Shape representation and analysis of 2D compact sets by shape diagrams

Séverine Rivollier, Johan Debayle, Jean-Charles Pinoli

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Séverine Rivollier, Johan Debayle and Jean-Charles Pinoli
CIS/LPMG, UMR CNRS 5148
Ecole Nationale Supérieure des Mines de Saint-Etienne,
158 cours Fauriel, 42023 Saint-Etienne cedex 2, France.
e-mail: rivollier@emse.fr, debayle@emse.fr, pinoli@emse.fr

***Abstract*— Shape diagrams are shape representations in the Euclidean plane introduced for studying 3D and 2D compact sets. A compact set is represented by a point within a shape diagram whose coordinates are morphological functionals defined from geometrical functionals and inequalities. Classically, the geometrical functionals for 2D sets are the area, the perimeter, the radii of the inscribed and circumscribed circles, and the minimum and maximum Feret diameters. The purpose of this paper is to present a particular shape diagram for which mathematical properties have been well-defined and to analyse the shape of several families of 2D sets for the discrimination of convex and non convex sets as well as the discrimination of similar sets.**

***Keywords*— Shape diagrams, Pattern analysis, Convex and no-convex sets.**

I. Introduction

The Blaschke's shape diagram [1], [2] allows to represent a 3D compact set by a point in the Euclidean 2D plane from three geometrical functionals: the volume, the surface area and the integral of mean curvature. The axes of this shape diagram are defined from geometric inequalities relating these functionals. These geometric inequalities do not provide a complete system: for any range of numerical values satisfying them, a compact convex set with these values for the geometrical functionals does not necessarily exist (in other words, a point within the 2D Blaschke shape diagram does not necessarily describe a 3D compact convex set).

Following the approach of Blaschke, Santalo [12] considered the shape diagrams of 2D compact sets from six geometrical functionals: the area, the perimeter, the radii of the inscribed and circumscribed circles, and the minimum and maximum Feret diameters [6]. Several studies on these shape diagrams have been performed [12], [4], [5].

The purpose of this paper is to present a particular shape diagram for which properties have been well-defined [5] and to analyse several families of 2D sets for shape discrimination. The geometrical and morphological functionals (morphological functionals are invariant under similitude transformations and are defined as ratios between geometrical functionals; for instance, their concrete meanings are circularity, thinness, ...) of 2D compact sets are exposed in the next section. From these functionals, a specific shape diagram is studied for the discrimination of convex and non convex sets as well as the discrimination of similar sets. The final section offers an application related to this study.

II. Shape representation

In this paper, the non-empty compact convex sets in the Euclidean 2D space $\mathbb{E}^2$ are considered. Several geometrical functionals are determinated in order to characterize the sets. They are related by the so-called geometric inequalities, which allow to define morphological functionals and thereafter shape diagrams.

A. Geometrical functionals and inequalities

For a compact set in $\mathbb{E}^2$, let A, P, r, R, ω, d, denote its area, its perimeter, the radii of its inscribed and circumscribed circles, its minimum and maximum Feret diameters [6], respectively. Figure 1 illustrates some of these geometrical functionals.

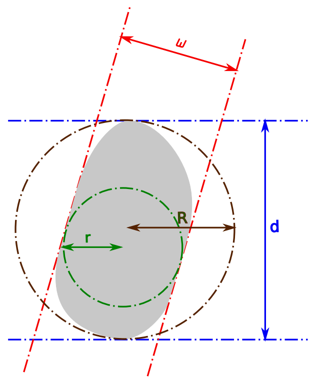


Fig. 1. Geometrical functionals of a compact set: radii of inscribed (r) and circumscribed (R) circles, minimum (ω) and maximum (d) Feret diameters.

For a compact set, these six geometrical functionals are greater than zero. The line segments and curves provide null values for A, r and ω, and the points for P, R and d.

For a compact set in $\mathbb{E}^2$, the relationships between these geometrical functionals are constrained by the geometric inequalities [3], [13], [7] referenced in the second column of Table 1. Some geometric inequalities (that are not considered in this paper) are restricted to convex sets. These inequalities link geometrical functionals by pairs and determine the so-called extremal compact sets that satisfy the corresponding equalities (Table 1, fourth column). Futhermore, they allow to determinate morphological functionals.

B. Morphological functionals

The morphological functionals are invariant under similitude transformations (consequently, they do not depend on the global size of the compact set) and are defined as ratios between geometrical functionals. In these ratios, the units of the numerator and the denominator are dimensionaly homogeneous and the result has therefore no unit. Moreover, a normalization by a constant value (scalar multiplication) allows to have a ratio that ranges in $[0,1]$. For each morphological functional, the scalar value depends directly on the associated inequality. These morphological functionals are referenced in the third column of Table 1. They are classified according to their concrete meanings namely:

- roundness: $4\pi\,\mathrm{A}\,/\,\mathrm{P}^2$, $4\,\mathrm{A}\,/\pi\,\mathrm{d}^2$, $\pi\,\mathrm{r}^2\,/\,\mathrm{A}$ and $\mathrm{A}\,/\pi\,\mathrm{R}^2$;
- circularity: $2\pi\,\mathrm{r}\,/\,\mathrm{P}$, $\mathrm{r}\,/\,\mathrm{R}$, $2\,\mathrm{r}\,/\,\mathrm{d}$ and $\omega\,/2\,\mathrm{R}$;
- diameter constance: $\pi\,\omega\,/\,\mathrm{P}$ and $\omega\,/\,\mathrm{d}$;
- thinness: $2\,\mathrm{d}\,/\,\mathrm{P}$ and $4\,\mathrm{R}\,/\,\mathrm{P}$.

The morphological functional $\sqrt{3}\,\mathrm{R}\,/\,\mathrm{d}$ expresses both the equilateral triangularity and the diameter constance. The ratios $2\,\mathrm{r}\,/\,\omega$ and $\mathrm{d}\,/2\,\mathrm{R}$ do not have concrete meaning, they are equal to one for many different compact sets.

Table 1. Shape functionals for compact sets. A, P, r, R, ω, d, denote the area, perimeter, radii of the inscribed and circumscribed circles, minimum and maximum Feret diameters [6], respectively.

Geometrical functionals	Geometric inequalities	Morphological functionals	Extremal sets
r,R	$\mathrm{r} \leq \mathrm{R}$	$\mathrm{r}\,/\,\mathrm{R}$	C
ω,R	$\omega \leq 2\,\mathrm{R}$	$\omega\,/2\,\mathrm{R}$	C
A,R	$\mathrm{A} \leq \pi\,\mathrm{R}^2$	$\mathrm{A}\,/\pi\,\mathrm{R}^2$	C
d,R	$\mathrm{d} \leq 2\,\mathrm{R}$	$\mathrm{d}\,/2\,\mathrm{R}$	Y
r,d	$2\,\mathrm{r} \leq \mathrm{d}$	$2\,\mathrm{r}\,/\,\mathrm{d}$	C
ω,d	$\omega \leq \mathrm{d}$	$\omega\,/\,\mathrm{d}$	W
A,d	$4\,\mathrm{A} \leq \pi\,\mathrm{d}^2$	$4\,\mathrm{A}\,/\pi\,\mathrm{d}^2$	C
R,d	$\sqrt{3}\,\mathrm{R} \leq \mathrm{d}$	$\sqrt{3}\,\mathrm{R}\,/\,\mathrm{d}$	Z
r,P	$2\pi\,\mathrm{r} \leq \mathrm{P}$	$2\pi\,\mathrm{r}\,/\,\mathrm{P}$	C
ω,P	$\pi\,\omega \leq \mathrm{P}$	$\pi\,\omega\,/\,\mathrm{P}$	W
A,P	$4\pi\,\mathrm{A} \leq \mathrm{P}^2$	$4\pi\,\mathrm{A}\,/\,\mathrm{P}^2$	C
d,P	$2\,\mathrm{d} \leq \mathrm{P}$	$2\,\mathrm{d}\,/\,\mathrm{P}$	L
R,P	$4\,\mathrm{R} \leq \mathrm{P}$	$4\,\mathrm{R}\,/\,\mathrm{P}$	L
r,A	$\pi\,\mathrm{r}^2 \leq \mathrm{A}$	$\pi\,\mathrm{r}^2\,/\,\mathrm{A}$	C
r,ω	$2\,\mathrm{r} \leq \omega$	$2\,\mathrm{r}\,/\,\omega$	X

Extremal sets are the sets for which an inequality becomes an equality.

- C the disks
- W the constant width compact convex sets
- L the line segments
- X many compact convex sets
- Y many simply connected compact sets
- Z every compact convex set of diameter d containing an equilateral triangle of side-length d

C. Shape diagrams

From these morphological functionals, 2D shape diagrams can be defined. They enable to represent the morphology of any compact set in the Euclidean 2D plane from two morphological functionals (that is to say from three geometrical functionals because the two denominators use the same geometrical functionals).

Let be any 3-uplet of the considered six geometrical functionals (A, P, r, R, ω, d) and (M_1, M_2) be some particular morphological functionals valued in $[0,1]^2$. A shape diagram $\mathcal{D}$ is represented in this plane domain $[0,1]^2$ (whose axis coordinates are the morphological functionals M_1 and M_2) where any 2D compact set S is mapped onto a point (x,y). Note that if M_1 or M_2 is in $\{\pi\,\mathrm{r}^2\,/\,\mathrm{A}, 2\,\mathrm{r}\,/\,\omega\}$, the line segments and curves can not be mapped onto a point because they provide null values for A and ω. Mathematically, a shape diagram $\mathcal{D}$ is obtained from the following mapping.

$$\mathcal{D} : \begin{cases} \mathcal{K}(\mathbb{E}^2) \rightarrow [0,1]^2 \\ \quad S \quad\; \mapsto (x,y) \end{cases}$$

where $\mathcal{K}(\mathbb{E}^2)$ denotes the compact sets of the Euclidean 2D plane. Using the morphological functionals listed in Table 1, twenty-two shape diagrams are defined, denoted $(\mathcal{D}_k)_{k\in[\![1,22]\!]}$, respectively (Table 2).

Table 2. The twenty two shape diagrams axes coordinates for 2D compact sets.

Shape diagrams	Axes coordinates	
$\mathcal{D}_1 : (\omega,\mathrm{r},\mathrm{R})$	$x = \omega\,/2\,\mathrm{R}$	$y = \mathrm{r}\,/\,\mathrm{R}$
$\mathcal{D}_2 : (\omega,\mathrm{A},\mathrm{R})$	$x = \omega\,/2\,\mathrm{R}$	$y = \mathrm{A}\,/\pi\,\mathrm{R}^2$
$\mathcal{D}_3 : (\mathrm{r},\mathrm{A},\mathrm{R})$	$x = \mathrm{r}\,/\,\mathrm{R}$	$y = \mathrm{A}\,/\pi\,\mathrm{R}^2$
$\mathcal{D}_4 : (\mathrm{A},\mathrm{d},\mathrm{R})$	$x = \mathrm{A}\,/\pi\,\mathrm{R}^2$	$y = \mathrm{d}\,/2\,\mathrm{R}$
$\mathcal{D}_5 : (\omega,\mathrm{d},\mathrm{R})$	$x = \omega\,/2\,\mathrm{R}$	$y = \mathrm{d}\,/2\,\mathrm{R}$
$\mathcal{D}_6 : (\mathrm{r},\mathrm{d},\mathrm{R})$	$x = \mathrm{r}\,/\,\mathrm{R}$	$y = \mathrm{d}\,/2\,\mathrm{R}$
$\mathcal{D}_7 : (\omega,\mathrm{r},\mathrm{d})$	$x = \omega\,/\,\mathrm{d}$	$y = 2\,\mathrm{r}\,/\,\mathrm{d}$
$\mathcal{D}_8 : (\omega,\mathrm{A},\mathrm{d})$	$x = \omega\,/\,\mathrm{d}$	$y = 4\,\mathrm{A}\,/\pi\,\mathrm{d}^2$
$\mathcal{D}_9 : (\mathrm{r},\mathrm{A},\mathrm{d})$	$x = 2\,\mathrm{r}\,/\,\mathrm{d}$	$y = 4\,\mathrm{A}\,/\pi\,\mathrm{d}^2$
$\mathcal{D}_{10} : (\mathrm{A},\mathrm{R},\mathrm{d})$	$x = 4\,\mathrm{A}\,/\pi\,\mathrm{d}^2$	$y = \sqrt{3}\,\mathrm{R}\,/\,\mathrm{d}$
$\mathcal{D}_{11} : (\omega,\mathrm{R},\mathrm{d})$	$x = \omega\,/\,\mathrm{d}$	$y = \sqrt{3}\,\mathrm{R}\,/\,\mathrm{d}$
$\mathcal{D}_{12} : (\mathrm{r},\mathrm{R},\mathrm{d})$	$x = 2\,\mathrm{r}\,/\,\mathrm{d}$	$y = \sqrt{3}\,\mathrm{R}\,/\,\mathrm{d}$
$\mathcal{D}_{13} : (\omega,\mathrm{r},\mathrm{P})$	$x = \pi\,\omega\,/\,\mathrm{P}$	$y = 2\pi\,\mathrm{r}\,/\,\mathrm{P}$
$\mathcal{D}_{14} : (\omega,\mathrm{A},\mathrm{P})$	$x = \pi\,\omega\,/\,\mathrm{P}$	$y = 4\pi\,\mathrm{A}\,/\,\mathrm{P}^2$
$\mathcal{D}_{15} : (\mathrm{r},\mathrm{A},\mathrm{P})$	$x = 2\pi\,\mathrm{r}\,/\,\mathrm{P}$	$y = 4\pi\,\mathrm{A}\,/\,\mathrm{P}^2$
$\mathcal{D}_{16} : (\mathrm{A},\mathrm{R},\mathrm{P})$	$x = 4\pi\,\mathrm{A}\,/\,\mathrm{P}^2$	$y = 4\,\mathrm{R}\,/\,\mathrm{P}$
$\mathcal{D}_{17} : (\omega,\mathrm{R},\mathrm{P})$	$x = \pi\,\omega\,/\,\mathrm{P}$	$y = 4\,\mathrm{R}\,/\,\mathrm{P}$
$\mathcal{D}_{18} : (\mathrm{r},\mathrm{R},\mathrm{P})$	$x = 2\pi\,\mathrm{r}\,/\,\mathrm{P}$	$y = 4\,\mathrm{R}\,/\,\mathrm{P}$
$\mathcal{D}_{19} : (\mathrm{A},\mathrm{d},\mathrm{P})$	$x = 4\pi\,\mathrm{A}\,/\,\mathrm{P}^2$	$y = 2\,\mathrm{d}\,/\,\mathrm{P}$
$\mathcal{D}_{20} : (\omega,\mathrm{d},\mathrm{P})$	$x = \pi\,\omega\,/\,\mathrm{P}$	$y = 2\,\mathrm{d}\,/\,\mathrm{P}$
$\mathcal{D}_{21} : (\mathrm{r},\mathrm{d},\mathrm{P})$	$x = 2\pi\,\mathrm{r}\,/\,\mathrm{P}$	$y = 2\,\mathrm{d}\,/\,\mathrm{P}$
$\mathcal{D}_{22} : (\mathrm{d},\mathrm{R},\mathrm{P})$	$x = 2\,\mathrm{d}\,/\,\mathrm{P}$	$y = 4\,\mathrm{R}\,/\,\mathrm{P}$

More specifically, the mathematical properties of the shape diagram $\mathcal{D}_8$ have been well-studied [5] (the choice of this shape diagram is based on the results synthetized in [8], [9], [10]). For example, this shape diagram is based on a complete system of geometrical inequalities: for any 3-uplet $(\mathrm{r},\mathrm{A},\mathrm{d})$ satisfying those conditions, a 2D compact convex set with these geometrical functionals values exists [12], [4]. In other words, the mapping that associates a 2D compact convex set in $\mathbb{E}^2$ to a point in this shape diagram can be surjective by restricting the arrival set. Each of the two associated inequalities determines a part of the convex domain boundary (the domain in which all compact convex sets are mapped). These two inequalities determine the whole boundary of the convex domain. The compact convex sets mapped onto the boundary points are the extremal compact convex sets of each considered inequality. Figure 3c shows the convex domain of the shape diagram (complete system) $\mathcal{D}_8$.

D. Illustration

Figure 3 illustrates the shape diagram $\mathcal{D}_8$ for the family $\mathcal{F}_1$ of compact sets (Figure 2). Note that the two axes represent the diameter constance (x-axis) and the roundness (y-axis) of a shape. The compact sets of the family $\mathcal{F}_1$ are chosen according to the faculty to compute analytically their geometrical (and morphological) functional values.

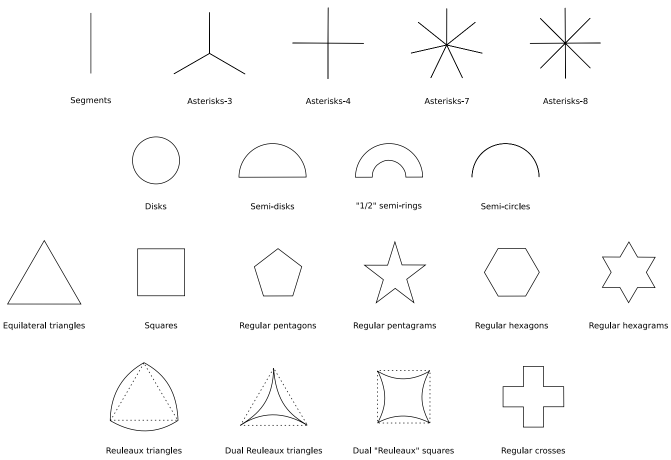
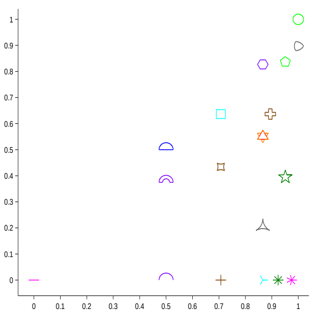
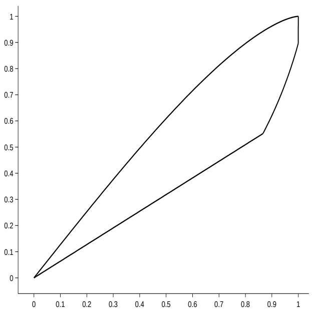


Fig. 2. Family $\mathcal{F}_1$ of 2D compact sets.

(a) $\mathcal{F}_1$ sets



(b) Shape diagram $\mathcal{D}_8 : (\omega, \mathrm{A}, \mathrm{d})$



(c) Convex domain of $\mathcal{D}_8$

Fig. 3. Shape diagram $\mathcal{D}_8$ (b) for the compact sets (a) of the family $\mathcal{F}_1$ with the boundaries of the convex domain (c).

In the next section, different families of compact sets will be studied through this shape diagram $\mathcal{D}_8$.

III. Shape Analysis

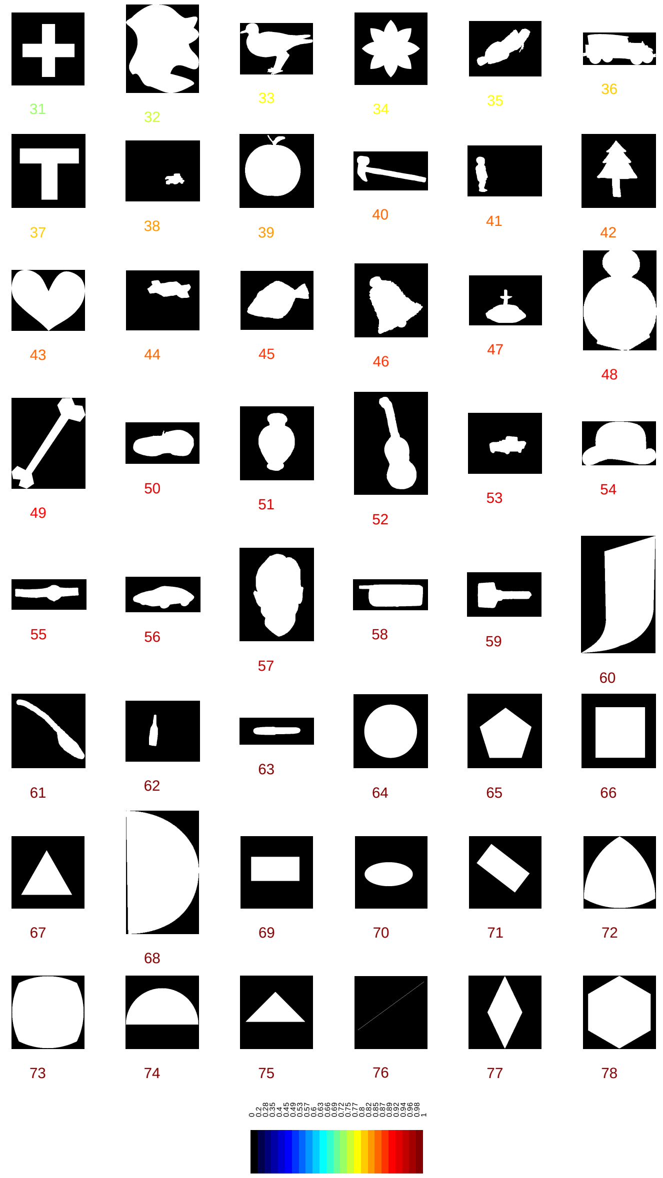
In this section, shape discrimination is studied for convex and non convex sets and also for sets with similar (visually) shapes.

A. Discrimination of convex and non convex sets

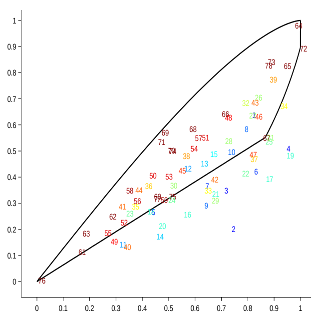
The following study on the convexity discrimination first requires the definition of the shape convexity. A set is convex when the line segment which joins any two points in it lies totally within the set. The parameter of Zunic and Rosin [14] is used for computing the shape convexity. It ranges between 0 and 1.

Figure 4 illustrates seventy-eight binary images constituting the family $\mathcal{F}_2$. For each image, the white object represents a discretized compact set and the black pixels represents the background. The binary images are numerated from one to seventy-eight. The image number is mapped to its proper point in the shape diagram $\mathcal{D}_8$ (Figure 4b). The color of the number is related to the convexity parameter value $c \in [0, 1]$ of the associated set (dark red for a high c, dark blue for a low c). This convexity range will enable to analyze the convexity discrimination within the shape diagram.

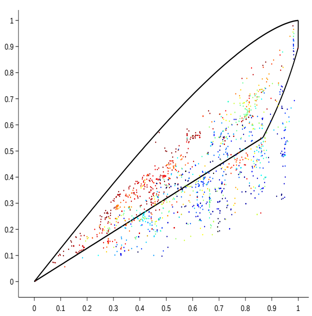
A wider family $\mathcal{F}_3$ of 1370 binary images from the Kimia database [11] is also considered and analyzed (Figure 4c).



(a) Range of convexity values



(b) Shape diagram of $\mathcal{F}_2$ sets



(c) Shape diagram of $\mathcal{F}_3$ sets

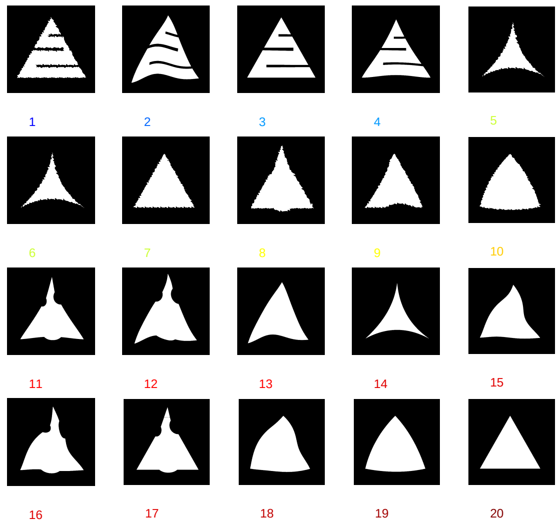
Fig. 4. Family $\mathcal{F}_2$ of seventy-eight 2D compact sets, represented in white on binary images. The color of each pattern number is related to the convexity parameter value using non-linear color tones (a). Convexity analysis for compact sets of the families $\mathcal{F}_2$ (b) and $\mathcal{F}_3$ (c) mapped onto the shape diagram $\mathcal{D}_8$.

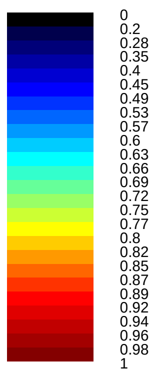
For the two families of sets, the colors are rather irregularly distributed according to the color range. Blue colors (resp. red colors) appear inside (resp. outside) the convex domain. In addition, the artifacts of the discretization can interfer with the results. In conclusion, the shape diagram $\mathcal{D}_8$ shows a moderate discrimination of convex and non convex sets.

B. *Discrimination of similar sets*

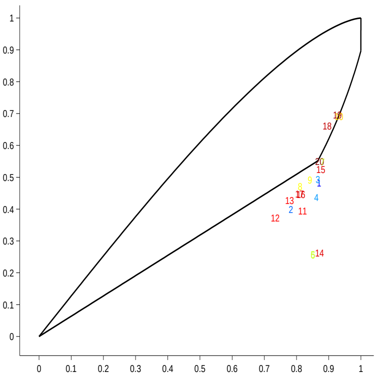
This section focuses on the discrimination of shapes that are visually similar enough, so that they can be considered as the same global shape.

Let be two families $\mathcal{F}_4$ and $\mathcal{F}_5$ of binary images representing triangles (Figure 5) and disks (Figure 6), respectively, that have undergone minor transformations, modifications, deformations. The morphological parameters are computed, and the images are located by a point in the shape diagram $\mathcal{D}_8$. The color of the number is related to the convexity parameter value c, and the convex domain boundary is illustrated with black lines.



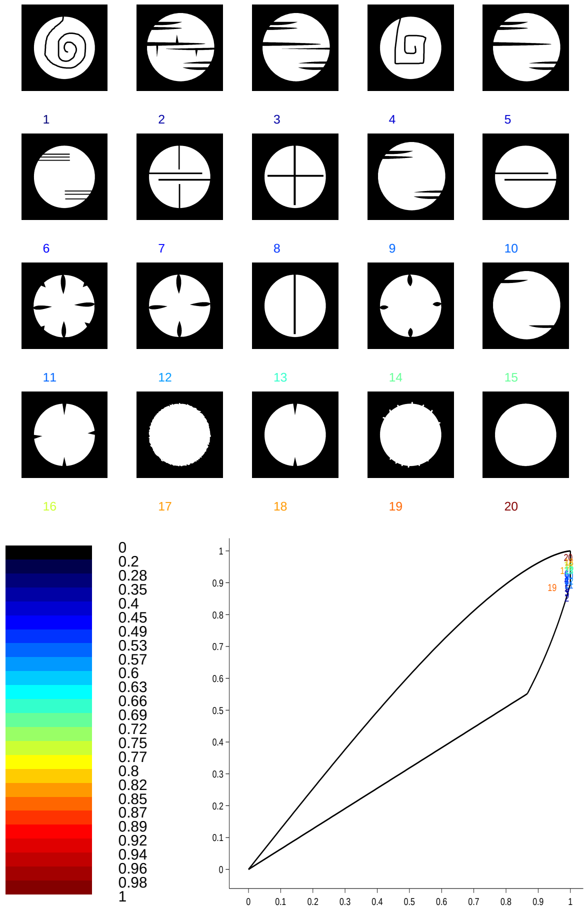


(a) Convexity



(b) Shape diagram of $\mathcal{F}_4$ sets

Fig. 5. Family $\mathcal{F}_4$ of twenty 2D compact sets, represented in white on binary images. All these sets have a 'triangle' shape. The color of the number is related to the convexity parameter value (a). Discrimination of similar sets for this family $\mathcal{F}_4$ (b) mapped onto the shape diagram $\mathcal{D}_8$.



(a) Convexity (b) Shape diagram of $\mathcal{F}_5$ sets

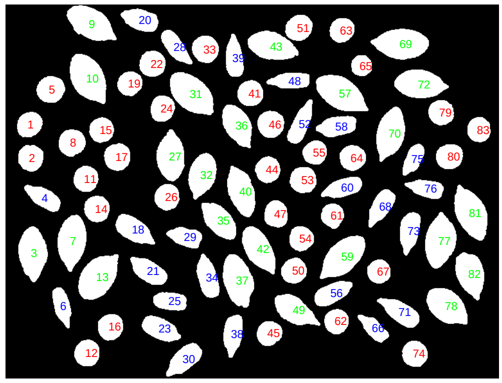
Fig. 6. Family $\mathcal{F}_5$ of twenty 2D compact sets, represented in white on binary images. All these sets have a 'disk' shape. The color of the number is related to the convexity parameter value (a). Discrimination of similar sets for this family $\mathcal{F}_5$ (b) mapped onto the shape diagram $\mathcal{D}_8$.

The results show that the spatial distribution of the $\mathcal{F}_5$ sets is dense. Each set is located around the same point. On the contrary, the locations of the $\mathcal{F}_4$ sets are more sparse (the shape roundness of the different sets varies a lot!).

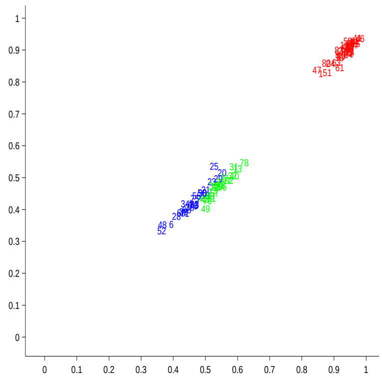
IV. Application

Let be a binary image with three classes of seeds (Figure 7): circular seeds, small and elongated seeds, and almond-shaped seeds. The connected component corresponding to the seeds are numbered. These discretized compact sets are mapped onto the shape diagram $\mathcal{D}_8$. The color of the set number represents the group to which the associated seed belongs: red for the circular seeds, blue for the small and elongated seeds, and green for the almond-shaped seeds.

In the shape diagram $\mathcal{D}_8$, the red locations form a well distinct cluster from the blue and green locations. It is usual as green and blue locations are a little confused. The associated seed shapes have similar shapes. They differ mainly in size, but remember that the shape diagrams axes are morphological functionals that do not depend on the global size of the set.



(a) Seeds image



(b) Shape diagram of seeds

Fig. 7. Seeds represented in white on the binary image (a). The seeds are numbered and the color of a number is related to the group to which the seed belongs: red for the circular seeds, blue for the small and elongated seeds, and green for the almond-shaped seeds. Discrimination of seeds (b) mapped onto the shape diagram $\mathcal{D}_8$.

This application shows that the shape diagram can be used for shape classification.

Let be four families of binary images representing apples, camels, sea-snakes and watches that have undergone minor transformations, modifications, deformations. These binary images are not illustrated here. They are extracted from the Kimia database and they are similar enough to the images 39, 13, 14 and 55 respectively of the figure 4. The morphological parameters are computed, and the images are located by a point in the shape diagram $\mathcal{D}_8$ (Figure 8). The color of the point is related to the group to which the set representing in the image belongs: red for a set of 'watch' shape, blue for a set of 'apple' shape, green for a set of 'sea-snake' shape, and black for a set of 'camel' shape.

The results show that the spatial distribution of each family form a well distinct cluster from each other. Thus, the shape diagram can be used for shape classification.

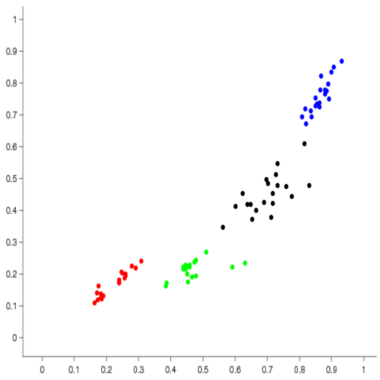


Fig. 8. Discrimination of sets mapped onto the shape diagram $\mathcal{D}_8$. The set locations are points whose the color is related to the group to which the set belongs: red for the sets of 'watch' shape, blue for the sets of 'apple' shape, green for the sets of 'sea-snake' shape, and black for the sets of 'camel' shape.

V. Conclusion and perspectives

This paper has dealed with shape diagrams of 2D compact sets built from geometrical functionals and inequalities. Each set is represented by a point within a shape diagram whose coordinates are morphological functionals defined as normalized ratios of geometrical functionals. Such a diagram enables to define a shape representation for 2D sets in the 2D Euclidean plane (bounded region).

A particular shape diagram for which mathematical properties have been well-defined has been studied for shape representation and analysis of several families of 2D compact sets. First, it was shown that the shape diagram allowed a moderate discrimination of convex and non convex sets. Secondly, the discrimination of similar sets was not always relevant in relation with the studied family of sets. Lastly, an application related to this study is offered.

Pattern recognition and classification could be investigated using the proposed shape diagrams defining a morphological signature of sets. Also, a more detailed study has to be performed in order to evaluate the quality of these signatures in relation with other standard ones using shape databases.

Currently, the authors are studying the shape discrimination of the twenty-one other possible shape diagrams. A comparative study is being performed [8], [9], [10].

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