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Capacity and material requirement planning modelling by comparing deterministic and fuzzy models**

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Summary

A model for the capacity and material requirement planning problem with uncertainty in a multi-product, multi-level and multi-period manufacturing environment is proposed. An optimization model is formulated which takes into account the uncertainty that exists in both the market demand and capacity data, and the uncertain costs for backlog. This work uses the concept of possibilistic programming by comparing trapezoidal fuzzy numbers. Such an approach makes it possible to model the ambiguity in market demand, capacity data, cost information, etc. that could be present in production planning systems. The main goal is to determine the master production schedule, stock levels, backlog, and capacity usage levels over a given planning horizon in such a way as to hedge against the uncertainty. Finally, the fuzzy model and the deterministic model adopted as the basis of this work are compared using real data from an automobile seat manufacturer. The paper concludes that fuzzy numbers could improve the solution of production planning problems.

Keywords: Production planning, Uncertainty modelling, Fuzzy methods, Possibility theory.

1 Introduction

Many possible forms of uncertainty could be present in the decision environment of a production planning system such as market demand, capacity data, cost information, etc. The consideration of uncertainty in manufacturing systems means a great advance, in terms of describing reality, but it may pose problems for solving the model. Models for production planning which do not recognize

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4 uncertainty can be expected to generate inferior planning decisions as compared to models which
5 explicitly take into account uncertainty. Over the years, many researches and applications aimed to
6 formalize the uncertainty in production planning systems. Mula *et al.* (2006b) review some of the
7 existing literature (82 citations from 1983 to 2004) about uncertainty modelling in production
8 planning problems.
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14 Different stochastic modelling techniques based on probability distributions have been
15 successfully applied to production planning problems with randomness, which comes from the
16 random nature of events, and deals with uncertainty regarding membership or non-membership of
17 an element in a set. Stochastic programming is appropriate if parameters are given as random
18 variables. Bitran and Yanasse (1984) propose deterministic approximations to a non-sequential
19 capacity planning model and analyze its effectiveness when the demand is characterized by
20 standard probability distributions. Paraskevopoulos *et al.* (1991) use a robust approach to solve a
21 production capacity planning problem with a stochastic demand. Escudero *et al.* (1993) analyze
22 different modelling approaches for the production and capacity problem using stochastic
23 programming. Gfrerer and Zäpfel (1995) present a multi-period hierarchical production planning
24 model with two planning levels, i.e. aggregate and detailed, and with uncertain demand. Kira *et al.*
25 (1997) propose a hierarchical approach to model the multi-period and multi-product production
26 programming problem with a finite set of demands through stochastic linear programming. Gupta
27 and Maranas (2003) and Sodhi (2005) address the problem of supply chain production planning
28 under demand uncertainty using a stochastic programming based approach.
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44 When considering uncertainty in models, some difficulties can arise to estimate parameters such
45 as high cost in acquiring data information and lack of statistical observations. Since sufficient data
46 are not always available for predicting uncertain parameters, the choice of the fuzzy set theory is
47 more logical and convincing for the expression of the uncertainty by the expert knowledge. Also,
48 probability distributions derived from evidences recorded in the past are not always reliable
49 because of market changes, which influence demand or backlog costs, or technological innovation,
50 which in turn has an influence on capacity data. In such situations, the fuzzy set theory has been
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4 used to model systems that are difficult to define accurately (Bellman and Zadeh, 1970; Dubois and
5 Prade, 1980; Zimmermann, 1996). This theory represents an attractive tool to support the
6 production planning research when the dynamics of the manufacturing environment limits the
7 specification of the model objectives, constraints and parameters. Guiffrida and Nagi (1998) provide
8 an exhaustive literature survey on the fuzzy set theory applications in production management
9 research.

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12 It is necessary to distinguish between the fuzziness or flexibility in constraints and goals and the
13 uncertainty of the data or epistemic uncertainty. Flexibility is modelled by fuzzy sets and may reflect
14 the fact that constraints or goals are linguistically formulated; their satisfaction is a matter of
15 tolerance and degrees, or fuzziness (Bellman and Zadeh, 1970). Epistemic uncertainty is
16 concerned with ill-known parameters modelled by fuzzy intervals in the possibility theory setting
17 (Zadeh 1978; Dubois and Prade 1988).

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20 A few research works have been carried out in the area of fuzzy production planning. Some
21 applications of flexible programming in production planning problems can be found in Miller *et al.*
22 (1997), Pendharkar (1997), Itoh *et al.* (2003), Melian and Verdegay (2005) and Mula *et al.* (2006a).
23 Other applications of possibilistic programming in production planning problems can be found in
24 Gen *et al.* (1992), Inuiguchi *et al.* (1994), Hsu and Wang (2001), Wang and Fang (2001), Lodwick
25 and Bachman (2005) and Wang and Liang (2005).

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28 With the aim to demonstrate the usefulness and significance of the fuzzy programming, in this
29 paper a possibilistic approach is applied to a capacity and material requirement planning problem
30 with uncertain data. The main contribution of this paper is that it shows an example where fuzzy
31 numbers could improve the solution. This work uses a fuzzy linear programming approach based on
32 the possibility theory to generate an optimal fuzzy solution. In this context, the survey works by
33 Kacprzyk and Orlovsky (1987), Delgado *et al.* (1994), Rommelfanger (1996) and Zimmermann
34 (2000) show some possibilities of how the fuzzy set theory can be accommodated within linear
35 programming.

This paper is arranged as follows: firstly in Section 2, we present a linear programming (LP) model for production planning with a deterministic structure. In Section 3, the LP model is transformed into a fuzzy model for production planning with uncertain data. Section 4 uses a real-world application to illustrate the potential savings which can be attained by using fuzzy models in a fuzzy decision-making environment. Section 5 offers some conclusions.

2 Formulation of the linear programming model

This linear programming (LP) model, originally proposed by Mula *et al.* (2006a), is adopted as the basis of this work. It is a model for the optimization of the production planning problem in a capacity constrained, multi-product, multi-level and multi-period manufacturing environment. The LP model is formulated as follows.

Decision variables and parameters for the model are defined in Table 1.

Table 1. Decision variables and model parameters.

$$\text{Min } Z = \sum_{i=1}^I \sum_{t=1}^T (cp_i P_{it} + ci_i INVT_{it} + cb_i B_{it}) + \sum_{r=1}^R \sum_{t=1}^T (ctun_r Tun_{rt} + ctov_r Tov_{rt}) \quad (1)$$

s.t.

$$INVT_{i,t-1} + P_{i,t-TS_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^I \alpha_{ij} (P_{jt} + SR_{jt}) + B_{it} = d_{it}, \quad i = 1, \dots, I, t = 1, \dots, T \quad (2)$$

$$\sum_{i=1}^I AR_{ir} P_{it} + Tun_{rt} - Tov_{rt} = CAP_r, \quad r = 1, \dots, R, t = 1, \dots, T \quad (3)$$

$$B_{it} = 0, \quad i = 1, \dots, I \quad (4)$$

$$P_{it}, INVT_{it}, B_{it}, Tun_{it}, Tov_{it} \geq 0, \quad i = 1, \dots, I, r = 1, \dots, R, t = 1, \dots, T \quad (5)$$

Equation (1) shows the total costs to be minimized: the costs of the inventories ci_i , the costs of the extra time used by resources, $ctov_r$, and the costs of the idle time of resources, $ctun_r$. The LP model includes a plan to satisfy the backlogs penalized with a cost, cb_i . It is assumed that this cost is linear to the number of backlogs in every period.

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4 The balance equations for the inventory are given by the group of constraints (2). These
5 equations take into account the backlogs which behave as a negative inventory. It is important to
6 highlight the consideration of parameter SR_{it} that guarantees the continuity of the production plan
7 throughout the successive runs carried out during a given planning horizon.
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12 Production in each period is limited by the availability of a group of shared resources. Equation
13 (3) considers the capacity limits of these resources. This equation has been thought in a similar way
14 to the model proposed by Billington *et al.* (1983), although the fixed costs/setup times have been
15 ignored since we are performing an aggregate production planning. The decision variables Tun_{rt} and
16 Tov_{rt} are not limited by any established parameter but are penalized with the corresponding costs in
17 the objective function. This is to provide the model with the best generality possible. The constraint
18 of these variables for specific applications could be easily considered taking into account that if
19 those constraints are exceeded, the solution of the model might not be feasible.
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24 A constraint has also been added (4) to finish the backlogs in the last period, T , of the planning
25 horizon.
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29 The model also contemplates the non negativity constraints (5) for the decision variables.
30 Finally, the decision variables P_{it} , $INVT_{it}$ and B_{it} will be defined as continuous or integer variables,
31 depending on the manufacturing environment where the model is applied.
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35 The LP model includes a small number of constraints with the objective of providing a model that
36 is as generic as possible. Other types of more specific constraints from the manufacturing
37 environment where the model is applied can be easily added, such as, alternative production
38 processes for some products, recruiting variables and manpower discharges for resources planning
39 and production, overtime or stock levels, etc. Some of these constraints are included in the model
40 proposed by Escudero (1994).
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50 The solution that satisfies the constraints and minimizes the objectives specified previously
51 should be used dynamically, i.e. only decisions related to the first periods of the planning horizon
52 will be executed. Then, when new information on demand, availability of resources, bill of materials,
53 lead times, costs, etc. arrive; the model should be upgraded and re-executed.
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3 A possibilistic programming approach

When the backlog costs, the market demand, the required capacity or the available capacity are uncertain data for the planner, the deterministic model becomes insufficient to describe the situation and we try to model and solve the mid-term production planning problem with a possibilistic programming approach.

This model adopts the fuzzy numbers comparison approach proposed by Negi and Lee (1993). The authors use the concept of possibilistic programming to compare fuzzy numbers and overcome the difficulties to obtain the possibility distribution due to the non-linearity by assuming a level of attainment of the possibility. Therefore the problem becomes linear.

We consider fuzzy technological coefficients, fuzzy cost coefficients and fuzzy right-hand-side numbers represented by a trapezoidal fuzzy number (*TrFN*), although different types of fuzzy numbers might be used.

A *TrFN*, \tilde{b} , is defined as follows, $\tilde{b} = \{\underline{b}, b_1, b_2, \bar{b}\}$, where \underline{b} is the smallest possible value, b_1 and b_2 are the main values and \bar{b} is the largest possible value (Negi and Lee, 1993).

The so-called membership function associated with \tilde{b} is $\mu_{\tilde{b}}(b, \theta)$, $\theta \in [0, 1]$, where θ is the maximum value of the membership function, thus when $b_2 \geq b \geq b_1$, then:

$$\mu_{\tilde{b}}(b, \theta) = \begin{cases} 0, & \text{if } b < \underline{b} \text{ or } b > \bar{b}, \\ \frac{(b - \underline{b})\theta}{b_1 - \underline{b}} & \text{if } b_1 \geq b \geq \underline{b}, \\ \theta & \text{if } b_2 \geq b \geq b_1, \\ \frac{(\bar{b} - b)\theta}{\bar{b} - b_2} & \text{if } \bar{b} \geq b \geq b_2, \end{cases} \quad (6)$$

where $\theta \geq \mu_{\tilde{b}}(b, \theta) \geq 0$ (see Figure 1).

Figure 1. Trapezoidal fuzzy number.

The comparison of two fuzzy numbers depends on the type of fuzzy number. Iskander (2002) uses the comparison of two triangular fuzzy numbers (*TFNs*): *TrFN* and *TFN*. Here, the comparison approach is used for two *TrFNs* according to Negi and Lee (1993).

If \tilde{A} and \tilde{B} are two *TrFNs*, with the same θ , where $\tilde{A} = \{a, a_1, a_2, \bar{a}\}$ and $\tilde{B} = \{b, b_1, b_2, \bar{b}\}$, the possibility that $\tilde{A} \geq \tilde{B}$ is defined as follows:

$$Pos(\tilde{A} \geq \tilde{B}) = \begin{cases} \theta, & \text{if } a_2 \geq b_1, \\ \frac{(\bar{a} - \underline{b})\theta}{(\bar{a} - a_2) + (b_1 - \underline{b})}, & \text{if } b_1 \geq a_2, \bar{a} \geq \underline{b}, \\ 0, & \text{if } \underline{b} \geq \bar{a}, \end{cases} \quad (7)$$

While the possibility that $\tilde{A} > \tilde{B}$ is defined as:

$$Pos(\tilde{A} > \tilde{B}) = \begin{cases} \theta, & \text{if } a_2 \geq \bar{b}, \\ \frac{(\bar{a} - b_2)\theta}{(\bar{a} - a_2) + (\bar{b} - b_2)}, & \text{if } \bar{b} \geq a_2, \bar{a} \geq b_2, \\ 0, & \text{if } b_2 \geq \bar{a}, \end{cases} \quad (8)$$

We include *TrFNs*, i.e. $\tilde{c}b_i$, \tilde{d}_{it} , $\tilde{A}R_{it}$ and $\tilde{C}A\tilde{P}_{rt}$, representing the fuzzy backlog cost, market demand, required capacity and available capacity, respectively, in Equations (1), (2) and (3) as follows:

$$\text{Max } Z = - \sum_{i=1}^I \sum_{t=1}^T (cp_i P_{it} + ci_i INVT_{it} + \tilde{c}b_i B_{it}) - \sum_{r=1}^R \sum_{t=1}^T (ctun_{rt} Tun_{rt} + ctov_{rt} Tov_{rt}) \quad (9)$$

s.t.

$$INVT_{i,t-1} + P_{i,t-TS_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^I \alpha_{ij} (P_{jt} + SR_{jt}) + B_{it} \leq \tilde{d}_{it}, \quad i = 1, \dots, I, t = 1, \dots, T \quad (10)$$

$$- INVT_{i,t-1} - P_{i,t-TS_i} - SR_{it} + INVT_{i,t} + B_{i,t-1} + \sum_{j=1}^I \alpha_{ij} (P_{jt} + SR_{jt}) - B_{it} \leq -\tilde{d}_{it}, \quad i = 1, \dots, I, t = 1, \dots, T \quad (11)$$

$$\sum_{i=1}^I \tilde{A}R_{ir} P_{it} + Tun_{rt} - Tov_{rt} \leq \tilde{C}A\tilde{P}_{rt}, r = 1, \dots, R, t = 1, \dots, T \quad (12)$$

$$-\sum_{i=1}^I \tilde{A}R_{ir} P_{it} - Tun_{rt} + Tov_{rt} \leq -\tilde{C}A\tilde{P}_{rt}, r = 1, \dots, R, t = 1, \dots, T \quad (13)$$

The crisp objective function to minimize in the LP model (1) has become the fuzzy objective function to maximize (9).

The equality equation of the inventory balance of the LP model (2) has become Equations (10) and (11) in the fuzzy model. In the same way, the equality equation of the available capacity of the LP model (3) has become the two inequality Equations (12) and (13) in the fuzzy model.

The non fuzzy constraints (4) and (5) should be incorporated into the fuzzy model, remaining unalterable.

In accordance with the fuzzy numbers comparison approach proposed by Negi and Lee (1993), and using the concept of the α -level by Zadeh (1965), i.e. the ordinary set (non fuzzy) A_α of elements that belong to the fuzzy set \tilde{A} at least with the α grade, is denominated the α -level set of \tilde{A} , or α -cut:

$$\tilde{A}_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha, 0 \leq \alpha \leq 1\} \quad (14)$$

In order to solve the problem, α settles down parametrically to obtain the value of the objective function for each of these $\alpha \in [0, \theta]$. The result is, however, a fuzzy set and the planner has to decide which pair (α, z) considers the optimum if a crisp solution is to be obtained. Let α be determined by the planner for any given value of θ , $\theta \in [0, 1]$. The equivalent crisp model is represented as follows:

$$\begin{aligned} \text{Max } & Z \\ \text{s.t.} & \end{aligned} \quad (15)$$

$$INVT_{i,t-1} + P_{i,t-TS_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^I \alpha_{ij} (P_{jt} + SR_{jt}) + B_{it} \leq \left(\frac{\theta}{\alpha} - 1 \right) \bar{d}_{it} + d_{it2}, \quad (16)$$

$$i = 1, \dots, I, t = 1, \dots, T$$

$$INVT_{i,t-1} + P_{i,t-TS_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^I \alpha_{ij} (P_{jt} + SR_{jt}) + B_{it} \leq \bar{d}_{it}, \quad (17)$$

$$i = 1, \dots, I, t = 1, \dots, T$$

$$-INVT_{i,t-1} - P_{i,t-TS_i} - SR_{it} + INVT_{i,t} + B_{i,t-1} + \sum_{j=1}^I \alpha_{ij} (P_{jt} + SR_{jt}) - B_{it} \leq \left(\frac{\theta}{\alpha} - 1 \right) d_{it} - d_{it1},$$

$$i = 1, \dots, I, t = 1, \dots, T \quad (18)$$

$$-INVT_{i,t-1} - P_{i,t-TS_i} - SR_{it} + INVT_{i,t} + B_{i,t-1} + \sum_{j=1}^I \alpha_{ij} (P_{jt} + SR_{jt}) - B_{it} \leq -d_{it},$$

$$i = 1, \dots, I, t = 1, \dots, T \quad (19)$$

$$\sum_{i=1}^I \left(\frac{\theta}{\alpha} AR_{ir2} - AR_{ir2} + \overline{AR}_{ir} \right) P_{it} + Tun_{rt} - Tov_{rt} \leq \left(\frac{\theta}{\alpha} - 1 \right) \overline{CAP}_{rt} + CAP_{rt2},$$

$$r = 1, \dots, R, t = 1, \dots, T \quad (20)$$

$$\sum_{i=1}^I \left(\frac{\theta}{\alpha} AR_{ir2} - AR_{ir2} + \overline{AR}_{ir} \right) P_{it} + Tun_{rt} - Tov_{rt} \leq \overline{CAP}_{rt}, \quad r = 1, \dots, R, t = 1, \dots, T \quad (21)$$

$$-\sum_{i=1}^I \left(\frac{\theta}{\alpha} AR_{ir2} - AR_{ir2} + \overline{AR}_{ir} \right) P_{it} - Tun_{rt} + Tov_{rt} \leq \left(\frac{\theta}{\alpha} - 1 \right) \overline{CAP}_{rt} - CAP_{rt1},$$

$$r = 1, \dots, R, t = 1, \dots, T \quad (22)$$

$$-\sum_{i=1}^I \left(\frac{\theta}{\alpha} AR_{ir2} - AR_{ir2} + \overline{AR}_{ir} \right) P_{it} - Tun_{rt} + Tov_{rt} \leq -\overline{CAP}_{rt}, \quad r = 1, \dots, R, t = 1, \dots, T \quad (23)$$

$$z \leq -\sum_{i=1}^I \sum_{t=1}^T \left(cp_i P_{it} + ci_i INVT_{it} + (\overline{cb}_i - \frac{\alpha}{\theta} \overline{cb}_i + \frac{\alpha}{\theta} cb_{i2}) B_{it} \right) - \sum_{r=1}^R \sum_{t=1}^T (ctun_{rt} Tun_{rt} + ctov_{rt} Tov_{rt})$$

$$(24)$$

$$z \geq -\sum_{i=1}^I \sum_{t=1}^T (cp_i P_{it} + ci_i INVT_{it} + cb_{i2} B_{it}) - \sum_{r=1}^R \sum_{t=1}^T (ctun_{rt} Tun_{rt} + ctov_{rt} Tov_{rt}),$$

$$(25)$$

Additionally, the non fuzzy constraints (4) and (5) should be incorporated into this model, remaining unalterable.

When applying the modifications proposed by Iskander (2002) to the approach developed by Negi and Lee (1993), Equations (24) and (25) are eliminated and the objective function (15) is replaced by Equation (26) as follows:

$$\text{Max } Z = -\sum_{i=1}^I \sum_{t=1}^T \left(cp_i P_{it} + ci_i INVT_{it} + (\overline{cb}_i - \frac{\alpha}{\theta} \overline{cb}_i + \frac{\alpha}{\theta} cb_{i2}) B_{it} \right) - \sum_{r=1}^R \sum_{t=1}^T (ctun_{rt} Tun_{rt} + ctov_{rt} Tov_{rt}) \quad (26)$$

The approach by Iskander (2002) reduces the number of decision variables. Furthermore, a sensitivity analysis of the values θ and α can be carried out. If the planner does not participate in the definition of θ and α , these values can be fixed at 1, eliminating the redundant constraints. Finally, the adopted approach can be applied if different values of α (different levels of possibility) are given to different constraints. The main disadvantage of this model in relation to the LP model is

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4 that it requires the planner to specify more data , such as the values of θ and α . Nonetheless, it
5 allows the uncertain data (backlog cost, demand, required capacity and available capacity) to be
6 formalized through the definition of the *TrFNs*: $c\tilde{b}_i$, \tilde{d}_{it} , $A\tilde{R}_{ir}$, and $CA\tilde{P}_{rt}$.
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10 11 **4 Application in an automobile seat manufacturer**

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13 The proposed models have been evaluated using data from an automobile seat manufacturer.
14 The company is an assembly plant for car seats belonging to a multinational group leader in the
15 supply of seats in the automobile sector. Currently, the company uses a production planning system
16 that is integrated in an ERP system based on a standard MRP II. Nevertheless, the company only
17 uses the MRP II module for material supply planning. The company considers it has no capacity
18 problems given that the car seat assembly plant is a flexible plant, designed according to
19 automobile assembler requirements and with the possibility of using extra capacity from other plants
20 of the industrial group. Therefore, the main objective of the generation of a production plan is to
21 supply material at the minimum cost which is subordinated to the constraint of avoiding stock
22 shortages.
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33 The company receives the demand information from the automobile assembler on a weekly
34 basis (every Monday) with a planning horizon for six months. However, these demand forecasts are
35 rarely accurate (Mula *et al.* 2005). Therefore, in this section we will validate if the fuzzy model for
36 production planning that we propose in this paper can be a useful tool for the decision-making
37 process of the production planners in an uncertain decision environment.
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43 **4.1 Assumptions**

44 The assumptions to carry out the computational experiment are summarized as follows:

- 45 • The study considers a representative single part called *RPN* (Representative Part Number).

46 The research of this representative part allows us to generalize its behaviour to any item in
47 the system. Moreover, it is important to understand the behaviour of a single part as the
48 basis for additional research of the proposed model.
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- 54 • The decision variables, P_{it} , $INVT_{it}$ and B_{it} are considered integer. Despite this being a mixed
55 integer linear programming problem, the mathematical relationships established in the
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4 section between the fuzzy model and the equivalent linear programming models are the
5 same as if it were a linear programming problem.
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- 8 • Only the final product has external demand.
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- 10 • A six-monthly planning horizon with a weekly period planning has been considered.
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- 12 • Firm orders from the automobile maker cannot be rejected.
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- 14 • Backlog for the final product *RPN* are considered but with a high penalization cost, since
15 the required service level for a sequenced and synchronized supplier of seats for
16 automobiles is 100%.
17
- 18 • The company, which has implemented a JIT philosophy, officially states it does not use
19 safety stocks but uses safety times for some of the components to be supplied to them.
20
- 21 • Once the seat assembler sends the purchase orders to its suppliers, these cannot reject
22 them. Moreover, all the materials are supplied according to the production program, so lead
23 times for supplied materials are assumed as constant values.
24
- 25 • The production stage is considered to be single, i.e. all the materials are supplied based on
26 demand forecasts and the final assembly of seats begins after receiving firm orders, in
27 other words, a production strategy denominated Assembly-to-Order (Fumero and Vercellis,
28 1994).
29
- 30 • A single productive resource restricts production: the assembly line.
- 31
- 32 • The decision variable of overtime (ToV_{rt}) for the *RPN* part refers to the hours of assembly to
33 be subcontracted externally, rather than the overtime used to carry out this work at the
34 factory itself. Therefore, overtime limits are not considered. It is assumed that the
35 subcontracted products will be ready just when required without lead time changes.
36
- 37 • The unit costs of undertime ($ctun_{rt}$) are considered null.
- 38
- 39 • For the fuzzy model, the parameter θ has been established at 1 while the parameter α is an
40 aspiration level ($0 \leq \alpha \leq \theta$) with a step of 0.1. In this particular case, we will select the α
41 value whose best results provide most of the evaluated indicators (see the next section).
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- Moreover, the fuzzy cost coefficient, $\tilde{c}b_i = \{\underline{c}b_i, cb_{i1}, cb_{i2}, \overline{c}b_i\}$, the fuzzy right-hand-side number, $\tilde{d}_{it} = \{\underline{d}_{it}, d_{it1}, d_{it2}, \overline{d}_{it}\}$, and the fuzzy technological coefficients, $\tilde{A}R_{ir} = \{\underline{A}R_{ir}, AR_{ir1}, AR_{ir2}, \overline{A}R_{ir}\}$ and $\tilde{C}A\tilde{P}_{rt} = \{\underline{C}A\tilde{P}_{rt}, CA\tilde{P}_{rt1}, CA\tilde{P}_{rt2}, \overline{C}A\tilde{P}_{rt}\}$ required by the fuzzy model, have been defined following the company's criteria. For instance, in the case of the demand information:
 - d_{it2} is considered to be the demand information received by the company,
 - d_{it1} is obtained by decreasing the value of d_{it2} by 5%,
 - \underline{d}_{it} is obtained by decreasing the value of d_{it2} by 10%, and
 - \overline{d}_{it} is obtained by increasing the value of d_{it2} by 10%.
 - Production variations due to quality or machine failures are not considered.
 - Planned orders are recalculated for every planning period, while the scheduled receipts of the components are considered to be 'firm'.
 - The execution measures for each model (LP model and fuzzy model) performance are: production, inventory, backlog and overtime costs.
 - The technical and economic information of the part RPN are considered, as is the demand information for a planning horizon of 30 weeks, from 11/11/2002 to 02/06/2003.

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Given the importance of production planning and the existing models with deterministic approaches, the experiment compares the performance of the deterministic approach proposed in Section 2, the LP model, with the possibilistic approach proposed in Section 3. The aim to this comparison is to demonstrate the usefulness and appropriateness of these fuzzy models operating in an uncertain decision-making environment. Thus, each model is executed for every period ($t = 1 \dots 30$) at a rolling horizon of initially 30 periods and later on 29, 28, 27, ..., 1, updating the demand values, the inventory, the backlog and the scheduled receipts of components. The main difference between the foreseen performances of both models is that the LP model will generate a production plan for every period and will assume certain values of all the model parameters, while the fuzzy

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4 model will do this by incorporating the existing uncertainty in demand, backlog costs and capacity
5 data. This is precisely the algorithm base we are comparing to.
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8 The experiment has been carried out on a PC, with an AMD Athlon processor at 2600 MHz and
9 with 256 MB of RAM memory, as follows (see Figure 2):
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15 **Figure 2.** *Computational experiment.*
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18 19 20 **4.2 Implementation and resolution**

21 The dynamic character of the production planning problem and the integrity requirements on
22 some variables are taken into consideration through the architecture developed for the production
23 planning of the proposed models, as illustrated in Figure 3.
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26 The models have been implemented with the modelling language MPL V4.11 (Maximal Software
27 Co. 2000). MPL can use different relational databases to manage the model data with only minor
28 adjustments. This provides very good data validation and data integrity tools, and is capable of
29 handling a large amount of data. Moreover, it provides easy access to the corporate data, which at
30 the same time facilitates the process of updating the data required by the models. The problem is
31 then solved by the mixed integer programming problem solver CPLEX 6.6 (CPLEX Optimization Co.
32 1994), and the solution is imported back into the database.
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46 **Figure 3.** *Models architecture.*
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48 The detailed data of this computational experiment and the MPL models can be found in Mula
49 (2004).
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4.3 Evaluation method

To evaluate the models, the following indicators have been defined to be measured: (i) the average service level; (ii) number of minimum inventory levels; (iii) planning nervousness respect to the planned period and the planned quantity; (iv) total costs, and (v) computational efficiency. These indicators have been selected in order to evaluate the model outputs given by the activity of the decision variables.

4.3.1 Average service level

The average service level, expressed as a percentage, is determined only for the final product *RPN* as follows:

$$\text{Average service level (\%)} = \sum_{t=1}^T \frac{\left(1 - \frac{B_{it}}{\sum_{t'=1}^t d_{it'}}\right) \times 100}{T} \quad \forall i = RPN \quad (41)$$

4.3.2 Minimum inventory levels

In order to evaluate the number of minimum inventory levels, the average total quantity of inventory of products (final product, raw materials and components) is determined for each period in the planning horizon. Then, we count the number of times that a model yields the lowest inventory.

4.3.3 Planning nervousness

Here we explain criterion (iii) in detail. A "nervous" or unstable planning is referred to a plan which undergoes important variations when incorporating the demand changes between what is foreseen and observed in successive plans, as defined by Sridharan et al. (1987). Whybark and Williams (1976) noted a difference between: (i) demand timing uncertainty, that is reflected by timing changes; and (ii) demand quantity uncertainty, that occurs when the demand quantities are increased or decreased. Therefore, the planning nervousness can be measured according to:

The demand changes in the planned period. The number of times that a planned order is rescheduled irrespectively of the planned quantity (Heisig, 1998). This case is directed to minimize disruptions in the production sequence, since this would imply extra setup time. Van Donselaar *et*

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a.l. (2000) propose a measure of the planning nervousness focused on the setup costs for a manufacturing company of trucks in an MRP environment. The rule is summarized as so:

At time t we check for each period $t + x$ ($x = 0, 1, 2, \dots, T-1$):

- If there is a planned order in $t + x$ and this order is not planned in the next planning run, we increase the number of reschedules by 1.
- If there was no planned order in $t + x$ and there is one in the next planning run, we increase the number of reschedules by 1.

The demand changes in the planned quantity. The number of times that the quantity of a planned order is modified (De Kok and Inderfurth, 1997). This case focuses on minimizing the stock shortages or excesses, since this would imply higher costs of backlog and a holding of inventories.

The rule is described as follows:

In the period $t = 1, \dots, T$, where T is the number of periods that forms the planning horizon, $t + x$ ($x = 0, 1, 2, \dots, T - 1$) is checked for every period:

- If a planned order exists in the period $t + x$, and if the quantity of the planned order is not the same as in the next planning run, we increase the number of reschedules by 1.

While computing planning nervousness, we measure the number of changes. Another way to compute would be to take into account the rate of the changes.

4.3.4 Total costs

Total costs are the sum of all the costs generated in each period of the considered planning horizon, derived from the production plans provided by the model.

4.3.5 Computational efficiency

The parameter computational efficiency measures the computational effort needed to resolve each model. After solving the model, MPL automatically creates a standard solution file with the following statistics: The number of barrier iterations before reaching the solution; the number of variables; the number of variables that are integer; the number of constraints; the number of nonzero elements for the full matrix that was stored; the matrix density of constraints; the CPU time it takes to obtain the solution.

4.4 Evaluation of the results

Table 2 presents the quantitative results of our application in terms of the indicators previously described. In the case of the fuzzy model that provides a fuzzy solution, the fuzzy set of the decision has been obtained. Then, it has been chosen as a crisp solution which obtains the best results in the highest number of the evaluated indicators. Only the selected crisp solution will be considered for this model throughout this section, where the parameter α was established at 0.9.

Table 2. *Evaluation of the results for all planning horizons.*

Both models present an average service level of between 99.44 and 99.49%. The fuzzy model provides a slightly better value at the service level (99.49%). Conversely, the fuzzy model generates a higher number of minimum inventory levels, i.e. 35 of the 46 evaluated items present lower inventories with the production plans obtained by the fuzzy model in contrast with the 8 items with lower inventories provided by the LP model.

The models presented a similar nervousness in relation to the planned time period. On the other hand, the fuzzy model presents the best value of nervousness in relation to planned quantity.

The fuzzy model also generates lower total costs than the LP model. These differences in the total costs are mainly due to two aspects:

- i) The consideration of possible future variations on the demand which may lead to a larger production and/or inventories with the objective of avoiding the strongly penalized backlogs.
- ii) The strict constraints of the deterministic model, where the required capacity and the available capacity of the assembly line are rigidly fixed.

Table 3. *Efficiency of computational experiments for only one execution (first week).*

With regard to the computational efficiency, both models can obtain the optimal solution of the mixed integer linear programming with no iterations in the first production plan execution. Obviously,

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4 the number of iterations can change in the rest of production plan executions, depending on the
5 input data. On the other hand, both models have the same number of variables and integer
6 variables, but the fuzzy model requires more non zero elements (thus, a larger array density) and
7 more constraints than the LP model, which could imply greater requirements of information storage.
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9 Finally, the LP model uses less CPU time than the fuzzy model.
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14 Therefore, the fuzzy model has provided freedom of action with regard to problems where
15 uncertain values appear with a minimum increment of information storage requirements and a
16 moderate increase of the required CPU time.
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20 From the empirical research carried out, we can draw the following conclusions:

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- 23 • Neither model behaves worse or better for all the evaluation criteria.
 - 24 • The fuzzy model behaves better than the LP model in four of the evaluation criteria.
 - 25 • The fuzzy model displays the best behaviour in relation to Service Level and Total Costs,
26 two of the most important evaluation criteria for the company.
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30 From an economic point of view, the fuzzy model obtained the best marks, i.e. it generates the
31 least Total Costs and, therefore, it would be selected as the most appropriate model for application.
32 On the other hand, if the company is looking for a model which behaves best in all the evaluation
33 criteria, the fuzzy model would also be selected.
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38 **5 Conclusions**

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40 In this paper, a linear programming (LP) model has been presented in order to solve the mid-
41 term production planning problem in a capacity constrained, multi-product, multi-level and multi-
42 period manufacturing environment. The LP model assumes that demand is known and that
43 production will follow the generated plan to a great extent. In many manufacturing environments
44 however, such as the automobile industry, these decisions have to be made under conditions of
45 uncertainty in parameters as important as costs, market demand or capacity data.
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52 A model based on fuzzy mathematical programming for production planning under conditions of
53 uncertainty has been proposed. It is important to highlight that the objective of the fuzzy model for
54 production planning proposed in this research is not to replace deterministic models, rather to
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4 provide a solid and efficient option for its application in environments with conditions of uncertainty
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6 where the use of deterministic models are not completely realistic.
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8 The proposed models have been applied to a company dedicated to the assembly of car seats.
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10 It is important to clarify that the decision makers from the company accept and understand the
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12 models and the differences between them, but only from the point of view of users; that is, they can
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14 not build them up or modify them.
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16 Generally speaking, the fuzzy model structure has been able to increase group satisfaction
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18 (level of service, inventory levels, planning nervousness and total costs) without causing an
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20 explosive growth of computational effort. Thus, this research work demonstrates that models based
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22 on fuzzy mathematical programming are sound for modelling production planning processes, and
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24 that an economic benefit can exist when these are used in comparison with traditional deterministic
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26 models. Fuzzy models can also be reformulated as equivalent crisp models that can be solved
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28 efficiently with standard software, such as CPLEX.
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30 Finally, the literature approaches to cope with different forms of uncertainty in production
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32 planning systems is considerable. Nevertheless, new approaches for production planning and
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34 control are still necessary to manage uncertainty. Our position is that the fuzzy sets theory is
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36 generally an adequate methodology that can mean a great advance in production planning
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38 research. Future plans are related to: (i) capacity, material and transport requirement planning
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40 modelling; (ii) supply chain production planning problems under uncertainty; and (iii) evolutionary
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42 algorithms for fuzzy optimization in production planning.
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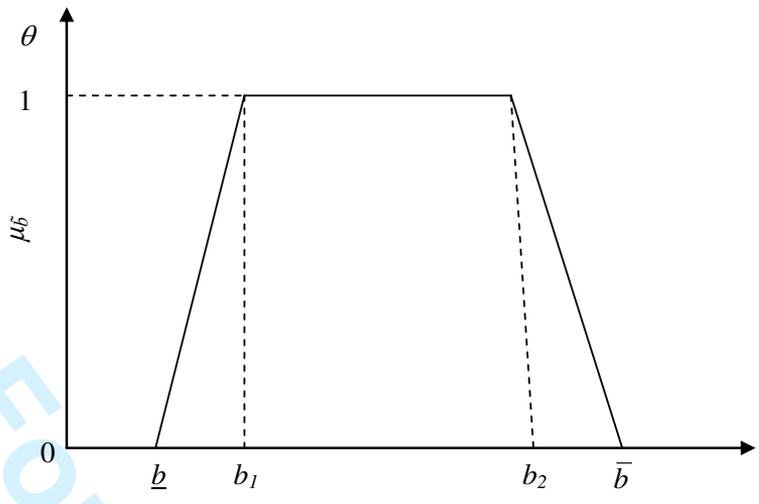


Figure 1. Trapezoidal fuzzy number.

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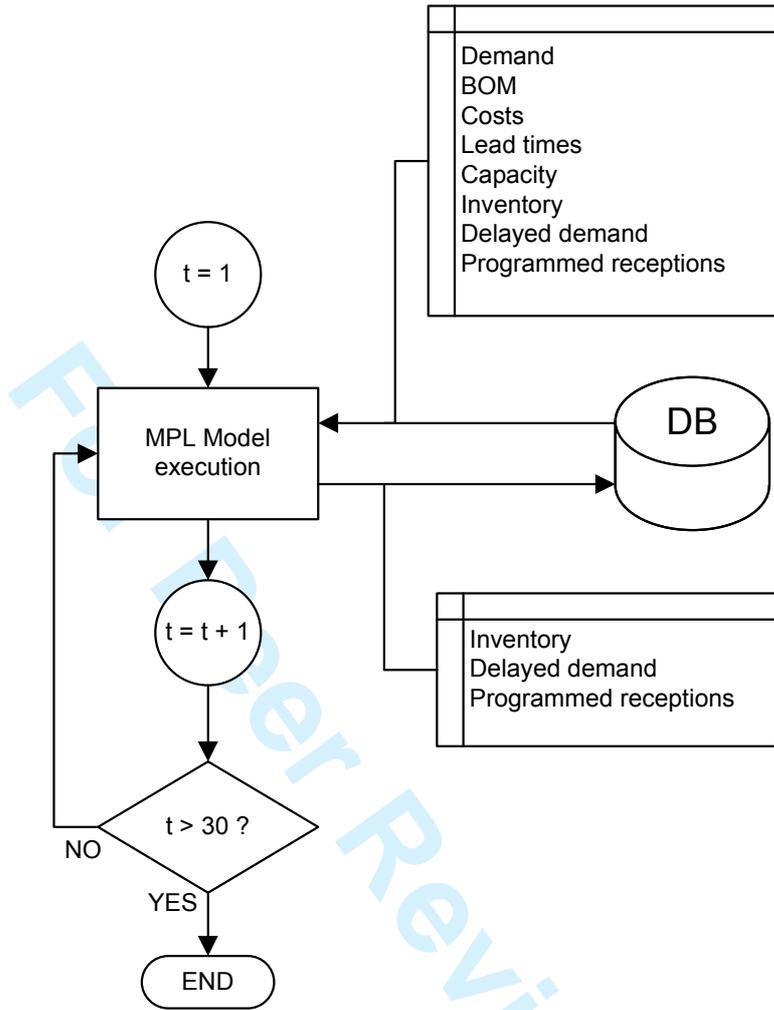


Figure 2. Computational experiment.

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Figure 3. Models architecture.

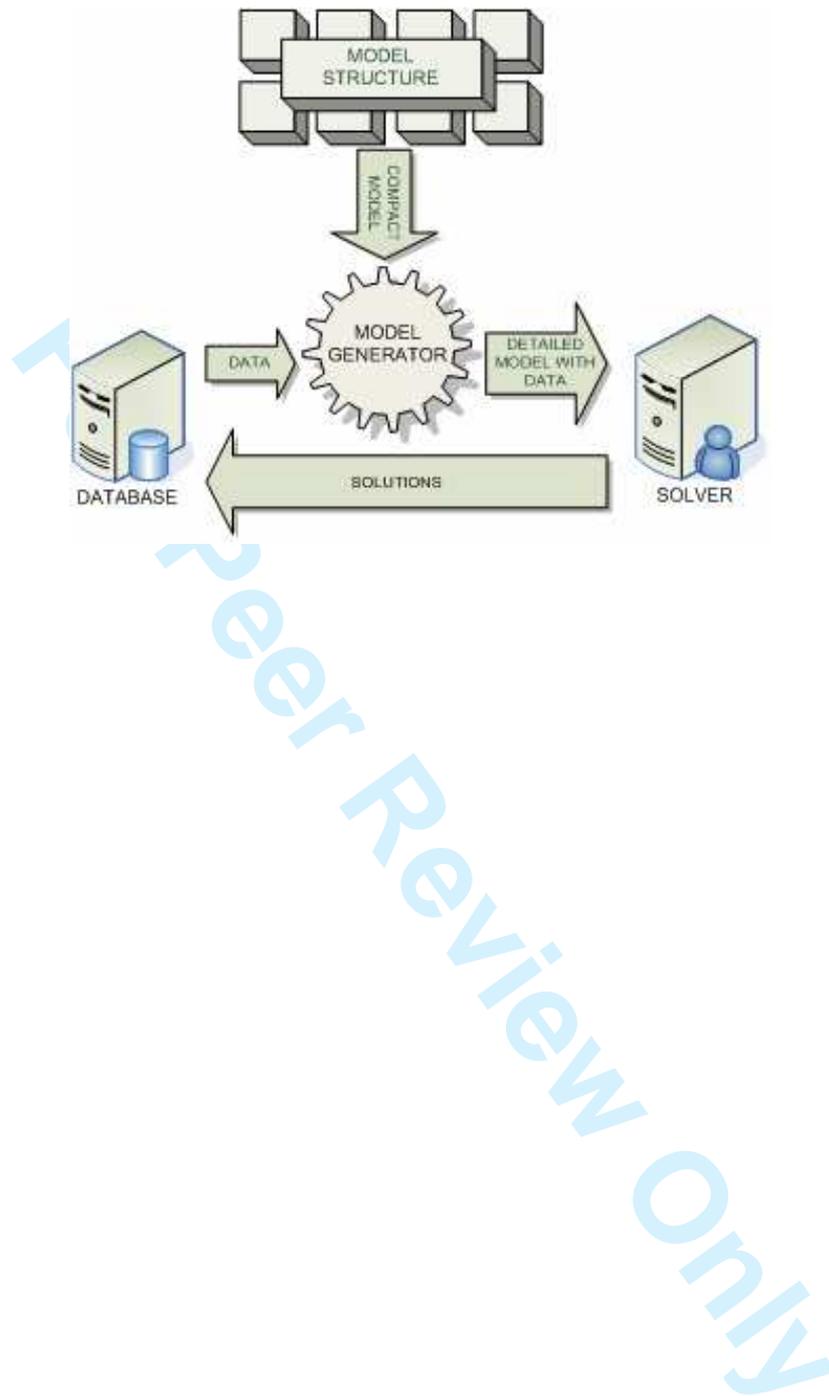


Table 1. Decision variables and model parameters.

Sets of Indices			
T	Number of periods in the planning horizon ($t = 1 \dots T$)		
I	Number of products ($i = 1 \dots I$)		
J	Number of the parent products in the bill of materials ($j = 1 \dots J$)		
R	Number of resources ($r = 1 \dots R$)		
Decision Variables		Data	
P_{it}	Quantity of product i to be produced in period t	d_{it}	Market demand of product i in period t
$INVT_{it}$	Inventory of product i at the end of period t	α_{ij}	Required quantity of i to produce one unit of product j
B_{it}	Backlog of product i at the end of period t	TS_i	Lead time of product i
Tun_{rt}	Undertime hours of the resource r in period t	$INVT_{i0}$	Inventory of product i in period 0
Tov_{rt}	Overtime hours of the resource r in period t	B_{i0}	Backlog of product i in period 0
Objective Function Cost Coefficients		SR_{it}	Scheduled receipts of product i in period t
cp_i	Variable cost of production of a unit of product i	Technological Coefficients	
ci_i	Inventory cost of a unit of product i	AR_{ir}	Required time of the resource r for one unit of production of product i
cb_i	Backlog cost of a unit of product i	CAP_{rt}	Available capacity of the resource r in period t
$ctun_{rt}$	Undertime hour cost of the resource r in period t		
$ctov_{rt}$	Overtime hour cost of the resource r in period t		

Table 2. Evaluation of the results for all planning horizons.

Model	Service Level (%)	Number of Minimum Inventory Levels	Planning Nervousness (period)	Planning Nervousness (quantity)	Total costs (€)
<i>LP model</i>	99.44	8	0.35	14.05	3,988,276.07
<i>Fuzzy model</i>	99.49	35	0.35	13.05	3,635,616.82

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Table 3. *Efficiency of computational experiments for only one execution (first week).*

Model	Iterations	Variables	Integer	Constraints	Elements non zero	Array Density (%)	CPU Time (seconds)
<i>LP model</i>	0	4237	5612	2797	8239	0.07	0.86
<i>Fuzzy model</i>	0	4237	5612	2977	8953	0.07	3.96

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