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Abstract

Effectiveness in the use of the work force is often the crucial advantage in a company's long-term success over its competitors, especially in technology driven and highly competitive branches as the one considered in this article. While employee scheduling and rostering has been the topic of extensive research over the past decades, usually it is assumed that the size of the work force is either given or can be obtained without difficulty. In this research we provide an integer programming model for long-term staffing decisions. The model is based on qualification profiles, the number of which grows exponentially in terms of the number of processes considered. We present empirical results for a company from the printing branch which highlight the potential of our approach. In particular, it will be shown that applying the model lowers the total cost of the work force in the range of 26% – 39%.

Keywords: Work force planning, qualification profile, printing company, empirical results

1 Introduction

Even though employee scheduling has been addressed by personnel managers, operations researchers and computer scientists for more than 40 years (see Burke et al. 2004), the rostering literature assumes that the demand for staff is either given or can be obtained without difficulty (see Ernst et al. 2004).

Work force planning or staffing is the highest level of personnel planning and determines the restrictions for lower levels like scheduling (see, e.g., Wijngaard 1983 and Pinedo 2005). Generally speaking, in the domain of work force planning and scheduling, three major areas can be distinguished: staffing, scheduling and reallocation. Usually the time horizon of staffing covers several months to years, while scheduling/rostering faces several weeks; see table 1. The staffing can again be decomposed into two stages: determination of temporal and total manpower requirements (see Tien and Kamiyama 1982). These belong to the set of strategic or long-term decisions made by the management. Scheduling or rostering is the process of constructing work timetables for the staff so that an organization can satisfy the demand for its goods or services (see Ernst et al. 2004) as well as legal limitations and the convenience of the individual. The last stage is to reallocate the staff on shift to the jobs to be processed.

area	denotation	time horizon
staffing	long-term	several month to years
scheduling/rostering	medium-term	several weeks
reallocation	short-term	hours to days

Table 1: Time horizons in work force planning

Wijngaard (1983) points out two extreme cases of organizations. One where each employee can do all kinds of jobs, where vacancies can be filled directly by recruitment and where firing is easily possible. The other extreme is the case of an organization with very specific functions and employees with low mobility which makes it difficult to fill vacancies directly by recruitment. In the first case all of the work force must be qualified at the same level causing the overall cost of employment to be high. In the latter case, the work force is much more heterogeneous so that low level activities can be processed by low qualified and cheaper jobholders, causing the overall costs of employment to be at a lower level. In this case the appropriate planning process is much more complex.

In the literature considerable work has been published on staff scheduling in service organizations. There typically demand varies throughout the day, seven days a week. Generally the goal is to determine a cost minimal complement of full-time, part-time or temporary workers in such a way that demand is met without violating labor rules and union agreements. Workers differ in their skills, speed, and/or quality. Without going into details here we refer to recent work of, e.g., Ahn et al. (2005), Bard et al. (2003) and Thompson and Goodale (2006).

The situation considered in the service organization literature differs from the one considered here in several aspects. In manufacturing companies demand does not vary throughout the day, seven days a week. To the contrary, goods are produced fife-days a week, one-shift per day and demand is relatively uniform throughout the year. This observation paths the way to a long-term, strategic analysis of the work force needed based on the average demand per period. This long-term analysis can nicely be done using qualification profiles as shown below. Surprisingly, long-term work force planning for manufacturing companies has only been addressed very rarely in the open literature, the only exception we are aware of being the work of Pesch and Tetzlaff (2005) where interactions between staffing and scheduling decisions for press machines in the automotive industry are studied.

In this paper we propose a general model for manufacturing-to-order companies which determines simultaneously the size and the qualification of the work force in order to meet predicted requirements with the goal of minimizing the total cost of employment.

The outline of the paper is as follows: The problem setting is detailled in section 2. In section 3 we formulate the long-term staffing problem as an optimization model. Section 4 is dedicated to algorithms used for solving the model. The aim of section 5 is to present empirical results. In section 6 we show how to calculate individual working times. Finally, section 7 summarizes some implications for management.

2 Problem setting

The model for work force planning was developed jointly with a German printing company with focus on the industrial workers of the printing branch. Consider a company, divided into a

number of departments. The employees work on a one shift schedule, five days a week. Within one department, the jobholders are qualified all on a similar level, handling heterogeneous tasks. There are no specialists and there are no assistants. This common scenario leads to the case, that jobholders may get overstrained by handling uncommon, infrequent tasks, resulting in longer processing times. On the other hand, jobholders have to "waste" their time with assistant tasks like filing, packing or doing errands. Now imagine the aggregated time of all assistant tasks is enough to justify the employment of a full time assistant. The accruing tasks could be reallocated, so that one regular employee can be replaced with a cheaper one, doing only assistant tasks. Furthermore the remaining regular jobholders do not have to have the same high level of qualification. It might pay to have a few specialists, handling uncommon hard tasks, when they occur but performing the same tasks as the regular jobholders otherwise. To adjust the qualification of the work force to the requirements broken down to a disaggregated level within a workflow, can reduce the overall costs of the work force and even improve its performance.

The focus is on the determination of the number and qualification of jobholders who are handling operational tasks on a day to day basis. The determination of the number and qualification of managers is not considered within this model, because their tasks are more complicated and their wages are arranged freely with the employer. The payment of jobholders is divided into a number of salary levels, depending on the qualification of the jobholder.

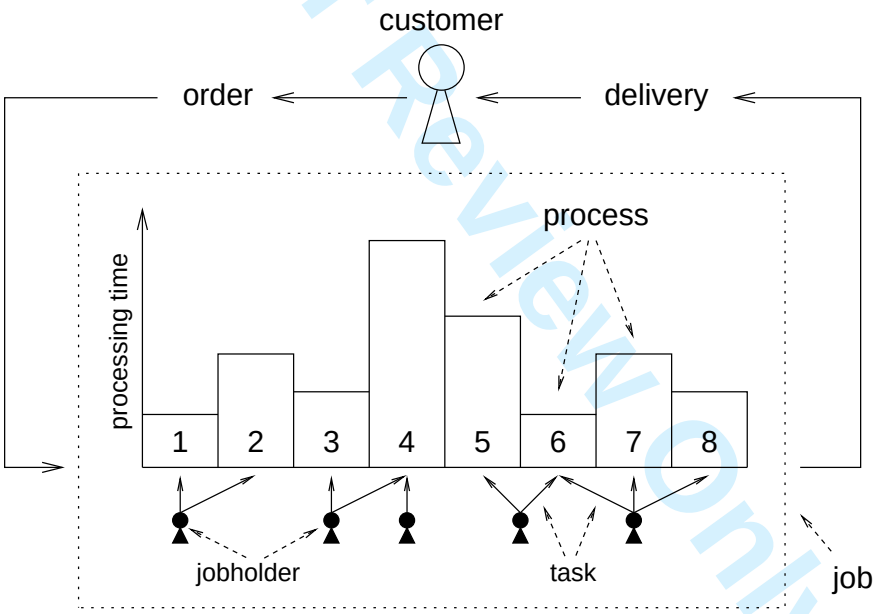

Figure 1: Schematic view of orders, jobs and processes

In the printing branch, a job is started by the order of a customer. Each job consists of a number of processes, from incoming order to delivery. These processes are handled in various tasks within different cost centers by jobholders. A task corresponds to the assignment of a process to a particular jobholder. Figure 1 illustrates the structure for one job consisting of eight processes. There are tasks requiring a high level of qualification and there are ones that can be handled by jobholders with low qualification. Since higher qualified jobholders go with higher costs for the company, the cost minimal set and number of qualification profiles has to be determined.

Because of heterogeneous orders, processing times of a process vary from job to job. To obtain

Occupational group	task i	qualification	wage
A	1	low	6
	2	high	10
B	3	low	5
	4	high	11

Table 2: Qualification level and wages

Qualification profile	wage
(1,1,1,0)	15
(0,1,1,0)	15
(1,0,1,0)	11
(0,0,1,0)	5
(1,1,0,0)	10
(0,1,0,0)	10
(1,0,0,0)	6
(1,1,1,1)	21
(0,1,1,1)	21
(1,0,1,1)	17
(0,0,1,1)	11
(1,1,0,1)	21
(0,1,0,1)	21
(1,0,0,1)	17
(0,0,0,1)	11

Table 3: Qualification profiles and wages

the overall time demand of each task within the considered time horizon, the aggregated processing time of each process for all jobs has to be calculated.

To determine the future demand of work force for a time horizon of for example one year, the demand of cumulated processing time for each process has to be predicted by using past data and by estimating marked demand for the next period. This data delivers the basis for our model to determine the work force, so that the time demand of a process is being covered by the time supply of a certain amount of jobholders, well qualified to handle all processes. Furthermore, there must be a minimum number of jobholders to be able to handle a certain process for backup reasons (e.g. vacation or disease) or to handle peak times which can result from short term reallocation.

The different processes, a job has to run through until its delivery, are being handled by jobholders belonging to different occupational groups such as, e.g., mercantile employees or type setters. Tasks like job accounting or acceptance of order are being handled by mercantile employees, while typesetting and graphical editing are handled by type setters. Within each occupational group there are grades of qualification going hand in hand with the money a jobholder earns. A mercantile employee who is only charged with filing documents is less qualified than someone in charge to purchase raw materials. The latter though is also able to handle tasks of lower qualification.

Let us take a look at the example in table 2 with two occupational groups A and B where each of them can be charged with a high and a low qualified task i to handle the related process. A high qualified employee always has the ability to handle low qualified tasks within the same occupational group. A qualification profile contains the binary information if a worker is qualified to handle a specific process or not. Overall in case of m processes 2^m qualification profiles do exist including the case that no qualification is available at all. Someone who is only charged to handle task $i = 1$, equivalent to the qualification profile (1,0,0,0), earns a

wage of 6 monetary units. Someone who is only charged to handle task $i = 2$ (0,1,0,0) earns a wage of 10. But someone who is charged to handle task $i = 1$ and $i = 2$ (1,1,0,0) also earns a wage of 10. Task $i = 2$ is superior in terms of qualification to task $i = 1$. Table 3 gives an overview of all the different qualification profiles of this example and the associated wages.

If in this model someone is charged to perform tasks from more than one occupational group, he is payed the sum of the wages of the tasks. A jobholder who is, e.g., charged to handle the processes $i = 1$ and $i = 4$ (1,0,0,1) would cost 17. Even though that doesn't seem to make sense in the first place, it penalizes certain qualification profiles and prevents the model to generate profiles where for example type setters are charged with the purchase of raw materials. Nevertheless, such profiles are not excluded explicitely but they will not be generated by the model (see section 5).

3 Optimization model

The model is based on the following assumptions: We assume that the annual number of jobs with specified processes, the minimum number of employees needed for each process and the annual cost of employee with a qualification profile is given. The objective is to determine the number of full-time employees with given qualification profiles which minimises the total annual cost of the work force.

More precisely, each job j, $j = 1, \ldots, n$, is manufactured by running through a number of processes i, $i = 1, \ldots, m$. The time a job j needs for process i is p_{ji}. Of course, $p_{ji} = 0$ indicates the case that a particular job j does not need process i and, hence, we can assume without loss of generality that each job uses the same number of processes m. The workers that process each job j can be skilled to execute a number of different processes. The different combinations of abilities are called qualification profiles. q denotes the overall number of profiles and h is used in order to index the profiles from 1 to q. The binary parameter v_{ih} indicates if an employee with qualification profile h can handle process i ($v_{ih} = 1$) or not ($v_{ih} = 0$). A worker with qualification profile h costs c_h monetary units per year. The annual working time of each full time worker is b.

Given the number n of jobs j within a time horizon of one year, the objective is to determine the number of workers x_h with qualification profile h such that the total annual cost of manpower is minimized. Doing so the aggregated time required by process i must not exceed the time provided by all workers being qualified to handle this particular process. Moreover, at least S_i workers must be qualified to handle process i.

Now we get the following optimization model:

$$\min \quad \sum_{h=1}^{q} c_h x_h \tag{1}$$

$$\text{s.t.} \quad \sum_{h=1}^{q} v_{ih} \frac{b}{\sum_{j=1}^{m} v_{jh}} x_h \geq \sum_{j=1}^{n} p_{ji} \qquad i = 1, \ldots, m \tag{2}$$

$$\sum_{h=1}^{q} v_{ih} x_h \geq S_i \qquad i = 1, \ldots, m \tag{3}$$

$$x_h \geq 0 \text{ and integer} \qquad h = 1, \ldots, q \tag{4}$$

Objective function (1) minimizes the total annual cost of the work force. Constraints (2) assure that the available working time for each process i meets at least the demand of working time of process i for all jobs. Of course, p_{ji} and b have to be measued in the same time units. Constraint (3) assures that there is a minimum number S_i of jobholders being able to handle process i for backup reasons as outlined above. Note that high values for S_i make the manpower on-hand more flexible, since more jobholders will be qualified to handle process i. Low values lead to low flexibility of the jobholders but also to lower overall costs.

In this model we consider the case that all jobholders have the same annual working time. This assumption does not prevent to apply the model in practice where, of course, also part-time workers do exist.

4 Algorithms

The model (1) to (4), also called integer master problem, has an exponential number of columns, hence, there is no chance to solve it directly. In order to cope with this fact, we use column generation in order to solve the linar programming relaxation to optimality. This gives us a lower bound. The outcome of column generation then is used in order to compute feasible solutions, i.e., upper bounds for the optimal objective function value as well.

First we describe in section 4.1 how to compute a lower bound for the optimal objective function value. Section 4.2 details how to come up with feasible solutions, that is, upper bounds. The algorithms are illustrated by means of an example in section 4.3.

4.1 Lower bound

In order to compute a lower bound of the integer staffing problem introduced above, we replace the integrality requirements (4) through $x_h \geq 1$ for all h. Starting with an initial set of columns, say q, we have to cheque whether a column does exist which lowers the objective function (1). Hence, we look at the dual (5) to (8) of the linear programming relaxation of the integer master problem.

$$\max \quad \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ji}\pi_i + \sum_{i=1}^{m} S_i\tau_i \tag{5}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_{ih}\frac{b}{\sum_{j=1}^{m} v_{jh}}\pi_i + \sum_{i=1}^{m} v_{ih}\tau_i \leq c_h \qquad h = 1,\ldots,q \tag{6}$$

$$\pi_i \geq 0 \qquad i = 1,\ldots,m \tag{7}$$

$$\tau_i \geq 0 \qquad i = 1,\ldots,m \tag{8}$$

Using the dual variables $\pi_i \geq 0$ associated with constraint (2) and $\tau_i \geq 0$ associated with constraint (3) we try to identify a column $q+1$ for which

$$c_h < \sum_{i=1}^{m} v_{i,q+1}\frac{b}{\sum_{j=1}^{m} v_{j,q+1}}\pi_i + \sum_{i=1}^{m} v_{i,q+1}\tau_i \tag{9}$$

is valid (i.e. a dual constraint (6) is violated). If such a column exists it has to be added to the primal (that is, it prices out attractively). If no such column exists we can stop.

In order to cheque this condition we have to solve the following optimization problem:

$$\max \left\{ \sum_{i=1}^{m} v_{i,q+1} \frac{b}{\sum_{j=1}^{m} v_{j,q+1}} \pi_i + \sum_{i=1}^{m} v_{i,q+1} \tau_i : (v_{i,q+1}) \text{ is a qualification profile} \right\} \quad (10)$$

This optimization problem can be solved in polynomial time by means of iterative shortest path computations. In order to do so the set of qualification profiles is transferred into a complete, directed network $G = (V, E, w)$ with node set V, arc set E, and arc weights w, respectively. Each node stands for the qualification to handle a specific process. Moreover, a source and a sink node have to be added and, hence, overall we have $m + 2$ nodes. The nodes are labeled uniquely such that node 0 is the source node, node $m + 1$ is the sink node and nodes 1 to m correspond to the m processes. The arc e_{ki} between node k and node i represents the option of combining process k and i within one qualification profile. The weight w_{ki} of arc e_{ki} is the extra payment needed. Figure 2 illustrates the structure of the network for the example introduced above.

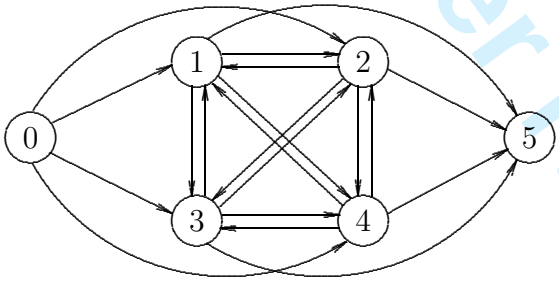

Figure 2: Network topology

$$\begin{pmatrix} 6 & 10 & 5 & 11 & - \\ - & 4 & 5 & 11 & 0 \\ 0 & - & 5 & 11 & 0 \\ 6 & 10 & - & 6 & 0 \\ 6 & 10 & & - & 0 \end{pmatrix}$$

Table 4: Arc weights $(w_{ki})_{k=0,\ldots,4}^{i=1,\ldots,5}$

The set of arcs E consists of four types of arcs $E = E^1 \cup E^2 \cup E^3 \cup E^4$ which are defined as follows (see table 4 also):

- Arcs E^1 from the source node 0 to every node i. The weight w_{0i} of these arcs is the wage for the qualification to handle process i.
- Arcs E^2 from every node i to the sink node $m+1$. The weight of these arcs is $w_{i,m+1} = 0$.
- Arcs E^3 connecting node k with node i within the same occupational group. The weight w_{ki} of these arcs equals 0, if the qualification to handle process k is higher than the one to handle process i. Otherwise the weight w_{ki} is the difference of the wage for the qualification to handle process i and the wage for the qualification to handle process k.
- Arcs E^4 connecting node k with node i not belonging to the same occupational group. The weight w_{ki} of these arcs is the wage for the qualification to handle process i.

The total wage is equal to the shortest path from the source to the sink via all nodes that are supposed to be within a certain qualification profile. All in all there are $m \cdot (m-1) + 2m$ arcs. The costs c_h of an employee qualified to handle, e.g., processes 1 and 2, that is the profile

(1,1,0,0), is determined by the shortest path from the source to the sink via nodes 1 and 2, that is, $c_h = w_{0,1} + w_{1,2} + w_{2,5} = 6 + 4 + 0 = 10$.

The network is designed that way, that there are multiple paths from the source to the sink. By setting up rules for the network topology, half of the arcs connecting the process nodes can be eliminated.

- *Rule 1* (labeling of nodes within occupational groups): Within one occupational group, the nodes have to be ordered by the level of qualification. This means that node $i+1$ is associated to a process $i+1$ that needs a higher level of qualification than process i.
- *Rule 2* (labeling of occupational groups): The occupational groups can be in arbitrary order as long as the processes associated to these groups satisfy rule 1.

After applying these rules, only the upper triangle of the matrix (w_{ki}) is needed, reducing the network G to $G' = (V, E', w')$ with $E' \subset E$ as shown in Figure 3. For notational simplicity we will omit the prime, i.e. keep the notation $G = (V, E, w)$ for the reduced network also.

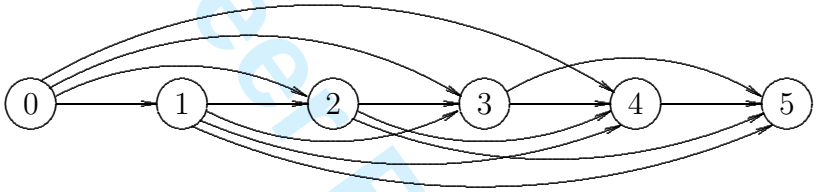

Figure 3: Reduced network

In order to compute a lower bound, first of all we have to define an initial set of colums, such that a feasible solution can be achieved. In our case this can easily be done by using the identity matrix.

Then the network is set up to generate column $q+1$ for the master problem. To this end the weights w_{ki} of the arcs are updated using the dual variables π_i and τ_i of the master problem according to

$$\bar{w}_{ki} = w_{ki} - \frac{b}{\sum_{j=1}^{m} v_{j,q+1}}\pi_i - \tau_i.$$

Since the weights $\bar{w}_{ki}$ of the network are a function of $\sum_{j=1}^{m} v_{j,q+1}$ we now have to deal with multiple arc weights. With every node included in the path from the source to the sink, the weights of all arcs change. Referring to figure 3, the weight of the arcs of a path from the source to the sink via one node only, would be $\bar{w}_{ki} = w_{ki} - \frac{b}{1}\pi_i - \tau_i$. The weight of the arcs of a path including two nodes would be $\bar{w}_{ki} = w_{ki} - \frac{b}{2}\pi_i - \tau_i$ and so on. But since the number of nodes included in the shortest path from the source to the sink in G is not known beforehand, we have to deal with the case that each arc has m weights. Equivalently we may consider m networks $G^z = (V, E, \bar{w}^z), z = 1, \ldots, m$, where the weight of each arc is calculated for a predefined number of nodes, that is,

$$\bar{w}^z_{ki} = w_{ki} - \frac{b}{z}\pi_i - \tau_i. \tag{11}$$

For each of these $z = 1, \ldots, m$ networks G^z, the shortest path from the source to the sink, containing a maximum of z nodes, has to be determined by using the Bellman-Ford algorithm

(see, e.g., Minieka 1978), a modification of the Dijkstra algorithm. This algorithm computes the shortest path between a pair of nodes covering at most z nodes. Note that we do not count the source and the sink node (which are contained in the path anyway) and, hence, a maximum of z nodes is equivalent to a maximum of $z+1$ arcs. If the number of nodes contained in the shortest path from the source to the sink in network G^z is smaller than z, then the network is discarded, because the weights of the arcs are not consistent with the number of nodes, contained in that path.

We restrain ourself from giving a formal description of the algorithm along with a formal proof of its correctness and refer the reader to Drexl and Mundschenk (2005) for technical details.

The lower bound LB_1 computed by means of the column generation technique sketched above can easily be improved. Let (x_h^{LP}) denote the optimal fractional variables of the linear programming relaxation of the integer master problem (1) to (4). These variables represent the non-integer number of jobholders, needed for qualification profile h. Then $\Phi = \sum_{h=1}^{q} x_h^{\mathsf{LP}}$ defines the sum of the non-integral amount of work force needed, to perform all processes within the considered period of time. Hence, the minimum work force size of the integer model must be at least $\lceil \Phi \rceil$. The costs of the difference $\lceil \Phi \rceil - \Phi$ can be evaluated with the minimum costs, possible to accrue for the missing work force. So the lower bound LB_1 can be tightened to $LB_2 = LB_1 + (\lceil \Phi \rceil - \Phi) \cdot \min_{h=1}^{q} c_h$ where q is the number of columns generated and c_h is the cost of column h.

4.2 Upper bounds

Upon termination of column generation we have a valid lower bound and a fractional solution (x_h^{LP}). An integral, feasible solution (x_h^{IP}) and a valid upper bound UB_1 can easily be computed by means of solving the mixed-integer program which is defined through the columns generated. Of course, it might be too time consuming to solve this mixed-integer program to optimality. Fortunately, a feasible solution is sufficient and, hence, computation can be aborted after a certain amount of time.

The gap between UB_1 and LB_2 usually allows much improvement. This is because the columns generated to solve the relaxed master problem might not be suitable in order to achieve a good integer solution. We use the solution corresponding to the upper bound UB_1 as initial solution for local search, producing a second upper bound denoted as UB_2. Among the variety of available local search algorithms (see, e.g., Aarts and Lenstra 1997) we decided to use simulated annealing in order to improve the upper bound.

For each qualification profile h a number of x_h^{IP} jobholders have to be employed and the total number of manpower needed to execute all processes is $u = \sum_{h=1}^{q} x_h^{\mathsf{IP}}$. Now we construct a matrix $A = (a_{ij})_{i=1,\ldots,m}^{j=1,\ldots,u}$. For every $x_h^{\mathsf{IP}} > 0$, we add x_h^{IP} columns to A, each containing the binary entries of the corresponding qualification profile (v_{ih}). Apparently, matrix $A = (a_{ij})$, or a for short, represents a feasible solution of the integer staffing problem.

Simulated annealing (SA) is a generic probabilistic heuristic approach for global optimization. Usually, SA locates a "good" approximation of the global optimum of a given objective function F in a large search space. At each iteration, SA considers some neighbors of the current solution a, and probabilistically chooses either to accept a new solution a' or keeping a. The probabilities are chosen so that the problem ultimately tends to move to solutions with better

objective function value. Typically this process is repeated until a solution which is "good enough" has been determined, or until a given time limit has been reached.

SA uses several basic concepts:

Neighborhood concept. At each iteration μ the neighborhood $N(a)$ of solution a is specified (usually problem-specific). $N(a)$ represents the subset of solutions a' which can be reached in one iteration emanating from a. Generally, it is not possible to store the neighborhood structure, because the set of feasible solutions has an exponential size.

Probabilistic acceptance of a new neighborhood solution. In each iteration μ solution a' is accepted with probability $P(a, a', T_\mu) = \min\left\{1, exp\left(-\frac{F(a')-F(a)}{T_\mu}\right)\right\}$. This probability is decreased during the course of the algorithm. In other words, we can escape a local optimum, but the probability for doing so is low after a large number of iterations. It guarantees that there is a high probability to locate a global optimum while avoiding to be trapped in local optima.

Parameter (temperature) dependent acceptance probability. The probability of making the transition to the new solution depends on a global time-varying parameter T called the annealing temperature: $(T_\mu)_{\mu=1}^{\infty}$ is a sequence of positive control parameters with $\lim_{\mu\to\infty} T_\mu = 0$.

Cooling schedule. Generally, the sequence $(T_\mu)_{\mu=1}^{\infty}$ is created by a function g, i.e. $T_{\mu+1} = g(T_\mu)$ for all μ. The initial annealing temperature T_0 has to be defined in advance.

Termination criterion. One has the freedom to introduce different stopping criteria. Typically, SA is repeated until the system reaches a state which is "good enough", or until a given time limit has been reached. The annealing temperature decreases to (nearly) zero short before termination.

In order to apply SA to a particular problem, we must specify the search space, the neighborhood search moves, the acceptance probability function, the cooling schedule and the termination criterion. These choices can significantly affect the method's effectiveness. Unfortunately, there is no unique choice that will be good for all problems, and there is no general way to find the best choice for a given problem (see, e.g., van Laarhoven and Aarts 1987, Wolpert and Macready 1997, Droste et al. 2002).

The local search is performed by swapping entries of the matrix (a_{ij}) from 1 to 0 or vice versa, following a set of rules. That is, the search mechanism is to examine members of the swap neighborhood of the starting solution a, and then move to neighbor a' and so on. To be more precise, let A_i and A^j denote row i and column j of matrix A, respectively.

- *Row rule:* Given row A_i a move is defined by chosing an entry $a_{ij}\,(j = 1, \ldots, u)$ at random and flipping it from 1 to 0 or vice versa.
- *Column rule:* Given column A^j a move is defined by chosing an entry $a_{ij}\,(i = 1, \ldots, m)$ at random and flipping it from 1 to 0 or vice versa.

We have modified the general purpose SA algorithm described in Abramson et al. (1996) to match the needs of our special case. To be more precise we have implemented the following algorithm. First, we scan the rows $i = 1, \ldots, m$ of matrix A cyclically in this order until no neighbor has been accepted within a certain number of trials. Given row A_i the row rule

is applied a certain number of times. If a neighboring solution a' is feasible and improves the current best known upper bound or is accepted probabilistically we go to the next row. Afterwards, we scan the columns $j = 1, \ldots, u$ of matrix A cyclically in this order until no neighbor has been accepted within a certain number of trials. Given column A^j the column rule is applied a certain number of times. Again, if a neighboring solution a' is feasible and improves the current best known upper bound or is accepted probabilistically we go to the next column.

According to preliminary computational tests not further documented here the initial temperature to start SA was set to $T_0 = 200$ and the cooling schedule is $T_\mu = 0,98^\mu \cdot T_0$. The algorithm terminates after one million swaps or if the solution gap is less than 2%.

SA terminates with a as the best feasible solution found. The objective value associated with a is denoted as UB_2. According to Abramson et al. (1996), we also implemented a function to restore feasibility of a' for the case that a move induces infeasibility. Because of the computational burden involved, the overall performance of SA worsened in terms of computation time and we did not keep this implementation.

4.3 Illustrative example

Consider a case with 50 jobs and 20 processes each job has to go through upon delivery. The basic data for this instance are provided in table 5. Column one identifies the process number i, column two shows the aggregated time demand P_i to process all $n = 50$ jobs, column three gives the minimum number S_i of jobholders needed to handle process i, column four identifies the occupational group each process belongs to where in this example we assume that processes 1 to 7 are being handled by commercial clerks (group 1), processes 8 to 13 by creative workers (group 2) and processes 14 to 20 by production workers (group 3). Finally, column five provides the wage for each process. Note that the level of qualification to handle process i and the associated wage are positively correlated. Finally, in this example the annual working time of each jobholder is set to $b = 70\ 000$.

In case of $m = 20$ processes, $2^{20} - 1 = 1\ 048\ 575$ different qualification profiles do exist. Column generation delivers the columns to obtain the optimal solution of the linear programming relaxation of (1) to (4). Table 7 gives an overview of the column generation process. An initial set of 10 columns has been constructed as outlined above. Column one of the table identifies the latest column considered, column two shows the corresponding optimal objective function value (OFV), column three provides the profile associated to the most recent column, column four gives the cost of the profile, column five tells us how attractive the column is in terms of the weights $\bar{w}_{ki}$ calculated according to equation (11) and the last column shows the corresponding cost difference.

Overall we can see that only 58 columns have to be considered and that the optimal objective function value of the linear programming relaxation is $LB_1 = 1\ 0881.98$. The minimum number of jobholders needed to handle all processes is $\Phi = \sum_{h=1}^{58} x_h = 8.488229$. Hence, the minimum integer size of the work force is $\lceil \Phi \rceil = 9$ and LB_1 can be improved as follows:

$$LB_2 = LB_1 + (\lceil \Phi \rceil - \Phi) \cdot \min_{h=1}^{58} c_h = 10\ 881.98 + (9 - 8.48) \cdot 600 = 11\ 189.05$$

i	P_i	S_i	group	wage
1	30 149	1	1	600
2	29 365	2	1	600
3	30 841	1	1	600
4	28 494	1	1	800
5	29 801	1	1	900
6	28 937	3	1	1 000
7	28 305	1	1	1 200
8	29 601	1	2	840
9	30 517	1	2	840
10	29 091	2	2	1 080
11	28 744	3	2	1 200
12	29 994	1	2	1 200
13	31 052	1	2	1 320
14	28 039	1	3	780
15	29 864	2	3	1 040
16	29 939	1	3	1 040
17	33 677	1	3	1 040
18	29 326	1	3	1 040
19	30 014	2	3	1 300
20	28 426	2	3	1 430

Table 5: Illustrative example

$$\begin{pmatrix}
1&0&0&0&0&1&0&1&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&1&0&1&0&0&0&0\\
1&0&0&0&0&0&0&1&0&0&0&0\\
1&0&0&0&0&1&0&1&0&0&0&1\\
1&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&1&1&1&0\\
0&0&0&0&0&0&0&0&1&1&1&0\\
0&0&0&0&0&0&0&0&1&1&1&0\\
0&0&0&0&0&0&1&0&1&1&1&0\\
0&0&0&0&0&0&0&0&1&1&1&0\\
0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&1&1&1&0&0&0&0&0&0&0\\
0&0&1&1&1&0&0&0&0&0&0&0\\
0&0&1&1&1&0&0&0&0&0&0&0\\
0&1&1&1&1&0&0&0&0&0&0&0\\
0&0&1&1&1&0&0&0&0&0&0&0\\
0&1&1&1&1&0&0&0&0&0&0&0\\
0&0&1&1&1&0&0&0&0&0&0&0
\end{pmatrix}$$

Table 6: Matrix $A = (a_{ij})_{i=1,\dots,m}^{j=1,\dots,u}$

#	OFV	profile	c_h	$\bar{c}_h$	$\bar{c}_h - c_h$
11	20 880.00	01100110011010101010	3 820.00	12 330.00	8 510.00
12	19 780.00	10010101111001100101	3 630.00	12 330.00	8 700.00
13	17 600.45	00000001000000000000	840.00	4 080.00	3 240.00
14	17 511.01	00000000000010000000	1 320.00	4 200.00	2 880.00
15	17 413.23	01101110101001011011	3 830.00	6 266.67	2 436.66
16	16 441.24	00000000000001000000	780.00	2 880.00	2 100.00
17	15 842.00	00000010000000000000	1 200.00	3 240.00	2 040.00
18	15 264.74	10010101111101011011	3 630.00	5 063.84	1 433.83
⋮	⋮	⋮	⋮	⋮	⋮
51	11 060.47	00000001111100000000	1 200.00	1 268.87	68.86
52	10 931.66	00000000111110000000	1 320.00	1 446.00	125.99
53	10 928.97	10110110000000000000	1 200.00	1 268.57	68.57
54	10 923.13	01000000000000000000	600.00	660.00	60.00
55	10 920.61	00000000000000111111	1 430.00	1 464.67	34.66
56	10 884.82	01100110000000000000	1 200.00	1 228.57	28.57
57	10 883.34	11000110000000000000	1 200.00	1 228.30	28.30
58	10 881.98	00001110000000000000	1 200.00	1 200.00	0.00

Table 7: Columns generated

If we solve the reduced integer master problem with the 58 columns computed before, an upper bound $UB_1 = 14\ 910$ is obtained. This implies that the solution gap is $\frac{UB_1 - LB_2}{LB_2} = 0.33$. The upper bound UB_1 corresponds to the solution shown in table 8. Thus 12 jobholders have to be employed with total costs of 14 910.

x_h^{IP}	profile	cost
1	10101110000000000000	1 200
1	00000000000000001010	1 300
3	00000000000001111111	1 430
1	11010100000000000000	1 000
1	00000000001010000000	1 320
1	10011100000000000000	1 000
3	00000001111100000000	1 200
1	01100110000000000000	1 200
12		14 910

Table 8: Feasible solution – UB_1

#	profile	cost
1	11100000000000000000	600
2	00000000000000011111	1 430
3	00000000000000100111	1 430
4	00000000000001000000	780
5	01111100000000000000	1 000
6	00000000101110000000	1 320
7	00000001111100000000	1 200
8	10011100000000000000	1 000
9	00000000000000111000	1 040
10	00000001011010000000	1 320
11	00000110000000000000	1 200
		12 320

Table 9: Improved feasible solution – UB_2

Now we use the feasible solution corresponding to the upper bound UB_1 as initial solution for the local search algorithm. The number of manpower needed is $u = \sum_{h=1}^{58} x_h^{\mathsf{IP}} = 12$. The matrix A with dimension (20×12) is given in table 6. Each column of A represents a qualification profile, that is, the entries $(a_{ij})_{i=1,\ldots,m}$ correspond to the parameter $(v_{ih})_{i=1,\ldots,m}$ computed during column generation. In particular we can see that the two rows 3 and 7 of table 8 imply the three identical profiles (columns) 3 to 5 and 9 to 11 of table 6. The cost vector associated with the $u = 12$ profiles is (1 200, 1 300, 1 430, 1 430, 1 430, 1 000, 1 320, 1 000, 1 200, 1 200, 1 200, 1 200).

After applying SA, we obtain an improved solution, shown in table 9. The improved upper bound now is $UB_2 = 12\ 320$ leading to a solution gap of $\frac{UB_2 - LB_2}{LB_2} = 0.101076$. The total number of manpower needed is reduced to 11. The total cost of the work force is 12 320.

5 Empirical results

The approach was tested in cooperation with a medium-size printing company. The data was extracted out of a computer-based job tracking system (see [11]), providing the complete data from 1999 to 2004. First, we perform a retrospective analysis and compare the overall empirical costs of the work force with the results of our model. Second, a prospective analysis is given by predicting the data for future periods.

The models and algorithms have been implemented in ANSI C. We used the open source LP/IP solver lp_solve (see [10]). The computations were performed on a PowerPC G4 with 1.67 GHz running Mac OSX. The results of an in-depth study of the algorithms in terms of deviations of lower and upper bounds and CPU times required can be found in Drexl and Mundschenk (2005).

year	n	m	annual workload (h)	cost (€)	# of jobholders	# of FTE
1999	3 667	33	85 723.88	3 276 664.95	132	91.43
2000	3 231	33	84 303.38	3 363 326.07	131	90.06
2001	3 052	33	85 064.68	2 884 708.38	110	89.71
2002	2 791	33	81 194.30	3 396 041.32	146	92.68
2003	2 318	33	70 852.68	2 853 577.81	119	80.97
2004	2 291	33	69 192.68	2 741 759.52	111	73.69

Table 10: Overview of empirical data sets

i	cost center	job description	group	S_i	costs(€)
1	Linotype-Topas-Scanner	Setzer	1	2	35 449
2	Plattenkopie konventional	Druckvorlagenherst.	1	2	39 868
3	Plattenkopie Zeitung Elfasol	Druckvorlagenherst.	1	2	39 868
4	DTP (Bogen u. Endlos)	Setzer	1	2	39 921
5	DTP-Zeitungsverlag	Setzerin	1	2	39 921
6	Zeitungsmontage	Setzer	1	2	39 921
7	Konventionelle Montage	Druckvorlagenherst.	1	2	39 921
8	CTP-Krause Laserstar 110	Setzer	1	1	39 921
9	Digitale Montage	Druckvorlagenherst.	1	0	47 223
10	Korrektorat	Korrektor	2	2	47 223
11	Heidelberger Tiegel	Drucker	3	1	39 921
12	MAN Roland 702	Drucker	3	2	46 780
13	MAN Roland 704	Drucker	3	0	46 780
14	MAN Roland 706	Drucker	3	2	46 780
15	Opti 3	Endlosdrucker	3	2	47 223
16	Opti 4	Endlosdrucker	3	2	47 223
17	Opti 5	Endlosdrucker	3	2	47 223
18	Speed SM 52	Drucker	3	2	47 223
19	Schobercollator	Buchbinderei Hilfskr	4	1	33 310
20	Hunkeler Snapband-Automat	Buchbinderei Hilfskr	4	1	33 310
21	Falzmaschinen	Buchbinderei Hilfskr	4	2	33 310
22	Sped-Clect Zusammentragmasch.	Buchbinderei Hilfskr	4	2	34 585
23	Müller-Martini Sammelhefter	Buchbinderei Hilfskr	4	2	34 585
24	Tisch/kl.Maschinen/Vers.Bogen	Buchbinderei Hilfskr	4	2	34 585
25	Fotolabor	Fotolaborantin	4	1	39 868
26	Lager Allg. (Bogen/Endlos/ZTG)	Lagerarbeiter	4	1	39 868
27	Bielomatic Rollencollator	Endlos Fachkraft	4	2	39 921
28	Planschneider	Buchbinderei Hilfskr	4	2	39 921
29	Logistik-Lager Feuersozietät	Drucker	4	2	39 921
30	MAN-Plamag-Ecoman-Rota(Offset)	Rotationsdrucker	5	4	47 223
31	Beilagen-/Einsteckmaschine	Packer	6	2	23 061
32	Tisch/Versand/Packerei Zeitung	Packer	6	2	23 061
33	Zeitungsauslieferung	Fahrer	7	2	23 061

Table 11: Cost centers of empirical data

Comparison with past data In the considered company, there are 33 different cost centers, every job can run through until its completion. The jobholder scans the barcode attached to the job ticked coming with each job, to register the checkin time of that job in that cost center, into the job tracking system. The same procedure is done, when the whole job has been performed within that cost center and the job has to be checked out. Since different kinds of processes are aggregated within each cost center, we cannot determine the requirements to the individual skills of the jobholders on the level of each single process. However, the data is still well enough to expect suitable results. An extract of the data is given in table 10. Since not all jobholders are working full-time, the number of full-time equivalents (FTE) are given in column 7. The cost centers and the classification to the different occupational groups are listed in table 11.[1]

The time unit used within the available data is measured in hours. Let us determine the annual working time b for a jobholder, working on a full-time shift. The relevant data is extracted out of the standard tariff for the printing industry (see [13]). Thus the working time for every jobholder in the printing industry is 35 hours a week, or 5 hours per day. Every jobholder can claim 30 days for vacation within one year. Finally let's assume that there are an average of six days of holiday within the annual working time, so the net capacity of every jobholder reduces to an annual time amount of

$$b = 52 \cdot 35 - 30 \cdot 5 - 6 \cdot 5 = 1\,640$$

hours. In the considered printing company, there are seven occupational groups involved in the production process. The wages of the different salary levels are shown in column 6 of table 11. Basically, the annual wages can be determined on the basis of the standard tariff. But because of bonuses and overpayment, the average wage of each occupational group of the company is used.

The parameters for the minimum number of jobholders S_i needed have been determined in cooperation with the management. The values for 1999 are also listed in table 11.

The approach was applied to each of the annual data sets, extracted from the job tracking system, with the appropriate parameters. Table 12 contains the annual workload of each cost center. Aggregated results are displayed in table 13. In the last column the solution gap between UB_2 and LB_2 with GAP $= \frac{UB_2 - LB_2}{LB_2}$ is displayed.

By generating the appropriate qualification profiles, it can be seen, that the total annual workload (see table 10) could have been handled with about 60% of the work force. Thus applying the model lowers the total cost of the work force in the range of 26% – 39% where this percentage is defined as $\frac{\text{old cost} - \text{improved cost}}{\text{old cost}}$.

Work force size needed in the future The data for 2005 and 2006, shown in table 12 have been determined using linear regression by minimizing $\sum_{i=1}^{n} u_i^2 = \sum_{i=1}^{n} (y_i - (\alpha + \beta x_i))^2$. The constant α and the slope β of this equation have been estimated for each cost center. The results are displayed in table 14. Since the total workload will be further on declining, the total size of the work force can further be reduced. By adjusting the qualification profile of each jobholder, the total cost of the work force can be reduced to about 1.7 million €. The qualification profiles of the 49 jobholders needed to handle the predicted workload in 2006 are listed in table 15 row by row.

[1] Even though advanced translators are available in the internet (see, e.g., [9]) it seems to be hopeless to try to translate these German specifics into English.

i	1999	2000	2001	2002	2003	2004	2005	2006
1	3088.50	3184.42	2813.20	2387.30	3180.35	3027.43	2872.52	2851.28
2	1457.88	982.58	786.48	400.27	151.03	76.25	0.00	0.00
3	497.42	30.67	0.00	0.00	0.00	0.00	0.00	0.00
4	4135.73	4776.98	5024.80	4915.17	3729.48	3295.25	3567.45	3354.46
5	13026.12	12166.07	11345.88	10028.50	7878.85	8173.75	6592.44	5494.13
6	5859.92	5792.82	5545.58	5621.93	5181.35	5127.80	4979.70	4824.89
7	4051.90	2185.67	1159.62	550.47	165.63	53.45	0.00	0.00
8	70.48	666.98	1688.35	2121.08	1699.20	1154.92	2128.66	2384.42
9	0.00	0.00	0.00	0.00	804.08	1676.07	1492.62	1800.98
10	3753.97	3891.88	4291.30	4424.32	4157.33	3213.37	3778.00	3727.32
11	147.18	179.48	179.73	91.17	175.30	91.92	106.39	95.60
12	737.82	899.93	300.18	0.00	0.42	0.00	0.00	0.00
13	0.00	0.00	803.08	1254.67	1367.18	1740.35	2186.37	2565.08
14	1713.02	1883.40	2715.62	2057.15	1940.42	1491.33	1807.24	1761.64
15	1234.52	1199.02	1100.25	966.17	483.58	297.25	183.46	0.00
16	1300.55	1276.27	1049.93	1018.77	858.35	846.00	702.54	600.90
17	1640.18	1645.48	1291.97	1222.75	964.22	950.30	729.57	570.65
18	1774.58	1640.10	1378.12	1328.98	994.80	903.32	702.51	521.33
19	37.47	63.18	2.22	0.00	0.00	0.00	0.00	0.00
20	157.60	163.58	103.07	87.93	150.73	106.13	97.07	88.19
21	2808.22	2419.25	2435.37	2332.30	2138.97	1974.53	1840.21	1694.14
22	463.38	379.72	267.62	193.57	115.90	88.22	0.00	0.00
23	1029.62	1106.82	722.88	596.87	614.70	592.00	398.10	289.80
24	4323.53	4890.53	4314.80	4690.73	3751.42	3994.87	3859.17	3725.32
25	165.40	78.78	65.33	121.00	68.17	41.50	30.46	13.44
26	1.17	0.00	3.33	0.00	0.00	2.42	1.44	1.53
27	1945.98	1710.73	1597.22	1469.13	1070.23	901.93	722.22	514.51
28	1357.15	1567.38	1690.95	1578.97	1274.00	1148.75	1232.79	1174.67
29	1080.47	1170.65	1148.27	961.87	822.02	1091.70	928.21	894.61
30	1924.17	1796.25	1810.90	1775.20	1805.88	1814.50	1765.64	1749.78
31	7151.38	7387.13	8438.68	8536.72	9455.13	9106.35	9953.59	10412.93
32	10852.20	12224.23	13417.92	12990.88	8213.10	7836.42	8168.52	7381.68
33	6846.50	6943.38	7572.03	7470.45	7640.85	8374.62	8437.78	8712.96
$\sum$	84634.00	84303.38	85064.68	81194.30	70852.68	69192.68	69264.68	67206.24

Table 12: Annual workload of cost centers

year	# of FTE	cost (€)	improvement	GAP
1999	58	2 214,625	32%	0.114
2000	56	2 152,561	36%	0.092
2001	59	2 143,392	26%	0.094
2002	56	2 055,859	39%	0.099
2003	51	1 828,938	36%	0.094
2004	50	1 775,936	35%	0.083

Table 13: Aggregated results

Year	# of FTE	cost (€)	gap
2005	49	1 806 299	0.112
2006	49	1 731 793	0.107

Table 14: Predicted work force size and associated costs

By analyzing the resulting qualification profiles, it can be seen, that the model does not create profiles, so that a jobholder has to be qualified to handle processes from more than one occupational group (see section 2 also).

Let's for example take a look at group 1, where the processes of the cost centers 1 to 9 are being handled. Cost center 1, the Linotype-topas scanner, will have an annual demand of processing time of 2 851 hours. Jobholder 1 will be qualified and charged to handle only that process, satisfying the demand of processing time with his total annual working time of 1 640 hours. To satisfy the remaining 1 221 hours of that cost center, the jobholders 3, 9 and 11 are also qualified to handle that process. Since, e.g., jobholder 11 is a typesetter, qualified to handle the processes of the cost centers 4 and 5, the cost for the company does not increase by this extra qualification to handle the processes of cost center 1. The reason is that handling of processes of the cost centers 4 and 5 asks for a higher qualification than to only operate the scanner in cost center 1.

6 Individual working times

In the previous sections we have provided a means to compute those qualification profiles which are needed in order to satisfy long-term staffing requirements in a cost efficient way. Afterwards, of course, the question arises how much time each worker should spend on a specific process. This question can easily be addressed by linear programming.

Using the additional parameters

P_i : demand of processing time for process i, that is, $P_i = \sum_{j=1}^{n} p_{ji}$
u : number of jobholders, index l
z_{il} : vector of qualification profile of jobholder l
b_l : annual working time of jobholder l

and the decision variables y_{il} which indicate the individual working time of jobholder l spent on process i we get the following model:

$$\min \quad \sum_{i=1}^{m} \sum_{l=1}^{u} y_{il} \tag{12}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} z_{il} y_{il} \leq b_l \qquad l = 1, \ldots, u \tag{13}$$

$$\sum_{l=1}^{u} z_{il} y_{il} \geq P_i \qquad i = 1, \ldots, m \tag{14}$$

$$y_{il} \geq \begin{cases} \delta & \text{if } z_{il} = 1, \\ 0 & \text{otherwise} \end{cases} \qquad i = 1, \ldots, m, l = 1, \ldots, u \tag{15}$$

Table 15: Qualification profiles of jobholders to be employed in 2006

group									1	2								3									4	5		6	7	
i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	wages
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	35 449
2	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
3	1	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
4	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
6	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
7	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
8	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
9	1	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
10	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
11	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	39 921
12	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	47 223
13	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	47 223
14	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	47 223
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	47 223
16	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	46 780
17	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	46 780
18	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	46 780
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	33 310
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	34 585
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	47 223
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	47 223
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	47 223
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	47 223
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	23 061
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	23 061
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	23 061
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	23 061
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	23 061
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	23 061
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	23 061
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	23 061
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	23 061
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	23 061
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	23 061
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	23 061
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	23 061
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	23 061
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	23 061
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	23 061
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	23 061
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	23 061
43	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	47 223
44	0	0	0	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	47 223
45	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	47 223
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	1	1	1	0	0	0	0	39 921
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	34 585
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	39 921
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	39 921
∑	4	0	0	6	7	7	0	5	2	3	1	0	3	4	1	2	2	2	0	1	2	0	1	3	1	1	2	3	3	4	10	8	6	1 731 793

Table 16: Time y_{il} spent on process $i = 1, \ldots, m$ by jobholder $l = 1, \ldots, u$

i \ l	1	2	3	4	5	6	7	8	9	10	11
1	9 814	0	0	0	0	0	0	52 758	0	0	0
2	29 355	0	0	0	10	0	0	0	0	0	0
3	30 831	0	0	0	10	0	0	0	0	0	0
4	0	0	0	0	11 272	0	0	17 222	0	0	0
5	0	0	0	0	29 791	0	0	10	0	0	0
6	0	0	0	0	28 917	0	0	10	0	0	10
7	0	0	0	0	0	0	0	0	0	0	69 990
8	0	0	0	0	0	0	10	0	0	60 592	0
9	0	0	0	0	0	10	30 507	0	0	0	0
10	0	0	0	0	0	0	19 703	0	0	9 388	0
11	0	0	0	0	0	8 964	19 770	0	0	10	0
12	0	0	0	0	0	29 984	10	0	0	0	0
13	0	0	0	0	0	31 042	0	0	0	10	0
14	0	0	0	70 000	0	0	0	0	0	0	0
15	0	0	10	0	0	0	0	0	58 608	0	0
16	0	18 557	0	0	0	0	0	0	11 382	0	0
17	0	33 667	0	0	0	0	0	0	10	0	0
18	0	10	29 316	0	0	0	0	0	0	0	0
19	0	10	30 004	0	0	0	0	0	0	0	0
20	0	17 756	10 670	0	0	0	0	0	0	0	0

The value δ in constraint (15) has to be chosen out of the range $0 < \delta < b_l$. To set the lower bound of y_{il} to a positive value in case of $z_{il} = 1$ assures constraint (3) of the staffing model.

For illustrative purposes we apply this model to the instance considered in section 4.3 (see table 5). If we assume that a total amount of $b = 70\ 000$ time units is available for every jobholder per year we get the distribution of working time shown in table 16.

7 Implications for management

The work force size and the associated qualification profiles for the jobholders needed to handle the predicted workload in 2006, deliver a target for the management. Though the work force inventory of the company does not have to be restructured drastically within the next two years, the results of the model should influence the personnel policy. New results have to be computed for every future period, after reliable data is available to perform valid forecasts. Since the optimal configuration of the work force changes every period, because of the dynamic environment of the company, a "true optimum" can never be achieved. Nevertheless, the work force needed to handle the workload predicted for 2006, can be cut down to about 60% of the work force inventory today. Jobholders have to be requalified and reassigned to different cost centers. The freed amount of manpower can be applied to work in new areas of the company, dealing with new products in new markets. This way, the marginal revenue of each jobholder can be raised, leading to a higher turnover of the company.

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