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## **(18th ICPR) INTEGRATING INSPECTION POLICY DESIGN IN ASSEMBLY LINE BALANCING**

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1  
2 **INTEGRATING INSPECTION POLICY DESIGN IN ASSEMBLY LINE**  
3  
4 **BALANCING**  
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## ABSTRACT

This paper addresses the problem of assembly line balancing and selection of the best inspection policy. Different inspection policies yield different manufacturing, reworking, inspection and external failure costs. While in literature assembly line balancing and inspection policy design have usually been addressed as different problems, they are strongly interrelated. Inspection tests requires time to be performed and thus their assignation directly influences the results of the assembly line balancing procedure, that is the number of stations that are necessary and hence the line installation cost. Provided what said above, the advantage of integrating inspection policy design in assembly line balancing appears clear, but to our knowledge there is not such a model yet. In this paper a new model that defines simultaneously inspection policy and assembly line balancing is presented.

Keywords:

Assembly Line Balancing, Inspection Policy, Quality Management, Linear Programming, Concurrent Engineering

## 1. INTRODUCTION

The last years have seen an increasing competition among companies in a context which is changing very rapidly. Quality has progressively become one of the key competitive priorities to compete in the market hence companies have paid more and more attention to this issue.

Quality has a relevant impact on the costs a company has to afford. Indeed not only the external failure cost has to be considered (the cost that a company incurs when a

1  
2 defective product reaches the market) but also internal quality costs connected to the  
3  
4 cost of inspection, manufacturing and reworking. Thus all the above mentioned costs  
5  
6 should be considered when defining the inspection policy for a production system.  
7

8  
9 This paper is focused on assembly line design and aims at integrating inspection policy  
10  
11 design into assembly line balancing. In this way indeed it is possible to consider already  
12  
13 at a design stage all the costs connected with quality: the external costs, connected to  
14  
15 the defective units that reach the customer, and internal costs, i.e the cost of  
16  
17 manufacturing, reworking and inspection taking into account the impact that the  
18  
19 allocation of inspections has on assembly line balancing. Indeed, as tests requires time  
20  
21 to be performed, their allocation strongly influences the line balancing procedure and  
22  
23 the results of the balancing procedure itself, that is the total number of workstations that  
24  
25 is necessary to set up .  
26  
27  
28

29  
30 This paper consists of 7 sections. Following the introduction is the review of the  
31  
32 background literature in section 2. Section 3 is dedicated to the definition of the  
33  
34 problem faced. Section 4 describes the mathematical model, its characteristics,  
35  
36 variables, objective function and constraints. In section 5 the design of the simulation  
37  
38 experiments that have been run to evaluate the performance of the method proposed is  
39  
40 explained. The results of those experiments have been reported in section 6. At the end  
41  
42 some conclusions are drawn. An appendix with some details follows up.  
43  
44  
45  
46  
47  
48

## 49 **2. LITERARY BACKGROUND**

### 50 51 52 53 54 *2.1 Inspection allocation in serial multistage production systems*

55  
56 Lindsay and Bishop (1964) were the first to formulate the problem of inspection  
57  
58 allocation in serial production systems. The problem is solved minimizing an objective  
59  
60 function which accounts for inspection, lost production and quality loss costs. Every

1  
2 step of the production system has a known probability of generating a defect and the  
3  
4 quality tests are available after each processing stage. The defective parts are removed  
5  
6 from the production system.  
7

8  
9 Pruzan and Jackson (1967), besides the variable inspection cost, introduced the fixed  
10  
11 cost as quota of the amortisation and they made the assignation of tests depend upon the  
12  
13 history of the piece, that is the sequence of operations previously assigned. Eppen and  
14  
15 Hurst (1974) proposed the first model that considers two types of inspection errors:  
16  
17 acceptance of defective units and rejection of conforming parts.  
18  
19

20  
21 The analysis of inspection allocation in multistage production system has been  
22  
23 gradually refined and several contributions on this issue were proposed loosening some  
24  
25 of the original hypothesis and proposing different solution methods.  
26  
27

28  
29 Ballou and Pazer (1982) studied the impact of inspection errors on the final solution and  
30  
31 demonstrated that type I errors (rejection of conforming parts) have a greater effect than  
32  
33 type two errors (acceptance of defective units). Lubicz (1983) developed the first model  
34  
35 to account for non-conformity costs that vary according to the stage where the non-  
36  
37 conformity has appeared. Yum and Mc Dowell (1987) proposed a model that permits to  
38  
39 consider different management policies for defective units that can be either repaired,  
40  
41 replaced, reworked or discarded with costs that vary according to the stage in the  
42  
43 manufacturing process. Considering that multiple or repeated inspections can be a way  
44  
45 to limit the effects of inspection errors, Raz and Kaspi (1991) proposed a non-linear  
46  
47 integer programming model for an integrated approach to sequencing and locating  
48  
49 multiple inspection operations which distinguishes itself for the presence of a transfer  
50  
51 function that links incoming and outgoing level of quality and costs for each testing and  
52  
53 manufacturing operation. Lee and Unnikrishnan (1998) proposed a model that considers  
54  
55 the presence of several parts, each one having a different technological cycle defined  
56  
57 through the sequence according to which they have been worked on every machine.  
58  
59  
60

1  
2 This model is characterized by the fact that a constraint on the maximum time for the  
3  
4 availability of inspection machines is set. Shiao (2002) formulated a model based on the  
5  
6 finite inspection resource constraint (as Lee and Unnikrishnan) and an inspection error  
7  
8 model: the inspection error is not constant or a specified probability, but accounts for  
9  
10 inspection capability, manufacturing capability and tolerance. Rau and Chu (2005)  
11  
12 presented an heuristic to solve the inspection allocation planning problem with two  
13  
14 types of workstation, workstation of attributable data (WAD) and workstation of  
15  
16 variable data; in addition this study considers three possibilities for the treatment of  
17  
18 detected nonconforming units, namely repair, rework and scrap.  
19  
20  
21  
22  
23  
24  
25

## 26 *2.2 Inspection allocation in non-serial multistage production systems*

27  
28 Britney (1972) was the first one to consider non-serial systems: in his model, that finds  
29  
30 the best solution minimising the internal and external cost of quality and the inspection  
31  
32 cost, every operation has a defined probability of generating a defective unit, a perfect  
33  
34 test is available after every operation and reparation is perfect as well.  
35  
36

37  
38 Following a path similar to the one described for serial production systems, the models  
39  
40 proposed for inspection allocation in non-serial multistage production systems have  
41  
42 been gradually refined.  
43

44  
45 Yum and Mc Dowell (1981) developed Brytney's model by introducing first type and  
46  
47 second type inspection error. Garzia-Diaz, Foster and Bonyquet (1984), taking into  
48  
49 account the reduction in inspection error thanks to repeated inspection, developed a  
50  
51 dynamic programming model having as variables the position of inspections and the  
52  
53 number of repetitions (defined as inspection degree). Also for non-serial system, Kaspi  
54  
55 and Raz (1994) considered the problem of allocating both manufacturing and inspection  
56  
57 operations whose relative job order is not assigned beforehand but defined through  
58  
59 precedence relations defined in an assembly diagram. The probabilities to generate  
60

1  
2 defective units are known but depend upon the specific operation and the operations  
3  
4 previously assigned; I type and II type inspection error are considered and defective  
5  
6 parts are discarded.  
7  
8  
9

### 10 11 *2.3 Assembly line balancing*

12  
13 Assembly line balancing is a very lively research field. The fundamental line balancing  
14  
15 problem is to assign the tasks to assemble a product to an ordered sequence of stations,  
16  
17 such that the precedence relations are satisfied and some measure of effectiveness is  
18  
19 optimised (e.g. minimize the balance delay or minimise the number of workstations),  
20  
21 (Gosh and Gagnon, 1989).  
22  
23

24  
25 Salveson (1955) was the first one to give a mathematical form to the problem and  
26  
27 proposed a linear integer programming model to solve it.  
28  
29

30  
31 Since then a lot of articles have been published detailing the advances in this field and  
32  
33 periodically articles reviewing the models proposed are published .  
34

35  
36 According to Gosh and Gagnon (1989) the Assembly Line Balancing problem (ALB)  
37  
38 and the accompanying research and literature can be classified into four categories:  
39  
40 Single Model Deterministic (SMD), Single Model Stochastic (SMS), Multi/Mixed  
41  
42 Model Deterministic (MMD), and Multi/Mixed Model Stochastic (MMS). The SMD  
43  
44 version of the ALB problem assumes dedicated single-model assembly line where task  
45  
46 times are known deterministically, the SMS model introduces the concept of task time  
47  
48 variability. The MMD problem formulation assumes deterministic task times, but  
49  
50 introduces the concept of an assembly line producing multiple products. The MMS  
51  
52 problem perspective differs from its MMD counterpart in that stochastic times are  
53  
54 allowed. For each category they distinguish between Simple Case and General Case.  
55  
56 The General Case refers to situations in which restrictions or other factors are  
57  
58 introduced into the model (e.g. parallel stations, zoning restrictions etc.).  
59  
60

1  
2 Rekiek, Dolgui, Delchambre and Bratcu (2002) distinguish between simple assembly  
3  
4 line balancing and general assembly line balancing problems. Within simple assembly  
5  
6 line balancing they classify models considering optimization methods (linear  
7  
8 programming, dynamic programming, branch and bound etc.), processing time  
9  
10 (deterministic time, stochastic time, dynamic time...), equipment selection, additional  
11  
12 constraints (operators, buffers etc.). As to the general problem, they classify methods  
13  
14 considering line configuration (U-Line, parallel stations, parallel lines...), design  
15  
16 objectives, use of simulation.  
17  
18  
19

20  
21 We-Min Chow (1990), Scholl (1995), and Nof, Wilhelm and Warnecke (1997) are good  
22  
23 references for deepening the knowledge of alternative line balancing methods.  
24  
25

26  
27 The most recent reviews are the ones by Scholl and Becker (2006) and Becker and  
28  
29 Scholl (2006). The former is a review of methods to solve simple assembly line  
30  
31 balancing problem and the latter is focused on the generalized assembly line balancing  
32  
33 problem.  
34

### 35 **3. PROBLEM FORMULATION**

36  
37 An assembly line consists of a finite set of tasks characterised by a processing time and  
38  
39 precedence relationships that specify the permissible ordering of the tasks. The  
40  
41 balancing line problem aims at assigning the tasks necessary to assemble a product to  
42  
43 workstations so to minimize a certain objective function, usually the total number of  
44  
45 stations and thus the total investment and running cost.  
46  
47

48  
49 Defining the inspection policy consists in deciding which tasks to control and where to  
50  
51 control them.  
52

53  
54 Several costs are connected to the inspection policy definition: manufacturing cost,  
55  
56 reworking cost, inspection cost and external failure cost. As an item is subject to  
57  
58 different operations on an assembly line, it can incur defects as a result of processing. If  
59  
60 no tests to detect defective operations are allocated, these parts will continue their

1  
2 working cycle on the line nourishing manufacturing costs; furthermore if the items  
3  
4 eventually reach the customer, additional failure cost has to be considered. On one hand  
5  
6 if inspections were allocated after every assembly task, defective units would be  
7  
8 detected as soon as they are generated reducing manufacturing and reworking cost; on  
9  
10 the other, tests imply additional costs for inspection equipment and, as they require time  
11  
12 to be performed, they impact on the number of stations that is necessary to set up and  
13  
14 thus on the line installation cost.  
15  
16

17  
18 Besides a processing time, tests are characterised by precedence relationships, exclusion  
19  
20 relationships and position added costs.  
21  
22

23  
24 Precedence relationships take into account the fact that in order to make a test, the  
25  
26 performing of some other tasks might be necessary.  
27

28  
29 Exclusion relationships account for the fact that a test can be no more available after  
30  
31 some other tasks have been performed. For example it might be no more possible to  
32  
33 check if a component or a group of components work well after some other parts  
34  
35 covering the components to be tested have been assembled.  
36

37  
38 The position added cost takes into account the relative position of a test and the tasks it  
39  
40 checks.  
41

42  
43 Indeed the cost connected to repairing a certain task can increase if the task is not  
44  
45 checked, and thus repaired, straight after it is performed. Indeed having performed some  
46  
47 assembly tasks between a task and the test that checks this task can imply extra  
48  
49 reparation cost if the task is found to be defective by the inspection. This extra  
50  
51 reparation cost can be due to extra-disassembly and re-assembly activities or material  
52  
53 loss (for example gases and fluids that are used to fill a circuit).  
54  
55

56  
57 Thus the definition of the inspection policy influences several costs (manufacturing  
58  
59 cost, reworking cost, external failure cost, position added cost) as well as the line  
60  
installation cost.

1  
2 Nevertheless, while assembly line balancing and inspection policy design are strongly  
3  
4 interrelated, to our knowledge, there are no models in literature addressing at the same  
5  
6 time these two issues.  
7

8  
9 The models focused on assembly line balancing do not consider the impact of  
10  
11 inspection policy.  
12

13  
14 Models that consider inspection allocation in multi-stage production system neglect the  
15  
16 distinctive characteristics that an assembly line problem presents: the cycle time  
17  
18 (constraining the amount of work that can be allocated to a station), the precedence  
19  
20 relationships for tasks and tests and the exclusion relationships for tests. Indeed in an  
21  
22 assembly line problem, tests are first characterised by a processing time as the time  
23  
24 required to perform a test directly influences the number of workstations that is  
25  
26 necessary to set up. Furthermore in these models, a test controlling all the tasks  
27  
28 previously assigned is available after every production stage, thus the precedence and  
29  
30 exclusion relationship cannot be considered as well as the position added cost.  
31  
32

33  
34 To conclude, existing models considering balancing problems do not consider the  
35  
36 inspection policy. The models allocating tests in multistage production systems, even  
37  
38 though offer some important references for inspection positioning, neglect some factors  
39  
40 that are fundamental for balancing purposes.  
41  
42

43  
44 Hence the need for a model that allows to define the inspection policy while balancing  
45  
46 an assembly line tackling specifically all the issues connected to a balancing problem,  
47  
48 appears clear. Several works show that the integrated and simultaneous approach gives  
49  
50 relevant benefits when it integrates different functional areas, for instance  
51  
52 manufacturing and quality or design and quality (Portioli-Staudacher, 1999; Portioli-  
53  
54 Staudacher, Landeghem, Mappelli, Redaelli, 2003).  
55  
56  
57  
58  
59  
60

#### 4. THE MODEL

This paper presents a model to balance an assembly line concurrently assigning inspection tests.

The assembly procedure (with the precedence relationships) is specified in an assembly graph (see figure 1). A set of in-line tests are available, each test controls one task or a set of tasks and is characterised by a processing time  $\tau_k$ , precedence relationships, exclusion relationships and position added cost. A final test is also available to check all assembly tasks. The final test is done out of the line. Whenever a defective product is detected, it is repaired or substituted. The defective one is repaired offline and then ready to substitute the next defective one. In this case it is necessary to afford reparation/replacement costs and then the product continues the assembly cycle along the line. Perfect reparability of the product is assumed.

The model proposed permits to balance the line to manufacture the product concurrently defining which tests to use and the position where they have to be performed.

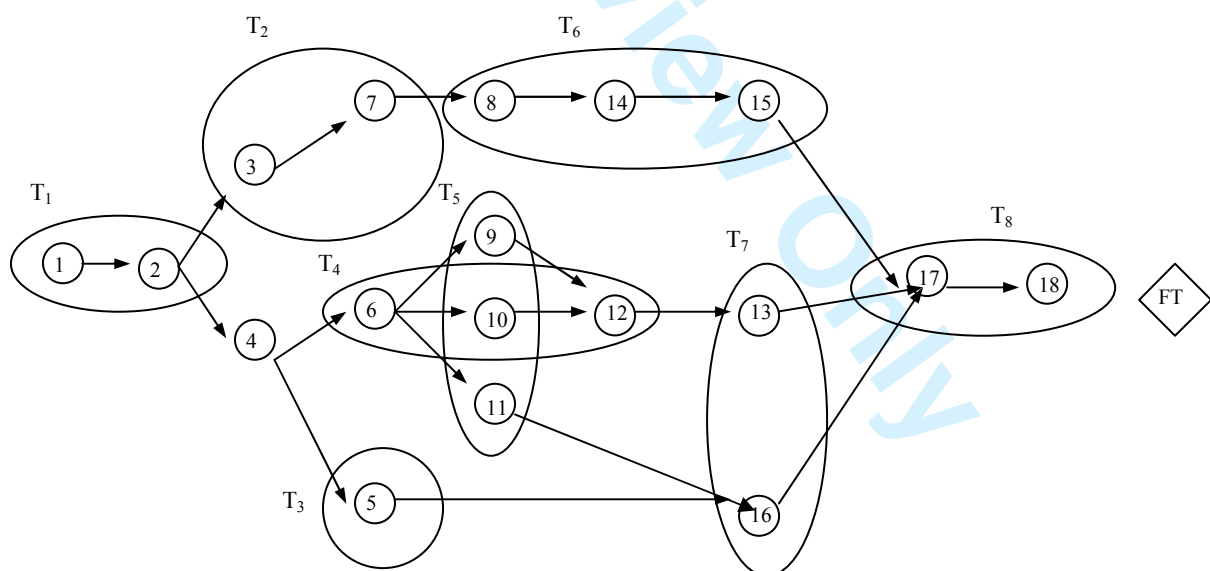


Figure 1: An assembly graph with a set of possible inspections

#### 4.1 Characteristics of the model

Here the characteristics of the model proposed are presented

- Assembly task time is known deterministically and is not affected by the allocation sequence (the tasks and tests already assigned and their relative position)
- Test time is known deterministically and is not affected by the allocation sequence
- Defect rate of each assembly task is known, constant and is not affected by the allocation sequence
- Tests are perfect: no first type or second type inspection error
- Each task can be controlled just by one test in line
- The final test is not in line, and therefore its processing time is not relevant
- The final test checks all tasks
- The reparation cost increases with the distance of the test from the tested task

The first hypothesis, i.e. the fact that assembly tasks time do not depend on the sequence of operations is not always true in reality. Nevertheless it is possible to take into account this by restraining the task sequence. The same can be said for the second hypothesis. The assembly task defect rate has been considered deterministic as it does not impact on the total output and is not the central issue of the problem.

The same holds for the fourth hypothesis.

The hypothesis that each task can be controlled by a test in line only, has been introduced to reduce computational difficulties.

The model considers the fact that reparation cost increases with the distance of the test from the tasks it checks. This is modeled in the position added cost that has been explained above.

## 4.2 Nomenclature

Here follows a definition of the symbols that have been used

### Indexes

$n$  number of nodes of the assembly diagram, it is the same as the number of tasks to assemble the product

$i$  task index ( $i \in [1, n]$ )

$K$  number of available inspection tests (except the final test); each control can check more than one defect

$k$  inspection test index ( $k \in [1, K]$ )

$n_{\min}$  theoretical minimum number of stations (see appendix)

$\text{Max}W$  maximum number of stations (see appendix)

$j$  station index ( $j \in [1, \text{Max}W]$ )

### Data

$CE_i$  external failure cost associated to defect  $i$

$CF_i$  repair/replacement cost if task  $i$  is controlled at the final test

$CFT$  final test cost

$CI$  station installation cost (portion of the station installation cost allotted to a unit of output)

$Clin_i$  repair/replacement cost for in line inspections associated to test  $i$  (see appendix)

$Clin_k$  repair/replacement cost for in line inspections associated to test  $k$  (see appendix)

$CO_k$  cost of test  $k$

1		
2	$CP_{i,k}$	position cost, it is the added reparation cost that has to be afforded if task i
3		
4		has already been performed in a previous station when test k is done and the
5		
6		product is found to be defective by test k
7		
8		
9	CT	assembly line cycle time
10		
11	$d_i$	defect rate of assembly task i (see appendix)
12		
13	$G_k$	probability that the tasks controlled by test k were performed correctly (see
14		
15		appendix)
16		
17		
18	H	binary $[K,n]$ matrix of non feasibility of the test
19		
20	$H_{k,i}$	component of H: $H_{k,i}=1$ if the execution of operation i makes test k no more
21		
22		feasible, 0 otherwise
23		
24		
25	$I_k$	set of operations controlled by test k (e.g. $I_3 = \{ 1, 4, 6 \}$ , test 3 controls the
26		
27		tasks 1, 4 and 6)
28		
29		
30	M	a very large number
31		
32	OT	binary $[n,K]$ matrix indicating which tasks are controlled by the tests
33		
34	$OT_{i,k}$	component of OT matrix; $OT_{i,k}=1$ if task i is controlled by test k, 0
35		
36		otherwise
37		
38		
39	PR	$[n,K]$ binary matrix of the test-assembly task position
40		
41	$PR_{i,k}$	component of the PR matrix: $PR_{i,k}=1$ if having performed operation i before
42		
43		test k is done, determines an increase in reparation cost if the product is
44		
45		found to be defective by test k.
46		
47		
48		
49	PT(i)	set of immediate predecessors of task i
50		
51	PI(k)	set of all the predecessors (tasks) of test k
52		
53		
54	$t_i$	time required to perform task i
55		
56	$\tau_K$	time required to perform test k
57		
58		
59		
60		

## Variables

FT	binary variable that equals 1 if the final test is allocated, 0 otherwise
$L_i$	auxiliary binary variable needed to make the product $FT \cdot TE_i$ linear ( $=FT \cdot TE_i$ )
$NOP_i$	number of the station to which task i is assigned
$NTEST_k$	number of the station to which test k is assigned
$Q_k$	index of allocation of test k: it equals 1 if test k has been allocated to a station, 0 otherwise
$T_{k,j}$	binary variable that equals 1 if test k is assigned to station j ( $k \in [1, K]$ , $j \in [1, MaxW]$ ), 0 otherwise
$TE_i$	index of assembly task inspection: $TE_i=1$ if task i is controlled, (i.e. if one of the tests available to control it is assigned), 0 otherwise
$W_j$	binary variable that indicates the opening of station j, it equals 1 if at least one operation (task or test) is allocated to the station, 0 otherwise
$Z_{i,j}$	binary variable that equals 1 if task i is assigned to station j ( $i \in [1, n]$ , $j \in [1, MaxW]$ ), 0 otherwise
$\Psi_{i,k}$	binary variable which equals 1 if a position added cost has to be calculated as concerns test k and operation i, 0 otherwise

### 4.3 Objective function

$$\text{Min: SIC} + \text{InC} + \text{EFC} + \text{LRC} + \text{FRC} + \text{TPAC} \quad (1)$$

where

$$\text{SIC} = \text{Station Installation Cost} = CI \cdot \sum_{j=1}^{MaxW} W_j \quad (2)$$

$$\text{InC} = \text{Inspection Cost} = \sum_{k=1}^K CO_k \cdot Q_k + \text{CFT} \cdot \text{FT} \quad (3)$$

$$\text{EFC} = \text{External Failure Cost} = \sum_{i=1}^n CE_i \cdot (d_i - d_i \cdot TE_i - d_i \cdot FT + d_i \cdot L_i) \quad (4)$$

$$\text{LRC} = \text{In line Repair/Replacement Cost} = \sum_{k=1}^K Clin_k \cdot Q_k \cdot (1 - G_k) \quad (5)$$

$$\text{FRC} = \text{Final Test Repair Cost} = \sum_{i=1}^n CF_i \cdot (d_i \cdot FT - d_i \cdot L_i) \quad (6)$$

$$\text{TPAC} = \text{Test Position Added Cost} = \sum_{i=1}^n \sum_{k=1}^K CP_{i,k} \cdot \Psi_{i,k} \cdot (1 - G_k) \quad (7)$$

The value of the objective function represents the unit cost associated to the manufacturing of one product. This cost takes into account relevant costs for our model, that is the Station Installation Cost, the Inspection Cost, the External Failure Cost, the In line Repair/Replacement Cost, the Final Test Repair Cost and the Test Position Added Cost. Other costs that would not be modified by the application of our model (as the cost for materials) are not included.

The station installation cost is given by the number of stations times the unit station installation cost (CI) which accounts for soil occupation cost, heating, personnel (if present) and all the other fixed costs. CI has been calculated as the portion of the station installation cost allotted to a unit of output.

The inspection cost is made up of two terms. The first one is the cost to perform a test due to the need of tools and fixtures. The second one represents the cost of the final test, in case it is performed. The cost of time consumption due to performing the test is already considered in the station installation cost, because it affects the number of stations needed.

The external failure cost represents the cost incurred when defective units reach the market. In (4) it is simply the external failure cost times the outgoing defectiveness. It is necessary here to give some explanations about the variable  $L_i$ . It is an auxiliary

variable that represents the product of  $TE_i$  and  $FT$ . As it will be explained afterwards, constraints (18) and (19) force  $L_i$  to 1 just when both  $TE_i$  and  $FT$  equal 1. In table 1 the outgoing defectiveness is indicated as a function of the value of  $TE_i$ ,  $FT$  and  $L_i$ .

Table 1: Outgoing defectiveness as a function of  $TE_i$ ,  $FT$  and  $L_i$

$TE_i$	$FT$	$L_i$	Outgoing defectiveness $(d_i - d_i \cdot TE_i - d_i \cdot FT + d_i \cdot L_i)$
0	0	0	$d_i$
1	0	0	0
0	1	0	0
1	1	1	0

The outgoing defectiveness associated to operation  $i$ , equals  $d_i$  if task  $i$  is controlled neither in line nor at the final test, otherwise it is zero.

The in line reparation/substitution cost has been evaluated as the units entering the reparation station times the reparation/substitution cost. The way the variable  $G_k$  can be inferred from the defect rates of the single assembly task ( $d_i$ ) is explained in the appendix.

Equation (6) calculates the reparation cost after the final test as the product of the detected defectiveness at the final test (associated to all the tasks except the ones already tested in-line) and the reparation cost after the final test. In table 2 the detected defectiveness at the final test has been reported as a function of  $TE_i$ ,  $FT$  and  $L_i$ .

Table 2: Detected defectiveness at the final test as a function of  $TE_i$ ,  $FT$  and  $L_i$ 

$TE_i$	$FT$	$L_i$	Detected Defectiveness at the final test $(d_i \cdot FT - d_i \cdot L_i)$
0	0	0	0
1	0	0	0
0	1	0	$d_i$
1	1	1	0

As said before the final test is an overall inspection that can find all the defects of a product. The detected defectiveness at the final test is used to evaluate the reparation cost after the final test. If the final test is not performed ( $FT=0$ ), since defective products would not be found, the detected defectiveness at the final test equals 0. The same happens if the task has already been controlled in line ( $TE_i=1$ ): in this case the defectiveness is 0 whether the final test is performed or not. The detected defectiveness at the final test equals  $d_i$  if task  $i$  is controlled just at the final test.

Equation (7) represents the test position added cost. As said before this cost keeps into account the fact that if a task is not controlled straight after it is done and some other tasks have already been done between the moment in which the task is done and the moment in which it is checked, some extra reparation cost might arise if the product is found to be defective by the test. This is due to the fact that reparation may require partial disassembly and then re-assembly of tasks performed in stations downstream of task inspected but before the test is performed. The test position added cost is computed as the product of the defective units entering a reparation station times the position cost ( $CP_{i,k}$ ). Constraints as in equation (20) forces  $\Psi_{i,k}$  to become 1 whenever having performed operation  $i$  before test  $k$  implies extra-reparation costs if the product is found

to be defective by test  $k$ . Through the matrix CP the model considers all the tasks that are performed before test  $k$  is done and the different impact that they can have on reparation cost.

#### 4.4 Constraints

The constraints of the model are the following:

$$\sum_{j=1}^{MaxW} Z_{i,j} = 1 \quad i = 1, \dots, n \quad (8)$$

$$Z_{i,j} \leq \sum_{h=1}^j Z_{v,h} \quad i = 1, \dots, n; \quad j = 1, \dots, MaxW; \quad \forall v \in PT(i) \quad (9)$$

$$T_{k,j} \leq \sum_{h=1}^j Z_{v,h} \quad k = 1, \dots, K; \quad j = 1, \dots, MaxW; \quad \forall v \in PI(k) \quad (10)$$

$$T_{k,j} \leq 1 - \sum_{h=1}^j \sum_{i=1}^n Z_{i,h} \cdot H_{k,i} \quad k = 1, \dots, K; \quad j = 1, \dots, MaxW \quad (11)$$

$$\sum_{i=1}^n t_i \cdot Z_{i,j} + \sum_{k=1}^K \tau_k \cdot T_{k,j} \leq CT \cdot W_j \quad j = 1, \dots, MaxW \quad (12)$$

$$W_{j+1} \leq W_j \quad j = 1, \dots, MaxW-1 \quad (13)$$

$$Q_k = \sum_{j=1}^{MaxW} T_{k,j} \quad k = 1, \dots, K \quad (14)$$

$$NTEST_k = \sum_{j=1}^{MaxW} j \cdot T_{k,j} \quad k = 1, \dots, K \quad (15)$$

$$NOP_i = \sum_{j=1}^{MaxW} j \cdot Z_{i,j} \quad i = 1, \dots, n \quad (16)$$

$$TE_i = \sum_{k=1}^K OT_{i,k} \cdot Q_k \quad i = 1, \dots, n \quad (17)$$

$$TE_i + FT - L_i \leq 1 \quad i = 1, \dots, n \quad (18)$$

$$- TE_i - FT + 2 \cdot L_i \leq 0 \quad i = 1, \dots, n \quad (19)$$

$$M \cdot \Psi_{i,k} \geq PR_{i,k} \cdot (NTEST_k - NOP_i) \quad k = 1, \dots, K; \quad i = 1, \dots, n \quad (20)$$

1  
2 Each task has a constraint of the form of equation (8). These constraints ensure that  
3  
4 each task is assigned to exactly one workstation.  
5

6  
7 Restrictions as in equation (9) forces adherence to precedence restrictions: to allocate an  
8  
9 assembly task to a station, it is necessary that all the tasks that are immediate  
10  
11 predecessors of this task were previously assigned.  
12

13  
14 The next restriction expresses the same concept in test precedence relationships.  
15

16  
17 Exclusion relationships for the tests are satisfied by equation (11): if a task that makes  
18  
19 the test unfeasible is allocated, the variable  $T_{k,j}$  is forced to zero implying that the test  
20  
21 cannot be assigned.  
22

23  
24 Constraints as in equation (12) ensure that the sum of task time and test time assigned to  
25  
26 each workstation does not exceed the cycle time. Besides, equations as in (12) force  $W_j$   
27  
28 to 1 if at least a task or a test is allocated to station  $j$ .  
29

30  
31 To avoid that a station is open unless the previous station has already been opened,  
32  
33 restrictions as in (13) are set.  
34

35  
36 Equations as in (14) and (17) define  $Q_k$  and  $TE_i$ . As  $TE_i$  is a binary variable, constraints  
37  
38 as in equation (17) impose that every task can be controlled just by one test (in other  
39  
40 words it is possible to allocate just one test that controls operation  $i$ ).  
41

42  
43 Equations as in (15) and (16) are used to evaluate the test position added cost together  
44  
45 with constraints as (20).  $NTEST_k$  represents the number of the station to which test  $k$  is  
46  
47 assigned (0 if test  $k$  is not assigned) and  $NOP_i$  represents the number of the station to  
48  
49 which test  $i$  is assigned.  $NTEST_k - NOP_i$  is either 0 (if test  $k$  and operation  $i$  are assigned  
50  
51 to the same station),  $\geq 1$  (if test  $k$  is assigned in a station after  $i$ ) or  $\leq 1$  (if test  $k$  is not  
52  
53 assigned or is assigned to a station before the one in which  $i$  is assigned). In equations  
54  
55 as (20),  $\Psi_{i,k}$  is forced to 1 when test  $k$  is done after task  $i$  has already been assigned in a  
56  
57 previous station and  $PR_{ik}$  equals 1 (i.e. having performed operation  $i$  before test  $k$  is  
58  
59 performed is a cause of extra reparation cost if the product is found to be defective by  
60

1  
2 test k). When  $\Psi_{i,k}$  is forced to 1, test position added cost is computed in the objective  
3  
4 function (see equation 7). It is worth noting that equation (7) considers all tasks for each  
5  
6 test thus allowing to compute the contribution to TPAC of all tasks already assigned to  
7  
8 stations between a task and its test. This allows to consider an increase in reparation  
9  
10 cost as the test is done downstream of the tested task.  
11

12  
13 Each task has constraints as (18) and (19) necessary to make the product of the two  
14  
15 binary variables  $TE_i$  and  $FT$  linear. They force the binary variable  $L_i$  to equal 1 just  
16  
17 when  $TE_i$  and  $FT$  are both equal to 1 and to be zero otherwise.  
18

19  
20 Further constraints could be added to the model to take into account other requirements  
21  
22 connected both to assembly tasks and inspection tasks. For instance on one hand it  
23  
24 could be necessary to allocate two assembly tasks to the same station (for example  
25  
26 because they require the same expensive equipment); on the other, some assembly  
27  
28 operations could be incompatible and thus have to be allocated to different stations (for  
29  
30 example an arc welding task and a task requiring the use of an inflammable solvent).  
31  
32 Similar considerations could be made for inspection tasks. In all these cases specific  
33  
34 constraints can be added to the model.  
35  
36  
37  
38

## 39 40 **5. EXPERIMENT DESIGN**

41  
42 A set of simulation runs has been performed to evaluate the advantages of the integrated  
43  
44 approach to assembly line balancing and quality management proposed in the model  
45  
46 (named as LBQ) in comparison with a sequential approach achievable adapting the  
47  
48 models present in literature.  
49  
50

### 51 52 *5.1 Benchmark selection*

53  
54 In order to evaluate the real benefits of the model presented, it has been compared with  
55  
56 models already existing in literature. In literature there are no models that concurrently  
57  
58 address the problem of assembly line balancing and inspection policy definition,  
59  
60

1  
2 therefore the benchmarks used to evaluate the model derives from the sequential  
3  
4 combination of two different algorithms derived from the literature. The first algorithm  
5  
6 defines the inspection policy (which tasks to control); the second algorithm takes as  
7  
8 input the output of the first algorithm and balances the line by allocating tasks and tests  
9  
10 to workstations (see also figure 2).  
11  
12

13  
14 Based on the above explained principle, two different benchmark models have been  
15  
16 designed, they have been named CONF-P and CONF-KL.  
17

18  
19 CONF-P model first defines the inspection policy by using the same approach proposed  
20  
21 by Brytney (1972) or Yum and Mc Dowell (1981, 1987). Indeed these models, among  
22  
23 the ones proposed in literature for inspection allocation in multistage production  
24  
25 systems, are the ones that suite better the problem proposed. Nevertheless a slight  
26  
27 modification of these models has been necessary. Indeed in the above mentioned model,  
28  
29 an inspection is available after every production step and it checks all the tasks that has  
30  
31 been done until that stage, while in the problem proposed there is a set of possible  
32  
33 inspections that control one task or a set of tasks. The costs that are considered while  
34  
35 defining the inspection policy are inspection cost, reparation/replacement cost, and  
36  
37 external failure cost. The test position added cost is neglected at this stage as it is  
38  
39 assumed that, if a test is done, it is performed straight after the task it checks. Using as  
40  
41 input the output of the first model, Askin and Standridge's model is used to balance the  
42  
43 line thus computing the line installation cost. At the end, the test position added cost is  
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45 calculated as a result of the position assigned to the tests by the second algorithm.  
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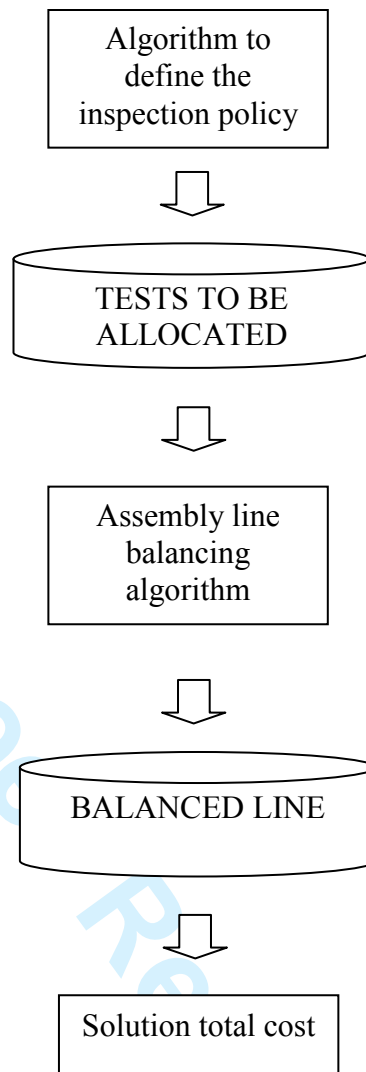


Figure 2: Logical schema of the benchmark models

CONF-KL has the same approach described for CONF-P except for the fact that the costs used in the first algorithm are slightly different. Using the same idea at the basis of Kottas and Lau algorithm (1973) the costs of the tests has been defined as in equations (21).

$$C_{TEST_k}^* = C_{TEST_k} + \frac{\tau_k}{CT} * CI \quad (21)$$

where

k=test index

1  
2 CTEST<sub>k</sub>=test cost as used in CONF-P model

3  
4  $\tau_k$ = time required to perform test k

5  
6  
7 CT= cycle time

8  
9 CI= unit station installation cost

10  
11 Thus the cost of the tests used in the CONF-KL model equals the cost of the tests used  
12 in the CONF-P model plus a percentage of the station installation cost based on the rate  
13 of time taken to perform the test and the cycle time. CONF-KL should therefore account  
14 for the time to perform the test and thus favor, *other things being equal*, short time tests  
15 which have a less relevant impact on line balancing.  
16  
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### 23 *5.2 Experimental factors*

24  
25 The models have been compared in different situations in order to test their  
26 characteristics. In particular the following factors have been identified as the most  
27 relevant: the graph type, the station installation cost and the test position cost and have  
28 been varied as indicated in table 3.  
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Table 3: Experimental factors

Parameter	Levels #	Levels
Graph type (GT)	3	Graph A Graph B Graph C
Station installation cost (CI)	3	Low (0,3) Medium (1) High (4)
Position cost ( $CP_{i,k}$ )	3	Low (0,1) Medium (1) High (4)

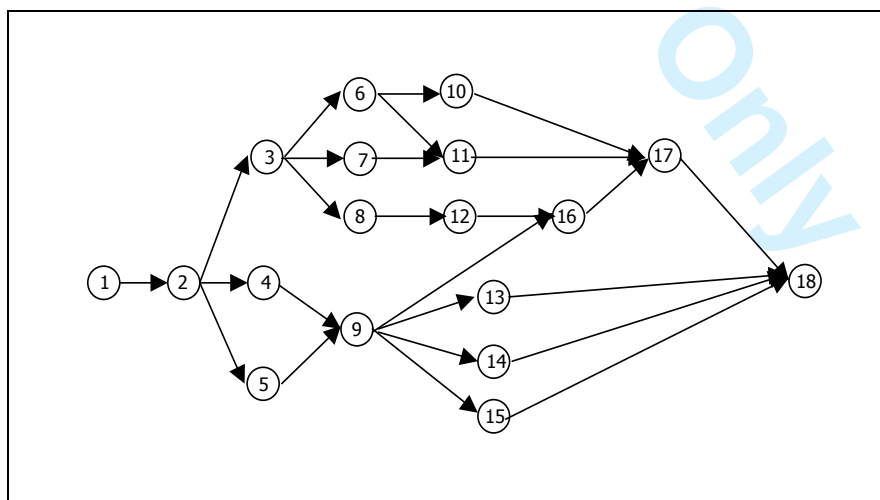


Figure 3: Precedence Graph type A

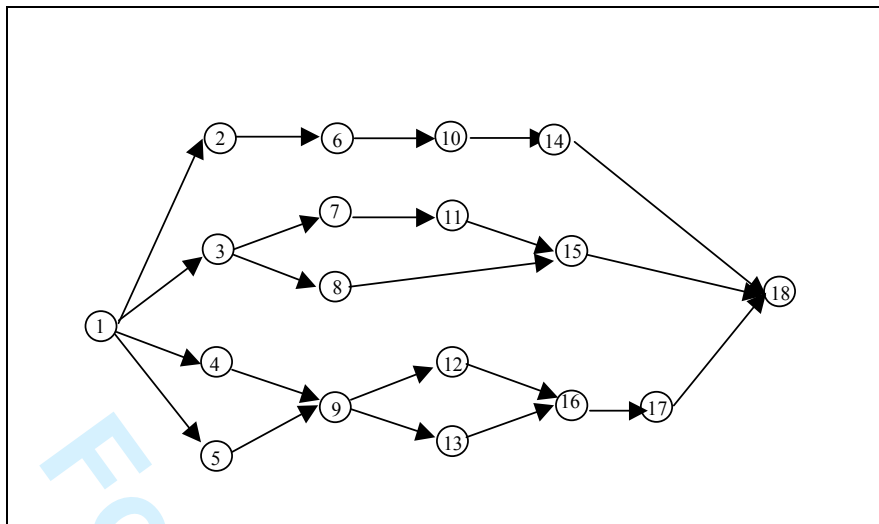


Figure 4: Precedence Graph type B

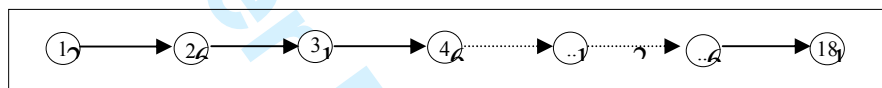


Figure 5: Precedence Graph type C

Table 4: Details about tests

TEST	1	2	3	4	5	6	7	8	9	10	11	12	13	14
TIME	72	56	47	34	71	79	38	65	66	53	30	49	65	45
TASKS CHECKED	2,3,5	7,9,11	2,3	8,16	13,14, 15,17	2,3, 4,5	4,6	10,11, 12,15	6,12, 13	6,7,8	15,17	10,11	5,7,9	15,16, 17

Graph A and graph B are described in figure 3 and 4 respectively. They represent different precedence relationships but have the same number of assembly operations and the same overall work content. (i.e. the sum of the duration of the 18 assembly tasks is the same for all the graphs). Finally figure 5 represents graph C which is an extreme situation with a serial sequence (the sequence of tasks is fixed).

All the graphs have 18 tasks and 14 possible inspections. The number of tasks and tests

1  
2 influences the complexity of the problem. The number of tasks and tests used for the  
3  
4 simulation are consistent with the models in literature concerning allocation of tests in  
5  
6 multistage production systems; for instance Kaspi and Raz (1994) considered a diagram  
7  
8 with 8 tasks and 5 tests and in Yum and McDowell (1981) the number of tasks is 10 as  
9  
10 well as the number of tests.  
11

12  
13 As to the station installation cost, it is a relevant parameter because it is connected with  
14  
15 the investment necessary to install the line. High value of the station installation cost are  
16  
17 typical of a product that requires high expenses for handling equipment during the  
18  
19 assembly procedure, or in a situation in which the personnel, the soil occupation cost  
20  
21 etc. are very expensive.  
22  
23

24  
25 A high value for the position cost is used to represent a situation where not finding the  
26  
27 defect right after having performed a task, is very costly due to the cost of disassembly  
28  
29 and the presence of unrecoverable components etc.  
30  
31

32  
33 In order to simulate low, medium and high value of the installation cost and the position  
34  
35 cost the following procedure has been followed. For each one of those parameters a  
36  
37 value has been taken as baseline representing the medium value. High value and low  
38  
39 values have been calculated by multiplying the medium value for the parameters in  
40  
41 Table 3. Thus, for example, low value and high value of the station installation cost are  
42  
43 respectively 30% and 400% of the medium value.  
44  
45

46  
47 To improve statistical confidence 9 replications have been carried out changing the time  
48  
49 needed to perform each task by re-allocating randomly the processing times among the  
50  
51 18 assembly tasks (assembly task times vary from 19 to 159). The overall assembly  
52  
53 time is thus maintained constant, so making easier to compare the different  
54  
55 performances. Cycle time has been set to 190.  
56  
57

58  
59 As to the other parameters of the model connected to the tasks, the defect rate of  
60  
assembly task varies from 0.021 and 0.132, reparation/replacement cost at the final test

range from 138 to 205, the external failure cost associated to a defect varies from 511 to 691. As concerns other costs, the cost of a test varies between 5 and 25, in line repair cost varies between 126 and 181 while the cost of the final test is 761. The installation cost is 523.

### 5.3 Performance measures

The different models tested have been compared on the basis of the total cost comprising station installation cost, inspection cost, external failure cost, in line repair/replacement cost, repair cost after the final test and test position added cost as they are defined in equation (1).

## 6. EXPERIMENT RESULT

In figure 6 the overall average total cost (considering all simulation runs) has been reported.

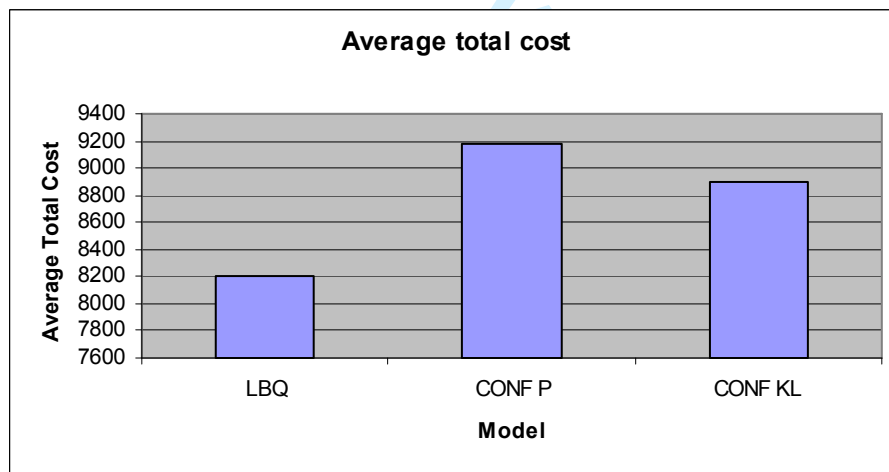


Figure 6: Average total cost

The proposed integrated model LBQ yielded the lowest cost, whilst CONF-P had the worst performance.

CONF-P has been taken as benchmark to evaluate the relative performance of the other two models.

A statistical test on total costs shows that both LBQ and CONF-KL are significantly better than CONF-P at  $\alpha=0.05$  (and thus the finding has 95% chance of being true).

The average percentage improvement is 8.7 % for LBQ and 1.3% for CONF-KL.

The average percentage improvement has been calculated in the following way:

$$\%improv = \frac{\sum_{i=1}^{total\_number\_runs} \frac{Cost(Conf - P) - Cost(LBQ)}{Cost(CONF - P)}}{total\_number\_runs} \quad (22)$$

The gap for LBQ is quite relevant if we consider that this percentages has been calculated considering the solution total cost. Indeed the number of stations in a line cannot be inferior to the minimum number calculated dividing the sum of all the assembly task processing times by the cycle time. This cost component, which is not negligible, cannot be ascribed to the model. Comparing the improvement achieved to the avoidable costs only, the improvement is much bigger. Anyway, being the total cost easier to understand as an objective value, the performance will be considered always in term of total cost.

It is interesting to highlight that LBQ has performed better than CONF-P in all the simulations and the percentage improvement has varied from a minimum value of 1.6% to a maximum value of 13%.

In order to understand better the influence of the experimental factors, the disaggregated data have been reported hereafter.

### 6.1 Graph type

In figure 7 the average percentage improvement has been reported as a function of the graph type.

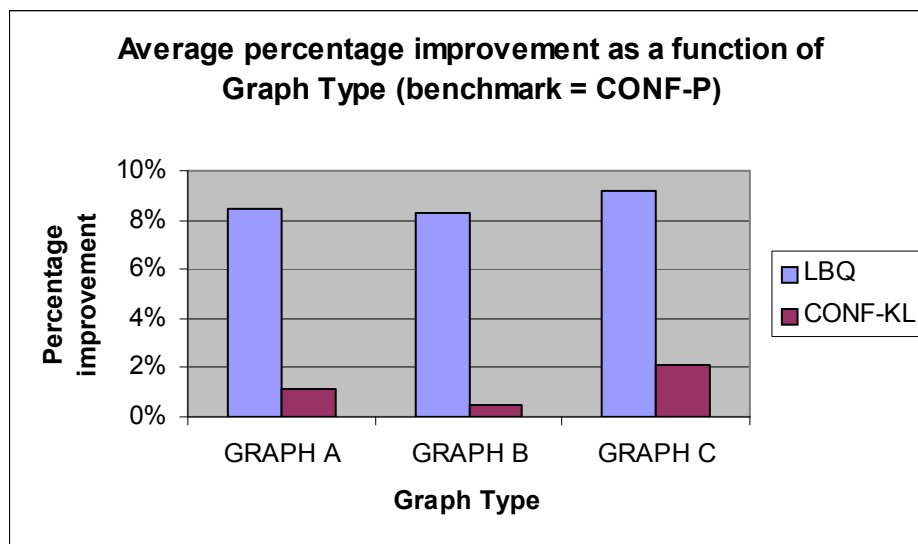


Figure 7: Average improvement in costs as a function of graph type

In all the cases LBQ and CONF-KL performed better than CONF-P and the improvements are statistically significant at  $\alpha=0.05$ .

The test on the percentage differences calculated on the graph type shows that these differences are not statistically significant. This is true both for LBQ and CONF-KL.

LBQ and CONF-KL are better than CONF-P but this does not depend upon the graph type.

### 6.2 Station installation cost

In figure 8 the percentage improvement has been reported as a function of the station installation cost.

The percentage improvement of LBQ on CONF-P varies from 5.1% for low level of the installation cost to 11.4% for high level of this factor maintaining a value of 9.4% for medium level.

Statistical tests show that LBQ is better than CONF-P and the gap is enhanced when the installation cost raises. CONF-P bad performance is due to the fact that it does not solve the problem globally; LBQ indeed can in some situations allocate the tests so to guarantee an inferior number of stations and thus, the more the installation cost

increases, the bigger savings are possible.

The same can be inferred about CONF-KL that improves its performance as the installation cost increases. This can be explained considering the way tests are appraised in CONF-KL, indeed CONF-KL somehow anticipates the effect of inspection time by attributing to the tests an extra-cost proportional to the processing time and to the installation cost itself.

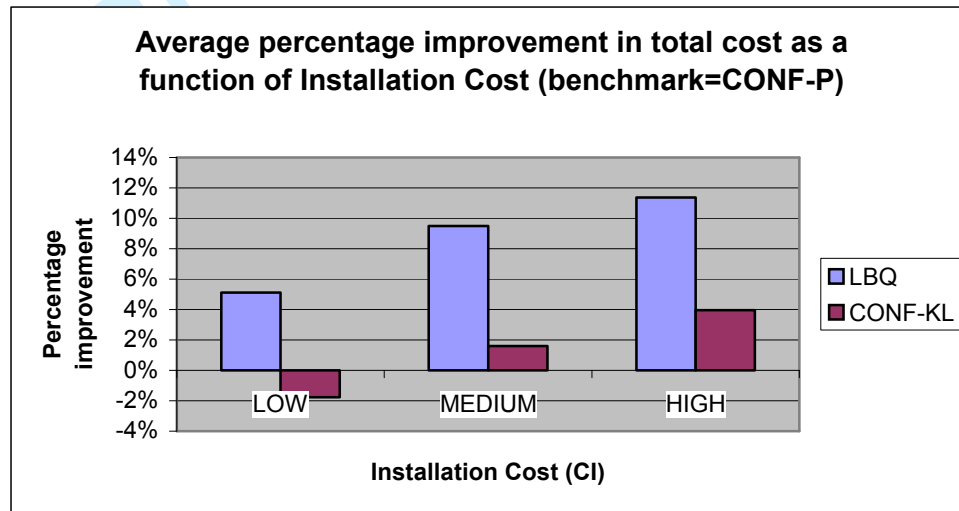


Figure 8: Average improvement in costs as a function of installation cost

### 6.3 Position cost

In figure 9 the percentage average improvement has been reported as a function of the position cost.

The savings made possible by LBQ range from 7.8% to 9.8%.

The difference between high value of the position cost and medium value is statistically significant whilst the difference between low and medium level is not. We can conclude that LBQ advantage increases as the position cost becomes sufficiently higher.

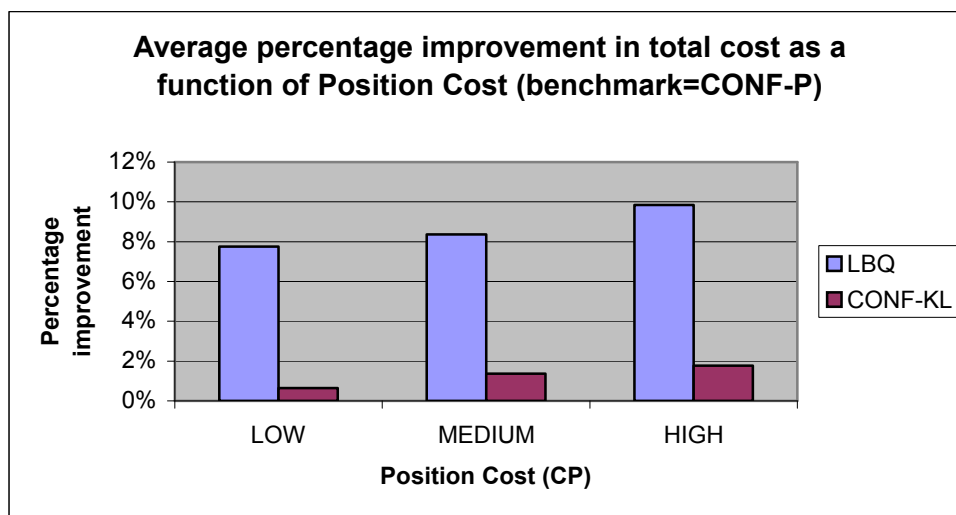


Figure 9: Average improvement as a function of test position cost

The advantage of LBQ lies in the optimisation of the trade off between good line balancing and the costs due to the allocation of tests not straight after the assembly tasks they check. This is not possible for the other two models that address line balancing and inspection policy in a sequential way.

Also considering the test position cost, CONF-KL performs slightly better than CONF-P. The percentage difference is significant only between low value of the station installation cost and high value. A possible explanation of the better performance of CONF-KL compared to CONF-P can be the fact that CONF-KL, *all the rest being equal*, favours short time tests. It is then more likely that these tests can be allocated to the same station of the tasks they control.

## 7. CONCLUSIONS

In literature, line balancing and quality control policies are addressed as two different problems, nevertheless the definition of the inspection policy has a very deep impact on line balancing. Thus an integrated approach to line balancing and inspection policy definition can give relevant benefits. In this paper a model for integrating line balancing and quality control policy selection has been proposed and tested.

1  
2 The model permits to balance an assembly line concurrently allocating assembly tasks  
3 and inspection tasks so to minimise the total cost of quality and installation. The model  
4 takes into account the cycle time, inspection tasks times, tasks defect rate, precedence  
5 and exclusion relationships for tests. Objective of the model is to minimize an objective  
6 function that considers inspection costs, reparation/substitution cost for in line controls  
7 and the final test, external failure cost, position added cost (to account for extra  
8 reparation costs connected to the relative position of a test and the tasks it checks) and  
9 installation cost (connected with the total number of workstations that is necessary to set  
10 up).

11  
12 The model has been tested by comparing it with two benchmark models that address the  
13 problem of inspection policy definition and assembly line balancing in a sequential way  
14 so that first the inspection policy is defined and then tests and tasks are allocated to  
15 different workstations. The result of the simulation has showed that the method  
16 proposed always achieves a better performance than the benchmark models. Statistical  
17 analysis shows that the savings made possible by the model proposed increase as the  
18 position cost and the installation cost raise. In this situation the model is especially  
19 suitable. Nevertheless also for low values of this factor, savings are significant.

20  
21 The model could be developed so to adapt it to multi-model lines, consider stochastic  
22 times for assembly tasks and inspections. Moreover heuristics could be developed to  
23 solve quickly large problems integrating inspection policy design into assembly line  
24 balancing.

25  
26 The model proposed considers several factors: Station Installation Cost, Inspection  
27 Cost, External Failure Cost, In line Reparation/Replacement Cost, Final Test Reparation  
28 Cost, Test Position Added Cost. All these factors reflect different objectives and they  
29 have been weighted estimating their economical impact (e.g. the impact of selling a  
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defective item), so to reach an objective function which is the sum of a number of different costs. Other types of weights could be adopted.

On a broader basis the model could also be developed to consider the possibility to evaluate alternative graphs and/or task duration, for the same product, so to consider alternative product design options and/or alternative process/technology solutions; this approach indeed is also very promising (Portioli-Staudacher, Singh, 1996).

## APPENDIX

*Estimation of  $n_{min}$  and MaxW*

$$n_{min} = \left\lceil \frac{\sum_{i=1,n} t_i}{CT} \right\rceil \quad (23)$$

$$MaxW = n+k \quad (24)$$

$n_{min}$  is used only for computational purposes and can be estimated assuming to allocate only assembly tasks.

MaxW is estimated by assuming to allocate, besides all the tasks, all the tests and considering the worst condition for balancing purposes, that is supposing that it is necessary to open a new station for every operation.

*Relationships between  $Clin_i$  and  $Clin_k$*

$Clin_k$  (in-line reparation cost associated to test k) can be calculated as follows:

$$Clin_k = FC_k + \sum_{i \in I_k} Clin_i * d_i \quad (25)$$

$FC_k$ =fixed cost associated to test k

1  
2  
3  
4 *Estimation of  $d_i$  and relationships between  $d_i$  and  $G_k$*   
5

6 We will analyze here in details how we could estimate the defect rate of an assembly  
7 task, one of the primary parameters of the model.  
8

9 Let us call  $A_{j,i}$  the  $j$  cause of defectiveness for the operation  $i$  and  $CS_i$  the total number  
10 of defectiveness causes. Assuming that the causes of defectiveness are independent, the  
11 probability  $d_i$  could be written as:  
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13

$$14 \quad d_i = P(\text{operation } i \text{ leads to a defect}) = 1 - P(\text{no defect causes happen}) = 1 - \prod_{j=1}^{CS_i} (1 - p_{A_{j,i}})$$

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23 (26)  
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25 If we have an estimation of the probability associated to all the causes of defectiveness,  
26 it is then possible to assess the defect rate of every assembly task.  
27

28 As a result it is possible to estimate the probability  $G_k$  that gives the probability that the  
29 assembly tasks controlled by test  $k$  were been performed properly. Using the hypothesis  
30 of independence of the defect rate from the allocation sequence,  $G_k$  is expressed by the  
31 following formula:  
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$$38 \quad G_k = \prod_{\forall i \in I_k} (1 - d_i) \quad (27)$$

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