
8	0.73	1	2	2	8	2	45.5	14.0	211	211	0	422
9	0.65	1	2	2	8	2	45.0	9.52	213	213	0	427

Table 9. End solutions with $FC = 100$ SEK/hour.

V	ρ	f_A	f_B	f_C	f_D	f_E	T^*	T_{inf}	TSC	THC	TFC	C
6	0.97	1	2	1	8	2	173	173	54	808	600	1462

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7	0.83	1	4	4	4	4	39.6	38.3	235	235	700	1170
8	0.73	1	2	2	8	2	45.5	14.0	211	211	800	1222
9	0.65	1	2	2	8	2	45.0	9.52	213	213	900	1327

Table 10. End solutions with $FC = 500$ SEK/hour.

V	ρ	f_A	f_B	f_C	f_D	f_E	T^*	T_{inf}	TSC	THC	TFC	C
6	0.97	1	2	1	8	2	173	173	54	808	3000	3862
7	0.83	1	4	4	4	4	39.6	38.3	235	235	3500	3970
8	0.73	1	2	2	8	2	45.5	14.0	211	211	4000	4422
9	0.65	1	2	2	8	2	45.0	9.52	213	213	4500	4927

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