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## Accepted Manuscript

2002 AA<sub>29</sub>: Earth's recurrent quasi-satellite ?

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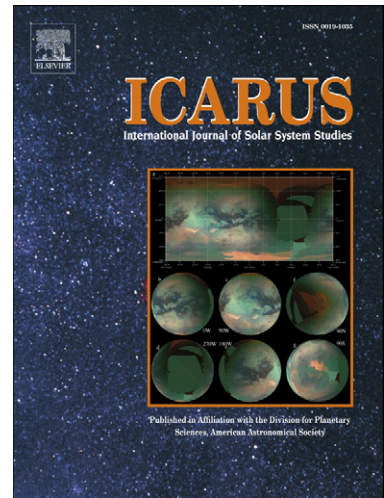
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## Abstract

We study the dynamical evolution of asteroid 2002 AA<sub>29</sub>. This object moves in the co-orbital region of the Earth and is the first known asteroid which experiences recurrent horseshoe - quasi-satellite transitions. The transitions between the HS and QS states are unique among other known Earth co-orbital asteroids and in the QS state 2002 AA<sub>29</sub> remains very close to Earth (within 0.2 AU for several decades; Connors *et al.* 2002). Based on results obtained analytically by Brasser *et al.* (2004b) we developed a simple analytical method to describe and analyze the motion of 2002 AA<sub>29</sub>. We distinguish a few moments in time crucial for understanding its dynamics. Near 2400 and 2500 this object will be close to going through the maxima of the averaged disturbing function and it will either change its co-orbital regime by transition from the HS into QS state, or leave the librating mode. These approaches generate instability in the motion of 2002 AA<sub>29</sub>. By means of 66 observations, covering a two-year interval, we extend the analysis of the long term evolution of this object presented by Connors *et al.* (2002) and Brasser *et al.* (2004a). Our analysis is based on a sample of 100 cloned orbits. We show that the motion of 2002 AA<sub>29</sub> is predictable in the time interval  $[-2600, 7100]$  and outside of this interval the past and future orbital history can be studied using statistical methods.

*Key Words:* Asteroids, 2002 AA<sub>29</sub>; Dynamics

## 1 Introduction

Asteroid 2002 AA<sub>29</sub> (hereafter AA<sub>29</sub>) was discovered on January 9, 2002 during a close encounter with the Earth. At that time the asteroid was within 0.0423 AU from the Earth and its visual magnitude was 19.8. The asteroid has a diameter of approximately 40-100 m and absolute visual magnitude of  $\sim 24$  (<http://newton.dm.unipi.it/cgi-bin/neodys/neoibo>). It belongs to the group of asteroids co-orbital with the Earth, due to the similarity of its semimajor axis to the Earth's semimajor axis. AA<sub>29</sub> was the first known member of this group to have such a small eccentricity that it moves around the Sun in an Earth-like orbit (Connors *et al.*, 2002; Ostro *et al.*, 2003). Its orbital behavior is intriguing. Already the first calculations of the orbit of AA<sub>29</sub> indicated that it is a co-orbital companion of the Earth, which from time to time transits to the quasi-satellite state (Connors *et al.*, 2002). In this state, the asteroid remains within 0.2 AU from the Earth for several years (Fig. 1). Connors *et al.* (2002) used the observations from an interval of 28 days to study the evolution of its orbit. They calculated the co-orbital motion of the asteroid as well as the motion of 300 clones (generated from the nominal solution), taking into account perturbations from all planets and the Moon. According to their calculations, AA<sub>29</sub> may enter quasi-satellite state near 550, 2600 and 3900 AD. Brassier *et al.* (2004a) performed a more detailed analysis of the asteroid's motion. In particular, they used the averaged disturbing function (effective potential well) in Hill's three-body problem to model analytically the HS-QS transitions of this object.

[Fig. 1]

Generally speaking, the motion of a co-orbital asteroid can be considered as a motion in the effective potential well whose walls are located near the maxima

(or singularities) of this potential (Namouni, 1999; Namouni *et al.*, 1999). Since the orbital elements vary in time, the shape of the averaged disturbing function changes as well. Particularly, the asteroid can either cross the peaks of this function and change the co-orbital type of motion or leave the resonant region. In order to analyze the dynamics of AA<sub>29</sub> we developed a simple method based on the analytical study in the case of the circular restricted three-body problem given in Brasser *et al.* (2004b). Our method allows to identify and analyze the type of co-orbital motion for arbitrary values of eccentricity and inclination of the asteroid's orbit. Also, in this way we can obtain information about its stability. The motion of AA<sub>29</sub> is dominated by co-orbital interactions with the Earth, but in the longer term the small planetary perturbations changes significantly the dynamics of this asteroid. The aim of this work is to examine the short and long period evolution of AA<sub>29</sub> and to answer the question asked in the title of this paper. We show that in the future, the dynamics of this object will essentially change and we suggest that the recurrent QS states will disappear and AA<sub>29</sub> should leave the Earth's co-orbital region.

This paper is organized as follows: in Section 2 we give a theoretical analysis of the co-orbital region based on the averaged disturbing function. In Section 3.1 we present our numerical method used to solve the equations of motion. These equations are solved to investigate the dynamical evolution of the orbit of AA<sub>29</sub>. Both analytical and numerical approaches are used in Section 3.2 to determine the co-orbital modes of the motion of AA<sub>29</sub> and dynamical evolution. In Section 3.4 the long term evolution of its orbit is analyzed. Finally, the last section provides a summary and conclusions.

## 2 Theory of co-orbital motion for arbitrary eccentricities and orbital inclinations

In recent years the dynamics of co-orbital objects has been intensively studied both analytically and numerically. Namouni (1999) investigated the dynamics analytically using Hill's formulation of the three-body problem (for small and moderate values of eccentricity and inclination). He also numerically integrated the equations of motion in the circular restricted three-body problem for arbitrary values of the eccentricity and inclination of the asteroid's orbit. Namouni *et al.* (1999) discussed the theory of the motion in the 1 : 1 resonance for large eccentricity and high inclination. Nesvorný *et al.* (2002) developed such a theory in the planar elliptic three-body problem. In the present paper we show a simple analytical method to study the motion of AA<sub>29</sub> and to consider the qualitative stability of its orbit. We applied the results obtained analytically by Brassier *et al.* (2004b) in the circular restricted three-body problem. This approach is valid for arbitrary values of eccentricity and inclination of the asteroid's orbit.

In the following we use the standard denotations  $a$ ,  $e$ ,  $i$ ,  $\omega$ ,  $\Omega$ ,  $M$  for the semimajor axis, eccentricity, inclination, argument of perihelion, longitude of ascending node, and mean anomaly of the asteroid, respectively. We further introduce  $\lambda = M + \omega + \Omega$  for the mean longitude. In our analysis we assume that the orbit of the planet is circular, and that its mass,  $m_p$ , and semimajor axis,  $a_p$ , are equal to those of the Earth.



## 2.1 Theory

The motion of an asteroid in a co-orbital region is a superposition of two components: a short period three-dimensional epicyclic motion with amplitudes of  $\sim ae$  and  $\sim a \sin i$ , and a long-period term, corresponding to the slow evolution of the guiding center<sup>1</sup> of the asteroid (Namouni *et al.*, 1999). In such a situation one can expect that the resonant and long-period terms of the disturbing function  $\mathcal{R}$  determine the secular motion.  $\mathcal{R}$  is defined as:

$$\mathcal{R} = k^2 m_p \left( \frac{1}{|\mathbf{r}_p - \mathbf{r}|} - \frac{\mathbf{r}_p \cdot \mathbf{r}}{r_p^3} \right), \quad (1)$$

where  $\mathbf{r}$  and  $\mathbf{r}_p$  are the vector positions of the small body relative to the Sun and the planet, respectively. Thus, we can eliminate all short-period terms from  $\mathcal{R}$  by averaging this function with respect to the fast variable  $\lambda_p$ :

$$\bar{\mathcal{R}}(\sigma) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathcal{R}(\lambda_p, \lambda(\lambda_p, \sigma)) d\lambda_p, \quad (2)$$

where  $\sigma = \lambda - \lambda_p$  is the principal resonant angle, and  $\lambda_p$  the mean longitude of the planet, and then we can consider only the evolution of the guiding center (Namouni *et al.*, 1999).

A analysis of the shape of the function  $\bar{\mathcal{R}}(\sigma)$  is crucial because the extrema of this function define the locations of stationary points. The minima of this function define the stable equilibrium points, i.e. libration centers around which appear librations, while the maxima define the unstable equilibrium points (Gallardo, 2006).

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<sup>1</sup> which is the averaged position of the asteroid with respect to the planet (Christou, 2000)

The trajectories of the guiding center are described by the following first integral (Brasser *et al.*, 2004b):

$$\bar{\mathcal{R}}(\sigma) + k^2(\sqrt{(1+m_p)a} + \frac{1}{2a}) = C_A, \quad (3)$$

where the constant  $C_A$  specifies the shape and type of the asteroid trajectory. This equation can also be used to determine the range of libration of the resonant angle  $\sigma$ . Equation (3) is valid for arbitrary values of eccentricity and inclination in contrast to a similar equation, that describes the evolution of the guiding center, obtained by Namouni (1999) in Hill's three-body problem. In this approximation the averaged disturbing function  $\bar{\mathcal{R}}(\sigma)$  is an even function of  $\sigma$  and for this reason is insufficient to explain the existence of libration modes which are asymmetric with respect to the principal resonant angle of the asteroid (Namouni *et al.*, 1999). In this paper we applied the theory of circular restricted three-body problem that allows for asymmetrical maxima of the averaged disturbing function in contrast to Hill theory.

Since we consider co-orbital objects of the Earth, we can write  $a = 1 + \Delta a$ . Moreover, in case of the Earth:  $m_p \ll \Delta a \ll 1$ . Hence, it is possible to obtain an approximation of Eq. (3) in the form:

$$(\Delta a)^2 = \frac{8m_p}{3}(C - R(\sigma)), \quad (4)$$

where  $C$  is a constant, and the normalized averaged disturbing function  $R(\sigma)$  is given by:

$$R(\sigma) = \bar{\mathcal{R}}(\sigma)/(k^2 m_p). \quad (5)$$

Eq. (4) is used in Section 3.2 to analyze the types of co-orbital motion and transitions between them.

The function  $R(\sigma)$  is evaluated numerically for a fixed set of values  $(e, i, \omega)^2$ , treated as parameters, and assuming  $a = 1$ . The integration was performed using Simpson's method with a variable step-size.

## 2.2 Overview of co-orbital families of orbits

For small values of eccentricity and inclination the disturbing function  $R(\sigma)$  has a collision singularity at  $\sigma = 0^\circ$  and two stable libration centers at  $\sigma = 60^\circ$  and  $\sigma = -60^\circ$ , around which exist tadpole orbits (TP). Another libration center is located at  $\sigma = 180^\circ$  and associated with horseshoe orbits (HS).

In the case of co-planar eccentric orbits the libration centers of eccentric TP orbits are displaced with respect to the equilateral locations at  $\sigma = \pm 60^\circ$  (Namouni and Murray, 2000). The function  $R(\sigma)$  has two singularities and a stable libration center around  $\sigma = 0^\circ$  between them. This libration center defines quasi-satellite (QS) orbits, moving around the associated planets. The QS and HS (or TP) orbits are separated by singularities.

Namouni (1999) showed, using Hill's approximation, that the profile of the co-orbital region changes significantly when an asteroid does not share the plane of motion of the planet. In this approximation collision singularities of the averaged disturbing function occur only if  $\omega = \pm 90^\circ$  at  $\sigma = 2e$ . Namouni *et al.* (1999) presented the first analytic solutions of the full equations of the circular restricted three body problem. They concluded that, generally, the

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<sup>2</sup>  $R(\sigma)$  is independent of the longitude of ascending node,  $\Omega$  (Brasser *et al.*, 2004b)

averaged disturbing function  $R(\sigma)$  is not an even function of  $\sigma$ , which open a possibility of existence of new types of orbits. For example, they showed that asymmetrical HS orbits (compound HS-QS orbits), which are unions of the HS and QS orbits, can exist. The function  $R(\sigma)$  becomes singular for specific values  $\omega_c$  of the argument of perihelion, when intersections between the orbits of the asteroid and planet in the ascending (+) or descending node (-) of the asteroid orbit occur:  $\cos \omega_c^\pm \approx \pm e$  (Nesvorný *et al.*, 2002). This implies that the singularities of this function generally disappear.

### 2.3 Transient HS-QS orbits

So far we have described possible orbital families assuming constant values of elements  $e$ ,  $i$ , and  $\omega$ . However, as it was shown by Namouni (1999) (see also Christou 2000), secular evolution of the argument of pericentre is crucial to explain the HS-QS transitions, because the evolution of  $\omega$  modifies the values of the maxima of  $R(\sigma)$ . Below we illustrate the transition from a HS to QS orbit by reproducing the results from Christou (2000) and by generalizing them for the orbits with arbitrary values of eccentricity and inclination.

Let us assume that an asteroid moves in an HS orbit and the shape of  $R(\sigma)$  is as shown in Fig. 2. Then  $R(\sigma)$  has a local minimum at  $\sigma = \sigma_0 \approx 0$  (denoted as  $R_0$ ) and two local maxima denoted by  $R_-$  for  $\sigma = \sigma_- < \sigma_0$  and  $R_+$  for  $\sigma = \sigma_+ > \sigma_0$ . The horizontal arrows represent the energy level (i.e. the value of  $C$ ) of the asteroid, and the direction of motion (changes of  $\sigma$ ) is shown by the arrowheads. If the value of  $C$  is greater than both  $R_+$  and  $R_0$ , then the asteroid transits to the QS state ( $\sigma \in [\sigma_-, \sigma_+]$ ). The asteroid can be trapped in the QS state if the maxima of the function  $R$  increase so that  $C < R_-, R_+$ .

The values of  $R_-$  and  $R_+$  are not constant in time due to secular evolution of  $\omega$ . Especially,  $R_+$  (or  $R_-$ ) tends to infinity as  $\omega$  approaches  $\omega_c^-$  (or  $\omega_c^+$ ).

In the present paper we use the above analytical approach to describe the dynamical behavior of AA<sub>29</sub>. Our method is similar to that presented by Christou (2000), but in contrast to his analysis, we use full equations of the three-body problem and a more complete version of the averaged disturbing function. This function was calculated for the set of values of the orbital osculating elements of AA<sub>29</sub> obtained from numerical integrations at any moment of time. A comparison of the value of  $C$  with the two extrema  $R_-$  and  $R_+$  of the function  $R(\sigma)$  provides us with information on the dynamical behavior of the asteroid (see section 3.2).

### 3 Analysis and discussion of motion of asteroid AA<sub>29</sub>

#### 3.1 Initial conditions and method of numerical integration

The positional observations of AA<sub>29</sub> are taken from the NeoDys pages publicly available at <http://newton.dm.unipi.it/neodys>. The observational material contains 66 observations covering more than a two-year interval: from 9th January 2002 to 15th January 2004. Analysis of motion of AA<sub>29</sub> was performed based on a sample of 100 cloned orbits created from initial coordinates and velocities of the nominal osculating orbit. The cloned orbits were generated by Sitarski's orbital program package (Sitarski, 1998), which allows to create an arbitrary number of initial orbital element sets, fitting the observations within statistical uncertainties. Table 1 presents the osculating Keplerian elements of the nominal orbit of AA<sub>29</sub>.

[Table 1]

Numerical integrations of the asteroid’s motion were performed using the recurrent power series (RPS) method (Hadjifotinou and Gousidou-Koutita, 1998) by taking the perturbations by all eight planets, the Moon and Pluto<sup>3</sup> into account. The initial Solar System state was taken from the JPL ephemeris DE406. The asteroid and its clones were treated as massless test particles. We decided to use the RPS method because it allows to determine the optimum step-size at every step of integration, ensuring the desired accuracy of results of computations is obtained (Sitarski, 1979).

### 3.2 Overview

The heliocentric orbit of AA<sub>29</sub> is nearly the same as that of the Earth. A significant difference is visible only in the inclination (see Tab. 1). In a reference frame corotating with the Earth the asteroid occupies a horseshoe orbit. During every horseshoe loop, which lasts about 190 years, its semimajor axis alternates regularly from 0.992 AU to 1.008 AU. The regular pattern of changing of the semimajor axis is broken near 2580. At that moment the asteroid’s approach to the Earth leads to a quasi-satellite behavior (Connors *et al.*, 2002). In this state the cyclic change of the semimajor axis is interrupted by a number of faster oscillations over a smaller amplitude range (from 0.9975 AU to 1.0025 AU). When the asteroid enters the quasi-satellite regime, the eccentricity increases rapidly at a uniform rate from about 0.035 to 0.06. Alongside, the ascending node,  $\Omega$ , and the argument of pericentre,  $\omega$ , change uniformly at a rate of approximately  $\dot{\Omega} \approx -0.15^\circ/\text{yr}$  and  $\dot{\omega} \approx -2.2^\circ/\text{yr}$ . The inclination in the quasi-satellite state remains near its initial value.

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<sup>3</sup> Pluto was included to obtain compatibility with the ephemeris DE406.

In order to indicate the differences between the quasi-satellite state of AA<sub>29</sub> and satellite in the keplerian sense, we considered the geocentric orbit of this asteroid in a non-rotating frame. Fig. 3 shows the geocentric motion of the object during the quasi-satellite phase. In the QS state the asteroid remains near the Earth at a mean distance of approximately 0.14 AU (roughly 54 times the Earth-Moon distance) over 50 years. Since the binding energy with respect to the Earth<sup>4</sup> (per unit mass) of this object is positive, then although the Earth does play a significant role in the motion of AA<sub>29</sub>, the object in the QS state is not captured by the Earth - the instantaneous geocentric orbit of the asteroid is not elliptical. During the QS state the asteroid stays on the same side of the Earth, in accordance with theory (Mikkola et al., 2006). [Fig. 3]

### 3.3 The dynamics

The dynamical behavior of AA<sub>29</sub> can be better understood within the framework of analytical approach for the averaged disturbing function considered in section 2. We calculated the osculating elements for every ten days to derive the constant  $C$  (see Eq. (4)) and compared its value with the maximum values of function  $R(\sigma)$ .

In the motion of AA<sub>29</sub> we can distinguish a few crucial steps that determine its dynamical evolution. Figure 4 shows the time evolution of  $R_+$ ,  $R_-$ , and  $C$  for this asteroid. The asteroid is moving in a regular horseshoe orbit up to about 2380. The motion can be visualized as being deep inside the averaged potential well (i.e. the value of  $C$  is significantly smaller than the values of  $R_+$  and  $R_-$ ). [Fig. 4]

<sup>4</sup> the binding energy was calculated as:  $\frac{v^2}{2} - \frac{k^2 m_p}{r}$ , where  $v$  is the velocity of the asteroid with respect to the Earth and  $r$  is its distance.

Thus, we cannot expect that small perturbations caused by the planets and the Moon would change the regime of the asteroid's motion in the co-orbital region. Near the year 2007 one has  $\omega \approx 101^\circ$  and the argument of pericentre increases. It implies that the values of  $R_+$ ,  $R_-$  decrease (Fig. 5). The critical moments are near 2385 and later near 2482, when the asteroid approaches the Earth. In the former case we have:  $\sigma = 3.2^\circ$ ,  $R_+ = 9.5$ ,  $C = 8.4$ ,  $\sigma - \sigma_+ = 1.2^\circ$ , and in the latter one:  $\sigma = -2.6^\circ$ ,  $R_- = 8.8$ ,  $C = 8.7$ ,  $\sigma_- - \sigma = 0.3^\circ$ . Figure 6 shows that close to these dates the asteroid is located near the unstable equilibria of  $R$ . From a dynamical point of view, it implies that the motion of AA<sub>29</sub> becomes unstable. Another crucial moment in the time evolution of this asteroid occurs near 2579. At this time  $C > R_+$ , thus there appears a possibility of transit to the quasi-satellite state. During this HS-QS transition  $\omega = 123.4^\circ$  and the argument of pericentre decreases, and, as one can see in Fig. 5, the values of  $R_+$  and  $R_-$  increase, whenever  $\omega < \omega_c \approx 90^\circ$ . This causes the asteroid to be trapped into the QS state. About 2594 one has  $\omega < \omega_c$  and the values of  $R_+$  and  $R_-$  decrease. The asteroid can leave the QS state about 2622 because at this time  $C < R_+$ , and therefore the object can easily cross the maximum  $R_+$  of the averaged disturbing function. Then AA<sub>29</sub> begins to move in the HS regime.

In the restricted circular three-body problem the secular variation of  $\omega$  increases or decreases depending on a given co-orbital regime (HS or QS) of the asteroid (Namouni, 1999). The recurrent transitions to the QS state follows from the fact that  $\omega$  of the asteroid librates around  $\omega_c$  (see Brasser *et al.* 2004a for details). Thus, as long as the asteroid stays in the co-orbital regime, we can expect that the cycle of HS-QS transitions repeats itself. However, the realistic evolution of the argument of pericentre is also dictated by the planetary



perturbations; particularly, when the eccentricity of the asteroid is not small. To test the influence of planetary perturbations on the co-orbital behavior of AA<sub>29</sub> and to confirm our theoretical considerations we repeated and extended the analysis given by Brassier *et al.* (2004a), i.e. we included and excluded the planets and the Moon from our numerical integration of the equations of motion.

In general, we found that the influence of the planetary perturbations does not change the type of coorbital motion of the asteroid before 2380. After this time the motion of AA<sub>29</sub> is highly sensitive to planetary perturbations. To explain this fact we have to return to our analysis of the effective potential well. The perturbations slightly modify the values of the orbital elements of the asteroid, therefore they also change the maxima of the disturbing function  $R(\sigma)$  (i.e. the values of the maxima of  $R_+$  and  $R_-$ ). We have found that these maxima are usually significantly greater than the value of the energy level ( $C$ , see Eq. (4)), but after 2380, when the object approaches the Earth, one has  $C \approx R_{\pm}$ . Hence, small perturbations determine the transition conditions, i.e. the inequalities  $C < R_{\pm}$  or  $C > R_{\pm}$ . The influence of planetary perturbations is stronger, the closer  $R_+$  and  $R_-$  are to the value of  $C$ .

### 3.4 Long-term evolution

In this section we examine the long term evolution of AA<sub>29</sub>. Integration of the equations of motion shows that the nominal orbit and 100 cloned orbits present a very similar behavior within the time interval  $[-2600, 7100]$ , while outside this interval dynamical evolution starts to be unpredictable. Fig. 7 shows the evolution of the nominal orbit and the orbits of eight arbitrarily

[Fig. 7]

chosen clones from the set of 100 clones.

The asteroid stays in the HS regime for a few thousand years and from time to time it transits to the QS state. Our investigations confirm the HS-QS transitions of AA<sub>29</sub> near 580, 2580 and 3750 found by Connors *et al.* (2002) and Brassier *et al.* (2004a). Brassier *et al.* (2004a) also showed that this object can enter the QS state near either 6400 or 6500. However, this HS-QS transition is uncertain due to the different behavior of cloned orbits at this time. We show that the motion of AA<sub>29</sub> can be predictable up to 7100. Near 6400 the HS-QS transition does not appear in our simulations and AA<sub>29</sub> leaves the resonant motion. All 100 cloned orbits exhibit this behavior.

The dynamical evolution of the orbital elements is predictable from -2600 to 7100, but outside of this time interval, the behavior of the cloned orbits (i.e. their backward and forward evolution) is different. In the future, the osculating elements  $e$ ,  $i$ ,  $\omega$  and  $\Omega$  of the clones show only small discrepancies between each other. In the past, the behavior of cloned orbits is more chaotic. To determine the past and future of the asteroid, we performed statistical analysis. The results can be summarized as follows:

- (1) The orbits of 32 clones (excluding the nominal orbit) stay inside the Earth's co-orbital region within the time of integration, i.e. between -8000 to 12000. Thus, AA<sub>29</sub> is probably a temporary co-orbital companion of the Earth. It agrees with the accepted hypotheses concerning the origin of such objects as well as the long term integrations of other Earth co-orbital asteroids (Christou, 2000; Morais and Morbidelli, 2002).
- (2) After 4000 years the orbital eccentricity of all cloned orbits rapidly increases toward 0.15-0.19. In consequence, the perturbations from Venus

and Jupiter can play an essential role in the motion of the asteroid, causing the object to be expelled from the co-orbital region.

- (3) After about 8000 years, starting from 2007, the orbital inclination of all clones decreases to 1-3 degrees. In the past evolution, the inclination stays in the interval of 9-11 degrees for 64 of the 100 clones.
- (4) The argument of pericentre librates around  $\omega_c$  within the time interval of 500-6350 (see Fig. 8). The value of  $\omega_c$  varies near  $90^\circ$ , depending on the eccentricity, which slightly changes in the mentioned interval of time. After 6350, the argument of pericentre decreases from about  $135^\circ$  to  $100^\circ$ , not reaching  $\omega_c$ , and the cycle of HS-QS transitions breaks down.
- (5) The longitude of ascending node slowly decreases during the simulation period for all 100 clones. After 4500 years of back integration it is difficult to say something about the past evolution of the argument of this asteroid, but in the future, after a period of long-term librations it would seem that it slowly increases. All 100 test particles exhibit this behavior.
- (6) In the past, some extra HS-QS transitions were possible. Only a few clone orbits show such a behavior near -2630 and -2740, but almost all clones (85) transition from HS to QS state near -2840.

[Fig. 8]

#### 4 Summary and Conclusions

The orbital motion of AA<sub>29</sub> is an interesting example of complexity of the 1:1 resonance. Coexistence as well as specific dynamics of the horseshoe and quasi-satellite types of motion allow for regular transitions of the former type of the orbit into the latter one. In the HS type of the motion, the semimajor axis regularly changes between 0.992 AU and 1.008 AU every 192 years. The

pattern of regular approaches to the Earth can break down and lead to transition into a QS orbit if the maxima of the averaged disturbing function are high enough to trap the asteroid in the QS motion. However, the latter state is not stable and after some time the asteroid leaves this type of motion by transiting into a HS orbit.

In this paper we used the theory of the co-orbital motion developed by Brasser *et al.* (2004b). This theory extends the theoretical results of Namouni (1999) and can be applied to the analysis of motion of asteroids with large eccentricities and high inclinations. Based on this analytical theory, we developed a simple analytical method to better understand the dynamics of AA<sub>29</sub>. We showed that in the motion of this object we can distinguish a few crucial moments of time for understanding its dynamics. Near 2400 and 2500 this object will approach the maxima of the averaged disturbing function and change its co-orbital regime by transition from the HS into QS state (or cease the resonant motion). These approaches generate an instability in its orbit. To check our predictions we computed the orbit of AA<sub>29</sub> by including and excluding the planets and the Moon from the numerical integration. This experiment confirmed that the future motion of this asteroid depends sensitively on the variation of the argument of pericentre.

The numerical integrations of the equations of motion of AA<sub>29</sub> allow us to make the following conclusions. The asteroid moves inside the co-orbital region in a regular horseshoe orbit and from time to time it transits to the QS state. We found that the transitions into the quasi-satellite phase occur near 580, 2580 and 3750. These results are in agreement with Connors *et al.* (2002) and Brasser *et al.* (2004a). However, the other horseshoe–quasi-satellite transition suggested by Brasser *et al.* (2004a) near 6400 does not appear in our

simulations. According to our simulation, during the next approach to the Earth after 6400, i.e. around 6500 the semimajor axis will not change from  $a > 1$  to  $a < 1$  and the asteroid will leave the librating regime. The future evolution of objects such as AA<sub>29</sub> is unclear, but, probably, they are temporarily in the 1:1 mean motion resonance with Earth and in the future can leave the Earth's co-orbital region (Christou, 2000; Morais and Morbidelli, 2002)

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Table 1

Osculating orbital elements of asteroid AA<sub>29</sub>. Epoch: April 10, 2007 (JD 2454200.5),  
Equinox: J2000.0.

Parameter	Value	1- $\sigma$ error
Semimajor axis, $a$ (AU)	0.9939425476	$2.70 \cdot 10^{-8}$
Eccentricity, $e$	0.0130975063	$1.73 \cdot 10^{-7}$
Inclination, $i$ (deg)	10.7432393	$4.18 \cdot 10^{-5}$
Longitude of the Ascending Node, $\Omega$ (deg)	106.4494702	$1.57 \cdot 10^{-5}$
Argument of Perihelion, $\omega$ , (deg)	101.727977	$2.92 \cdot 10^{-4}$
Mean Anomaly, $M$ (deg)	2.702964	$3.38 \cdot 10^{-4}$



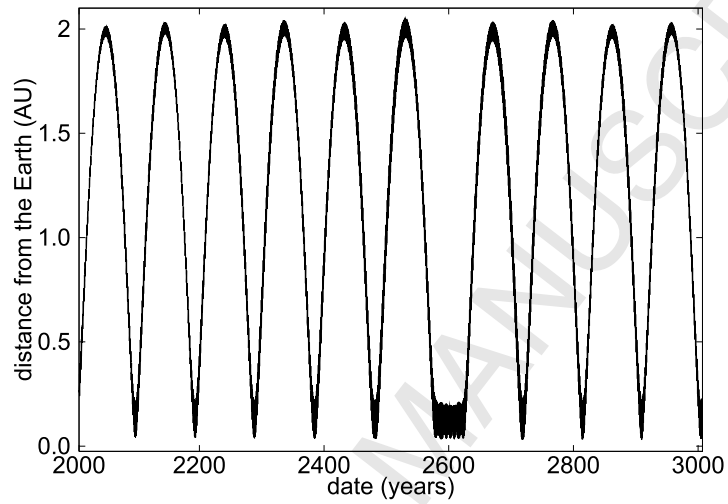


Fig. 1. Distance of AA29 asteroid from the Earth as a function of the time. For several hundred years this object moves in a horseshoe orbit and regularly approaches the Earth. The horseshoe motion is interrupted occasionally, transforming into the quasi-satellite motion. In this state, the asteroid stays within 0.2 AU from the Earth for several decades.

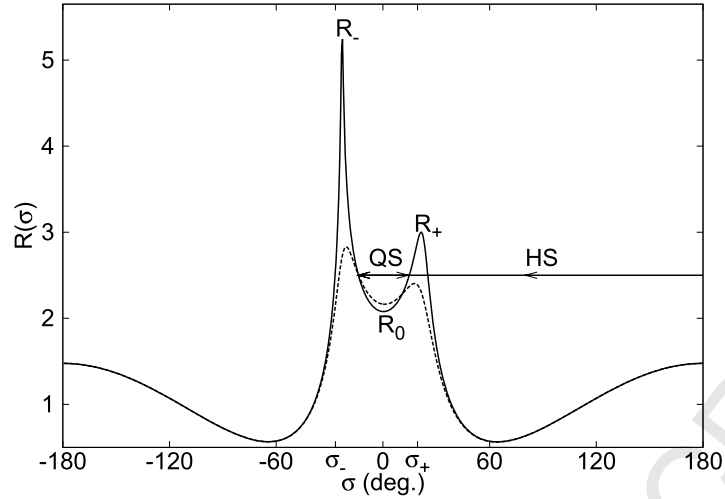


Fig. 2. Schematic illustration of the HS-QS transition. For simplification we assumed that the values of  $e$  and  $i$  are constant in the time. In both cases  $e = 0.2$  and  $i = 15^\circ$ . The averaged disturbing function is drawn with the dashed line for  $\omega = 50^\circ$  and with the thin line for  $\omega = 75^\circ$ . Initially,  $\omega = 50^\circ$ , the asteroid is in the HS regime and approaches the Earth. Because  $C > R_+$ , the object can enter the QS mode ( $\sigma \in [\sigma_-, \sigma_+]$ ). If the argument of pericentre is close to the values of  $\omega_c$  (in our example  $\omega_c$  equal to  $78^\circ$  and  $102^\circ$ ), the values of  $R_+$  and  $R_-$  rapidly increase (thin line) to obtain  $C < R_+$  and  $C < R_-$ , and, the asteroid is trapped in the QS phase.

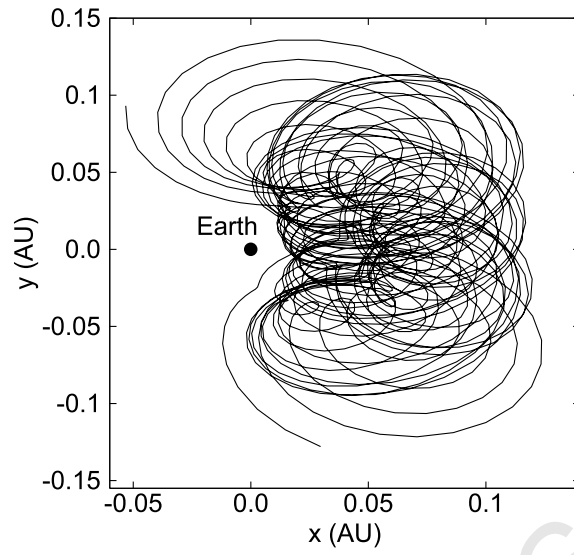


Fig. 3. Geocentric motion of 2002 AA29 during the QS state.

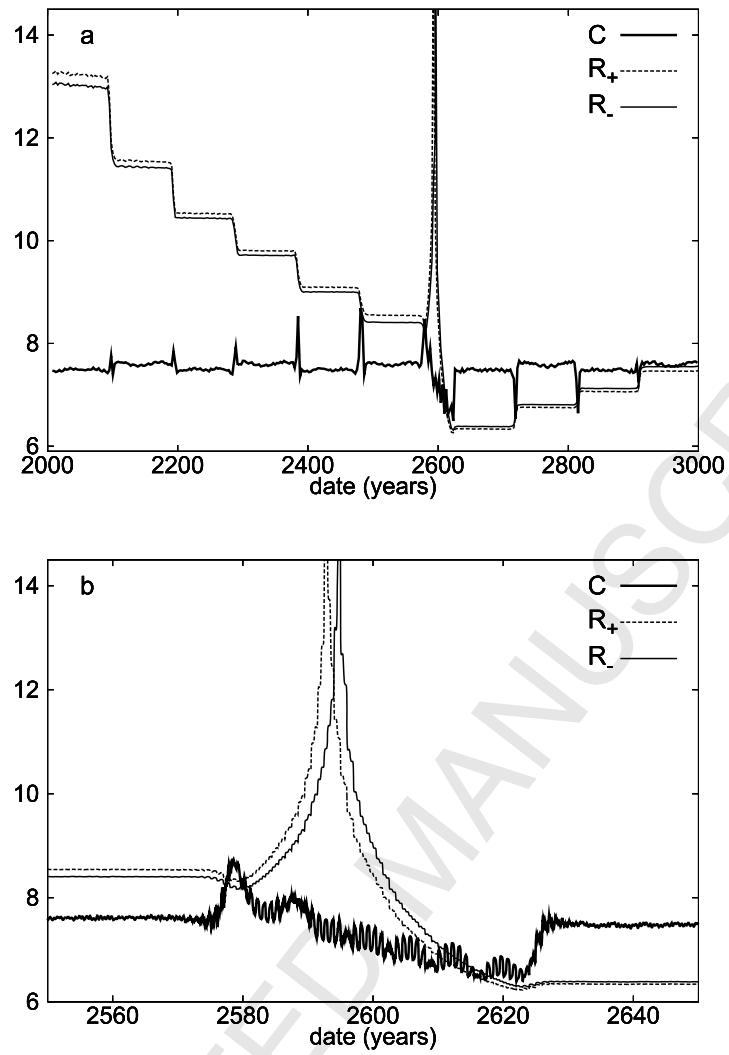


Fig. 4. (a)-Future evolution of  $C$  (thick line),  $R_+$  (dashed line), and  $R_-$  (thin line).  
 (b) same as in (a), but during the QS state.

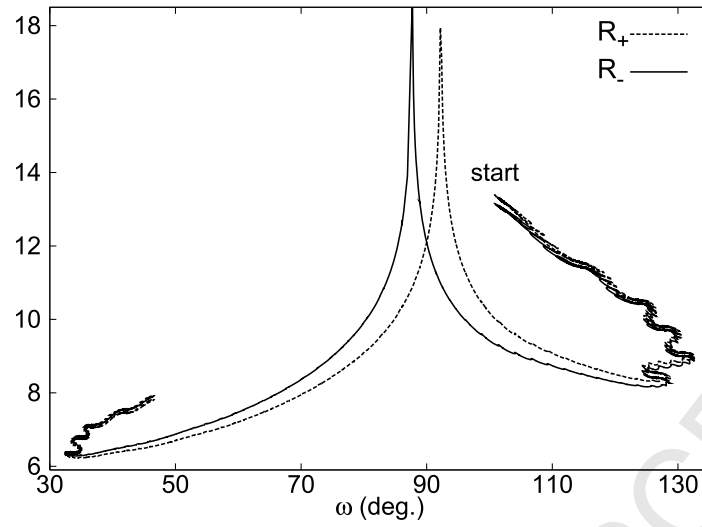


Fig. 5. The maxima  $R_+$  (dashed line) and  $R_-$  (thin line) as functions of the argument of pericentre within the time interval of 2007-3007. The value of  $R_+$  tends to infinity near 2593 and of  $R_-$  near 2595. In the former case one has  $\omega_c = 92.2^\circ$ , and in the latter one  $\omega_c = 87.7^\circ$ .

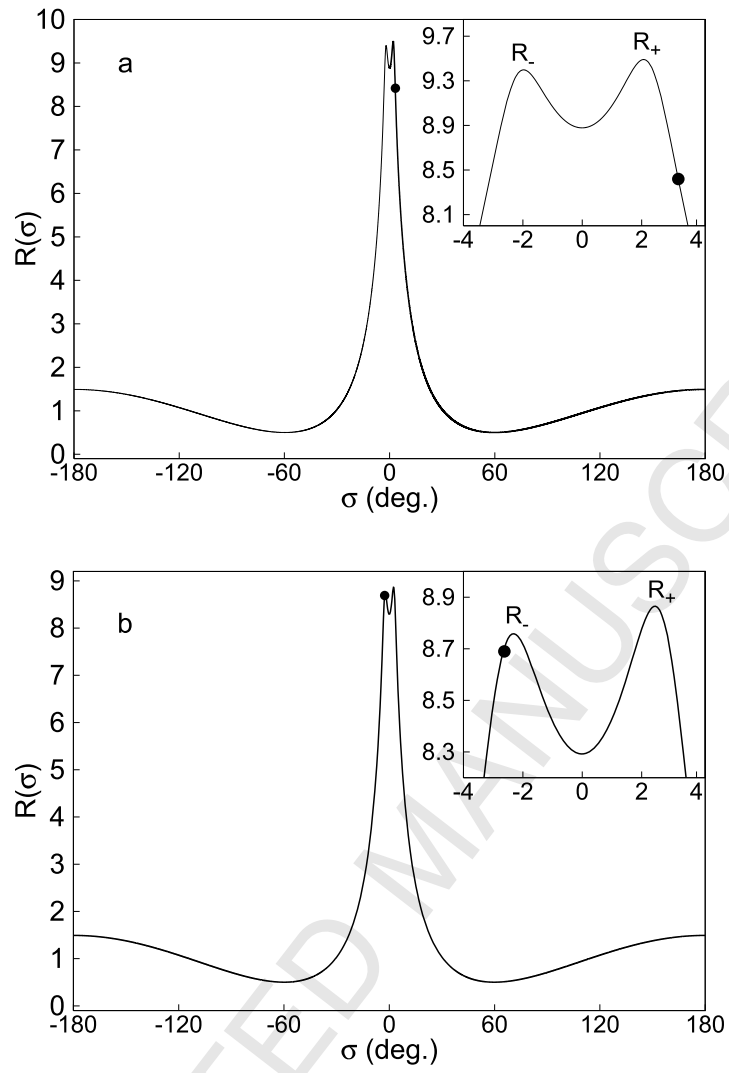


Fig. 6.  $R(\sigma)$  of the asteroid approaching the Earth near 2385 (a) and near 2482 (b). The positions of the guiding center of AA<sub>29</sub> in the potential well are marked by a full circle.

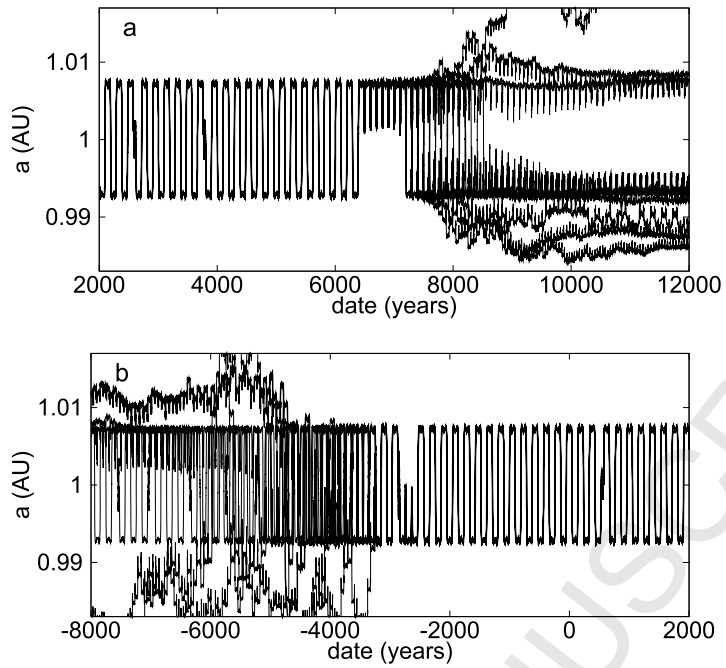


Fig. 7. Future (a) and past (b) evolution of the semimajor axis. The thick line indicates the evolution of  $a$  of the nominal orbit and the thin lines - the evolution of eight arbitrarily chosen cloned orbits.

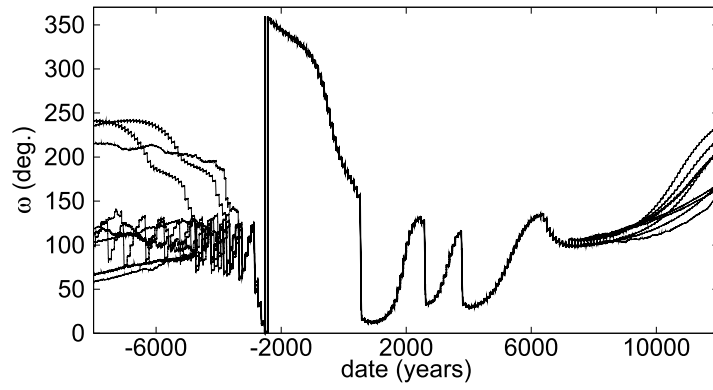


Fig. 8. Future and past evolution of the argument of pericentre. The thick line indicates evolution of  $\omega$  of the nominal orbit and the thin lines - the evolution of eight arbitrarily chosen cloned orbits.