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A Note on 'Modified Hamiltonian Chain: a Graph Theoretic Approach to Group Technology' after S.K.Mukhopadhyay, K. Ramesh Babu and K. V. Vijai Sai.

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Abstract

This note examines and compares a methodology, devised by Mukhopadhyay et al (2000), and offers possible solution approaches to a potential problem identified by the authors.

Introduction

In an adjunct to their other contributions Mukhopadhyay et al (1994, 1995a,1995b,1997), Mukhopadhyay (2000), suggest, and successfully employ, a modified Hamiltonian chain (MHC) graph theoretic approach to the group technology configuration problem (GTCP). The Mukhopadhyay et al (2000), algorithm, hereafter referred to as MBSMHCA, consists of a two stage process. Stage I forms a graphical depiction of the component flow routes (from given knowledge of the Facility Component Incidence Matrix (FCIM)), whilst Stage II generates a modified Hamiltonian chain (a linear arrangement of the facilities employed to process the given component set) The approach suggested is simple both to understand and apply and, in their conclusions, Mukhopadhyay et al (2000) show that the results obtained, in tests employing the suggested approach, compare favourably with other solution methods. This contribution examines the MBSMHCA, identifies possible difficulties and suggests modifications which could improve solution quality. The suggested modifications are then applied to a simple problem, taken from the existing research literature, the result of which is compared to the result obtained by employing another algorithmic methodology devised by Ho et al (1993).

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The MBSMHCA.

In their paper, Mukhopadhyay et al (2000) generate a schematic, in which facilities are represented as nodes or vertices and component flow paths as undirected edges. They then map the component process routes (Table 1) onto the graph to identify the number of components that require each of the created edges (Figure 1). With this information, they then tabulate each of the created edge connections and total the number of components which require a given edge. Once completed, the list is then ranked in descending order by the number of components which use a given edge and, starting with the highest ranked edge, is used to construct a modified Hamiltonian chain of the facility set (Figure 2(a-f)). Finally, once the chain is complete it can then be examined, in conjunction with the original FCIM, to directly obtain both the facility set and component order. In addition, if identifiable clusters of facilities exist in the original data set, the method can clearly present these in a re-ordered FCIM. In conclusion, Mukhopadhyay et al (2000) also state, in comment, that their method is particularly useful when the number of components is greater than the number of facilities.

A worked example of the MBSMHCA.

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The data set shown in Table 1 and the FCIM shown in Table 2, represent one of the example data sets employed by Ho et al (1993) to show the effectiveness of the Heuristic Pattern Matching Method they developed, and is used, in this contribution, to demonstrate the method of the MBSMHCA. The data comprises four components (1 – 4) which are processed on seven facilities (1 – 7). The first stage creates the Component/Facility Edge Flow graph (shown in Figure 1), which has been developed from the data contained in Tables 1 & 2.

Component	Facility Sequence	No. of Facilities
1	$2 \Rightarrow 4 \Rightarrow 7$	3
2	$1 \Rightarrow 2 \Rightarrow 5 \Rightarrow 6$	4
3	$1 \Rightarrow 3 \Rightarrow 2 \Rightarrow 4 \Rightarrow 7$	5
4	$2 \Rightarrow 3 \Rightarrow 4 \Rightarrow 5 \Rightarrow 6 \Rightarrow 7$	6

Table 1 Component Process Route Data.

	Components			
Facilities	1	2	3	4
1		1	1	
2	1	1	1	1
3			1	1
4	1		1	1
5		1		1
6		1		1
7	1		1	1

Table 2. Facility/Component Incidence Matrix (FCIM) for Table 1.

Stage 1. Create the Component/Facility Edge Flow graph.

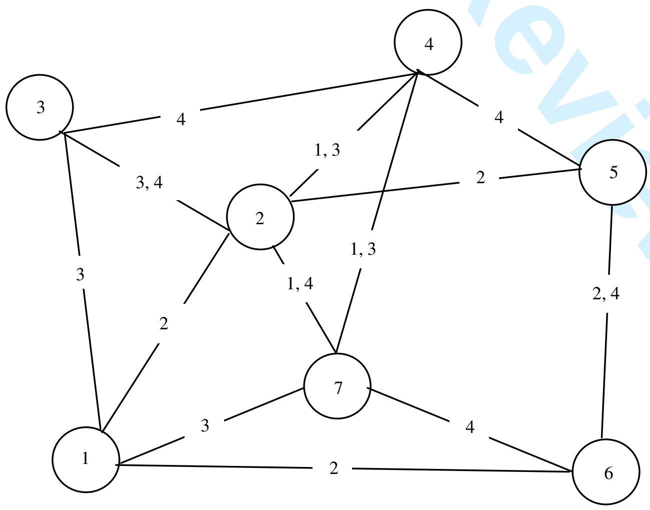

Figure 1. Component/Facility Edge Flow

Stage 2 Generate the Facility Pair Ranking Table for Ho et al.

Once the Component/Facility Edge Flows have been identified and mapped, it is then possible to generate the Facility Pair Ranking (Table 3). This tabulates the Edge Connections required by the each of the Component Flow paths: the Components (and number) that use a given Edge Connection: the ranked order of the Edge Connections (by the number of components that require that edge) and whether, in the subsequent Facility chain (Figure 2(a – f)), the Edge Connection is made.

Count	Edge Pair	Components	Sum	Rank	Edge Made
1	2 – 4	1, 3	2	1	Yes
2	4 – 7	1, 3	2	2	Yes
3	7 – 2	1, 4	2	3	No
4	1 – 2	2	1	10	No
5	2 – 5	2	1	11	No
6	5 – 6	2, 4	2	4	Yes
7	6 – 1	2	1	12	Yes
8	1 – 3	3	1	13	No
9	3 – 2	3, 4	2	5	Yes
10	7 – 1	3	1	6	Yes
11	3 – 4	4	1	7	No
12	4 – 5	4	1	8	No
13	6 – 7	4	1	9	No

Table 3.Edge Connection and Ranking.

Create the Hamiltonian Chain.

Having generated the Edge Connection and Ranking table, the next phase is to construct the Hamiltonian Chain. This is undertaken by successively admitting edge pairs into the chain. The selection priority, for admission to the chain, is based upon the ranking established in the Edge Connection and Ranking table.

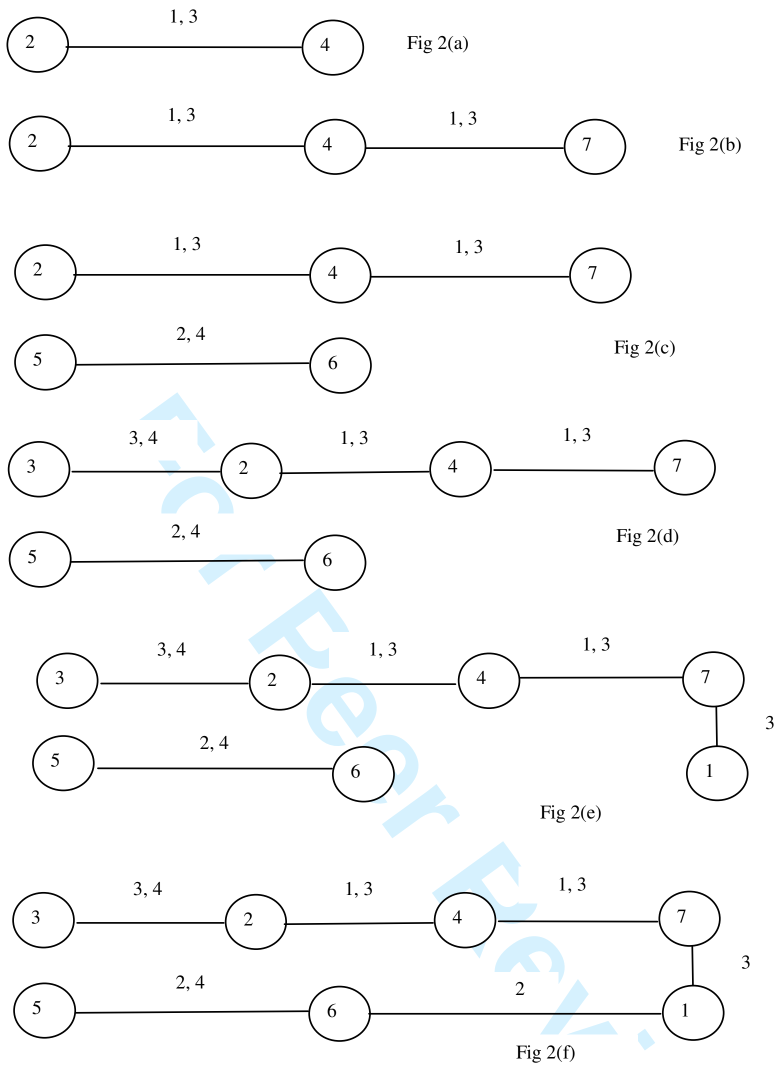

Figure 2. The Hamiltonian Chain for the data in Table 3

Comment on, and comparison of, the MBSMHCA.

The final Hamiltonian Chain, after Mukhopadhyay et al (2000), is shown in Figure 2(f). This can be represented by:-

$$3 \Rightarrow 2 \Rightarrow 4 \Rightarrow 7 \Rightarrow 1 \Rightarrow 6 \Rightarrow 5.$$

If this string is annotated, the flow paths shown in Figure 3(a-d) for each of the components can be drawn.

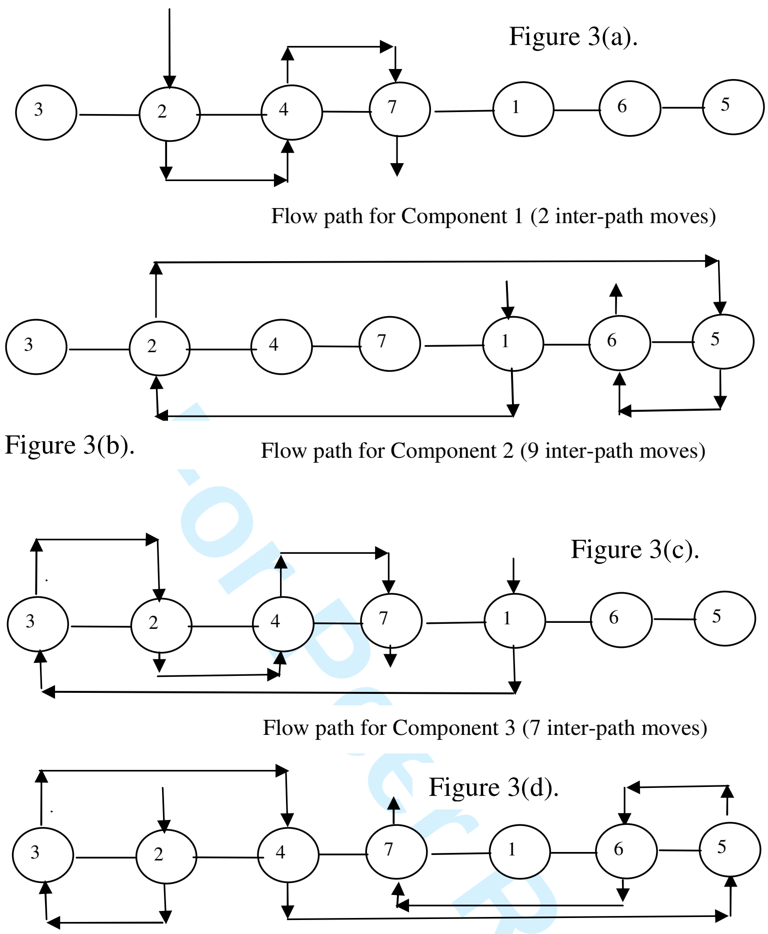

Figure 3(a).

Flow path for Component 1 (2 inter-path moves)

Figure 3(b).

Flow path for Component 2 (9 inter-path moves)

Figure 3(c).

Flow path for Component 3 (7 inter-path moves)

Figure 3(d).

Flow path for Component 4 (10 inter-path moves)

Figure 3. Flow Paths for Components 1 – 4 after Mukhopadhyay et al (2000). Total inter-path moves = 28

If the same exercise is undertaken, again using the data in Table 1, and employing the Ho et al (1993) Heuristic Pattern Matching algorithm two strings are generated, the best of which is:-

$$1 \Rightarrow 3 \Rightarrow 2 \Rightarrow 4 \Rightarrow 5 \Rightarrow 6 \Rightarrow 7$$

If this string is now annotated, as was the MBSMHCA, the flow paths shown in Figure 4(a-d) for each of the components can be drawn.

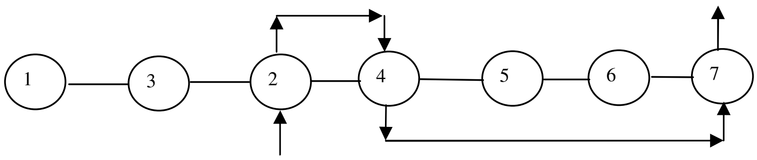

Figure 4 (a) Flow path for Component 1 (4 inter-path moves)

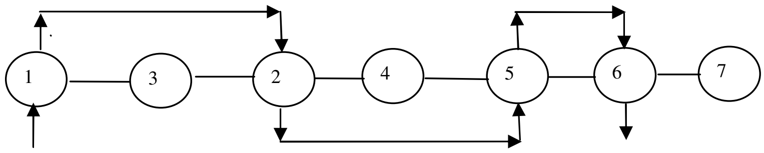

Figure 4 (b) Flow path for Component 2 (5 inter-path moves)

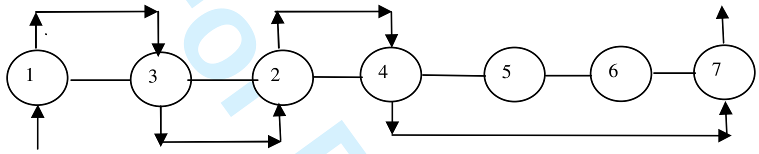

Figure 4 (c) Flow path for Component 3 (6 inter-path moves)

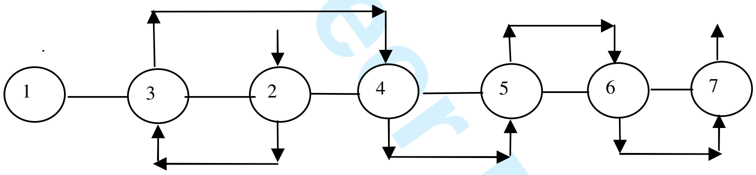

Figure 4 (d) Flow path for Component 4 (6 inter-path moves)

Figure 4 (a – d). Flow Paths for Components 1 – 4 after Ho et al (1993). Total inter-path moves = 21

However, if the two are compared, in terms of a simple measure of flow, i.e. that employed by Ho et al. (a count of the number of inter-stage moves necessary to process the component set) differences are evident. Figures 3 & 4 indicate that the solution to this particular problem, derived using the Ho et al. Heuristic pattern Matching algorithm, is, at least, ((28 – 21)/28) = 25% better than the MBSMHCA. Thus, given this discrepancy, it would be reasonable to question why, in this exemplar, the MBSMHCA performs so badly.

Analysis of the MBSMHCA.

There could be many reasons why the MBSMHCA appears to perform so badly in this instance, two of these could be:-

1. The arbitrary method by which the edge connections (Table 3).are ranked.

2. The making of edge connections (e.g. 6 – 1, 7 – 1), which although necessary to complete a Hamiltonian path, do not exist in the raw data set.

Given these problems, the following is suggested as one possible strategy for improving the quality of the MBSMHCA solutions.

1. Resolve any ties, which appear in Table 3, Column 4 (Sum), by considering the total number of process stages which those components require. The logic for such an approach is reflective of the Ho et al method, which selects the component with the greatest number of process stages as the initial iteration of their solution methodology.

2. Other research effort (John and Hammond (2000)) would suggest that it can be beneficial, at this stage, to reduce the importance of edge connections in the rank order if they do not actually appear as process stage connections in the raw data set (Table 1).

If the foregoing suggestions are adopted, then Table 3 can be modified to that shown in Table 4. In this table the following suffix notation is included.
[1.] Edge connection is only forward sequential flow in raw data.
[2.] Edge connection includes non-sequential forward flow in raw data.
[3.] Edge connection includes forward and backward flow in raw data.

(An 'X' in the 'Rank' column indicates that the edge does not appear in the raw data set.

Count	Edge Pair	Components	Total Number of Process Stages	Rank	Edge Made
1	$(2-4)^2$	1, 3	8	3	Yes
2	$(4-7)^2$	1, 3	8	4	Yes
3	$(7-2)$	1, 4	9	X	No
4	$(1-2)^2$	2	4	9	No
5	$(2-5)^2$	2	4	10	No
6	$(5-6)^1$	2, 4	10	2	Yes
7	$(6-1)$	2	4	X	No
8	$(1-3)^1$	3	5	8	Yes
9	$(3-2)^3$	3, 4	11	1	Yes
10	$(7-1)$	3	5	X	No
11	$(3-4)^2$	4	6	7	No
12	$(4-5)^1$	4	6	5	No
13	$(6-7)^1$	4	6	6	Yes

Table 4.Edge Connection and Ranking.

If the modified rankings, shown in Table 4, are now applied, using the MBSMHCA. The chain shown in Figure 5 can be generated.

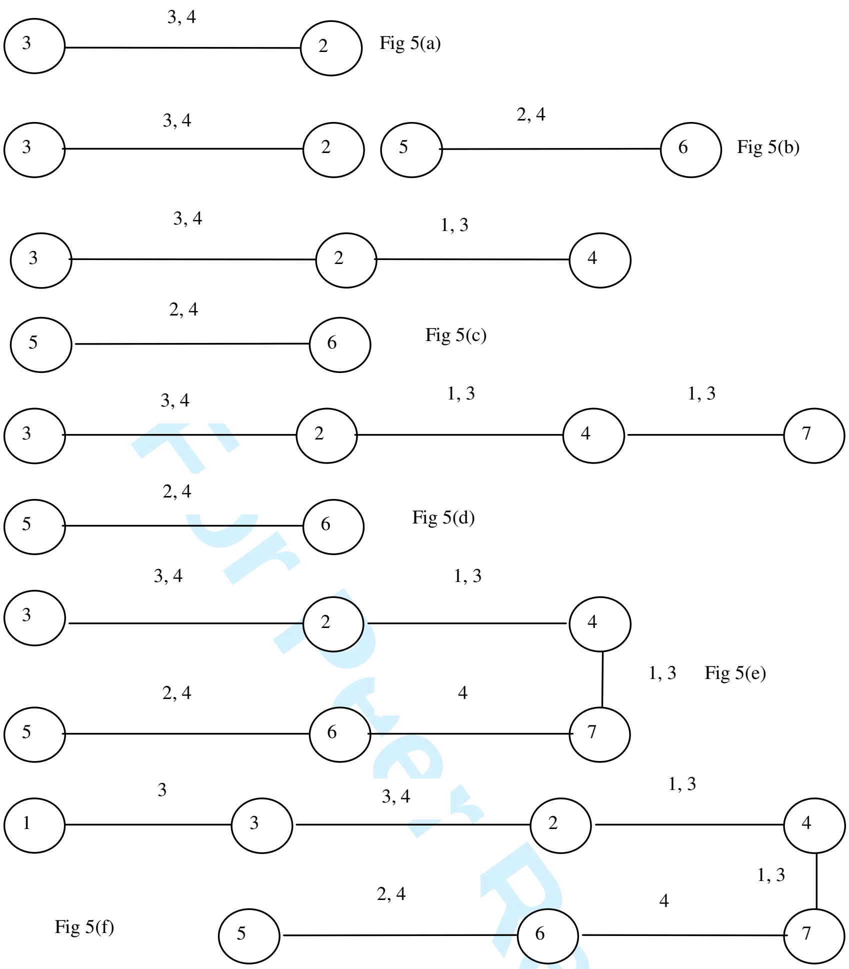

Figure 5. The Hamiltonian Chain for the data in Table 4

From the foregoing, the final chain, using the method modifications suggested in this contribution, is:-

$$1 \Rightarrow 3 \Rightarrow 2 \Rightarrow 4 \Rightarrow 7 \Rightarrow 6 \Rightarrow 5.$$

If a similar approach to measurement, employing this sequence to process the component set, is undertaken the inter-process move requirements for the components shown in Table 1 are shown in Figure 6.

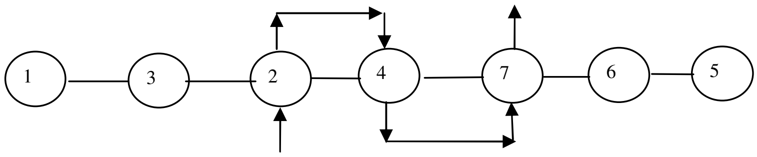

Figure 6 (a) Flow path for Component 1 (2 inter-path moves)

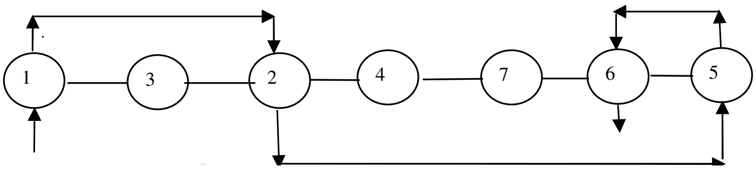

Figure 6 (b) Flow path for Component 2 (7 inter-path moves)

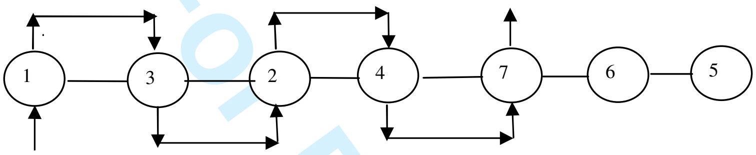

Figure 6 (c) Flow path for Component 3 (4 inter-path moves)

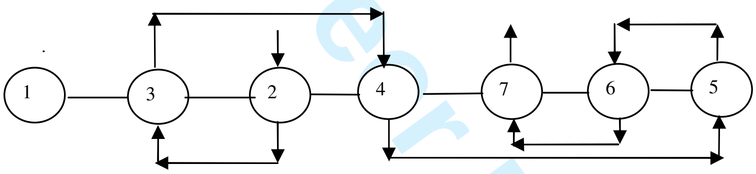

Figure 6 (d) Flow path for Component 4 (8 inter-path moves)

Figure 6. Flow Paths for Components 1 – 4 (modified algorithm)
Total inter-path moves = 21

Concluding Comment

There can be no doubt, from the evidence presented in the Mukhopadhyay et al (2000) contribution, that the MBSMHCA is an effective methodology for determining a linear arrangement of facilities. In that contribution, the performance of the MBSMHCA is tested against several different sized problems taken from the available research literature. The testing compares their approach with other well-known methods. These include both rank order clustering and multi-dimensional scaling procedures. For completeness, the results of these tests are included in this contribution, as Table 5. An examination of these results, clearly show that the MHC algorithm generates either the highest or joint highest Grouping Efficiency in six of the seven quoted cases. Thus, it can be argued that the MBSMHCA is a very effective solution approach. However given the disappointing result generated in this contribution it would also seem reasonable to question the cause of such a relatively poor performance and whether outcomes could be bettered if the modifications suggested in this contribution were incorporated into their approach.

Source	Problem Size (m x n)	Grouping Efficiencies		
		ROC	MDS	MHC
Chandraeskharen and Rajagopalan (1986)	8 x 20	86.66	95.83	95.83
Nair and Narendran (1996)	24 x 40	64.92	96.08	96.18
Mukhopadhyay and Gopalakrishnan (1995)	10 x 10	85.29	85.29	85.29
Chan and Milner (1982)	10 x 15	88.40	96.00	96.00
Boe and Cheng (1991)	5 x 18	91.51	91.51	91.51
Boe and Cheng (1991	20 x 35	60.42	80.88	80.88
Mukhopadhyay (1995)	7 x 9	91.10	88.13	88.13

Table 5 Performance of the MBSMHCA.

The seven facility simple problem used in this contribution has only 5,040 (7!) possible solution arrangements (although for each forward passing processing string e.g. 1 ⇒ 2 ⇒ 3, there exists a backward passing processing string e.g. 3 ⇒ 2 ⇒ 1 and thus the actual number of real solutions is in fact 2,520). Therefore, in order to explore the solution space, it was decided to perform a total enumeration exercise. First, the exercise showed that the best possible solution is (1 ⇒ 3 ⇒ 2 ⇒ 4 ⇒ 7 ⇒ 5 ⇒ 6). This particular sequence requires only 20 inter facility moves to process the four component sets and was not found either by the Ho et al (1993) heuristic, nor by the modifications to the MBSMHCA suggested in this contribution..

Secondly, and somewhat surprisingly, there are a total of 79 other solutions which bettered the 28 inter facility move sequence (3 ⇒ 2 ⇒ 4 ⇒ 7 ⇒ 1 ⇒ 6 ⇒ 5) derived using the MBSMHCA. Of the other possible solutions, the next best are three sequences (each requiring 21 inter process stage moves). One was generated by Ho et al (1 ⇒ 3 ⇒ 2 ⇒ 4 ⇒ 5 ⇒ 6 ⇒ 7). Another was developed using the methodology proposed in this contribution (1 ⇒ 3 ⇒ 2 ⇒ 4 ⇒ 7 ⇒ 6 ⇒ 5), whilst the remaining other solution (1 ⇒ 3 ⇒ 2 ⇒ 4 ⇒ 5 ⇒ 7 ⇒ 6) was not found, even as a possible alternative solution, by either procedure. Why this should be so is not clear, but it is evident that the edge couplet (5 – 7), which exists in this particular sequence and that of the best possible sequence, amongst others, does not even appear in the raw data set. It is also apparent that the triplet (1 ⇒ 3 ⇒ 2) is the predominant start sequence in the four solutions previously mentioned, does not appear in the MBSMHCA solution, but does appear frequently in several of the sequences where the number of inter facility moves is less than the 28 required by the MBSMHCA.

Finally, broader reflections upon this contribution would have to enquire whether there exists any correlation between the measures of Grouping Efficiencies used as measure of 'goodness' when evaluating comparative clustering algorithms and the inter facility flow measures used in this contribution to evaluate 'string goodness'.

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