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AN ALTERNATIVE METHOD TO TOLERANCE TRANSFER FOR PARTS WITH 2D BLUEPRINT

GONZÁLEZ F. AND ROSADO P.

The classic tolerance charting method to allocate working dimensions and tolerances, is a particular case of tolerance transfer between different dimension schemes. Tolerance transfer allows tolerance allocation for a new dimension scheme complying initials blueprint specifications.

~~The classic method of tolerance charting can be substituted by methods based on equations, denominated tolerance transfer techniques. These methods allow the evaluation of dimensions and their tolerance which are equivalent to the initial blueprint.~~

This paper suggests a new method to tolerance transfer that allows the evaluation of dimensions and their tolerance which are equivalent in 2D problems. Initially, a method that is able to consider dimensions, angles and orientation tolerances, is established. The first thing that is indicated in the proposed method is how to establish the equations for each 2D dimensional chain through 2D graphs. In an intermediate step a strategy to solve the 2D equations obtaining relations between the tolerances is proposed. Finally the equivalent tolerances are evaluated, considering both the variance accumulation theory and the deviation accumulation theory. The proposed methodology has been applied to different cases with satisfying results.

1. INTRODUCTION.

The study of tolerances is an important research area because tolerancing is very important in different stages of the development of a product (design, manufacturing, assembly and quality control). ~~The study of tolerances is an important research area in all the different stages of development of a product (design, manufacturing, assembling).~~ Different techniques regarding

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4 tolerances have been reported on papers about the state of art (Zhang and Huq 1992, Ngoi and
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6 Kuan 1995).
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9 At process planning, working dimensions and tolerances for the different setups are allocated.
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11 The set of working dimensions defines a new dimension scheme for the part (machining
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13 dimension scheme). If machining and design datums for dimensions are different, machining
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15 and design dimension scheme are different too (Li et al. 2000). At each setup of process
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17 machined surfaces must be related to machining datum directly or through a dimensional chain
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19 including only surfaces machined in the same setup. If design dimension scheme violates these
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21 conditions, blueprint specification must be converted into a valid machining dimension scheme,
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23 and so tolerance transfer is required.
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26 Among tolerancing techniques, the most frequently used one is tolerance charting, which is
27
28 very used a lot in manufacturing industries in order to determine the necessary process
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30 dimensions and tolerances. In order to compute tolerance accumulation, tolerance charting
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32 needs a dimensional chain tracing method. Tolerance transfer techniques allow obtaining the
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34 inequalities that establish tolerance accumulation at final dimension scheme. A tolerance chart
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36 represents the relations between process dimensions and blueprints throughout all the different
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38 manufacturing stages of a product (Li et al. 2000). Tolerance transfer techniques can substitute
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40 tolerance charting modelling the problem with equations.
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44 Other working lines regarding tolerances are related to tolerancing algebra (Hong and Chang
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46 2003), tolerancing in assembly (Jayaraman and Srinivasan 1989, Ngoi and Cheong 1998, Ngoi
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48 and Ong 1999), kinematic tolerance analysis (Joskowicz et al. 1997), tolerance information
49
50 models (Kulkarni and Pande 1996, Shah and Yan 1996, Serré et al. 2001, Zhou et al. 2002,) and
51
52 quality and cost tolerance chart optimisation (Jeang 1998).
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54
55 Tolerance charting is a technique that guarantees that the dimensions and tolerances of the
56
57 process comply with blueprint specifications. Tolerance charting is a particular application of
58
59 tolerance transfer in order for two dimensions schemes to be equivalent (Desrochers and
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Verheul 1999). From the very beginning this technique has mainly been used in 1D problems.

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4 According to this approach, 1D tolerance charting is applicable to square shouldered parts and is
5 applied in industries following the manual methodology described by Wade (1983). For these
6 kinds of parts 2D and 3D blueprints can be simplified to various 1D tolerance charts transfers,
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8 hypothesis under which the technique is valid.
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12 Mathematical relations between the dimensions and tolerances of the process and the
13 blueprint specification (Zhang and Huq 1992) can be found through dimensional chains in 1D
14 problems. By means of these relations one can calculate the equivalence between these two
15 groups of tolerances that represent both the design and manufacturing specifications
16 respectively. It is also necessary to use dimensional chains in 2D and 3D problems, but their
17 formulation is more complex than in 1D problems (Ji and Xue 2002, Rosado et al. 2002).
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26 Different methods to identify the dimensional chains have been proposed. Xiaoqing and
27 Davies (1988) proposed a matrix representation of the tolerance chart which, through matrix
28 operations, solves the problem parallel to that of the manual solution. Graphic methods have
29 also been proposed (Irani et al. 1989). Other approximations are based on a representation tree
30 (Ji, 1993, Ngoi and Fang 1994). All these works permit to work with 1D problems.
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36 **Tolerance charting in 2D and 3D cases is equivalent to a multidimensional tolerance transfer.**
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38 Some studies dealing with 2D and 3D tolerance chart problems like the one carried out by Ngoi
39 and Kuan (1995) apply a double tolerance chart to parts that have blueprint in two orthogonal
40 directions, the dimensions of which are independent. Thus, this case is only a double 1D
41 problem, not really a 2D problem. Ji and Xue (2002), on another hand, have developed a
42 method to integrate angular features that relate, for example, the radial and axial dimensions in a
43 revolution part. Their analysis faces the problematic of a 2D problem by using real 2D
44 dimensional chains.
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54 Studies representing dimensional and geometric tolerances have been carried out in 3D
55 tolerancing (Serré et al. 2001). Shah and Yan (1996), suggest a model of representing 3D
56 tolerances through design features. Gupta and Turner (1999) have developed a three-
57 dimensional geometric model with which to analyse tolerances. Finally the works of Villeneuve
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4 et al. (2001) and Vignat and Villeneuve (2003) should be highlighted. Here tolerances and
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6 deviations are modelled in the drilling and turning process by applying the small displacements
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8 torsor. Thus, the relationship between design tolerances and the deviations and capacities of the
9
10 productive resources is established.
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13 This paper proposes a new approach for tolerance transfer in 2D parts with real 2D blueprint.
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15 The method proposed is based on 2D graphs of tolerances to obtain 2D chains equations. This is
16
17 a general approach, and it has been applied both to dimensional and orientation tolerances.
18
19 Through this method it is possible to obtain the equations that relate design dimensions and
20
21 tolerances (blueprints) to process dimensions and tolerances. Relations between the tolerances
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23 of the process and the blueprint of the part (inequalities) can be obtained by applying
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25 equivalence conditions. In order to solve inequalities allocating the new tolerances, an objective
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27 function and additional tolerance constraints must be added. Usually objective function is the
28
29 manufacturing cost and the process tolerances are used as additional constraints. In order to
30
31 solve in the best possible way to determine the processes with sufficient capacity and minimum
32
33 cost, the suggestions given in the bibliography regarding the issue can be followed (Jeang 1998,
34
35 Tseng and Terng 1999, Li et al. 2000).
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40 2. THEORETICAL FORMULATION.

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42 The tolerance transfer problem can be defined as the search for tolerances T_{A_j} that should
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44 affect dimensions A_j so that their compliance guarantees that the part fulfils its design
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46 specifications defined by dimensions C_i and its tolerances T_{C_i} . A dimension scheme A which
47
48 fulfils the previous condition will be equivalent to dimension scheme C.
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51 In order to establish the value that tolerances T_{A_j} should have, the C_i initial dimensions should
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53 be expressed according to the final dimensions:
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$$56 C_i = f_i(A_j) \quad (1)$$

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If we assume that tolerances are centred in the nominal values of dimensions (bilateral system), such tolerances or variation values admissible in the dimensions are represented by the absolute value of the blueprint differentials, thus:

$$\begin{aligned} T_{C_i} &= |d(C_i)| = |C_i'| \\ T_{A_j} &= |d(A_j)| = |A_j'| \end{aligned} \quad (2)$$

so if $\partial f_i / \partial A_j$ can be known results

$$C_i' = \sum_j \frac{\partial f_i}{\partial A_j} A_j' \quad (3)$$

Being V_{C_i} the maximum variation in dimension of C_i due to the variations of dimensions A_j therefore

$$V_{C_i} = |C_i'| \leq \sum_j \left| \frac{\partial f_i}{\partial A_j} \right| |A_j'| = \sum_j \left| \frac{\partial f_i}{\partial A_j} \right| T_{A_j} \quad (4)$$

Since these variations must be lower or equal to those admitted in the initial dimensions scheme

$$T_{C_i} \geq V_{C_i} \quad (5)$$

and using (4), we can establish:

$$T_{C_i} \geq \sum_j \left| \frac{\partial f_i}{\partial A_j} \right| T_{A_j} \quad (6)$$

These equations formulate the constraints to assign T_{A_j} using an objective function, usually the manufacturing process cost.

Since operations among dimensions are sums and subtractions, in the case of a 1D dimensional transfer, the values of $|\partial f_i / \partial A_j|$ are always 1. However, in a 2D or 3D dimensional scheme the values of $|\partial f_i / \partial A_j|$ will have to be calculated (Xue and Ji 2004).

From a different point of view, the dimensions transfer problem can be treated as the accumulation of the uncertainty of dimensions. The tolerances of dimensions are related to their

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4 **uncertainty**, that have an uncertainty which is established by its tolerances. Thus, the tolerance
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6 of a dimension is only its uncertainty, which can be expressed as a variance (U^2) by a factor of
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8 uncertainty (k) that depends on the level of confidence. Thus:
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$$\begin{aligned} T_{C_i} &= kU_{C_i} \\ T_{A_j} &= kU_{A_j} \end{aligned} \quad (7)$$

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15 By applying the law of accumulation of variances to expression (1), and variables A_j being
16
17 independent, the variance of dimensions C_i ($U_{C_i}^2$) can be evaluated as

$$U_{C_i}^2 = \sum \left(\frac{\partial f_i}{\partial A_j} \right)^2 U_{A_j}^2 \quad (8)$$

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25 which, taking the same uncertainty factor for all of the dimensions and (5), leads us to

$$T_{C_i}^2 \geq \sum_j \left(\frac{\partial f_i}{\partial A_j} \right)^2 T_{A_j}^2 \quad (9)$$

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32 The application of (9) in a 1D problem leads us to the quadratic composition of tolerances.
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34 Similarly to that presented previously, for 2D problems it is necessary to evaluate $\partial f_i / \partial A_j$.

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36 **Other accumulation methods can be proposed but they are formulated as function of terms**
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39 $\partial f_i / \partial A_j$.

40 41 42 **3. DEFINITION OF THE 2D TRANSFER PROBLEM.**

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45 Transfer between two 2D dimension schemes can include different types of dimensions and
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47 tolerances or specifications. Among the different types, dimensional tolerances and
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49 form/geometric tolerances should be distinguished. Whereas the first one establishes the
50
51 possible variations in the distances or angles between sides and/or points of the part, the second
52
53 one limits the possible variations in the shapes and orientation of its sides. In the suggested
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55 methodology a homogeneous treatment of both types of tolerances, dimensional and
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57 geometrical, is followed. However, restrictions that are presented next should be taken into
58
59 consideration.
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3.1. Constraints to dimensional schemes.

3.1.1. Types of specifications.

The methodology presented can work both with dimensional tolerances and geometric tolerances. All dimensional blueprints and its corresponding tolerances can be included in the dimension scheme. Nevertheless, not all geometric tolerances are liable to be treated. Geometric tolerances can be classified into ~~five~~ ~~three~~ basic types: form, **profile**, orientation, **runout** and location/**position**. ~~Form and individual profile with no datum~~ tolerances limit the variation that a surface or side of the part can have, for example straightness. This type of tolerances can not be transferred because **they involve only individual features and so** they limit the intrinsic form of the surface. Something similar happens with the run-out tolerances, which imply functional conditions of radial and/or axial run-out as regards an axis of reference (Tseng and Kung 1999). Location/position tolerances will not be dealt with in this study, although the authors are working on them at present.

3.1.2. Unique and complete dimensional schemes.

In order to do the **tolerance** transfer between two dimensional schemes of a part, both schemes must only define the part and its geometry. Thus, the group of dimensions of each one of them must define the situation and orientation of all of its sides and/or points with at least another side of the part. This restriction implies that the blueprints of a dimension scheme as a whole should keep the geometry of the part from loosing its nominal shape indefinitely when fulfilling the tolerances. (Clement et al, 1999).

The necessary number of dimensions and tolerances to define a 2D geometry sufficiently can be established according to the number of nodes or vertices of the dimension scheme. A node is the intersection between the geometry lines. A sufficient dimensional scheme should establish the position of each of its nodes, and thus restrict its two degrees of freedom (Serré et al. 2001). Nevertheless, considering the part as a rigid solid, we should eliminate the degrees of freedom

corresponding both to the global position of the part and to its rotation. If we refer to the number of nodes of the part as n , in accordance with that mentioned previously, the number of dimensions and its tolerances (N_T) will be defined by

$$N_T = ((n - 1) * 2) - 1 \quad (10)$$

Each of the dimension, in accordance to its type, eliminates a number of degrees of freedom. Thus, a dimension which is between sides eliminates two degrees of freedom. The rest of dimensions that have been mentioned eliminate one degree for freedom each one.

If the number of dimensions is not sufficient, additional dimensions and tolerances will have to be established. The values of which will be set in accordance with the general drawing office specifications.

3.1.3. Principle of independence.

On another hand, the blueprints of a dimensional scheme will be interpreted according to the principle of independence (ISO 8015), according to which its compliance is independent from the compliance or not of the rest of the specifications of the part (Clement et al. 1999).

3.2. Formulation of the dimension schemes.

All the different types of dimensions considered will be formulated according to longitudinal and angular variables, the basic working variables.

Figure 1 and 2 show the expressions assumed for each dimension C and its corresponding tolerance T_C , depending on their type. These expressions indicate the used necessary variables to model the problem. for their definition. One must notice how the tolerance of a distance between sides is formed by two contributions: one that assesses the situation among the sides ($(L \sin \alpha)$) and the other that assesses its orientation or parallelism ($d a'$).

Figure 1 [Insert about here]

Figure 2 [Insert about here]

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4 Figure 3 shows the variables that are necessary to represent orientation tolerances. In the
5 three cases the value of tolerance is **formulated** ~~represented~~ as the differential of the angle
6 multiplied by the distance of the affected side. With this **formulation** these orientation tolerances
7 are dealt with in the same way as the rest of dimensional tolerances.
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12 Figure 3 [Insert about here]
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15 16 **4. 2D DIMENSIONAL TRANSFER METHODOLOGY.** 17

18 The proposed methodology to carry out the 2D dimensional transfer is as follows. This
19 methodology obtains what we denominate fundamental transfer equations (3). The methodology
20 can be summarised in three steps:
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- 24 1. Formulate Dimensional Chains. Dimensional Chains are a group of equations that relate
25 initial and final dimensions, both of them expressed according to longitudinal and/or
26 angular variables. Two kinds of variables can be distinguished: variables defining the
27 problem, which correspond to tolerances, and 'virtual' variables, which do not correspond
28 to any tolerances.
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- 35 2. Auxiliary equations. These are a group of equations that relate 'virtual' variables to the
36 variables defining the problem of its dimension scheme. Obviously, the number of auxiliary
37 equations should be the same as the number of 'virtual' variables used in the previous step.
38 These equations are divided into two groups that correspond to the initial problem and to
39 the final one. Each one of them allows the elimination of 'virtual' variables according to the
40 variables of the dimension scheme.
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- 48 3. Fundamental equations. Fundamental equations are the result of combining the two
49 previous groups in order to establish the group of equations that relate the two dimension
50 schemes, in which 'virtual' variables have been eliminated. The equations (3) can be cleared
51 out of this last group of equations. **As result, the fundamental equations link the final**
52 **dimension scheme with the initial one.**
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Once the fundamental equations of the transfer have been obtained, $\partial f_i / \partial A_j$ are recognised and equations (6) and/or (9) are applied to establish the **equations system of inequalities** that will mark the values of the final dimension tolerances.

In order to facilitate the operation of the group of equations, these are transferred into their derived form, becoming linear equations due to the fact that the new unknown become the derivatives of the variables of the problem.

The solution to the problem is guaranteed through the dimensional scheme hypothesis carried out, which guarantees the unique definition of the degrees of freedom of all nodes. This insures both the existence of equations of equivalence (among different dimension schemes) and of auxiliary equations for any 'virtual' variable considered.

4.1 Dimensional Chains.

Dimensional chains **identify** ~~shape~~ mathematical relations between initial dimensions and tolerances and final ones. In order to determine these relations, some graphical representations which we will refer to as 'graphs', are used. These are of two types: dimensional graphs and angular graphs.

4.1.1 Dimensional graphs.

Dimensional graphs can represent graphically the dimensions established by the dimensional scheme according to the variables that define them. Dimensions normal to a direction of reference, marked by the dimensions themselves, are represented in a graph. The graph will show the direction of reference, marked by two nodes, and the projection of the rest of nodes in the normal direction. Dimension projections are showed as a connecting line between nodes. Each connecting line has an expression using the problem variables.

Graphs have two parts: left, where the relations of the initial dimension scheme are represented, and right, where the relations of the final dimension scheme are represented (Figure 4).

Figure 4 [Insert about here]

Dimensional graphs are proposed for the representation of all the dimensions of the initial dimension scheme. The number of graphs should be equal to the number of different directions that define these dimensions. If the dimensions do not imply generating at least two graphs, these two directions are established in an arbitrary way since, due to the fact that it is 2D dimension schemes, at least two graphs are necessary.

Considering the references of the dimensions as references for the graphs ensures finding the relation of the dimensions of the initial with that of the final (dimensional chains) in a direct way.

Each dimension in the initial dimension scheme is represented in only one graph. If the dimension affects one side, it is represented on the graph that has that side as a reference. If the dimension affects two sides, it is represented on the graph that has one of the sides as a reference, introducing the necessary additional 'virtual' variables. If the dimension only defines the distance between two points, its projection is represented on one of the graphs already created, introducing the necessary additional 'virtual' variables.

Dimensions on the final dimension scheme are represented on the right side of the graphs. Dimensions between two points will be represented, introducing the necessary additional 'virtual' variables. Dimensions affecting one side will be placed in a graph constructed from that sides' direction. If a graph to represent them does not exist, this will be contemplated subsequently in order to cause auxiliary equations.

From Parting from the example of Figure 5 and using the notation on Figure 6, the dimensional graphs will be those shown in Figures 7 and 8. The part blueprint specifications shown by the initial dimension scheme, will be transferred for a new specifications set. In the new dimension scheme the dimension B is replaced by the dimension F. The equivalence between tolerance schemes require that the other tolerances of initial blueprint must be newly valuated in the final tolerance scheme. These tolerances are G and E.

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Figure 5 [Insert about here]

Figure 6 [Insert about here]

Figure 7 [Insert about here]

Figure 8 [Insert about here]

4.1.2 Angular graphs.

Angular graphs show the angles between the directions defined by the **different** pairs of nodes of the dimension scheme, which do not have to coincide with the sides of the part. Two graphs are proposed, one for the initial node and another for the final one. These graphs have to be constructed using a unique criterion for angle measure (clockwise or anticlockwise). The angular graphs of the example are shown in Figure 9.

Figure 9 [Insert about here]

4.1.3 Formulation of dimensional chains.

Similarly to 1D problems, chains of dimensions are formulated following the suggested dimensional and angular graphs. In order to do that, an equivalent path is searched for in the final graph for each of the connecting lines of the initial graph. Thus, all relation on the left side of a dimensional graph **are** equalized to the relations that **have** the same path **constructed** with the available connecting lines on the right side. These equations are derived in order to obtain the equations of equivalence.

Similarly, equations of equivalence for angular dimensions are established by searching the equivalent path in the final angular graph for each of the angular dimensions appearing in the initial angular graph. If throughout this process it is necessary to consider new 'virtual' variables, those that most directly define the required path are added.

In the example the dimensional chains are:

$$B \sin \alpha_{1,2-1,3} = LL_{2,3} \sin G$$

$$L_{1,2} \sin C = LL_{1,2} \sin G$$

$$L_{1,4} \sin \alpha_{2,3-1,4} = LL_{1,4} \sin \beta_{2,3-1,4}$$

$$C = G$$

$$D = \beta_{3,4-1,2}$$

4.2 Auxiliary equations.

Transfer auxiliary equations permit the removal of 'virtual' variables that have been introduced in the dimensional chains. The number of independent auxiliary equations will be the same as that of introduced 'virtual' variables. The search of these equations will be carried out separately for the initial dimension scheme and for the final one. Its formulation is set out through three means:

- Equations of definition. The equations of definition of the dimensions themselves are added, as shown previously. These equations relate the dimensions to the variables used for their representation.
- Graph closed paths. All of the paths that can be closed with the defined connecting lines or by introducing new connecting lines to complete new paths, are expressed on the graphs. These paths are searched for dimensional and angular graphs for both the initial and the final dimension schemes.
- New graphs. When there are dimensions of the final dimension scheme which have not been able to be represented in any of the graphs due to not having a reference direction, a new graph is created, only with the part of the final dimension scheme, which will use that direction as a reference. Auxiliary equations are set out over this graph, as in the previous case.

The process of generating auxiliary equations can entail the consideration of new 'virtual' variables. The problem is defined when there are as many equations, linearly independent, as the number of 'virtual' variables considered in each of the initial and final dimensions schemes.

In order to automate the methodology, all of the above can be generalised, presenting all the possible closed paths for all of the graphs. This would entail introducing more ‘virtual’ variables, though there would be more auxiliary equations to clear them out. Finally, among those left, the linearly independent ones would be extracted, clearing out the whole of the ‘virtual’ variables depending on the dimensions.

On applying it to our example, due to the definition of dimensions, the following equations would remain in first place.

$$A = L_{1,2} \sin C + L_{1,4} \sin \alpha_{1,4-2,3}$$

$$E = LL_{1,2} \sin G + LL_{1,4} \sin \beta_{1,4-2,3}$$

$$F = LL_{2,3} \sin G + LL_{3,4} \sin \beta_{3,4-1,2}$$

Figure 10 [Insert about here]

Figure 11 [Insert about here]

Figure 12 [Insert about here]

Subsequently, forming graph closed paths, the dotted arrows would be added to the graphs (Figures 10, 11 and 12), and they would result in:

Auxiliary equations of the initial dimension scheme

$$A - L_{1,2} \sin C - L_{1,4} \sin \alpha_{2,3-1,4} = 0$$

$$B \sin \alpha_{1,2-1,3} + L_{3,4} \sin D - L_{1,4} \sin(C - \alpha_{2,3-1,4}) = 0$$

$$L_{1,2} \sin C + L_{1,4} \sin \alpha_{2,3-1,4} - L_{3,4} \sin(180 - C + D) = 0$$

$$L_{1,2} \sin C - B \sin(C + \alpha_{1,2-1,3}) = 0$$

Auxiliary equations of the final dimension scheme

$$E - LL_{1,2} \sin G - LL_{1,4} \sin \beta_{2,3-1,4} = 0$$

$$F - LL_{2,3} \sin G - LL_{3,4} \sin \beta_{3,4-1,2} = 0$$

$$LL_{1,2} \sin G + LL_{1,4} \sin \beta_{2,3-1,4} - LL_{3,4} \sin(180 - G + \beta_{3,4-1,2}) = 0$$

$$LL_{2,3} \sin G + LL_{3,4} \sin \beta_{3,4-1,2} - LL_{1,4} \sin(G - \beta_{2,3-1,4}) = 0$$

4.3 Fundamental equations.

Once these auxiliary equations have been obtained, the values of the differentials of the 'virtual' variables are differentiated and cleared up according to the values of the differentials that the dimensions represent.

Applying it to our example, the fundamental transfer equations (3) are:

$$B' = 0.8944E' + 0.4472F' + 89.4425(G' - \beta_{3,4-1,2}' - \beta_{2,3-1,4}')$$

$$D' = \beta_{3,4-1,2}'$$

$$A' = E'$$

$$C' = G'$$

$$\alpha_{2,3-1,4}' = \beta_{2,3-1,4}'$$

being

$$\begin{pmatrix} T_A \\ T_B \\ T_C \\ T_D \\ T\alpha_{2,3-1,4} \end{pmatrix} = \begin{pmatrix} A' \\ B' \\ L_{2,3}C' \\ L_{3,4}D' \\ L_{2,3}\alpha_{2,3-1,4}' \end{pmatrix} \quad \begin{pmatrix} T_E \\ T_F \\ T_G \\ T\beta_{3,4-1,2} \\ T\beta_{2,3-1,4} \end{pmatrix} = \begin{pmatrix} E' \\ F' \\ LL_{2,3}G' \\ LL_{3,4}\beta_{3,4-1,2}' \\ LL_{2,3}\beta_{2,3-1,4}' \end{pmatrix}$$

Using the tolerance notation the fundamental transfer equations are

$$\begin{aligned} T_B &= 0.8944T_E + 0.4472T_F + 0.8944(T_G - 0.5T\beta_{3,4-1,2} - T\beta_{2,3-1,4}) \\ T_D &= T\beta_{3,4-1,2} \\ T_A &= T_E \\ T_C &= T_G \\ T\alpha_{2,3-1,4} &= T\beta_{2,3-1,4} \end{aligned}$$

that applying (6) give the inequality system, formulating the constraints governing transference.

$$\begin{aligned} T_B &\geq 0.8944T_E + 0.4472T_F + 0.8944T_G + 0.4472T\beta_{3,4-1,2} + 0.8944T\beta_{2,3-1,4} \\ T_D &\geq T\beta_{3,4-1,2} \\ T_A &\geq T_E \\ T_C &\geq T_G \\ T\alpha_{2,3-1,4} &\geq T\beta_{2,3-1,4} \end{aligned}$$

that applying (6) can be solved to evaluate final tolerances

One solution is for example

$$\begin{pmatrix} T_A \\ T_B \\ T_C \\ T_D \\ T\alpha_{2,3-1,4} \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.4 \\ 0.1 \\ 0.1 \\ 0.4 \end{pmatrix} \triangleright \begin{pmatrix} T_E \\ T_F \\ T_G \\ T\beta_{3,4-1,2} \\ T\beta_{2,3-1,4} \end{pmatrix} = \begin{pmatrix} 0.198 \\ 0.298 \\ 0.1 \\ 0 \\ 0 \end{pmatrix}$$

The $T\alpha_{2,3-1,4}$ value representing a parallelism is compatible with the tolerance for dimension A. In the solution the zero value is assigned to $T\beta_{3,4-1,2}$ and $T\beta_{2,3-1,4}$. This assignment allows to increase the tolerances of dimensions E, F and G, and is possible because $\beta_{3,4-1,2}$ and $\beta_{2,3-1,4}$ are “virtual” variables and so they are not necessary to guarantee the final unique and complete dimension scheme. In the same way, $T\beta_{3,4-1,2}$ and $T\beta_{2,3-1,4}$ values are the parallelisms related to dimensions F and E.

5. RESULTS.

In order to give an example of the method suggested, its application on the dimensional transfer is going to be shown for the part of Figure 13-a. It is a 2D part with 2D blueprint. Through this example we intend to evaluate the machining dimensional scheme process dimensions and tolerances (Figure 13-b) that are equivalent to design blueprint. If the holes are machined locating part as show in figure 14, the tolerances and dimensions O, and R and their tolerance are needed. The equivalence between tolerance schemes require that tolerances of initial blue print must be newly valuated in the final tolerance scheme. These tolerances are M, K, S and Q. By transforming orientation tolerances into angular dimensions, the initial dimension scheme remains as shown in Figure 15-a with the indicated tolerances. Similarly, the final problem remains as indicated in Figure 15-b, the values of its tolerances remaining the unknown of the dimensional transfer problem.

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4 Figure 13 [Insert about here]
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8 Figure 14 [Insert about here]
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11 Figure 15 [Insert about here]
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13 The graphs necessary to represent blueprint initials, together with the necessary connecting
14 lines to formulate dimensional chains, are shown in Figure 16. The angular graphs of the initial
15 and final angular dimensions are shown in Figure 17. The ‘virtual’ variables considered for the
16 formulation of dimensional chains of the initial and final problem are shown in Figure 18.
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23 Figure 16 [Insert about here]
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26 Figure 17 [Insert about here]
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30 Figure 18 [Insert about here]
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32 In order to eliminate ‘virtual’ variables, auxiliary equations are formulated, and new
33 connecting lines in the dimensional graphs of both the initial and final (~~Figure 19~~) problems are
34 needed (**Figure 19**). Considering new connecting lines usually implies new ‘virtual’ variables.
35 In this example only length LL_{56} has been added for the final problem. When determining the
36 new connecting line it is necessary to search for angles between bars through the angular graphs
37 of the initial and final problem.
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47 Figure 19 [Insert about here]
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49 Next, the auxiliary equations of the transfer are shown, which will allow us to eliminate the
50 ‘virtual’ variables of the dimensional chains.
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52 Auxiliary equations of the initial dimension scheme are:
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$$\begin{aligned}
 F + G - \alpha_{5,6-1,2} &= 180 \\
 A &= L_{1,2} \sin F + L_{2,3} \sin \alpha_{1,4-2,3} \\
 E &= L_{5,4} \sin \alpha_{5,4-5,6} + L_{4,3} \sin \alpha_{4,3-5,6} \\
 L_{1,2} \sin F + L_{2,3} \sin \alpha_{1,4-2,3} &= L_{4,3} \sin(\alpha_{4,3-5,6} + G) \\
 L_{5,4} \sin(\alpha_{5,4-5,6} + G) + B &= 0 \\
 C + L_{5,4} \sin(\alpha_{5,4-5,6} + \alpha_{5,6-1,2}) + L_{4,3} \sin(\alpha_{4,3-5,6} + \alpha_{5,6-1,2}) &= L_{2,3} \sin(F - \alpha_{1,4-2,3})
 \end{aligned}$$

and auxiliary equations of the final dimension scheme:

$$\begin{aligned}
 K &= LL_{1,2} \sin Q + LL_{2,3} \sin \beta_{1,4-2,3} \\
 L_{4,3} \sin(180 - Q + P) &= LL_{1,2} \sin Q + LL_{2,3} \sin \beta_{1,4-2,3} \\
 S - N &= LL_{5,4} \sin(\beta_{5,4-5,6} + P - \beta_{4,3-5,6}) \\
 O - N &= LL_{5,6} \sin(P - \beta_{4,3-5,6}) \\
 R - M &= LL_{5,6} \sin(180 - Q + P - \beta_{4,3-5,6}) \\
 M + LL_{5,4} \sin(\beta_{5,4-5,6} + 180 - Q + P - \beta_{4,3-5,6}) &= 0 \\
 S + LL_{4,3} \sin P &= LL_{2,3} \sin(Q - \beta_{1,4-2,3})
 \end{aligned}$$

By working with the derivative form of auxiliary equations it is possible to obtain the expression of the derivatives of ‘virtual’ variables, according to the derivatives of the dimensions that define both the initial and final problem.

By transforming dimensional chains into their derivative form and substituting the derivatives of the ‘virtual’ variables, it is possible to obtain the derivatives of the dimensions of the initial problem according to the derivatives of the final problem. These are what we have called fundamental transfer equations (3), the coefficients of which are the expressions $\partial f_i / \partial A_j$.

$$\begin{pmatrix} A' \\ B' \\ C' \\ D' \\ E' \\ F' \\ G' \\ \alpha_{4,3-5,6}' \\ \alpha_{1,4-2,3}' \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.866 & 0.866 & -3.5 & 0 & 3.5 & 0 & -151.554 & 0 \\ 0 & 1.097 & -0.597 & -1.034 & 0.866 & -1.9 & 115.47 & -15.877 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0.5 & -0.5 & -0.866 & 0 & -0.866 & 0 & 37.5 & 0 \\ 0 & 1.155 & -1.155 & 2 & 0 & -2 & 115.47 & -28.867 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{pmatrix} K' \\ R' \\ M' \\ N' \\ S' \\ O' \\ P' \\ Q' \\ \beta_{1,4-2,3}' \end{pmatrix}$$

being:

$$\begin{pmatrix} T_A \\ T_B \\ T_C \\ T_D \\ T_E \\ T_F \\ T_G \\ T\alpha_{4.3-5.6} \\ T\alpha_{1.4-2.3} \end{pmatrix} = \begin{pmatrix} A' \\ B' \\ C' \\ D' \\ E' \\ L_{1.4}F' \\ L_{5.6}G' \\ L_{4.3}\alpha_{4.3-5.6}' \\ L_{1.4}\alpha_{1.4-2.3}' \end{pmatrix} \quad \begin{pmatrix} T_K \\ T_R \\ T_M \\ T_N \\ T_S \\ T_O \\ T_P \\ T_Q \\ T\beta_{1.4-2.3} \end{pmatrix} = \begin{pmatrix} K' \\ R' \\ M' \\ N' \\ S' \\ O' \\ LL_{4.3}P' \\ LL_{1.4}Q' \\ LL_{1.4}\beta_{1.4-2.3}' \end{pmatrix}$$

We can obtain the solutions for the new tolerances applying (9):

$$\begin{pmatrix} T_A \\ T_B \\ T_C \\ T_D \\ T_E \\ T_F \\ T_G \\ T\alpha_{4.3-5.6} \\ T\alpha_{1.4-2.3} \end{pmatrix} = \begin{pmatrix} 1 \\ 0.6 \\ 0.6 \\ 0.6 \\ 0.6 \\ 0.2 \\ 0.2 \\ 0.6 \\ 1 \end{pmatrix} \triangleright \begin{pmatrix} T_K \\ T_R \\ T_M \\ T_N \\ T_S \\ T_O \\ T_P \\ T_Q \\ T\beta_{1.4-2.3} \end{pmatrix} = \begin{pmatrix} 1 \\ 0.113 \\ 0.113 \\ 0.113 \\ 0.4 \\ 0.113 \\ 0.4 \\ 0.113 \\ 0 \end{pmatrix}$$

The $T\alpha_{4.3-5.6}$ and $T\alpha_{1.4-2.3}$ values representing parallelisms are compatible with the tolerance for dimensions E and A. As $\beta_{1.4-2.3}$ is a “virtual” variable, a zero value has been assigned to its tolerance, allowing a great value for the other tolerances. In the same way, $T\beta_{1.4-2.3}$ is the parallelism related to dimension K.

6. CONCLUSIONS.

This paper has presented a methodology which, based on dimensional and angular graphs, allows tolerance transfer between two 2D dimension schemes of a part. The methodology also allows the homogeneous treatment of dimensional and orientation tolerances by representing them with longitudinal and angular variables.

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4 Through this method the fundamental transfer equations (3) can be obtained, with which
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6 $\partial f_i / \partial A_j$ can be evaluated, enabling the application of the desired method of tolerance
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8 accumulation.
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11 A main characteristic of that differentiates this methodology from the rest is that allows to
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13 make tolerance transfers among complex 2D dimension schemes. It is based on graphs and
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15 equations and can be automated.
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18 Since it is a general methodology, it can be applied to all kinds of transfers between
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20 dimension schemes. For example it can be used in order to establish the dimension scheme of
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22 manufacturing, of editing or of verification, from the dimension scheme of design. Also, since
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24 dimension schemes are represented through variables, this methodology can be used in order to
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26 establish the dimension scheme of a part from its functional conditions.
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29 Finally we would like to point out that this methodology can be automated, and when
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31 following a similar working procedure it can be extended to problems of transfer among 3D
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33 dimension schemes, as well as to the consideration of location position tolerances.
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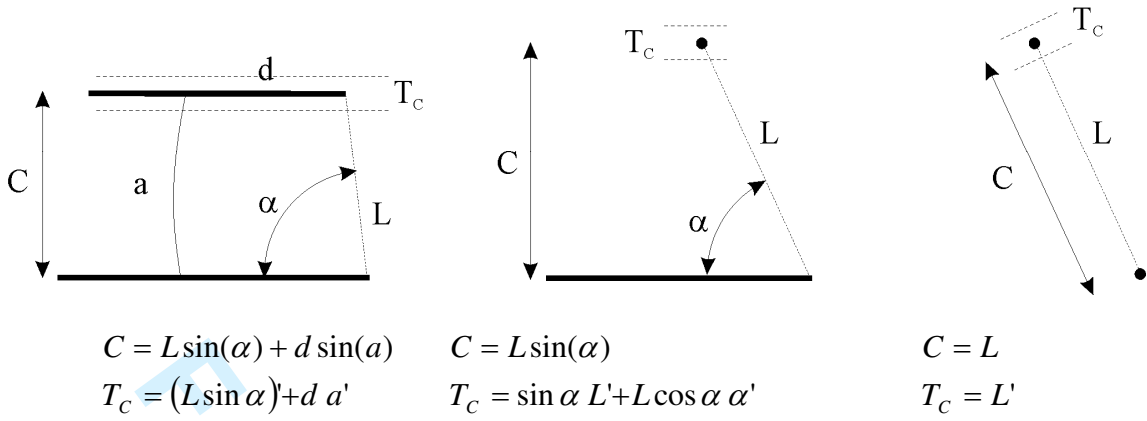


Figure 1. Variables that represent dimensional tolerances.

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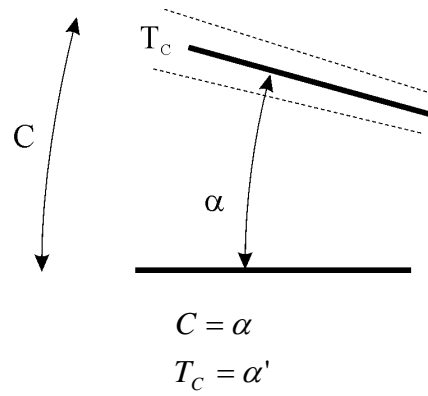


Figure 2. Variables that represent angular tolerances.

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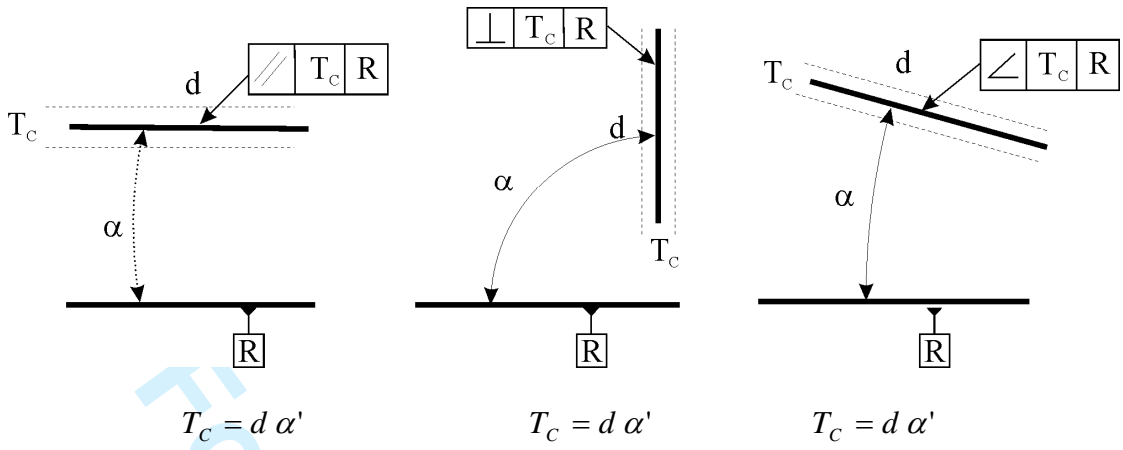


Figure 3. Variables that represent orientation tolerances.

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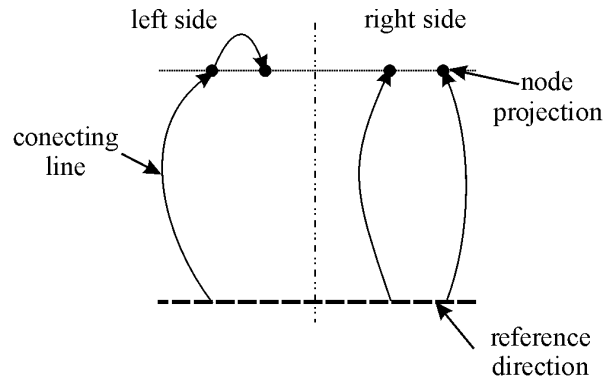
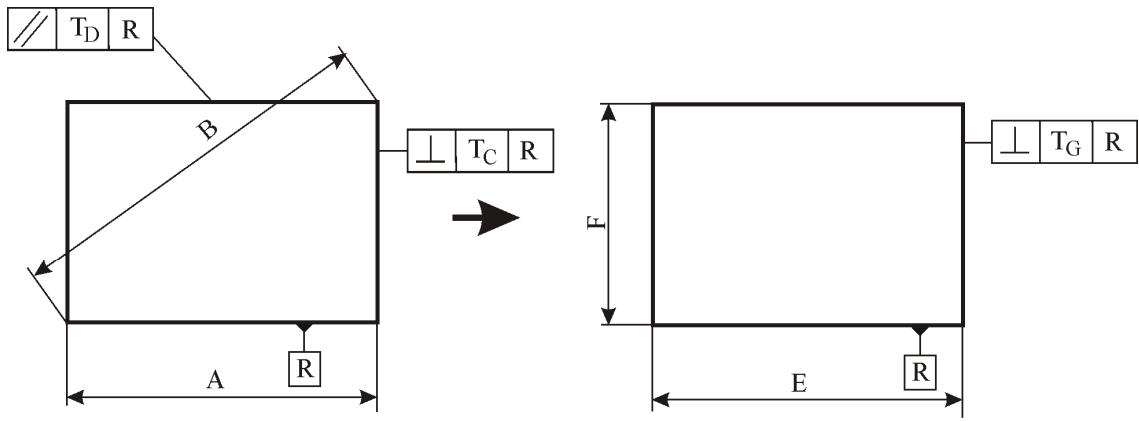


Figure 4. Dimensional graph.

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$$A = 200 \pm 0,2 \quad T_C = 0,1$$
$$B = 223,607 \pm 0,2 \quad T_D = 0,1$$

Figure 5. Example of 2D transference.

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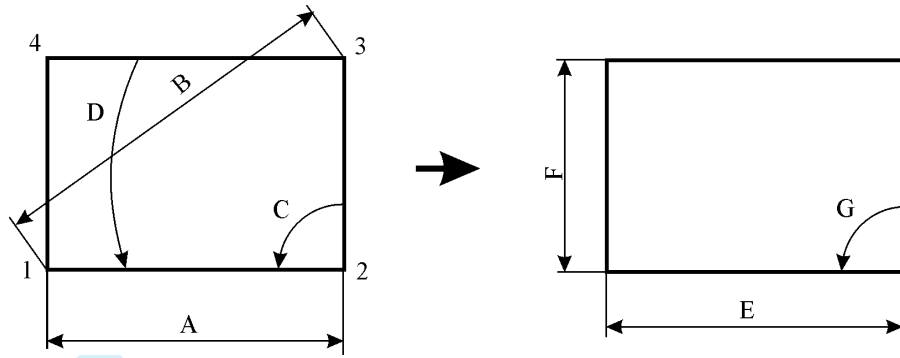


Figure 6. Numbering of nodes and variables of the problem

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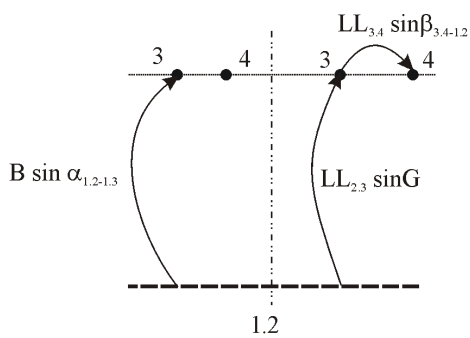


Figure 7. Dimensional graph normal to direction 1.2

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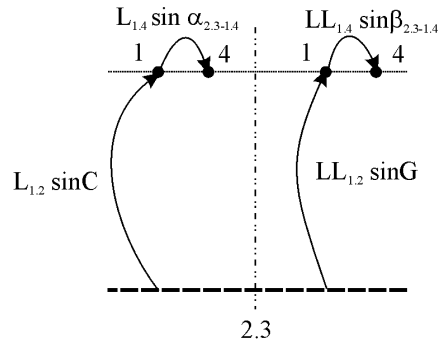


Figure 8. Dimensional graph normal to direction 2.3.

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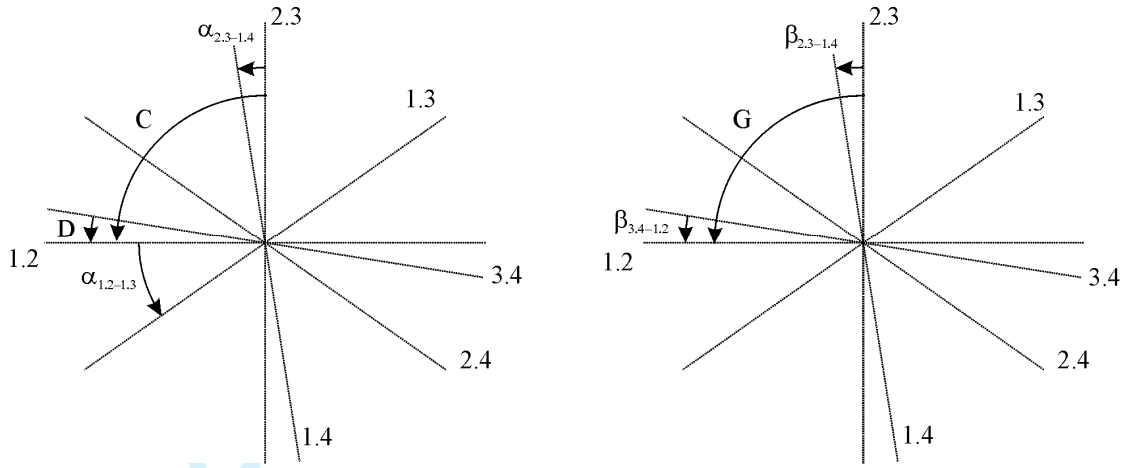


Figure 9. Angular graphs

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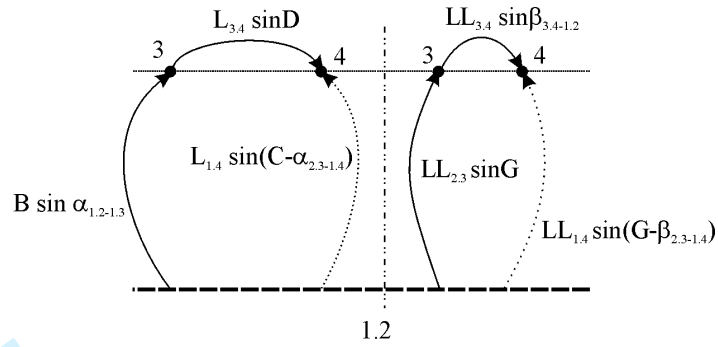


Figure 10. Closed paths on dimensional graph normal to direction 1.2

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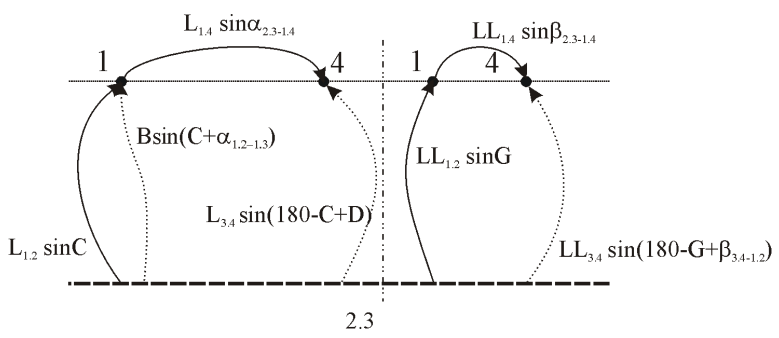


Figure 11. Closed paths on dimensional graph normal to direction 2.3

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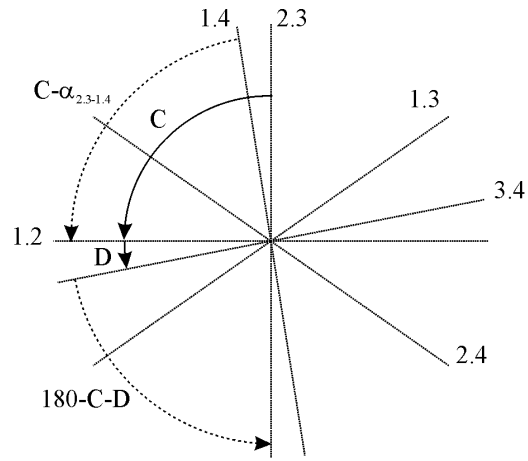


Figure 12. Closed paths on initial angular graph.

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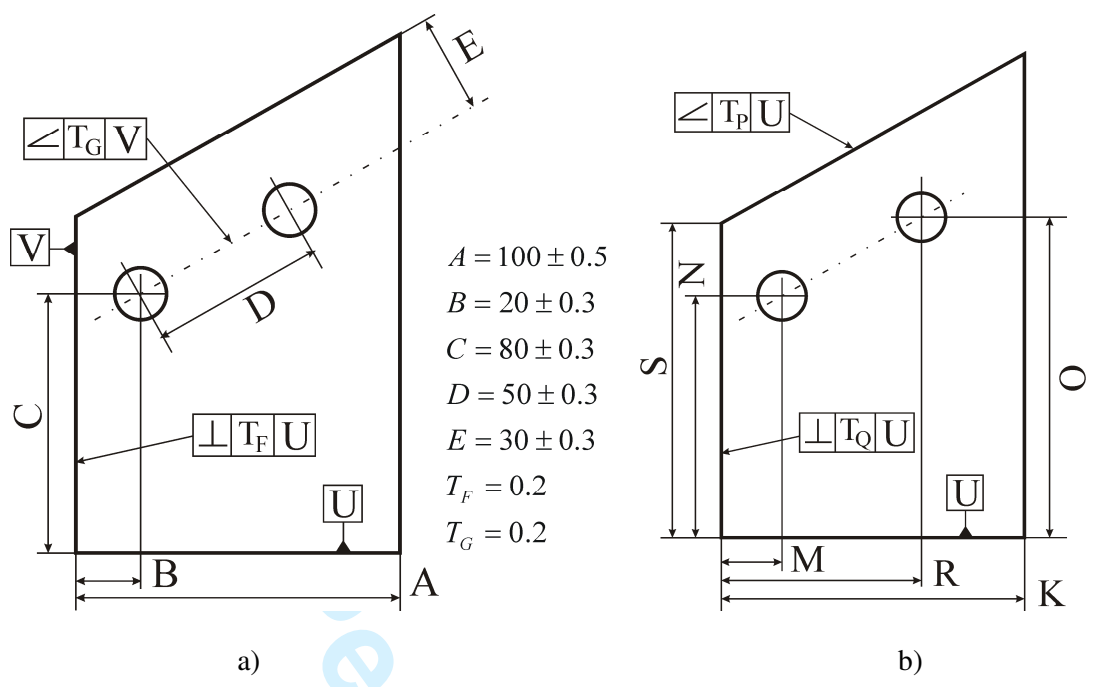


Figure 13. Blueprint (a) and process dimensions (b).

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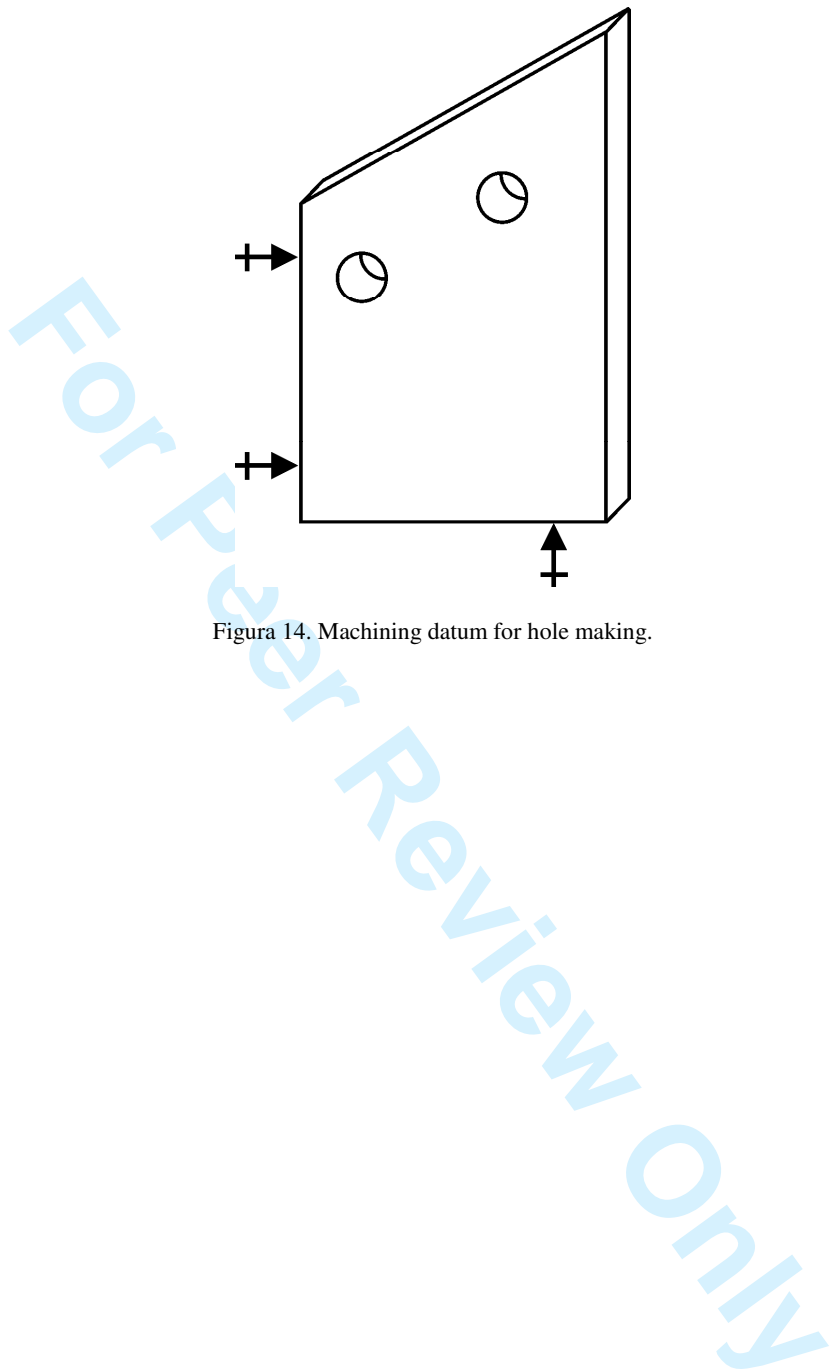
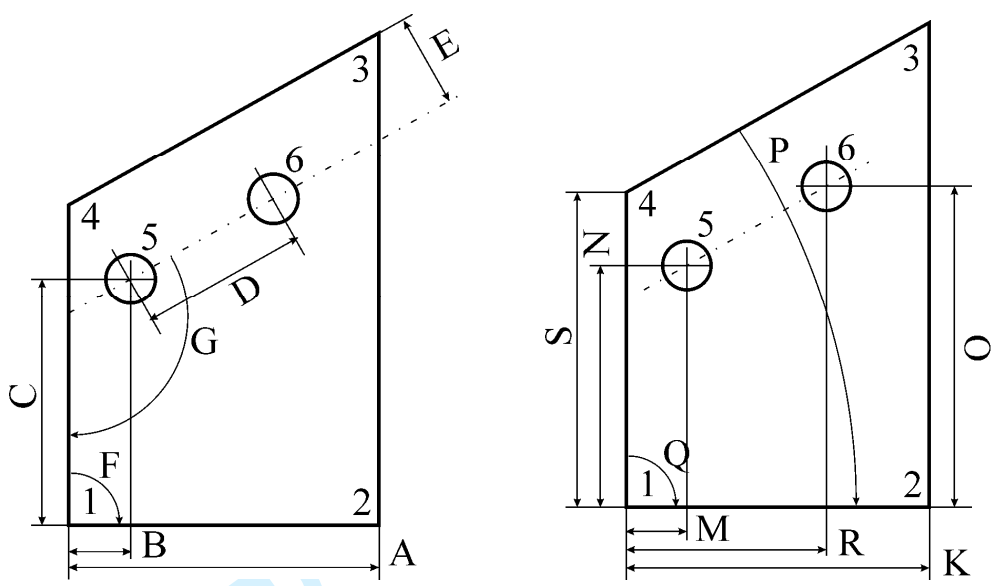


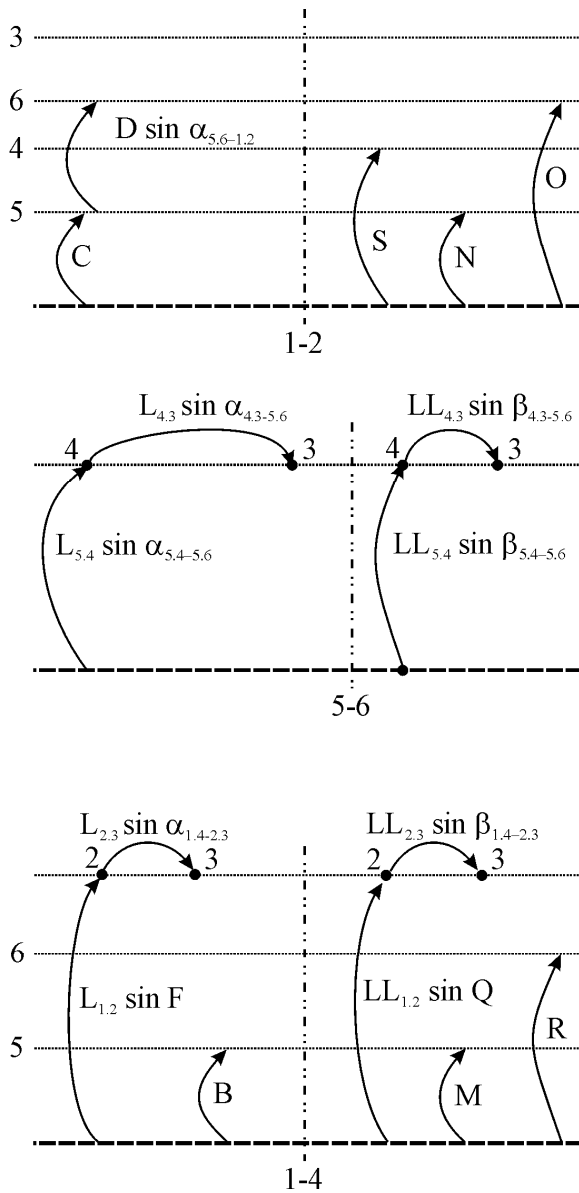
Figura 14. Machining datum for hole making.

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a) b)

Figure 15. Variables of initial (a) and final (b) problems.



$$D \sin \alpha_{5,6-1,2} = O - N$$

$$C = N$$

$$L_{5,4} \sin \alpha_{5,4-5,6} = LL_{5,4} \sin \beta_{5,4-5,6}$$

$$L_{4,3} \sin \alpha_{4,3-5,6} = LL_{4,3} \sin \beta_{4,3-5,6}$$

$$L_{1,2} \sin F = LL_{1,2} \sin Q$$

$$L_{2,3} \sin \alpha_{1,4-2,3} = LL_{2,3} \sin \beta_{1,4-2,3}$$

$$B = M$$

$$F = Q$$

$$G = Q - P + \beta_{4,3-5,6}$$

Figure 16. Dimensional graphs and dimensional chains.

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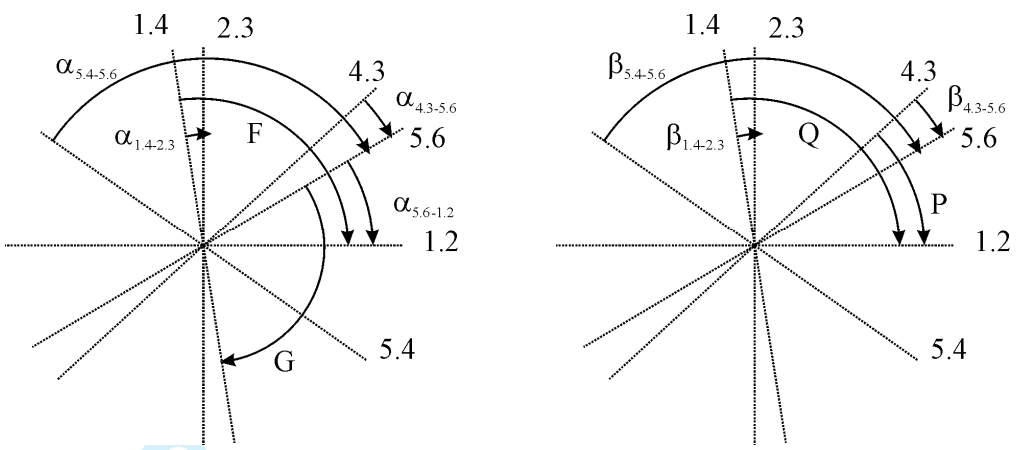


Figura 17. Angular graphs in order to formulate dimensional chains.

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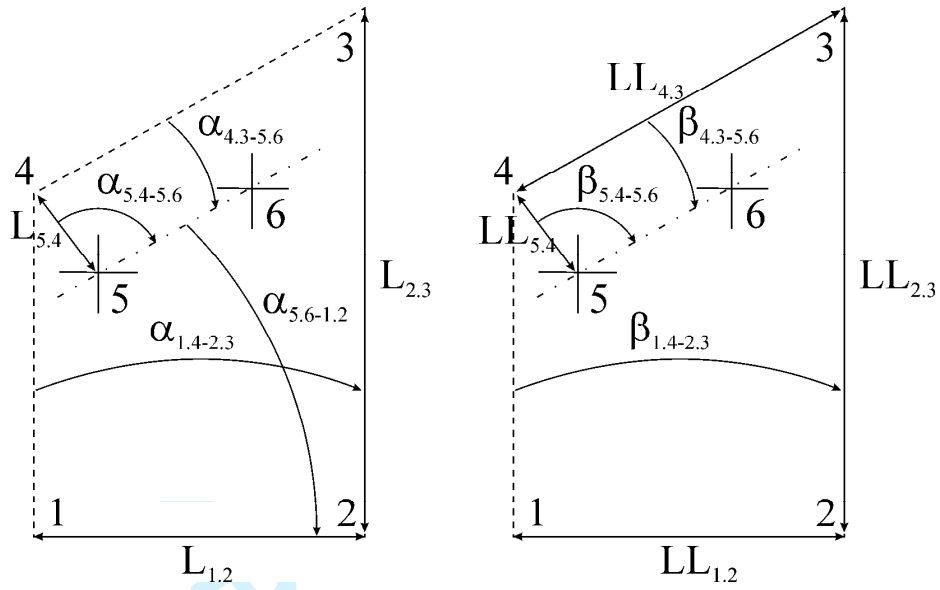


Figura 18. 'Virtual' variables added for dimensional chains.

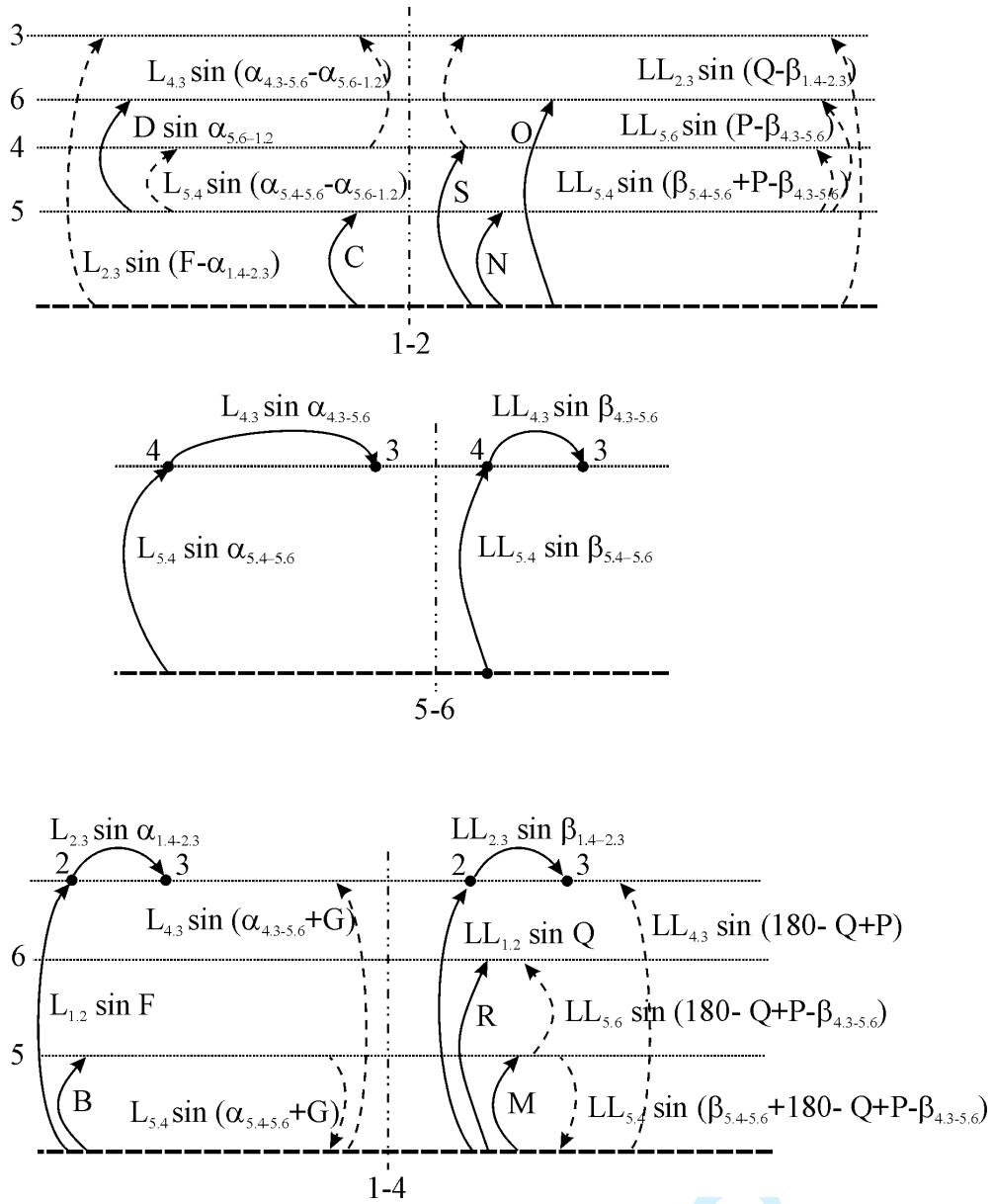


Figura 19. Completed dimensional figures and graphs for the auxiliary equations.