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## MULTI-OBJECTIVE PRODUCTION SCHEDULING IN MAKE-TO-ORDER MANUFACTURING

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### Abstract:

In this paper a multi-objective, long-term production scheduling in make-to-order manufacturing is considered and a lexicographic approach with a hierarchy of integer programming formulations is proposed. The problem objective is to allocate customer orders with various due dates among planning periods with limited capacities to minimize the number of tardy orders as a **primary optimality criterion**. **Then, the maximum level of the input and output inventory is minimized as a secondary criterion, and finally the aggregate production is levelled over the planning horizon as an auxiliary criterion.** A close relation between minimizing the maximal inventory and the maximum earliness of customer orders is shown and used to simplify the inventory levelling problem. Numerical examples modeled after a real-world make-to-order flexible flowshop in a high-tech industry are provided and some computational results are reported. The paper indicates that the maximum earliness of customer orders is an important managerial decision variable, and its minimum value can be applied to control the inventory of purchased materials and finished products, given the customer service level.

### Key words:

Production scheduling, Allocation of customer orders, Levelling production and inventory, Make-to-order manufacturing, Integer programming.

# 1 Introduction

In make-to-order manufacturing the performance of production planning and scheduling is evaluated by customer satisfaction and production costs. A typical measure of the customer satisfaction is customer service level, i.e., the fraction of customer orders filled on or before their due dates, and a typical customer due date related performance measure is the minimization of the number of tardy orders, e.g. Markland et al. (1990), Silver et al.(1998), Shapiro (2001). **In some practical situations, instead of single customer due dates a set of possible delivery dates is fixed, e.g. Lesaoana (1991), Hall et al. (2001).**

On the other hand to achieve low unit production cost, renewable production resources (e.g. machines and people) should be utilized highly and evenly and the aggregate production levelled over time, e.g. Leachman et al. (1996), Neumann and Zimmermann (1999). In addition, the inventory should be minimized, both the input inventory of purchased materials waiting for processing in the system and the output inventory of finished products waiting for delivery to the customers. **Furthermore, some companies produce and deliver to customers directly without holding output inventory, for example in the computer and food catering industries completed orders are delivered to customers within a short lead time, e.g. Chen and Vairaktarakis (2005).**

If demand on capacity in some period exceeds available capacity, then some customer orders with due date in such a period should be moved early or late to reach feasibility. To minimize the number of tardy orders or, if possible, to meet all customer due dates, some orders should be reallocated to earlier periods with excess of capacity. The smaller is the earliness of a customer order with respect to its due date, the later can be the delivery date of required materials. To reduce the required input inventory of purchased materials, the materials should be delivered as late as possible, i.e., the order earliness should be as small as possible, while achieving the minimum number of tardy orders or, if possible, meeting all customer due dates. On the other hand the smaller the earliness of customer orders, the smaller is the output inventory of finished products completed before customer required shipping dates and waiting for delivery to the customers.

An important managerial decision variable for a customer order is its latest ready period (material availability period) or, equivalently, the least earliness of its completion with respect to due date such that during the planning horizon the minimum number of tardy orders yet can be achieved or, if possible, all customer due dates yet can be met. If for some customer orders the earliness is smaller than the minimum earliness, i.e., ready periods and due dates are closer each other, then reallocation of orders to the earlier periods with surplus of capacity is restricted due to later material availability. As a result, the number of tardy orders may increase or even some orders may remain unscheduled during the planning horizon.

Accordingly, an important managerial decision-making problem is the minimization of the

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maximum earliness for all customer orders, i.e., the maximum length of the interval between order due date and its ready period. The maximum earliness of customer orders determines for each order the latest period of material availability such that the minimum number of tardy orders is yet achieved. The smaller the maximum earliness, the lower is the required input inventory of purchased materials and the output inventory of finished products.

The purpose of this paper is to present a lexicographic approach with a hierarchy of integer programming formulations for a multi-objective, long-term production scheduling in make-to-order manufacturing. The problem objective is to allocate customer orders with various due dates among planning periods with limited capacities to minimize the number of tardy orders and the maximum level of the input and output inventory, respectively as a primary and secondary optimality criterion and to level the aggregate production over the planning horizon as an auxiliary criterion.

The proposed integer programs are enhanced to consider the finite input, output or central buffers for holding purchased materials and finished products.

In the literature on production planning and scheduling the integer programming models have been widely used, e.g. Nemhauser and Wolsey (1988), Shapiro (1993), Silver et al.(1998), Markland et al.(1990), Kolisch (2000). In industrial practice, however, the application of integer programming for production scheduling is limited, in particular in make-to-order manufacturing. For example, a hierarchical approach and integer programs for production scheduling in make-to-order company are presented in Carravilla and Pinho de Sousa (1995), however computational results are based on developed heuristics. An integer goal programming formulation for production scheduling with a due date related criterion is also presented in Markland et.al (1990), and the focus is again on application of heuristic approaches. Bradley and Arntzen (1999) apply mixed integer programming for the simultaneous production, capacity and inventory planning in seasonal demand environments. A hierarchical framework for decision making (e.g. Schneeweiss, 1999) and the integer programming formulations for a long-term assignment of customer orders to planning periods and a short-term machine assignment and scheduling of production lots in a flexible flowshop with multi-capacity machines and due date related performance measures are proposed in Sawik (2006).

This paper shows how simple mixed integer programs can be used to find the optimal value of the maximum earliness and how to optimize long-term production schedules in make-to-order manufacturing. The major contribution of this paper is that it proposes a lexicographic approach with a hierarchy of integer programming formulations for the multi-objective production scheduling in make-to-order manufacturing, where both maximization of the customer satisfaction and levelling of production and inventory are integrated in the objective function. In addition, the close relation between the maximum level of input and output inventory and the maximum earliness of customer orders is shown and used to simplify the inventory

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levelling problem. The paper indicates that the maximum earliness of customer orders is an important managerial decision variable in scheduling of make-to-order manufacturing, and its minimum value can be applied to control the inventory of purchased materials and finished products so as to maximize the customer service level and to minimize the production cost.

The paper is organized as follows. In the next section the description of make-to-order production scheduling in a flexible flowshop is provided. The integer programming formulations for a lexicographic approach to the multi-objective production scheduling are presented in Section 3. In Section 4 the proposed integer programs are enhanced to consider the finite input, output or central buffers for holding purchased materials and finished products. Numerical examples modeled after a real-world, make-to-order assembly system in a high-tech industry and some computational results are provided in Section 5. Conclusions are made in the last section.

## 2 Problem Description

The production system under study is a flexible flowshop (e.g. **Kis and Pesch, 2005**) that consists of  $m$  processing stages in series and each stage  $i \in I = \{1, \dots, m\}$  is made up of  $m_i \geq 1$  identical, parallel machines. In the system various types of products are produced in a make-to-order environment responding directly to customer orders. Let  $J$  be the set customer orders that are known ahead of a planning horizon. Each order  $j \in J$  is described by a triple  $(a_j, d_j, s_j)$ , where  $a_j$  is the order arrival date (e.g. the earliest period of material availability),  $d_j$  is the customer due date (e.g. customer required shipping date), and  $s_j$  is the size of order (the quantity of ordered products of specified type). Denote by  $J(d)$  the subset of orders with the same due date  $d \in D$ , where  $D = \{d_j : j \in J\}$  is the set of distinct due dates of all customer orders. Each order requires processing in various processing stages, however some orders may bypass some stages. Let  $J_i \subset J$  be the subset of orders that must be processed in stage  $i$ , and let  $p_{ij} > 0$  be the processing time in stage  $i$  of each product in order  $j \in J_i$ . The orders are processed and transferred among the stages in lots of various size that depends on the ordered product type and let  $b_j$  be the size of production lot for order  $j$ .

The planning horizon consists of  $h$  planning periods (e.g. working days). Let  $T = \{1, \dots, h\}$  be the set of planning periods and  $c_{it}$  the processing time available in period  $t$  on each machine in stage  $i$ .

The following two types of the customer orders are considered:

1. Small size (single-period) orders, where each order can be fully processed in a single time period, e.g. during one day. The single-period orders are referred to as indivisible orders.
2. Large size (multi-period) orders, where each order cannot be completed in one period

and must be split and processed in more than one time period. The multi-period orders are referred to as divisible orders.

In practice, two types of customer orders are simultaneously scheduled. Denote by  $J1 \subseteq J$ , and  $J2 \subseteq J$ , respectively the subset of indivisible, and divisible orders, where  $J1 \cup J2 = J$ , and  $J1 \cap J2 = \emptyset$ .

It is assumed that each customer order  $j \in J1$  must be fully completed in exactly one planning period, and each customer order  $j \in J2$  must be completed in consecutive planning periods.

The maximum number of planning periods required to complete a multi-period order  $j \in J2$  is  $\max_{i \in I, t \in T} \lceil \frac{p_{ij} s_j}{c_{it}} \rceil$ , where  $\lceil \cdot \rceil$  is the least integer not less than  $\cdot$ .

For convenience, it is assumed that each multi-period order can be completed in at most two consecutive planning periods, however, this assumption can be easily relaxed (Sawik, 2005a). In addition the available processing capacity is assumed to be sufficient to schedule all the orders during the planning horizon, if all required materials are available at the beginning of the horizon.

The objective of the long-term production scheduling is to assign customer orders to planning periods to minimize the number of tardy orders and the maximum level of the total (input and output) inventory, or equivalently the maximum earliness of orders, respectively as a primary and secondary optimality criterion and to level aggregate production over the planning horizon as an auxiliary criterion. An implicit objective is to achieve a high customer service level by meeting customer due dates, and a low unit production cost by levelling production and the inventory of purchased materials and finished products.

A lexicographic approach is applied, where the primary objective of maximizing customer service level is reached at the top level. At the top level the customer orders are allocated among planning periods to find the minimum number of tardy orders, then the maximum level of the total inventory or equivalently the maximum earliness of orders is minimized at the medium level and finally the aggregate production is levelled over the horizon for the minimum number of tardy orders and the minimum value of the maximum earliness, see Fig. 1.

Figure 1: A lexicographic approach to multi-objective production scheduling.

### 3 Integer Programming Models for the Multi-Objective Production Scheduling

In this section the integer programming formulations are presented for a lexicographic approach to the multi-objective production scheduling. Decision variables are defined in Table 1.

Table 1: Notation

**Model M1:** *Customer orders assignment to minimize number of tardy orders*

Minimize

$$U_{sum} = \sum_{j \in J} u_j \quad (1)$$

subject to

1. *Order assignment constraints*

- each indivisible customer order is assigned to exactly one planning period,

$$\sum_{t \in T: t \geq a_j} x_{jt} = 1; j \in J1 \quad (2)$$

- each divisible customer order is assigned to at most two consecutive planning periods,

$$x_{jt} + x_{jt+1} \leq 2; j \in J2, t \in T: a_j \leq t \leq h-1 \quad (3)$$

$$x_{jt} + x_{jt'} \leq 1; j \in J2, t \in T, t' \in T: a_j \leq t \leq h-2, t' \geq t+2 \quad (4)$$

2. *Order allocation constraints*

- each order must be completed,

$$\sum_{t \in T: t \geq a_j} y_{jt} = 1; j \in J \quad (5)$$

- each indivisible order is completed in a single period,

$$x_{jt} = y_{jt}; j \in J1, t \in T: t \geq a_j \quad (6)$$

- each divisible order is allocated among all the periods that are selected for its assignment,

$$x_{jt} \geq y_{jt}; j \in J2, t \in T: t \geq a_j \quad (7)$$

- the minimum portion of a divisible order allotted to one period is not less than the batch size,

$$y_{jt} \geq b_j x_{jt} / s_j; j \in J2, t \in T: t \geq a_j \quad (8)$$

3. *Tardy orders constraints*

- an indivisible tardy order is completed after its due date,

$$u_j = \sum_{t \in T: t > d_j} x_{jt}; j \in J1 \quad (9)$$

- a divisible tardy order is partly assigned after its due date,

$$u_j \geq \sum_{t \in T: t > d_j} y_{jt}; j \in J2 \quad (10)$$

$$u_j \leq \sum_{t \in T: t > d_j} x_{jt}; j \in J2 \quad (11)$$

#### 4. Capacity constraints

- in every period the demand on capacity at each processing stage cannot be greater than the maximum available capacity in this period,

$$\sum_{j \in J_i} p_{ij} s_j y_{jt} \leq c_{it} m_i; i \in I, t \in T \quad (12)$$

#### 5. Variable nonnegativity and integrality constraints

$$u_j \in \{0, 1\}; j \in J \quad (13)$$

$$x_{jt} \in \{0, 1\}; j \in J, t \in T : t \geq a_j \quad (14)$$

$$0 \leq y_{jt} \leq 1; j \in J, t \in T : t \geq a_j \quad (15)$$

The objective function (1) represents the number of tardy orders to be minimized. The solution to **M1** determines the assignment of indivisible customer orders to single planning periods and the allocation of divisible orders among the pairs of consecutive planning periods.

In model **M1** all customer orders are assumed to be available for processing at the beginning of the planning horizon, i.e., the order ready period (material availability period) is  $a_j = 1, \forall j \in J$ . As a result the solution to **M1** (Fig. 1) determines the smallest possible number of tardy orders  $U_{sum}^*$ .

Given the minimum number of tardy orders, the next optimization step is to minimize maximum level of the total input and output inventory. In model **M2** presented below the input inventory of product-specific raw materials only is considered with no common materials for different product types taken into account. Furthermore, it is assumed that each product requires one unit of the corresponding product-specific material (e.g. one printed wiring board of a specific design per one electronic device of the corresponding type). As a result, for each order  $j$  the required quantity of product-specific material equals the quantity of the ordered products  $s_j$ .

**Model M2:** *Customer orders assignment to minimize maximum inventory level, given number of tardy orders*

Minimize

$$I_{max} \quad (16)$$

subject to (2) - (15) and

Order assignment constraints:

- the number of tardy orders is at minimum,

$$\sum_{j \in J} u_j = U_{sum}^* \quad (17)$$

Maximum earliness constraints

- for each early order  $j$  assigned to period  $t < d_j$ , its earliness  $(d_j - t)$  cannot exceed the maximum earliness  $E_{max}$ ,

$$(d_j - t)x_{jt} \leq E_{max}; j \in J, t \in T : t \geq a_j \quad (18)$$

Material availability constraints

- for each order  $j$  the required raw materials are available for processing  $E_{max}$  periods ahead of the order due date  $d_j$ , and at the latest in period  $d_j - 1$  if  $E_{max} = 1$ ,

$$r_{jt} \leq 1 + (t - d_j + E_{max})/h; j \in J, t \in T : a_j \leq t \leq d_j - 1 \quad (19)$$

$$r_{jt} \geq (1 + t - d_j + E_{max})/h; j \in J, t \in T : a_j \leq t \leq d_j - 1 \quad (20)$$

Inventory constraints

- in every period the total input inventory of raw materials and output inventory of finished products cannot exceed its maximum level  $I_{max}$  to be minimized,

$$\sum_{j \in J: a_j \leq t \leq d_j - 1} s_j r_{jt} + \sum_{j \in J: t \geq d_j} s_j - \sum_{j \in J, \tau \in T: a_j \leq \tau \leq t, t \geq d_j} s_j y_{j\tau} \leq I_{max}; t \in T \quad (21)$$

Variable nonnegativity and integrality constraints

$$r_{jt} \in \{0, 1\}; j \in J, t \in T : a_j \leq t \leq d_j - 1 \quad (22)$$

$$E_{max} \geq 1, \text{ integer} \quad (23)$$

$$I_{max} \geq 0 \quad (24)$$

The objective function (16) represents the maximum level of the total input and output inventory to be minimized, defined in the left hand side of (21). Implicitly, (16) tends to level the total inventory over the planning horizon. The first two summation terms  $\sum_{j \in J: a_j \leq t \leq d_j - 1} s_j r_{jt} + \sum_{j \in J: t \geq d_j} s_j$  in the left-hand side of (22) represent the required amount of product-specific materials. The third summation term  $\sum_{j \in J, \tau \in T: a_j \leq \tau \leq t, t \geq d_j} s_j y_{j\tau}$  represents the amount of the finished products that had already been shipped to customers by period  $t$ .

Material availability constraints (19), (20) are formulated such that binary variable  $r_{jt} = 1$ , if  $t \geq d_j - E_{max}$ , and  $r_{jt} = 0$ , if  $t \leq d_j - E_{max} - 1$ .

It should be noted that the actual input inventory of product-specific materials depends on material supply schedule (e.g., Silver et al., 1998, Sawik, 2005b) and may differ from the required amount calculated in (21), where the materials are assumed to be supplied exactly  $E_{max}$  periods before each customer order due date. In contrast, (21) accounts for the actual inventory level of finished products.

The solution to the mixed integer program **M2** determines for each order its assignment period and by this the latest period of availability the required materials. The corresponding earliness of each early order with respect to its due date is found such that the number of tardy orders remains at its minimum. As a result the maximum earliness  $E_{max}$  (the maximum length of the interval between order due date  $d_j$  and its ready period  $a_j$ , i.e., material availability period) is determined for all orders such that the maximum level  $I_{max}$  of the total input and output inventory is minimized and the number of tardy orders is at minimum,  $U_{sum}^*$ .

Given the minimum number of tardy orders, and the minimum value of the maximum earliness of orders, the next optimization step is to find production schedule such that levels aggregate production over the planning horizon for a minimum number of tardy orders and the minimum total inventory level.

The integer program **M3** for the base level problem is formulated below. In model **M3** each customer order is assumed to be available for processing  $E_{max}$  periods before its due date, i.e., for each order  $j$  ready period (material delivery period) is  $a_j = \max\{1, d_j - E_{max}\}$ .

**Model M3:** *Customer orders assignment to level aggregate production, given number of tardy orders and the latest periods of material availability*

Minimize

$$P_{max} \quad (25)$$

subject to (2) - (15), (17) and

*Production levelling constraints:*

- in every period the aggregate production cannot exceed the maximum production level to be minimized,

$$\sum_{j \in J} s_j y_{jt} \leq P_{max}; \quad t \in T \quad (26)$$

*Variable nonnegativity constraints*

$$P_{max} \geq 0 \quad (27)$$

The objective function (25) tends to level the aggregate production over the planning horizon.

The solution to **M3** determines the levelled production schedule, i.e., the optimal allocation of customer orders among planning periods,  $\{x_{jt}^*, y_{jt}^*\}$  such that the number of tardy

orders and the maximum inventory level are kept at minimum, and the aggregate production is levelled over the planning horizon.

The following simple cutting constraint can be added to models **M1**, **M2** or **M3**.

*Cumulative production and demand balancing constraint*

- In every period cumulative production is not less than cumulative demand minus tardy demand

$$\sum_{j \in J, \tau \in T: a_j \leq \tau \leq t} s_j y_{j\tau} \geq \sum_{j \in J: d_j \leq t} s_j (1 - u_j); t \in T \quad (28)$$

All the integer programs presented in this section are obviously NP-hard problems. In particular, M1 is a resource constrained problem of assignment divisible and indivisible tasks. It can also be viewed as a special type of ordered bin packing problem, where  $n$  ordered,  $m$ -dimensional items (customer orders ordered by due dates) must be packed into a sequence of  $h$ ,  $m$ -dimensional bins (planning periods with limited capacities) to minimize the number of items packed into the bins on positions later than due dates.

### 3.1 Maximum level of total inventory versus maximum earliness of customer orders

This subsection shows that for a given set of tardy orders, minimizing the maximal inventory of product-specific materials and finished products can be approximately achieved by minimizing the maximum earliness of customer orders.

The following formulae derived for  $a_j = \max\{1, d_j - E_{max}\}$  indicate that the smaller is the maximum earliness  $E_{max}$  for customer orders, the later the required materials can be delivered by suppliers and the lower is the output inventory of the finished products completed before due dates and waiting for the delivery to customers

- Cumulative supplies of product-specific materials by period  $t$ :  $\sum_{j \in J: d_j - E_{max} \leq t} s_j$ ,
- Cumulative production by period  $t$ :  $\sum_{j \in J, \tau \in T: d_j - E_{max} \leq \tau \leq t} s_j y_{j\tau}$ ,
- Cumulative deliveries to customers by period  $t$ :  $\sum_{j \in J, \tau \in T: d_j \leq t, d_j - E_{max} \leq \tau \leq t} s_j y_{j\tau}$ ,
- Input inventory in period  $t =$  Cumulative supplies by period  $t -$  Cumulative production by period  $t$ :  $\sum_{j \in J: d_j - E_{max} \leq t} s_j (1 - \sum_{d_j - E_{max} \leq \tau \leq t} y_{j\tau})$ ,
- Output inventory in period  $t =$  Cumulative production by period  $t -$  Cumulative deliveries by period  $t$ :  $\sum_{j \in J, \tau \in T: d_j - E_{max} \leq \tau \leq t < d_j} s_j y_{j\tau}$ ,

- Total (input and output) inventory in period  $t$  = Cumulative supplies by period  $t$  - Cumulative deliveries by period  $t$ :

$$\sum_{j \in J: d_j \leq t + E_{max}} s_j - \sum_{j \in J, \tau \in T: d_j \leq t, d_j - E_{max} \leq \tau \leq t} s_j y_{j\tau}.$$

The last formula can be rewritten as below.

$$\sum_{j \in J: d_j \leq t} s_j (1 - \sum_{d_j - E_{max} \leq \tau \leq t} y_{j\tau}) + \sum_{j \in J: t+1 \leq d_j \leq t + E_{max}} s_j \quad (29)$$

The first summation term in (29) is the inventory of product-specific materials for customer orders due by period  $t$ , and the second term is the inventory of product-specific materials and finished products of customer orders due after period  $t$ . The first term represents the input inventory in period  $t$  of product-specific materials for tardy orders and is greater than zero only if some customer orders are tardy, otherwise this term is equal to zero. The second term increases with the maximum earliness  $E_{max}$ . Given the tardy orders, the total inventory increases with  $E_{max}$ , i.e. both the input inventory of product-specific materials and the output inventory of finished products can be reduced when ready periods and due dates of customer orders are closer.

Therefore, the maximum level  $I_{max}$  of the total input and output inventory can be implicitly minimized by minimizing the maximum earliness  $E_{max}$  of early orders, given the minimum number  $U_{sum}^*$  of tardy orders. As a consequence, a complex problem of minimization the maximum inventory level  $I_{max}$  can be replaced with a much more simple problem of minimization the maximum earliness  $E_{max}$  such that the minimum number of tardy orders  $U_{sum}^*$  is yet achieved. Accordingly, a complex mixed integer program **M2** can be replaced with a much more simple integer program **M2a**, presented below.

In **M2a** for each early order  $j$  assigned to period  $t < d_j$ , its earliness  $(d_j - t)$  is determined, and the resulting maximum earliness over all early orders  $E_{max} = \max_{j \in J, t \in T} (d_j - t)x_{jt}$  is directly minimized.

**Model M2a:** *Customer orders assignment to minimize maximum earliness, given number of tardy orders*

Minimize

$$E_{max} \quad (30)$$

subject to (2) - (15), (17), (18), (23).

The objective (30) represents the maximum earliness of customer orders to be minimized or equivalently the maximum difference between order due date and its ready period, i.e., the latest period of material availability.

#### 4 Finite Capacity of Input, Output or Central Buffers

In many manufacturing systems purchased materials waiting for processing in the system and finished products waiting for the delivery to customers are stored in finite input buffer and finite output buffer, respectively or are stored in a common central buffer of limited capacity, e.g. Hall et al.(1988a, 1988b). Typically, the capacity of such buffers is not large to **limit material and finished product inventory** and to limit early supplies of purchased materials before their processing dates and early completion of customer orders before the customer required shipping dates.

When the finite buffers need to be considered, additional buffer capacity constraints should be added to the proposed integer programs.

*Input buffer capacity constraints:*

- in every period the total inventory of materials available for processing cannot exceed the input buffer capacity  $B1$ ,

$$\sum_{j \in J: t \geq a_j} s_j - \sum_{j \in J, \tau \in T: a_j \leq \tau \leq t} s_j y_{j\tau} \leq B1; t \in T \quad (31)$$

*Output buffer capacity constraints:*

- in every period the total inventory of finished products completed before their due dates and waiting for shipping to customers cannot exceed the output buffer capacity  $B2$ ,

$$\sum_{j \in J, \tau \in T: a_j \leq \tau \leq t < d_j} s_j y_{j\tau} \leq B2; t \in T \quad (32)$$

The minimum input and output buffer capacity, respectively  $B1_{min}(E_{max})$  and  $B2_{min}(E_{max})$ , required to begin processing of each order  $j \in J$  at its ready period  $a_j = \max\{1, d_j - E_{max}\}$  such that the smallest number of tardy orders is yet achieved can be determined as the optimal solutions to the following mixed integer programs:

$$B1_{min}(E_{max}) = \min\{B1 \geq 0 : (2) - (15), (17), (31)\} \quad (33)$$

$$B2_{min}(E_{max}) = \min\{B2 \geq 0 : (2) - (15), (17), (32)\} \quad (34)$$

Notice that the minimum capacity  $B1_{min}(E_{max})$  (or  $B2_{min}(E_{max})$ ) is determined assuming unlimited capacity of  $B2$  (or  $B1$ ) in (33) (or (34)), respectively.

If both the raw materials and the finished products are stored in a common central buffer, then the following central buffer capacity constraints should replace (31) or (32)

*Central buffer capacity constraints:*

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5 - in every period the total inventory of raw materials and finished products stored in the  
6 central buffer cannot exceed its finite capacity  $BC$   
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$$\sum_{j \in J: t \geq a_j} s_j - \sum_{j \in J, \tau \in T: a_j \leq \tau \leq t, t \geq d_j} s_j y_{j\tau} \leq BC; t \in T \quad (35)$$

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12 The minimum capacity  $BC_{min}(E_{max})$  of the central buffer required to begin processing of  
13 each order  $j \in J$  at its ready period  $a_j = \max\{1, d_j - E_{max}\}$  such that the smallest number of  
14 tardy orders is yet achieved can be determined as the optimal solution to the following mixed  
15 integer program:  
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$$BC_{min}(E_{max}) = \min\{BC \geq 0 : (2) - (15), (17), (35)\} \quad (36)$$

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22 Notice that the optimal capacity  $BC_{min}^*(E_{max})$  of the central buffer is identical with the  
23 optimal value of maximum inventory level  $I_{max}^*$  achieved for the same value of maximum  
24 earliness  $E_{max}$ , i.e., for ready periods  $a_j = \max\{1, d_j - E_{max}\}$ ,  $j \in J$ .  
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## 28 5 Computational Experiments

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31 In this section numerical examples and some computational results are presented to illustrate  
32 possible applications of the proposed lexicographic approach with a triple of integer programs  
33 **M1**, **M2** or **M2a** and **M3**. The examples are modeled after a real world distribution center  
34 for high-tech products, where finished products are assembled for shipping to customers.  
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37  
38 *Fig. 2.* Distribution center: a flexible flowshop.

39  
40 The distribution center can be modeled as a flexible flowshop made up of six processing  
41 stages **in series and parallel**, with parallel machines. In the distribution center 10 product  
42 types of three product groups are assembled. The processing stages are the following: material  
43 preparation stage, where all materials required for assembly of each product are prepared,  
44 postponement stage, where products for some orders are customized, three flashing/flexing  
45 stages **in parallel**, one for each group of products, where required software is downloaded,  
46 and a packing stage, where products and required accessories are packed for shipping.  
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50 Customer orders require processing in at most four stages: material preparation stage,  
51 postponement stage, one flashing/flexing stage, and packing stage (see, Fig. 2). However,  
52 some orders do not need postponement.  
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56 Customer orders are split into production lots of fixed sizes, each to be processed as a  
57 separate job. Each large size (multi-period) customer order must be completed in at most  
58 two planning periods (two days).  
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60 In the computational experiments four types of the test problems are constructed with  
the following four regular patterns of demand:

1. Increasing, with demand skewed toward the end of the planning horizon.
2. Decreasing, with demand skewed toward the beginning of the planning horizon.
3. Unimodal, where demand peaks in the middle of the planning horizon and falls under available capacity in the first and last days of the horizon.
4. Bimodal, where demand peaks at the beginning and at the end of the planning horizon and slumps in mid-horizon.

Pattern 1 requires some orders to be completed earlier, for pattern 2 a majority of orders must be moved later in time, whereas patterns 3 and 4 require that orders are moved both early and late to reach feasibility.

For each demand pattern, the following two scenarios will be considered with different tightness measure  $tcr$ , (37) of the capacity constraint (12):

- scenario I with medium tightness of capacity constraints:  $tcr = 0.762$ ,
- scenario II with high tightness of capacity constraints:  $tcr = 0.955$ .

where  $tcr$  (total capacity ratio) is defined below as the maximum over all processing stages of the total demand on capacity to total available capacity

$$tcr = \max_{i \in I} \left( \frac{\sum_{j \in J} p_{ij} s_j}{m_i \sum_{t \in T} c_{it}} \right) \quad (37)$$

A brief description of the production system, production process, products and customer orders is given below.

#### 1. Production system

- six processing stages: 10 parallel machines in each stage  $i = 1, 2$ ; 20 parallel machines in each stage  $i = 3, 4, 5$ ; and 10 parallel machines in stage  $i = 6$ .

#### 2. Products

- 10 product types of three product groups, each to be processed on a separate group of flashing/flexing machines,
- 100 customer orders, each consisting of several suborders (customer required shipping volumes), known ahead of a monthly planning horizon. Every suborder has a different volume ranging from five to 6345 products (scenario I) or from five to 7930 products (scenario II). The total number of suborders is ranging from 669 to 816 depending on demand pattern and the capacity scenario. The total demand for all products is 429685 and 537995, respectively for scenario I and II.

- production (and transfer) lot sizes: 200,200,300,100,100,100,200,200,300,100, respectively for product type 1,2,3,4,5,6,7,8,9,10.

3. Processing times (in seconds) for product types:

product type/stage	1	2	3	4	5	6
1	20	0	120	0	0	15
2	20	0	140	0	0	15
3	10	0	160	0	0	10
4	15	5	0	120	0	15
5	15	10	0	140	0	15
6	10	5	0	160	0	10
7	15	10	0	180	0	15
8	20	5	0	0	120	15
9	15	0	0	0	140	10
10	15	0	0	0	160	10

4. Planning horizon:  $h = 30$  days, each of length  $L = 2 \times 9$  hours.

The number of large size (multi-period) customer orders is not greater than 10 orders for each capacity scenario and demand pattern.

Notice that the suborders in the computational examples play the role of orders in the mathematical formulation. Now, the problem objective is to determine an assignment of customer suborders over the planning horizon to minimize number of tardy suborders as a primary criterion.

Table 2: Computational results for scenario I: Model **M1**.

Table 3: Computational results for scenario I: Model **M2**.

Table 4: Computational results for scenario I: Model **M2a**.

Table 5: Computational results for scenario I: Model **M3**.

The characteristics of integer programs **M1**, **M2**, **M2a** and **M3** for the two capacity scenarios and various demand patterns and the solution results are summarized in Tables 2 - 9. The size of each integer program is represented by the total number of variables, Var., number of binary variables, Bin., number of constraints, Cons., and number of nonzero elements in the constraint matrix, Nonz. **The counts presented in the tables are taken**

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**from the models after presolving.** The last two columns of each table present the solution values  $U_{sum}$  for **M1**,  $I_{max}$  for **M2**,  $E_{max}$  for **M2a**,  $P_{max}$  for **M3**, and CPU time in seconds required to prove optimality of the solution (**or % GAP if optimality is not proven within the limit of 3600 CPU seconds**). The solution value  $I_{max}$  for **M2** and  $E_{max}$  for **M2a** is presented along with the corresponding associate value of  $E_{max}$  and  $I_{max}$  (in brackets), respectively.

Table 6: Computational results for scenario II: Model **M1**.

Table 7: Computational results for scenario II: Model **M2**.

Table 8: Computational results for scenario II: Model **M2a**.

Table 9: Computational results for scenario II: Model **M3**.

If cutting constraint (28) is applied, the CPU time can be reduced by up to 15%. The greater is the number of tardy orders in the optimal solution, the more efficient is the constraint. **Tables 2-9 presents computational results without application of cut (28), except for model M2a and scenario II with decreasing demand pattern in Table 8.**

For the optimal values of maximum earliness  $E_{max}^*$ , various demand patterns and capacity scenario II, Fig. 3 shows the difference of cumulative aggregate production and cumulative aggregate demand, Fig. 4 shows the aggregate production schedules, and Fig. 5 shows the required input inventory of purchased materials and the output inventory of finished products. (For the total inventory, sum of the input and output inventories, see the corresponding charts for  $E_{max}^*$  in Fig. 7.)

The negative values in Fig. 3 indicate the tardy demand. Fig. 4 shows that the aggregate production is best levelled over time for the increasing demand pattern. Finally, Fig. 5 indicates that the required material inventory and the finished product inventory are varying over time similarly, following or anticipating the demand pattern.

*Fig. 3.* Cumulative difference of aggregate production and demand for scenario II and  $E_{max}^*$ .

*Fig. 4.* Levelled production schedules for scenario II and  $E_{max}^*$ .

*Fig. 5.* Input and output inventory for scenario II and  $E_{max}^*$ .

A comparison of the solution values  $I_{max}$  achieved for **M2** and **M2a** indicate that **M2a** generates the same optimal values for all demand patterns except of a slight difference for decreasing demand:  $I_{max}^* = 31970$  for **M2** versus  $I_{max} = 33280$  for **M2a**, for scenario I, and  $I_{max}^* = 95635$  for **M2** versus  $I_{max} = 97395$  for **M2a**, for scenario II. The difference is due to a different allocation of some orders among planning periods, in particular, a different tardiness of tardy orders.

Table 10: Minimum capacity of input, output, and central buffer for scenario II.

For a comparison, Table 10 presents the minimum capacity of the common buffer storage  $BC_{min}(E_{max})$  for the input and output inventory as well as the minimum capacity of separate input buffer  $B1_{min}(E_{max})$  and output buffer  $B2_{min}(E_{max})$ . The minimum capacities are obtained as solutions to the mixed integer programs (34), (35) and (37), for the optimal values of the maximum earliness  $E_{max}^*$  for each demand pattern. The solution results in Table 10 indicate that the minimum capacity of the central buffer  $BC_{min}(E_{max}^*)$  and the corresponding optimal value  $I_{max}^*$  of maximum inventory are identical for all demand patterns.

Table 11: Minimum number of tardy orders vs. maximum earliness for scenario II.

When order ready periods (i.e., material availability periods) and due dates are closer than  $E_{max}^*$ , the limited order earliness due to later material availability restricts reallocation of orders to the earlier periods with surplus of capacity, which may result in a greater number of tardy orders or even infeasible schedules with some customer orders unscheduled during the planning horizon. Table 11 shows how the number of tardy (or unscheduled) orders increases as the maximum earliness decreases below the optimal value  $E_{max}^*$  for various demand patterns and capacity scenario II. **For example, for the increasing demand pattern and  $E_{max}^* = 6$ , the number of unscheduled orders increases from 1 to 9 as the maximum earliness  $E_{max}$  decreases from 5 to 1.**

The difference of cumulative aggregate production and demand for various demand patterns and capacity scenario II is shown in Fig. 6 to illustrate examples with the maximum earliness  $E_{max} = 1$ . Now, for each demand pattern the number of tardy orders has increased, in particular, infeasible schedules with unscheduled orders have been obtained for increasing, unimodal and bimodal demand pattern.

On the other hand, both the input inventory of raw materials waiting for processing in the system and the output inventory of the finished products completed before due dates and waiting for delivery to the customers can be reduced when orders ready periods and due dates are closer. For a comparison, Fig. 7 presents total (input and output) inventory for the maximum earliness  $E_{max} = 1$ ,  $E_{max} = 10$  and  $E_{max}^*$ , for various demand patterns and capacity scenario II.

For the maximum earliness  $E_{max} \geq E_{max}^*$ , the total inventory is varying over time similarly to demand pattern, and is best levelled over the planning horizon for  $E_{max}^*$ . For  $E_{max} = 1$ , the ending inventory level is greater than zero due to remaining materials for the unscheduled customer orders, except for the decreasing demand pattern.

Fig. 6. Cumulative difference of aggregate production and demand for scenario II and

$$E_{max} = 1.$$

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*Fig. 7. Total inventory for scenario II and different maximum earliness  $E_{max}$ .*

The computational experiments were performed using AMPL programming language and the CPLEX v.9.1 solver on a laptop with Pentium IV at 1.8GHz and 1GB RAM. The results have indicated that the lexicographic approach is capable of finding proven optimal production schedules in a reasonable CPU time for large size problems with typical patterns of demand that can be encountered in the industrial practice.

Notice that the proposed approach is deterministic in nature and is capable of scheduling customer orders in a static environment, where a set of customer orders is known ahead of the planning horizon. Numerical examples presented in this section have been modelled after an assembly system in such a static environment, where virtually no new orders need to be considered during the horizon. The approach and the proposed mixed integer programming models, however, can also be used for reactive scheduling (e.g. Vieira et al., 2003) to iteratively update the schedule in a dynamic environment. A dynamic scheduling horizon can be used to successively solve the mixed integer programs when new orders arrive or old, yet uncompleted orders are cancelled or modified during the horizon. The fact, that the scheduled orders can be completed in one or at most in two consecutive periods, makes a reactive scheduling based on the proposed simple mixed integer programming models a valid approach for such a make-to-order dynamic environment.

## 6 Conclusions

In this paper a lexicographic approach with a hierarchy of mixed integer programming formulations for a multi-objective, long-term production scheduling in make-to-order manufacturing has been proposed. First, the customer orders are allocated among planning periods to find the minimum number of tardy orders, then the maximum level of the total inventory or equivalently the maximum earliness of orders is minimized, and finally the aggregate production is levelled over the horizon for the minimum number of tardy orders and the minimum value of the maximum earliness. An implicit long-term scheduling objective has been to achieve a high customer service level by meeting all customer due dates and a low unit production cost by levelling production and the inventory of purchased materials and finished products.

The mixed integer programs have been enhanced to consider the finite input, output or central buffers for holding purchased materials and finished products.

The proposed approach has been applied to optimize the long-term production schedule in a flexible flowshop. The computational experiments modeled after a real world make-to-order manufacturing environment in a high-tech industry have indicated that the approach is capable of finding proven optimal production schedules for large size problems in a reasonable computation time, using commercially available software for mixed integer programming.

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This paper has also indicated that the maximum earliness of customer orders is an important managerial decision variable, and its minimum value can be applied to control the inventory of purchased materials and finished products so as to maximize the customer service level and to minimize the production cost. To ensure that the materials are available for processing not later than it is required by the minimum value of the maximum earliness, a real-time monitoring of raw materials inventory in a factory as well as during transportation from a supplier to the factory should be applied, e.g., by using RFID technology. Such a solution combined with a Vendor Managed Inventory installed in the factory by the supplier could guarantee that the materials are available the required number of days in advance with respect to order due date, and, as a result, that the customer service level is maximized.

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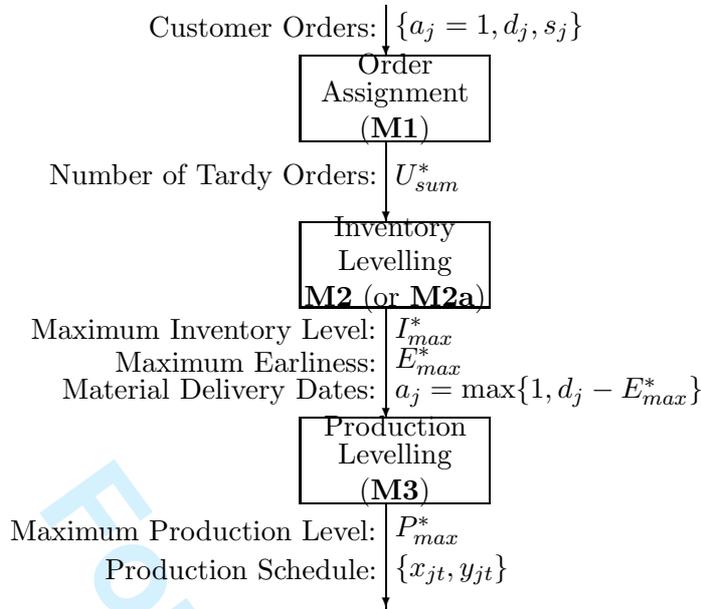


Figure 1: A lexicographic approach to multi-objective production scheduling.

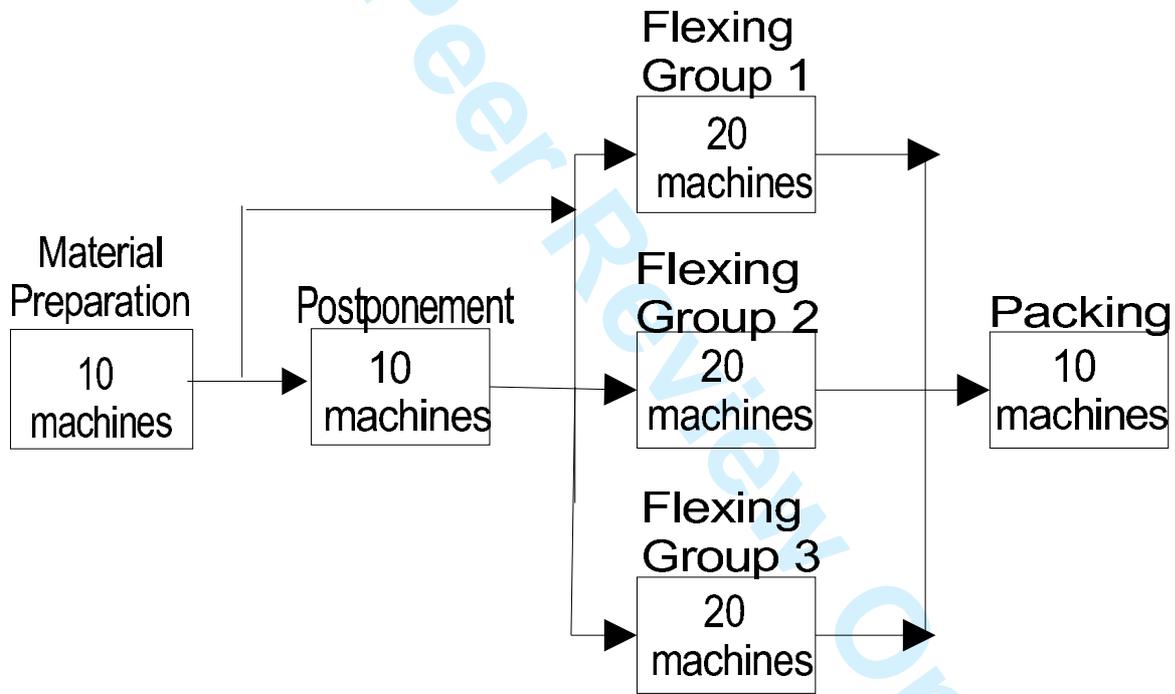


Figure 2: Distribution center: a flexible flowshop.

Table 1: Notation

<b>Indices</b>	
$i$	= processing stage, $i \in I = \{1, \dots, m\}$
$j$	= customer order, $j \in J = \{1, \dots, n\}$
$t$	= planning period, $t \in T = \{1, \dots, h\}$
<b>Input Parameters</b>	
$a_j, d_j, s_j$	= arrival date, due date, size of order $j$
$b_j$	= production lot for order $j$
$c_{it}$	= processing time available in period $t$ on each machine in stage $i$
$m_i$	= number of identical, parallel machines in stage $i$
$p_{ij}$	= processing time in stage $i$ of each product in order $j$
$D$	= $\{d_j : j \in J\}$ set of distinct due dates of all customer orders
$J1$	= subset of small (single-period) customer orders
$J2$	= subset of large (multi-period) customer orders
$J(d)$	= $\{j \in J : d_j = d\}$ subset of customer orders with identical due date $d$
$J_i$	= $\{j \in J : p_{ij} > 0\}$ subset of customer orders to be processed in stage $i$
<b>Decision variables</b>	
$r_{jt}$	= 1, if material required for processing order $j$ is available in period $t$ ; otherwise $r_{jt} = 0$ (material availability variable)
$u_j$	= 1, if order $j$ is completed after due date; otherwise $u_j = 0$ (unit penalty for tardy orders)
$x_{jt}$	= 1, if order $j$ is performed in period $t$ ; otherwise $x_{jt} = 0$ (order assignment variable)
$y_{jt} \geq 0$	= fraction of customer order $j$ to be processed in period $t$ (order allocation variable)
$E_{max}$	= maximum earliness of orders
$I_{max}$	= maximum level of total (input and output) inventory
$P_{max}$	= maximum level of aggregate production
$U_{sum}$	= number of tardy orders

Table 2: Computational results for scenario I: Model **M1**.

Demand pattern	Var.	Bin.	Cons.	Nonz.	$U_{sum}$	CPU <sup>†</sup>
Increasing	49759	24880	28696	217772	0	2
Decreasing	49813	24907	28750	223519	2	8
Unimodal	43253	21627	25052	191630	0	1
Bimodal	41089	20545	23833	182233	0	1

<sup>†</sup> CPU seconds for proving optimality on a PC Pentium IV, 1.8GHz/CPLEX v.9.1

Table 3: Computational results for scenario I: Model **M2**.

Demand pattern	Var.	Bin.	Cons.	Nonz.	$I_{max}, (E_{max})$	CPU <sup>†</sup>
Increasing	45464	29752	60915	360899	63380, (2)	274
Decreasing	58233	33327	53670	511969	31970, (1)	703
Unimodal	32146	20944	42908	299110	95250, (3)	650
Bimodal	29786	19394	39394	274899	102260, (3)	266

<sup>†</sup> CPU seconds for proving optimality on a PC Pentium IV, 1.8GHz/CPLEX v.9.1

Table 4: Computational results for scenario I: Model **M2a**.

Demand pattern	Var.	Bin.	Cons.	Nonz.	$E_{max}, (I_{max})$	CPU <sup>†</sup>
Increasing	31361	15680	33515	163339	2, (63380)	38
Decreasing	49747	24872	36700	240013	1, (33280)	86
Unimodal	22341	11170	23998	115402	3, (95250)	73
Bimodal	20721	10360	22264	106883	3, (102260)	38

<sup>†</sup> CPU seconds for proving optimality on a PC Pentium IV, 1.8GHz/CPLEX v.9.1

Table 5: Computational results for scenario I: Model **M3**.

Demand pattern	Var.	Bin.	Cons.	Nonz.	$P_{max}$	CPU <sup>†</sup>
Increasing	4791	2395	5127	59932	18495	9
Decreasing	32845	16421	20000	429639	20685	25
Unimodal	5503	2751	5185	80637	18565	38
Bimodal	5231	2615	4941	77577	17855	33

<sup>†</sup> CPU seconds for proving optimality on a PC Pentium IV, 1.8GHz/CPLEX v.9.1

Table 6: Computational results for scenario II: Model **M1**.

Demand pattern	Var.	Bin.	Cons.	Nonz.	$U_{sum}$	CPU <sup>†</sup>
Increasing	50004	25003	29298	219734	0	11
Decreasing	50060	25031	29354	225520	7	37
Unimodal	43437	21720	26085	194241	2	40
Bimodal	41334	20669	25365	185985	1	37

<sup>†</sup> CPU seconds for proving optimality on a PC Pentium IV, 1.8GHz/CPLEX v.9.1

Table 7: Computational results for scenario II: Model **M2**.

Demand pattern	Var.	Bin.	Cons.	Nonz.	$I_{max}, (E_{max})$	CPU <sup>†</sup>
Increasing	45674	29890	61389	363060	175355, (6)	2335
Decreasing	58592	33531	54949	942010	95635, (2)	>3600, ( <b>0.44%</b> <sup>‡</sup> )
Unimodal	53282	31533	55586	489773	146785, (4)	993
Bimodal	30105	19583	41856	281577	173775, (5)	240

<sup>†</sup> CPU seconds for proving optimality on a PC Pentium IV, 1.8GHz/CPLEX v.9.1

<sup>‡</sup> % **GAP** after **3600** seconds of CPU time

Table 8: Computational results for scenario II: Model **M2a**.

Demand pattern	Var.	Bin.	Cons.	Nonz.	$E_{max}, (I_{max})$	CPU <sup>†</sup>
Increasing	31507	15753	33863	164653	6, (175355)	247
Decreasing	50059	25029	<b>38676</b>	<b>613618</b>	2, (97395)	<b>1438</b> <sup>‡</sup>
Unimodal	43438	21720	36630	462309	4, (146785)	471
Bimodal	20985	10494	24403	150177	5, (173775)	972

<sup>†</sup> CPU seconds for proving optimality on a PC Pentium IV, 1.8GHz/CPLEX v.9.1

<sup>‡</sup> **solution cannot be proven within 3600 CPU seconds without cut (28)**

Table 9: Computational results for scenario II: Model **M3**.

Demand pattern	Var.	Bin.	Cons.	Nonz.	$P_{max}$	CPU <sup>†</sup>
Increasing	10813	5406	8248	146203	17935	440
Decreasing	34581	17290	21822	459781	22655	317
Unimodal	27848	13925	17834	349848	20245	512
Bimodal	7915	3959	7455	123848	20394	180

<sup>†</sup> CPU seconds for proving optimality on a PC Pentium IV, 1.8GHz/CPLEX v.9.1

Table 10: Minimum capacity of input, output, and central buffer for scenario II.

Demand pattern/ $E_{max}^*$	$B1_{min}(E_{max}^*)$	$B2_{min}(E_{max}^*)$	$BC_{min}(E_{max}^*)$
Increasing/6	54780	43830	175355
Decreasing/2	68715	17220	95635
Unimodal/4	58090	58351	146785
Bimodal/5	71580	55500	173775

Table 11: Minimum number of tardy orders vs. maximum earliness for scenario II.

Demand pattern/ $E_{max}^*$	Maximum earliness					
	1	2	3	4	5	6
Increasing/6	9 <sup>‡</sup>	4 <sup>‡</sup>	3 <sup>‡</sup>	1 <sup>‡</sup>	1 <sup>‡</sup>	0 <sup>†</sup>
Decreasing/2	9	7 <sup>†</sup>	-	-	-	-
Unimodal/4	11 <sup>‡</sup>	7	3	2 <sup>†</sup>	-	-
Bimodal/5	12 <sup>‡</sup>	8	3	2	1 <sup>†</sup>	-

<sup>†</sup> minimum number of tardy orders for  $E_{max}^*$

<sup>‡</sup> number of unscheduled orders (no feasible solution)

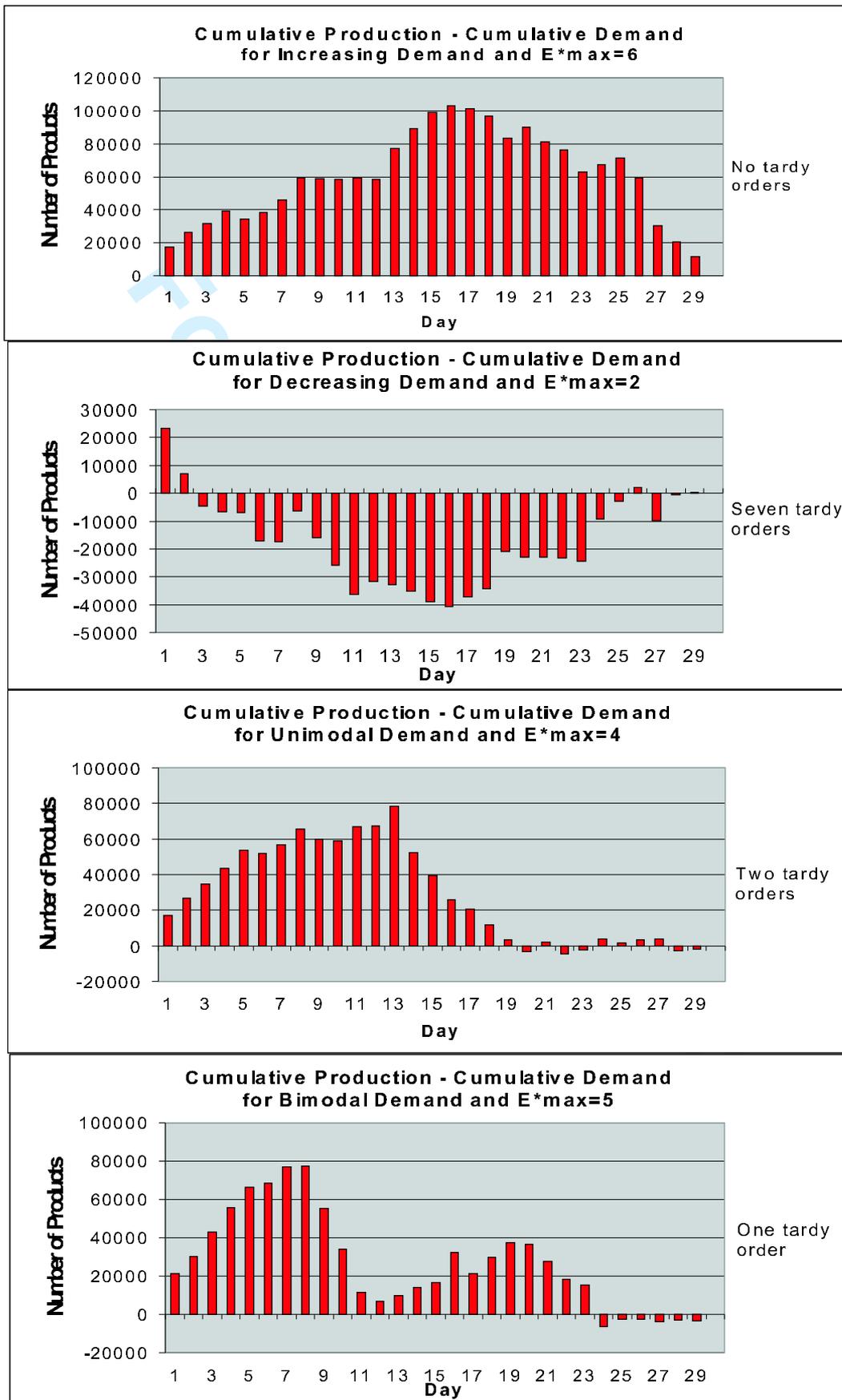


Figure 3: Cumulative difference of aggregate production and demand for scenario II and  $E^*_{max}$ .

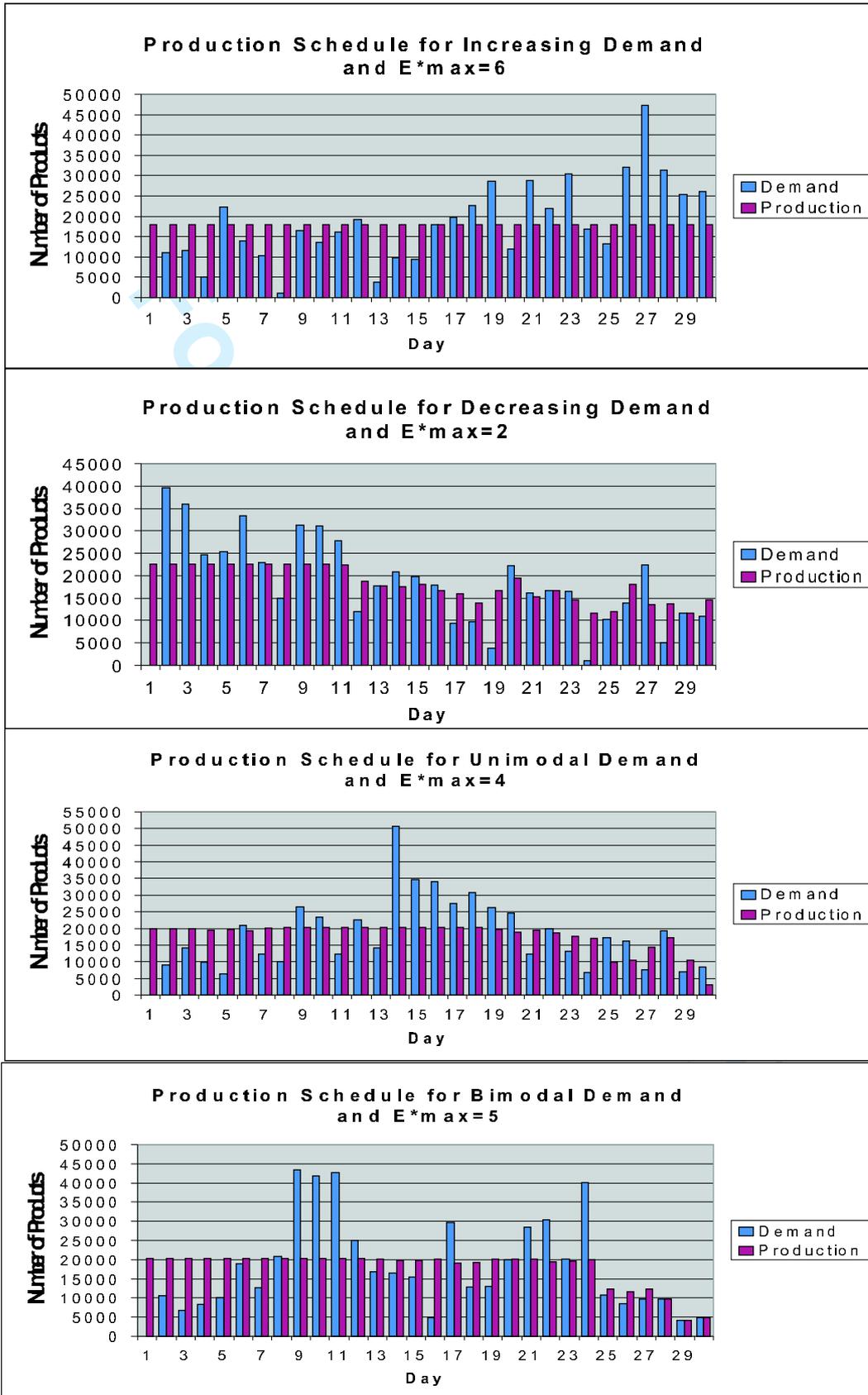


Figure 4: Levelled production schedules for scenario II and  $E^*_{max}$ .

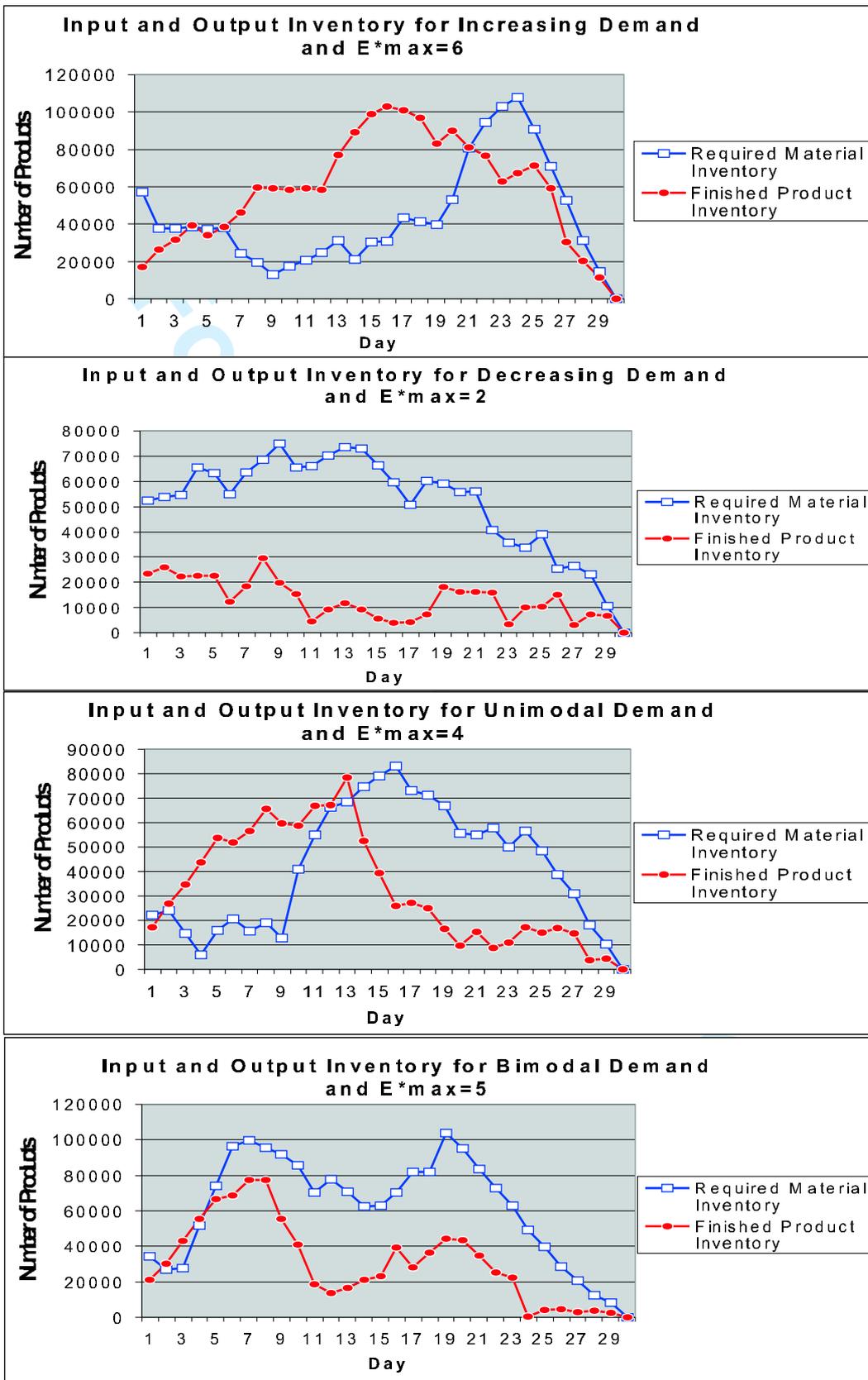


Figure 5: Input and output inventory for scenario II and  $E^*_{max}$ .

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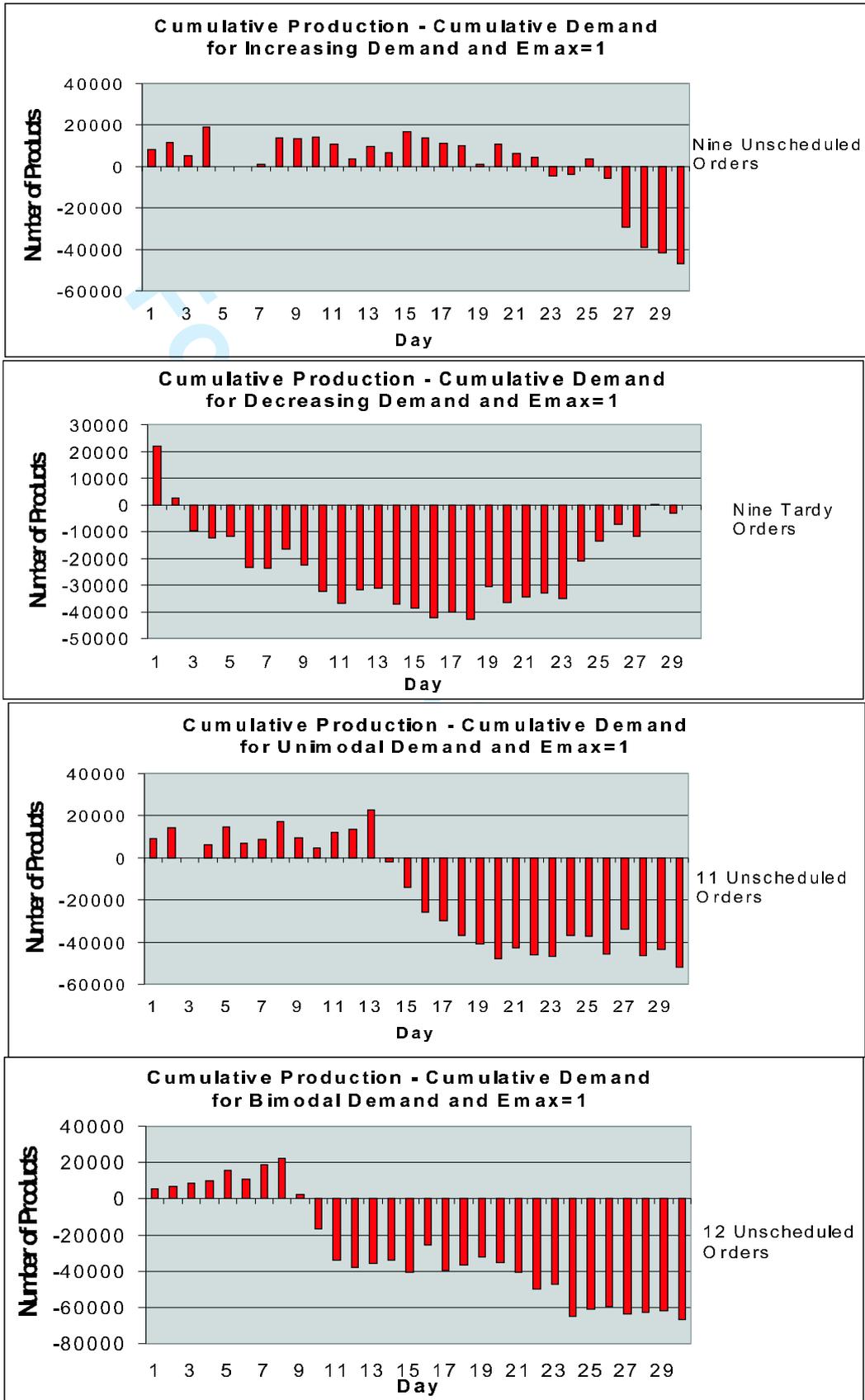


Figure 6: Cumulative difference of aggregate production and demand for scenario II and  $E_{max} = 1$ .

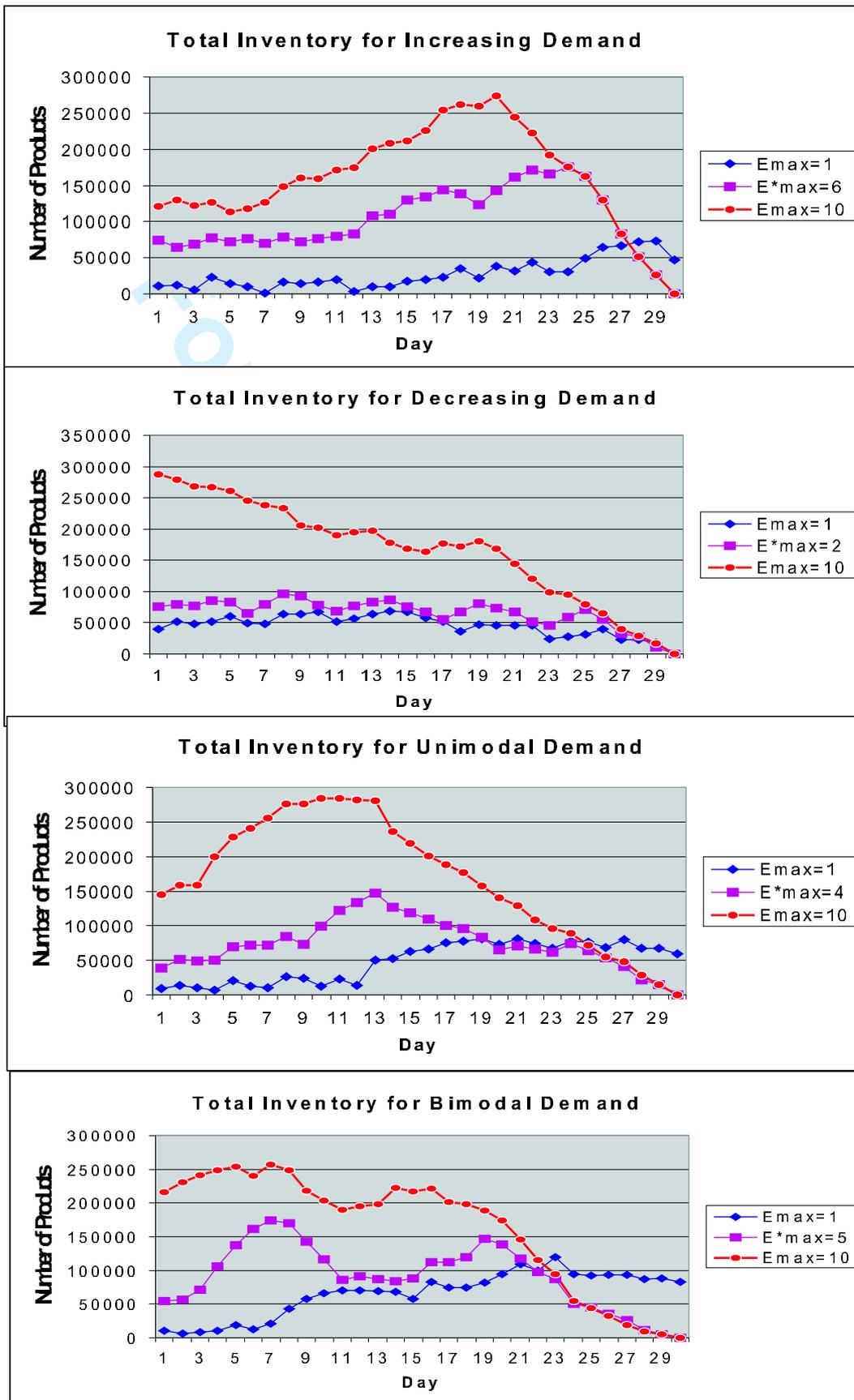


Figure 7: Total inventory for scenario II and different maximum earliness  $E_{max}$ .