



Distributed control of beams by electric transmission lines with PZT actuators

Francesco dell'Isola, Stefano Vidoli
Dipartimento di Ingegneria Strutturale e Geotecnica,
Università di Roma "La Sapienza",
Via Eudossiana 18, I-00184 Roma, Italy.
E-mail: isola@scilla.ing.uniroma1.it, vidoli@scilla.ing.uniroma1.it

ABSTRACT

The aim of the paper is to propose a distributed control technique based on the modal coupling between a beam and an electric transmission line with PZT actuators; these last behave as capacitances in the electric line and contribute to beam stiffness because their end-points are restrained along the beam; we call the described system a piezoelectro-mechanical (PEM) beam.

Structural control by means of concentrated devices often requires high voltages and high inductances; this fact is particularly serious when one has to deal with PZT actuators; in order to overcome the problem we propose a device in which the control action is distributed along the beam.

We found that the line inductance and the line resistance behave as electric "taps" respectively for the electro-mechanical modal coupling and for the damping efficiency: we found a bounded region in the line impedance plane in which the energy exchanged between the beam and the electric line is maximum and therefore the control strength is maximum. By means of already available PZT actuators we obtain very high damping ratios for mechanical vibrations ($\delta \simeq 0.1$)¹.

We only investigated a simple case of passive control in damping vibration, but several active control applications are conceivable: it seems to us that such control technique is more efficient than those already proposed in the literature characterized by the use of actuators supplying concentrated forces.

Keywords: one-dimensional continua, piezoelectro-mechanical coupling, vibration damping.

1 Introduction

We propose a new device to control extensional, torsional and bending waves in truss modular beams based on their coupling with electric transmission lines by adding in every module PZT actuators. These actuators behave as bars when regarded as mechanical components of the beam and as capacitances when regarded as electrical

¹This refers to the expansion for the wave amplitude A :

$$A(t) = A_0 e^{-2\pi\delta t/T}$$

where T is the wave period. This means that, if $\delta \simeq 0.1$, after two periods we have: $A(2T) \simeq 28\% A_0$.

component of the line.

We show that the PZT actuators now available (see [7]) allow an efficient energy exchange from its mechanical form to its electrical one. As a consequence the system which we conceive in this paper (i.e. PEM beam) seems to be suitable for space applications. Indeed when adding dissipative elements in the electrical circuitry (eventually charging batteries) we show that it is possible to damp those mechanical waves which in a frictionless environment would damage the structure.

The electro-mechanical system which we study is characterized by the following features: 1) the conceived placement of PZT actuators allows for a continuously distributed control of the mechanical structure, 2) the transmission line connecting the PZT actuators allows for a global electro-mechanical response of the structure to external loading, 3) the equations governing mechanical and electrical phenomena are of the same type and of the same differential order so showing the same modal behavior.

We think that a strong modal coupling represents the most efficient way to control mechanical structures by means of electrical devices.

The previous ideas are used to prove the feasibility of damping of extensional and torsional waves by a standard second order transmission line. However in [1] this second order line was proven not to be efficient for bending waves. Thus we were led to conceive a non-standard electrical transmission line adding a capacitance and a voltage driven current generator (see figure 3). The reason for this choice is very intuitive: we need a line whose governing hyperbolic equation has exactly the same form of the well known Euler-Bernoulli bending equation.

As a detailed description of the system would require huge numerical programming, to test qualitatively the efficiency of proposed electro-mechanical coupling we consider -using an identification procedure based on the principle of virtual power- a coarse continuum model of PiezoElectro-Mechanical (PEM) beams. Of course such a model describes carefully PEM beams only when truss module lengths are negligible with respect to the considered wave-lengths.

In particular we study the dispersions formula for different wave types; we find in the frequency-line inductance plane the locus of the "maximal electro-mechanical exchange" inductances.

For homogeneous boundary conditions and purely mechanical initial conditions we study time-evolution problems both for a dissipative and non-dissipative transmission line, the appropriate choice of the line inductance value being suggested by the aforementioned locus. Efficient piezoelectric damping is shown to be possible for PEM beams by means of already available PZT actuators.

Finally we find the modal damping ratios as functions of the line resistance: we prove that there exists a critical value for which the electric dissipation of mechanical energy is maximum.

2 Continuum model equations

In [3] truss beams are modelled using a continuum description generalizing the standard Bernoulli-Navier theory. The main result of the cited paper -based on the identification of the mechanical power on the refined level with that on the continuum one- allows for the determination of the constitutive equations for the contact and inertial actions explicitly in terms of the mechanical properties in the module. The identification of the mechanical response of the truss modular beam when modelled as a one-dimensional continuum is then possible by means of an algorithmic procedure.

The same approach applied to the electrical transmission line leads to a standard homogenization procedure

in which the electrical impedance is continuously distributed per unit length.

In [1] and [2] the identification procedure proposed in [3] was adapted to piezoelectro-mechanical truss beams with second and fourth order transmission lines (for a detailed treatment of this identification procedure we refer the reader to [1], [2] and [3]).

As done in the standard transmission line theory, we introduce a coarse model for the electric line by defining the function $\gamma(\varsigma, t)$ representing the electrical charge stored in the $[0, \varsigma]$ segment of the line at the instant t . The current field is given by its time derivative $\iota = \dot{\gamma}$.

Now on we consider two different modules in order to study separately the coupling of axial-displacement and bending waves with the electrical line. Indeed the symmetries in the first module (shown in fig. 1) and in the second (fig. 2) lead to separate bending and extension problems.

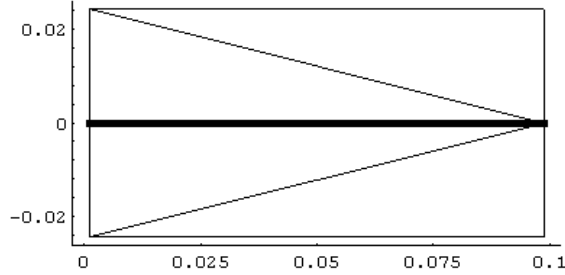


Figure 1: Module 1 (bold lines for PZT bars)

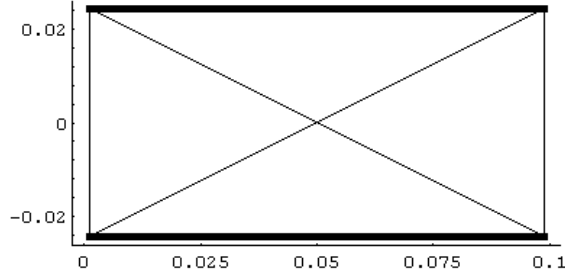


Figure 2: Module 2 (bold lines for PZT bars)

Considering the first module we get the following linearized equations for the axial displacement u and for the charge γ :

$$\begin{aligned} K_{mm} u'' + K_{me} \gamma'' &= \rho \ddot{u} \\ K_{em} u'' + K_{ee} \gamma'' &= K_L \ddot{\gamma} + K_R \dot{\gamma} \end{aligned} \tag{1}$$

where ρ is the linear mass density, K_L and K_R the line inductance and resistance; the other K s are material constants; more precisely they are derived by the identification procedure in terms of the geometrical and constitutive properties in the module: the index "m" stands for mechanical and "e" for electrical.

In eqns. (1) u represents the axial displacement but it can be replaced by the twist angle in order to obtain the

torsional-charge equations; this means that (as explicitly shown in [1]) the following results for the extensional-charge problem hold also for the torsional-charge one.

We remark that in the second module the two PZT bars are connected to two different transmission lines characterized by the same parameters; however it has been shown (see [1]) that the standard transmission line (whose governing equations presents a second spatial derivative) is not suitable to obtain a modal coupling with the spatial fourth order bending equation; this fact led us to introduce a fourth order transmission line obtained by the standard one simply adding a capacitance and a voltage driven current generator as shown in fig. 3 (the emphasized boxes represent the PZT actuators, the dashed elements the current generators).

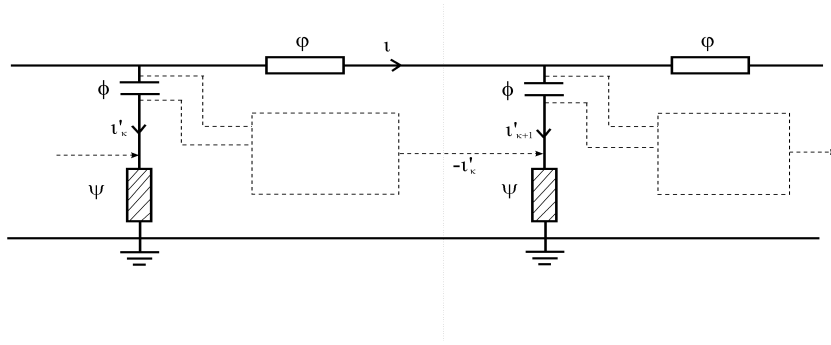


Figure 3: Non standard transmission line

Considering the second module connected with two fourth order transmission lines, we get the following linearized equations for the trasversal displacement v , for the charges difference $\delta := \gamma_1 - \gamma_2$ and for the charges sum $\sigma := \gamma_2 + \gamma_1$:

$$\begin{aligned} K_{mm} v^{IV} + K_{me} \delta^{IV} &= -\rho \ddot{v} \\ K_{em} v^{IV} + K_{ee} \delta^{IV} &= K_C \delta'' - K_L \ddot{\delta} - K_R \dot{\delta} \\ K_{ee} \sigma^{IV} &= K_C \sigma'' - K_L \ddot{\sigma} - K_R \dot{\sigma} \end{aligned} \quad (2)$$

where the symbols have the same meaning that in (1), but obviously different values (K_C is the inverse of the capacitance introduced in the fourth order line).

Eqns. (2) generalize the Euler-Bernouilli bending beam equations ([5], [6]) and constitute a system of linear fourth order hyperbolic equations; we remark that, due to the module 2 symmetry, the field σ is not coupled with v and δ .

3 Numerical results

In order to prove the applicability of the introduced electro-mechanical system the we examine different problems on eqns. (1) and (2)_{1,2}: in particular our interest is devoted to the electro-mechanical energy exchanges as functions of the electrical parameters (i.e. K_L and K_R) and on the mechanical damping efficiency.

3.1 Dispersion formulae and electro-mechanical energy exchange

Dispersion formulae associated to respectively to (1) and (2)_{1,2} (for $K_R \rightarrow 0$) are found out as usual searching solution in the form:

$$\begin{pmatrix} u \\ \gamma \end{pmatrix} = \begin{pmatrix} x_m \\ x_e \end{pmatrix} e^{\lambda s + j\omega t}, \quad \begin{pmatrix} v \\ \delta \end{pmatrix} = \begin{pmatrix} x_m \\ x_e \end{pmatrix} e^{\lambda s + j\omega t} \quad (3)$$

where $\omega \in \mathcal{R}$ and $\lambda \in \mathcal{C}$; we get respectively:

$$(K_{mm} \lambda^2 + \rho \omega^2) (K_{ee} \lambda^2 + K_L \omega^2) - K_{em} K_{me} \lambda^4 = 0 \quad (4)$$

$$(K_{mm} \lambda^4 - \rho \omega^2) (K_{ee} \lambda^4 - K_C \lambda^2 - K_L \omega^2) - K_{em} K_{me} \lambda^8 = 0 \quad (5)$$

The elastic energy associated to a given wave form is proportional to:

$$E = \begin{pmatrix} x_m \\ x_e \end{pmatrix}^T \cdot \begin{pmatrix} K_{mm} & K_{me} \\ K_{em} & K_{ee} \end{pmatrix} \cdot \begin{pmatrix} x_m \\ x_e \end{pmatrix} \quad (6)$$

The fraction of E related to the electro-mechanical exchange energy is given by:

$$R_{ex} = \frac{(K_{me} + K_{em}) x_m x_e}{E} \quad (7)$$

which depends on K_L , and ω by means of the eigenvector (x_m, x_e) .

For every ω we define as "maximal exchange" inductances K_L^* those which locally maximize R_{ex} ; in figure 4 is shown the graph of $R_{ex}(K_L, \omega)$ for the charge-bending equations: a two branches crest is observed defining the "maximal exchange" inductances locus as a function of frequency $K_L^*(\omega)$; however for high frequencies the two branches merge.

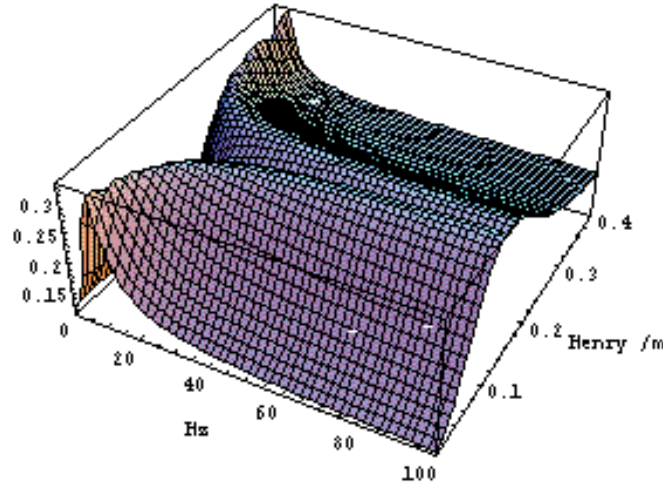


Figure 4: Electro-mechanical coupling (case 2)

On the contrary, for charge-extension waves the critical value of inductance does not depend on the wave frequency: figure 5 shows the electro-mechanical energy exchange in such a case: for a critical value of $\simeq 10^{-2}$ Henry/m the exchanged energy is more than 30% of the wave energy.

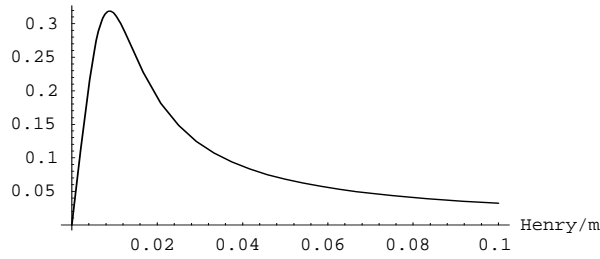


Figure 5: Electro-mechanical coupling (case 1)

Eqns. (4) and (5) for every K_L , and ω have a set of solutions, say $\Sigma := \{.. \lambda_i ..\}$, for λ . We partition Σ into pairs associating to every λ_i the closest (in the metric of the complex plane) $\lambda_{j \neq i}$. Physically this partition means to group together similar wave forms.

Our numerical simulations show that a critical K_L^* minimizes the distance of some of these pairs. This mathematical fact substantiates our intuition which leads us to expect electro-mechanical coupling when the wave forms, the wavelengths and the frequencies are close. This fact is strengthened by figure 6

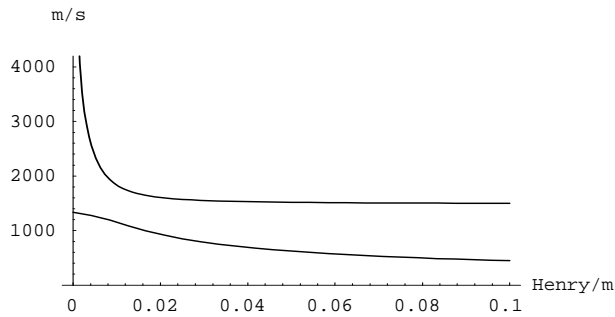


Figure 6: Wave speeds vs. line inductance (case 1)

in which a wave speeds veering is observed at the critical inductance value ($\simeq 10^{-2}$ Henry/m).

3.2 Time evolution problems

Now on we consider clamped-clamped, electrically insulated PEM beams: this means that eqns. (1) and (2)_{1,2} are solved with the following boundary conditions in $\zeta = 0, L$:

$$\begin{aligned} u &= \gamma = 0 \\ v &= v' = \delta = \delta' = 0 \end{aligned}$$

Figure 7 shows the solution of a time evolution problem for eqns. (2)_{1,2} starting from a purely initial data: a back and forth exchange of mechanical and electrical energies is observed (the line is assumed to be non dissipative, i.e. $K_R = 0$); this proves that the critical value K_L^* efficiently couples the electrical and mechanical modes.

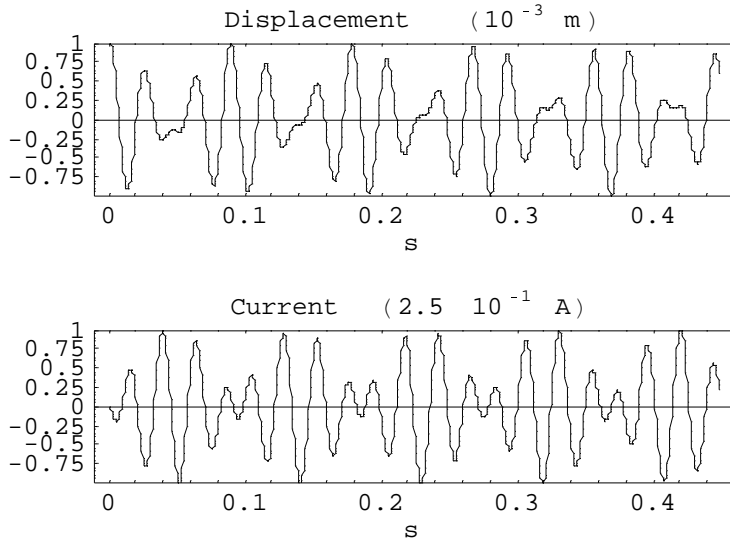


Figure 7: Graph of $v(L/2, t)$ and $\dot{\delta}(L/2, t)$ (non dissipative line)

Figure 8 shows the solution of a time evolution problem for eqns. (1) starting from a purely initial data; the critical value for inductance is chosen but with a dissipative line: the energy once transformed in an electrical form is now dissipated by Joule effect.

3.3 Damping ratios

With reference to the previous time-evolution problems, in figure 9 the energy dissipated in one period is calculated varying the line impedance.

We can easily locate an optimal bounded range \mathcal{I}_{LR} for the electrical parameters K_L and K_R ; they behave as control parameters respectively for the electro-mechanical coupling and for the dissipated energy.

Figure 10 shows the damping ratios obtained in the charge-extension problem as functions of K_R ; there is a critical value K_R^* for line resistance for which the mechanical damping ratio attains its maximum.

A physical interpretation for the existence of a critical value K_R^* may be the following: the line resistance has to be large enough to assure an electrical dissipation and small enough to allow the flow of the electro-mechanically induced current.

4 Concluding remarks

The basic physical idea is to couple a truss beam with an electrical transmission line by means of piezoelectric elements which act respectively as bars and capacitances, thus paralleling the mechanical wave equations with electrical ones.

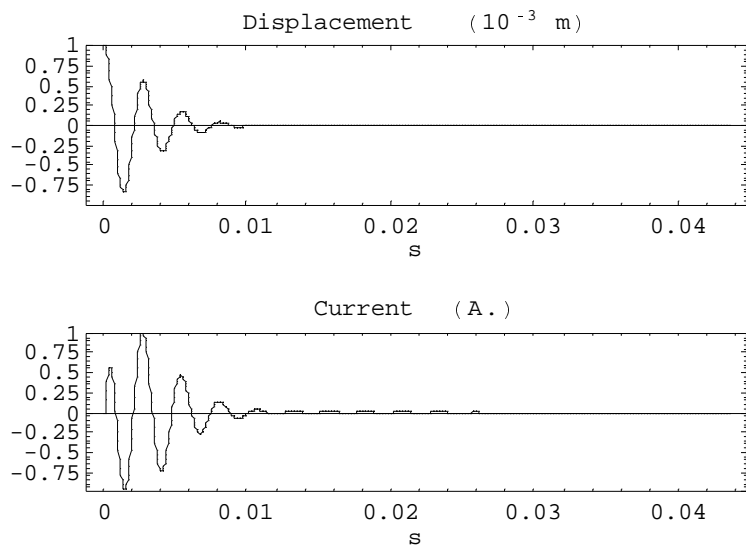


Figure 8: Graph of $u(L/2, t)$ and $\dot{\gamma}(L/2, t)$ (dissipative line)

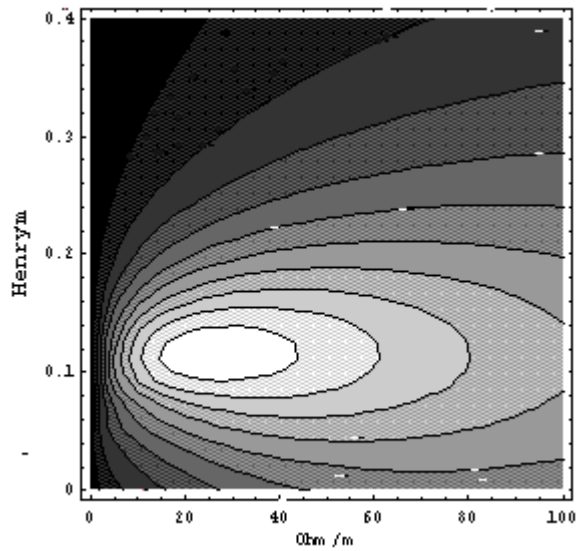


Figure 9: Dissipated energy vs. line impedance (case 2)

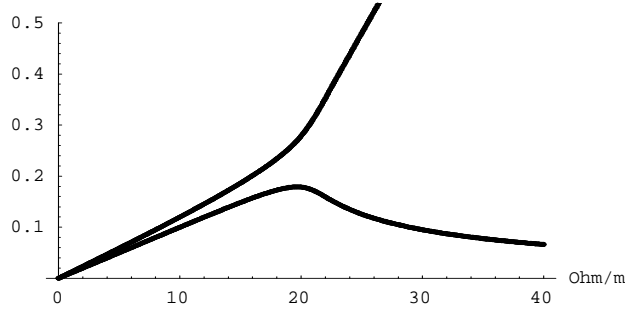


Figure 10: Damping ratio vs line resistance (case 1)

Two different modules have been studied in order to investigate coupling phenomena between capacitive charge and respectively axial displacement and bending. Our aim was to answer the following question: what happens when changing the line impedance (i.e. K_L and K_R)?

We found an interesting result: there is a critical line inductance range that maximizes the electro-mechanical modal coupling; this means a relevant exchange of energy between the beam and the electrical line and the possibility of an efficient control on the mechanical phenomena acting on electric parameters. We only investigated a simple case of passive control in damping vibration, but several active control applications are conceivable (for instance introducing in the line a voltage feedback); it seems to us that such damping technique is more efficient than those already proposed in the literature characterized by the use of actuators supplying concentrated forces.

5 Appendix

Next table shows the numerical parameters used in simulations (the module geometry may be evaluated by figures 1 and 2):

$L_T = 2. \text{ m}$	$\rho = 1.6 \text{ kg/m}$	$K_C = 10^3 \text{ Farad}^{-1}$
----------------------	---------------------------	---------------------------------

We consider PZT actuators characterized by the following constitutive parameters:

$k_{mm} = 1.75 \cdot 10^6 \text{ N}$	$k_{me} = -3. \cdot 10^6 \text{ N/Coulomb}$
$k_{em} = -1.3 \cdot 10^5 \text{ V}$	$k_{ee} = 2.5 \cdot 10^6 \text{ Farad}^{-1}$

which determine the axial force (N) and the potential drop (V) once known the elongation (ε) and the charge (Q) by the:

$$\begin{pmatrix} N \\ V \end{pmatrix} = \begin{pmatrix} k_{mm} & k_{me} \\ k_{em} & k_{ee} \end{pmatrix} \begin{pmatrix} \varepsilon \\ Q \end{pmatrix} \quad (8)$$

Acknowledgments

S.V. was supported by a research contract of Agenzia Spaziale Italiana, issued under the scientific responsibility of Prof. A. Di Carlo. Both authors acknowledge his friendly and accurate criticism.

6 REFERENCES

- [1] dell'Isola, F., Vidoli, S., Continuum Modelling of Piezoelectro-mechanical Truss Beams: an application to vibration damping, *to appear in Archive of Applied Mechanics*.
- [2] dell'Isola, F.; Vidoli, S.: Bending-Waves Damping in Truss Beams by Electrical Transmission Line with PZT Actuators, *to appear in Archive of Applied Mechanics*.
- [3] Di Carlo, A., Rizzi, N., Tatone, A., Continuum modelling of beam-like latticed truss: identification of the constitutive functions for the contact and inertial actions, *Meccanica* **25**, 1990, pp. 168-174.
- [4] Maugin, G.A., The method of virtual power in continuum mechanics: application to coupled fields, *Acta Mechanica* **35**, 1980, pp. 1-70.
- [5] Euler L., *Methodus inveniendi lineas curvas maximi minimive proprietati gaudentes*, Lausanne, 1744.
- [6] Bernoulli J., *Specimen alterum calculi differentialis ... una cum additamento quodam ad problema funicularum, aliisque*, Acta Eruditorum, 1691.
- [7] Near, C.D., Piezoelectric actuator technology, *Proceedings of SPIE Conference on "Smart Materials and Structures"*, San Diego, California, 24-29 Feb. 1996.