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## CATEGORY THEORY BASED APPROACH FOR IMS MODELLING

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Abstract: Main researches and developments in IMS, MAS (Multi-Agent System) and HMS (Holonc Manufacturing System) for manufacturing plant control and management as well as manufacturing enterprise networking are based on ICTs (Information & Communication Technologies) based approaches. Beyond these technology-driven approaches, the field of 'Intelligence in Manufacturing' requires sound scientific foundations in order to meet systematisation in modelling as originally addressed by (Yoshikawa, 1995). In this way, this paper deals with principles and elements of the Category Theory as a coherent mathematical framework in order to move to more prescriptive and 'bottom-up' modelling approaches able to better understand and model the dynamical emergent properties of new agile manufacturing systems.

Keywords: Manufacturing, IMS, Emergence Mechanism, Category Theory

### 1. INTRODUCTION

ICTs play an ever more prominent role in the networking manufacturing enterprise in order to integrate all the aspects of the product and process life cycle by the means of hardware/software systems ranging from MEMSs (Micro Electro-Mechanical System) through IMS to Enterprise Systems (Ollero, *et al.*, 2002). In particular, manufacturing plants become a place where interoperable and autonomous units embedding a digital intelligence to transform information flows into products flows (Van Brussel and Valckenaers, 1999). But the capability of these manufacturing organisations to be 'intelligent' leads to a dilemma between the uncertainty due to changing environments with unlimited functioning alternatives and the execution effectiveness of enterprise processes (Thannhuner, 2001). It means that ordering information for control, decision-

making and management purposes within intelligent manufacturing organisations needs to balance between dynamics/complexity and precision of enterprise processes. Consequently, (Morel and Zaremba, 2001) has shown that it is necessary to question, not only the traditional hierarchical architectures of these organisations, but also, to question the current 'top-down' approaches for modelling these intelligent manufacturing organisations in favour of more 'bottom-up' ones. This concern of the coherence between the modelling process and the model of the system has been also previously addressed by (Wortmann, 1997), (Neunreuther, *et al.*, 1997) and recently by (Valckenaers, *et al.*, 2002) for IMS modelling purposes.

In this way, this paper emphasizes the main principles and elements of the Category Theory (CT)

as a coherent mathematical framework for modelling the various aspects of IMS.

The next section states on the limits of the main IT-based modelling techniques, according to the requirement of modelling IMS by emergence in order to meet agility.

The third and the fourth sections mathematically define an IMS as a category and the mechanism of emergence as a colimit. This CT-based modelling framework is then partially applied in the fifth section on a Laboratory mock-up previously modelled with a MAS approach.

Finally, the conclusion of this paper states our ongoing researches towards practice and fully validation of this CT approach on IMS modelling purposes.

## 2. ORDERING OF INFORMATION IN IMS MODELLING

Ordering of information in information-intensive manufacturing organisations has been mainly addressed by ‘top-down’ approaches such as IDEF-0, CIM-OSA, ..., or by ‘bottom-up’ approaches such as object-oriented and MAS approaches, to satisfy the requirements of the ‘integration in manufacturing’ paradigm. Although standardized frameworks have been proposed to cope with the complexity to map the conceptual model of an integrated manufacturing system on a distributed ICT-based operational architecture, these approaches remain ‘too rigid’ to dynamically model emergent organisations required by the ‘intelligence in manufacturing’ paradigm.

(Mac Farlane, *et al.*, 2002) proposes an holonic approach based on the concept of ‘product driven control’ to ensure the coherence between the physical flows and the information flows throughout the entire client/manufacturer/supplier life cycle of the products, it means as a whole.

(Valckenaers, *et al.*, 2002) states that the emergent behavior of complex systems exceeds the mental capabilities of human designers when they combine hardware parts and their software counterparts into a larger system. (Dias, *et al.*, 2001) orders information along four orthogonal dimensions in engineering design organisations in the sense that structure emerges from the interacting behavior of individual actors or perturbations of previous structures rather than from a central source (figure 1).

(Top down)	Aspect	(Bottom up)
Pre-determined	← Strategy →	Emergent
Formal	← Style →	Informal
Macro	← Focus →	Micro
Centralized	← Source →	Distributed

Fig. 1: Various aspects of the top-down vs. bottom-up dimension of information ordering (Dias, *et al.*, 2001)

These concerns between ‘top-down’ versus ‘bottom-up’ approaches for complex system modelling has been addressed in many ways to state that their properties cannot be reduced to the sum of their parts and that new properties emerge from the combination of their parts (Simon, 1990). In this way, we have proposed to **informally** model such **emergent** properties of manufacturing systems as resulting from an interaction between **distributed** objects (product/customer) having their own (**micro**) properties (figure 2).

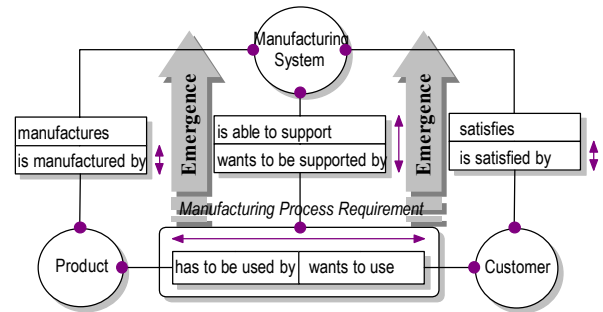


Fig. 2: N-dimension molecule of a manufacturing system (Mayer, *et al.*, 1996)

For that, we have used two IT-based mechanisms : the ‘nesting’ (or ‘objectification’) mechanism<sup>1</sup> of Object Role Modelling (Halpin, 1995) and the ‘association class’<sup>2</sup> of the Unified Modelling Language, in order to put into practice the modelling of an IMS by emergence (figure 3a and 3b).

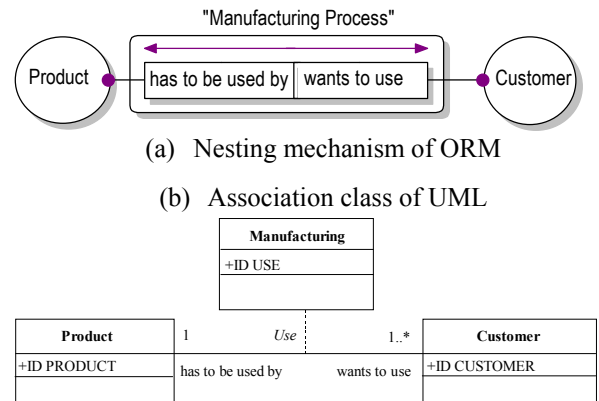


Fig. 3: IT-based mechanisms

Nevertheless, these mechanisms are not fully compliant with more theoretical (formal) definitions of emergence of system properties as, for example, the ‘computational’ emergence proposed by (Cariani, 1991) where global order of a system arises from local deterministic and computational interactions, or, as proposed by (Rosen, 1996) and (Ehresmann and Vanbreemersch, 1997), as a variation of behaviour between a real physical system and its model.

<sup>1</sup> In ORM, the ‘nesting’ mechanism allows relationships to be considered as objects.

<sup>2</sup> In UML, an association class has both association and class properties.

Moreover, the two previous authors emphasize the interest of the Category Theory to mathematically formalise the informal concept of emergence in natural complex systems. Based on these works, our contribution aims to demonstrate the efficiency of the Category Theory to model emergent properties in IMS.

### 3. IMS AS A THEORETICAL CATEGORY

Eilenberg and Mac Lane (1942) have introduced the notion of categories in order to transform complex problems of Topology (area of mathematics) into more understanding problems of Algebra.

Later, the Category Theory has been developed as a framework for various domain of mathematics and is now recognized as a powerful language for a universal semantics of mathematical structures.

In the fifties, Rosen proposed to use Category Theory to model emergence in Biology (Rosen, 1958). More recently, Ehresmann and Vanbremeersch (1987, 1997) use Category Theory for a formalisation of emergence in the framework of human neuronal networks and have proposed to model a complex system as an 'Evolutive System'.

According to those works, we have proposed in (Lavigne, 2000) (Mayer, *et al.*, 2001) this mathematical theory as a scientific framework to model Manufacturing System.

#### 3.1 Formal definition of a Category.

(Barr and Wells, 1995; Mac Lane, 1971) provide the necessary background for Category Theory by formally defining the concepts of category and functor. A Category C is characterised as follow:

- a set  $Ob(C)$  whose the elements are the objects or nodes of C
- a set  $Fl(C)$  whose the elements are the morphisms or arrows of C
- two mappings  $s$  and  $t$  from  $Fl(C)$  to  $Ob(C)$ , respectively named source and target.

A Category C must respect necessary the following composition law:

$$\forall (f, g) \in (Fl(C))^2, \exists g \circ f \Leftrightarrow [t(f) = s(g) \wedge s(g \circ f) = s(f) \wedge t(g \circ f) = t(g)] \quad (1)$$

A functor is another important concept of Category Theory. A functor F is a transformation, from a Category C to another one C', preserving the internal structure of these Categories. In other words, F is a mapping from  $Ob(C)$  to  $Ob(C')$  and a mapping from  $Fl(C)$  to  $Fl(C')$ . F respects the natural composition law.

#### 3.2 Manufacturing System as a Category.

The partition of a system between the semantial and the material, proposed by (Ducrocq, 1996) is

perfectly corresponding to the framework of Category Theory.

We consider a Manufacturing System, at an instant t, as a Category  $C_t$  (by using the notations of Ehresmann and Vanbremeersch):

- $Ob(C_t)$  contains the components of the system, e.g. the manufacturing resources.
- $Fl(C_t)$  contains the different interactions between the objects of  $C_t$ , e.g. the product flow or services.

A functor  $F_{tt'}$  describes the changes of state in the manufacturing system between the instants t and t'. The most important is that  $F_{tt'}$  is applied to objects as well as to their interactions. Of course, we impose to this functor a respect of composition law.

The dynamics of the manufacturing system can be formalised - mainly the temporal evolution of the resources used to produce the product - by introducing the element 'Zero' of the Category Theory which permits to express that:

- if a component or an interaction disappears at t', its image by  $F_{tt'}$  is zero of  $C_t$ ,
- if a component or an interaction appears at t', its antecedent by  $F_{tt'}$  is zero of  $C_t$ .

### 4. IMS MODELLING BASED ON A THEORETICAL CATEGORY MECHANISM

In order to model the mechanism of emergence, we introduce the concept of colimit in a Category.

#### 4.1 Formal definition of a colimit

Let C and C' two categories and F a functor from C to C'. A colimit (figure 4) of F is an object c in  $Ob(C')$ , defined both by some objects A of C and a family of morphisms  $\{u_A: F(A) \rightarrow c\}$  following these properties:

$$\begin{aligned} &\forall Y \in C', \forall (v_A)_{A \in Ob(C)}, v_A : F(A) \rightarrow Y, \text{ with} \\ &(\forall B \in Ob(C), \forall f : A \rightarrow B, v_B \circ F(f) = v_A) \quad (2) \\ &\exists ! v : c \rightarrow Y, \forall A \in Ob(C) v \circ u_A = v_A \end{aligned}$$

The most important property of such a colimit is the uniqueness of the morphism  $v$  : it is independent from the choice of the objects A and B of the category C.

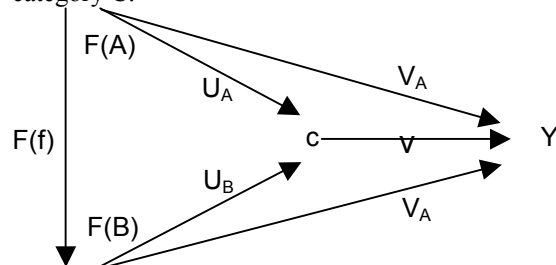


Fig.4: Colimit in a Category

#### 4.2 Emergence mechanism as a colimit

Let us consider a Manufacturing System as a Category at an instant  $t$  but with the properties of an Evolutive System.

Let  $P$  be a pattern constituted by some components  $A_i$  and  $A_j$  in interaction. This interaction is modelled by morphisms  $f_{ij}$ . We call collective link from  $P$  to an another object  $L'$ , a family of morphisms  $(v_i)$  from each  $A_i$  to  $L'$ . This collective link represents the collective actions of objects acting in co-operation, which cannot be realised by the objects acting loneliness. This combination of components  $A_i$  can be seen as an emergence of a new object  $L$  whose the order is higher. We model this mechanism of emergence by using the concept of colimit. We can consider  $L$  as the colimit of the pattern  $P$ , i.e.:

- . having a family of morphisms  $(a_i)$  from  $P$  to  $L$ ,
- . each collective link  $v_i$  from  $P$  to  $L'$  sticks together again in a unique morphism  $g$  from  $L$  to  $L'$

$$\forall i, v_i = g \circ a_i \quad (3)$$

**This colimit, if it exists, forgets the details of the pattern's organisation and keeps the collective actions, which the pattern can do.**

By using this mechanism, we can model in the same time, the heritage properties between  $(A_i)$  and  $L$  and the environmental properties between  $L$  and  $L'$  (figure 5).

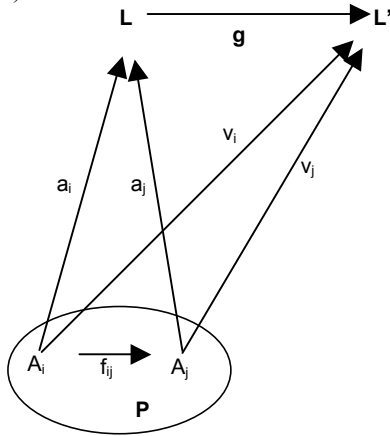


Fig. 5: Mechanism for emergence

This allows, when modelling a manufacturing system, to model new system properties emerging from the combination of its components.

#### 5. APPLICATION OF THE CT FRAMEWORK FOR IMS MODELLING

To illustrate the way in which our proposed modelling approach is applied, the lab 'SHIVA' MAS mock-up (Patriti, 1998) has been considered (figure 6).

The aim of this manufacturing system is to mill manufacturing parts. Spindles, table, tools storage, tools, gripper, axes, jack and an assembled mechanical support are the physical resources of the system. An operator is assigned to SHIVA. Note that SHIVA is a prototype of a MAS milling machine with parallel architecture.

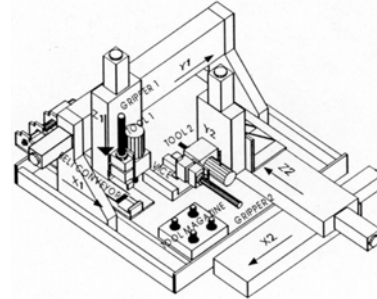


Fig. 6: 'SHIVA' MAS mock-up

#### 5.1 A MAS approach

In order to design a self-organised control of the SHIVA manufacturing system, Patriti (1998) has applied different MAS approaches in order to define significant agents and has experimentally validated the resulting models.

In such a context, SHIVA has been shown to be more a combination of agents, services, and requirements for these services, than a simple combination of agents. Agents and services have to integrate different constraints in order to respect the highest level of flexibility and reliability of the manufacturing system. The SHIVA resulting model consists in a control architecture able to self-organise agents and services (figure 7).

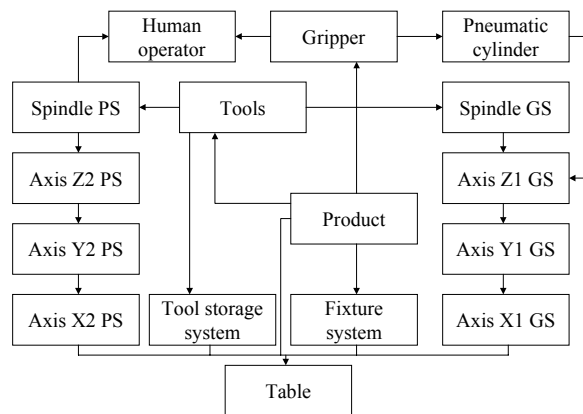


Fig. 7: 'SHIVA' architecture model

But, Patriti (1998) underlined some defaults of this approach. Particularly, he notes that this approach doesn't consider temporal evolution of the self-organised system. In this way, dynamics of interactions cannot be described, so it refers to a static vision of phenomena and, by so doing, emergent properties are not modelled in a satisfying

way. In addition, V. Patriti remarks that his approach doesn't offer a sufficient holonic framework to systematically design a control agent linked to its physical counterpart. We suggest, now a categorical approach like a response to these last remarks.

### 5.2 A Category Theory approach

We have proposed to introduce a Category Theory based modelling approach as a way to solve these above mentioned lacks.

We consider SHIVA system as a Category at an instant  $t$  where:

- . Objects are the components described at Fig. 7: table, tool storage system, tools, axis, gripper, pneumatic-cylinder ...
- . Morphisms are the services directly linked to the components to produce a product and the services of the interaction between components: rotation, move, roughing, machining... For instance, "an operator  $O$  moves a tool  $T$ " is representing by the morphism  $M: O \rightarrow T$ .

And, in order to describe the dynamics of the components to manufacture parts, we used categorical functors to model the changes of state between two instants  $t$  and  $t'$ .

By so doing, temporal evolutions are introduced in our model. In addition, we can consider the physical system (a set of physical agents) and the control one (a set of their control counterpart agents) as two categories, which are defined as the sub-categories of a global category. This global category proposes to model the entire intelligent manufacturing system and permits to really consider it as a holonic one.

The different emergent properties are modelled via the colimit principle, compliant with the emergence mechanism of a complex system (figure 8). In other mathematical words, a flow of manufacturing parts can be seen like the colimit of the different interacted components.

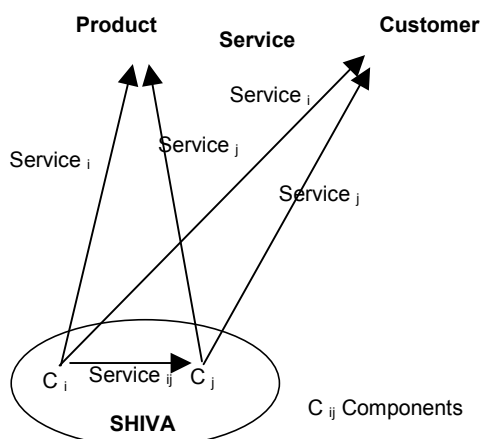


Fig. 8: Co-limit principle applied to 'SHIVA'

This application is used in our current works to demonstrate the interest of the Category Theory in complement of more practical but less formal modelling approaches as MAS ones.

## 6. CONCLUSION

Throughout this paper, we introduce CT as a promising theoretical approach for IMS modelling and, in particular, the CT co-limit as an emergence mechanism in order to extend 'bottom-up' approaches.

The perspectives of our work concern the use of a Functional Programming Language based on the Standard Meta Language (Rydeheard and Burstall, 1988) in order to execute a CT based model. In parallel, we need to use benchmarking techniques in order to compare MAS and HMS modelling approaches with the CT based approach.

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