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# Relative number squeezing in four-wave mixing of matter waves

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We report the demonstration of relative number squeezing in four-wave mixing of Bose-Einstein condensates of metastable helium. The collision between two Bose-Einstein condensates produces a scattering halo populated by pairs of atoms of opposing velocities, which we divide into several symmetric zones. We show that the atom number difference for opposing zones has sub-poissonian noise fluctuations whereas that of non-opposing zones is well described by shot noise. The atom pairs produced in a dual number state are well adapted to sub shot-noise interferometry and studies of Einstein-Podolsky-Rosen-type nonlocality tests.

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The creation of squeezed states of the electromagnetic field has been a major preoccupation of quantum optics for several decades [1]. Such states are not only inherently fascinating, but also have the potential to improve sensitivity in interferometers [1], going beyond the “shot noise” or standard quantum limit. In the field of atom optics, workers are beginning to use the intrinsic non-linearities present in a matter wave field to produce non-classical states, especially squeezed states [2–6]. Indeed, an atom interferometer using squeezed inputs was recently demonstrated [7]. In our case, we produce dual number states in four-wave mixing of Bose-Einstein condensates (BECs). These states form the basis of a very different proposal for atom interferometry beyond the standard quantum limit [8–10]. The atoms are also created in such a way as to permit macroscopic spatial separations and can therefore be adapted to studies of Einstein-Podolsky-Rosen-type nonlocality tests [11] with massive particles.

Correlated photon pairs can be generated using optical processes such as four-wave mixing [12] or parametric down conversion [13]. The matter wave analogs of these processes have recently been demonstrated [14, 15]. The spontaneous four-wave mixing process [15], which we use here, simply corresponds to the collision of two Bose-Einstein condensates during which binary collisions produce scattered pairs of atoms with correlated momenta. Correlations however, do not guarantee relative number squeezing (see Ref. [16] for an example of correlations without squeezing) nor entanglement. The success of proposals such as those of [8–10], in which dual number states are used in an interferometer, will likely be determined by the degree of squeezing. Thus, with a view towards using such correlated states in interferometry, it is important to verify that these processes do indeed produce squeezing. In this letter, we demonstrate and

quantify the relative number squeezing. One can think of our result as the matter wave analog of the experiment of Ref. [17] in which relative number squeezing was observed in the generation of twin light beams created by parametric down conversion.

Squeezing of atom samples may prove even more important than of light because the number of available atoms is often limited and therefore surpassing the standard quantum limit can be the only way to increase the signal-to-noise and improve performance. In interferometry proposals relying on dual number states, the observable corresponding to the relative phase is completely undetermined. But paradoxically, after passing through a beam splitter, the phase difference is no longer undetermined, but is peaked around two values of the phase [10]. It has been argued that such states can be more robust to loss processes than maximally entangled states [9]. The pairs we produce should also be entangled in a sense analogous to [18]. The fact that they are created with a large momentum difference means that after free flight, they can have significant spatial separations (several cm here) and are thus well suited to investigations of (non-local) EPR entanglement [11] and Bell’s inequalities using atoms.

We use metastable helium atoms which are detected by a micro-channel plate detector (MCP) with a delay line anode [19]. The detector allows three dimensional reconstruction of the momentum of each atom. Atoms in the  $2^3S_1$ ,  $m_x = 1$  state are evaporatively cooled in a vertically-elongated optical trap to produce a BEC with about  $10^5$  atoms and no discernible thermal component [20]. The use of an optical trap has resulted in substantially better shot to shot reproducibility than its magnetic antecedent [15]. The atomic angular momentum, which is due entirely to the electron spin, is defined relative to a 4 Gauss magnetic holding field in the  $x$ -

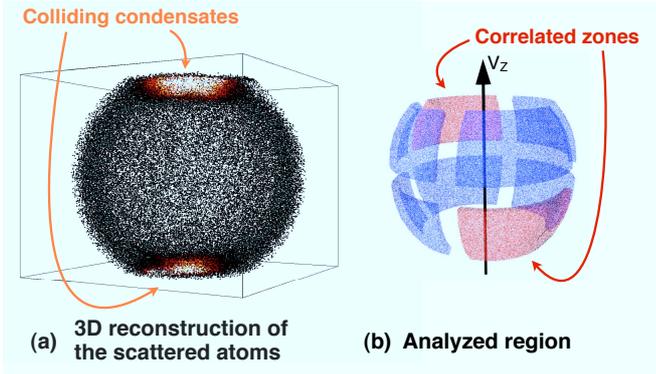


FIG. 1. (Color online) View of the approximately spherical halo after the collision of two BECs and a subsequent ballistic expansion. (a) Reconstruction of the experimental data, with each dot corresponding to an atom in momentum space. Atoms on the collision halo are black, while the colliding, pancake shaped BECs at the top and the bottom of the halo are very dense and are shown in orange/yellow. The collision axis, denoted  $z$ , and the optical trap axis are both almost vertical. (b) Schematic view of the analyzed part of the collision halo. To eliminate the condensates from the analysis, the analyzed region is limited to  $|v_z| < 0.5 v_{\text{rec}}$ . In this case the sphere is sliced into  $N_Z = 8$  zones that are separated from each other for better visualization. An example of two correlated zones is shown (red). The number difference between these two zones shows sub shot-noise fluctuations.

direction (orthogonal to the optical trap long axis). After cooling, the atomic spin is rotated away from the axis of the holding field by  $\pi/2$  using a 2 ms RF sweep [20]. The laser trap is then switched off and one microsecond later the condensate is split by applying counter-propagating laser beams for  $2.5 \mu\text{s}$ . These beams are blue detuned from the  $2^3P_0$  state by 600 MHz, inclined at a  $7^\circ$  angle to the vertical axis and linearly polarized along the quantization axis. About one third of the atoms are diffracted into each of two momentum classes traveling at  $\pm v_{\text{rec}}$ , where  $v_{\text{rec}} = 9.2 \text{ cm/s}$  is the recoil velocity. Most of the last third remain at zero velocity. Binary collisions take place between atoms of all three velocity classes producing 3 collision halos with centers of mass velocities  $\pm v_{\text{rec}}$  and zero. Since the atomic spin is orthogonal to the local field, 50% of the atoms are in the  $m_x = 0$  state with respect to the magnetic field axis [20], and these atoms fall to the detector, unperturbed by magnetic field gradients. The trajectories of atoms in the  $m_x = \pm 1$  states are perturbed by residual field gradients and we therefore apply an additional gradient that causes these atoms to miss the detector entirely. The analysis is only focused on the collision halo centered at  $+v_{\text{rec}}$  (see Fig. 1a).

The collision halo centered at  $v = 0$  has a radius  $2 v_{\text{rec}}$  and is too large to be entirely captured by the detector

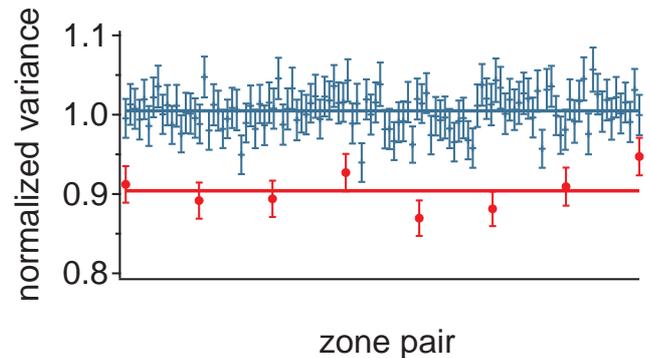


FIG. 2. (Color online) Variance of all possible pairs of zones for the halo cut into 16 zones ( $p = 4$ ), and summing 3600 shots. The normalized variance is  $V_{i,j}$  and the error bars reflect its standard deviation  $\delta V_{i,j}$ . Circles correspond to the 8 correlated zones and crosses to the 112 uncorrelated ones. The two horizontal lines correspond to the mean of each data set. The line thickness is twice the standard deviation of the mean of each data set, considering each pair of zones as independent.

while the two halos centered at  $\pm v_{\text{rec}}$ , with radii  $v_{\text{rec}}$ , are entirely detected. In addition to binary scattering events, these two latter halos can be populated by spontaneous photon scattering whenever an atom at  $v = 0$  scatters a photon from one of the diffraction laser beams. The diffraction efficiency depends on the product  $I_1 I_2$  of the two laser intensities, while the spontaneous scattering into a given halo depends on only one of these intensities. So to reduce this effect we introduce an intensity imbalance in the two laser beams such that the halo centered at  $+v_{\text{rec}}$  is populated by the weaker beam and contains fewer such optically scattered atoms.

If squeezing is present, we expect a sub-shot noise variance in the number difference of any two diametrically opposed volumes in the scattering halo. For any other pair of volumes, we expect a variance corresponding to shot noise. We define the halo as a spherical shell of radius  $v_{\text{rec}}$  and thickness  $\pm 0.15 v_{\text{rec}}$  and remove the areas on the halo containing the scattered BEC's. The excised regions correspond to vertical velocities  $|v_z| > 0.5 v_{\text{rec}}$ . We divide the remainder of the halo in half at the equator and then make  $p$  vertical cuts along the meridians, dividing the halo into  $N_Z = 4p$  equal zones as shown in Fig. 1b for  $p = 2$ .

We define a normalized number difference variance for zones  $i$  and  $j$ :

$$V_{i,j} = \frac{\langle (N_i - N_j)^2 \rangle - \langle N_i - N_j \rangle^2}{\langle N_i \rangle + \langle N_j \rangle} \quad (1)$$

The brackets  $\langle \dots \rangle$  denote the average over the 3600 shots, and  $N_i$  refers to the number of atoms detected in the  $i$ -th zone on a single shot. On average, we detect 150 atoms per shot on the whole analyzed region. If the zones  $i$

and  $j$  are uncorrelated, the normalized variance should be unity. Figure 2 shows the measured variances of all possible pairs of zones when the halo is cut into 16 zones [21]. The 8 pairs of correlated zones indeed show relative number squeezing ( $V < 1$ ) and the 112 pairs of uncorrelated zones do not.

Perfectly correlated pairs and perfect detection would result in a zero variance. This however is almost unattainable in practice because of various imperfections, the most significant of which is the non-unit quantum efficiency  $\eta$  of our MCP detector. The effect of the efficiency alone leads to a variance  $V = 1 - \eta$  of the correlated zones, and therefore we can immediately deduce a lower limit of 10% on the quantum efficiency, in agreement with estimates we have made in the past [22]. A second, less severe but intrinsic imperfection comes about because the momenta of the correlated atoms are not exactly equal and opposite, but have a width determined by the momentum spread of the atoms within the initial condensates, as confirmed by the finite width of the two-body correlation function in momentum space [15]. This function gives the conditional probability of finding some momentum for one particle, given the momentum of its partner. Thus it is possible for the two atoms of a correlated pair to end up in zones that are not diametrically opposed. We can study this effect by observing how the amount of relative number squeezing varies as we change the number of zones  $N_Z$  (Fig. 3). The smaller the zones into which we cut the sphere, the more likely that an atom will miss the zone diametrically opposed to that of its partner.

Since we have measured the correlation function for back-to-back momenta, we can develop a model to account for the trend seen in Fig. 3. The back-to-back correlation function was measured to have rms widths of  $0.17 v_{\text{rec}}$  in the radial ( $x$  and  $y$ ) directions, and  $0.02 v_{\text{rec}}$  in the axial ( $z$ ) direction. Making the approximation of neglecting the much smaller axial correlation width, we estimate the probability  $P(N_Z)$  that, given an atom hitting one zone, its partner will hit the diametrically opposite one. This probability decreases as the number of zones increases, and, neglecting all other effects, should result in a variance  $V = 1 - P$  in the number difference. Taking both quantum efficiency and the geometrical hit probability into account we expect  $V = 1 - \eta P$ . The function  $V(N_Z)$  is plotted as the solid line in Fig. 3. The approximate agreement of this simple model with the data leads us to conclude that the above two loss mechanisms account very well for the observed variance. We also get a slightly better lower limit on the quantum efficiency,  $\eta > 12\%$ .

The situation was also analyzed using a stochastic Bogoliubov simulation as in Ref. [23]. The result for the variance is shown as the dashed curve in Fig. 3. The curve is plotted assuming a detector quantum efficiency of 12% as in the simpler model. The simulation shows the

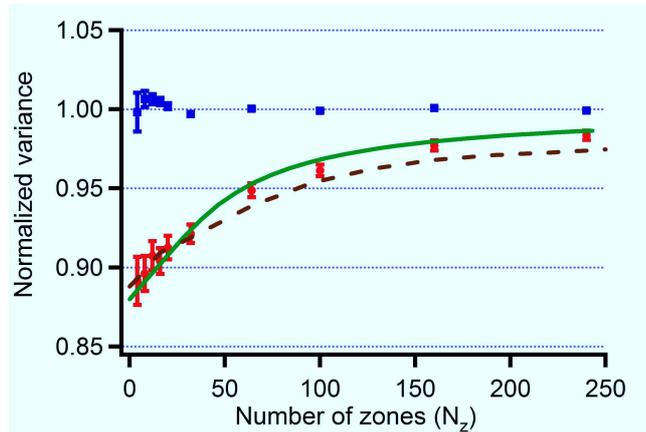


FIG. 3. (Color online) Observed variance, as a function of the number of zones into which we cut the halo. Red circles: average over all correlated zones, blue squares average over all uncorrelated zones. Error bars correspond to the standard deviation of the mean of the variances for a given  $N_Z$ . The solid curve is the predicted variation of the simple model discussed in the text and assuming a 12% quantum efficiency. The dashed curve results from a stochastic Bogoliubov simulation like that in Ref. [23], also assuming a 12% quantum efficiency.

observed trend, but agrees less well with the data than the simple model. The discrepancy arises because the simulation predicts a narrower back-to-back correlation function than was observed in the data which results in a slower approach to unity for the variance. The origin of the discrepancy remains to be resolved but the simulation nevertheless confirms the idea that the lack of perfect correlation in momentum determines most of the variation seen in Fig. 3.

Other known imperfections include the possible contamination of the sphere by atom pairs with one atom in the  $m = 0$  state and another in the  $m = 1$  state. These pairs contribute a single detected atom without a partner to the halo. We have no independent experimental estimate of the number of such collisions but they could account for as much as one half of the observed atoms on the halo. Their presence would mimic a loss in detector quantum efficiency and thus raise our lower limit on  $\eta$ . Spontaneous emission processes act in the same way, but independent measurements indicate that such processes contribute only about 1.5% of the detected atoms on the analyzed halo. As discussed above, the halo centered at  $-v_{\text{rec}}$  was more affected by spontaneous emission, though squeezing on it still also observed, albeit to a lesser degree. While one might hope to improve the quantum efficiency of the detector, or suppress unwanted scattering events, the stochastic Bogoliubov simulation with perfect detection efficiency predicts a limiting variance  $V \approx 0.1$  for a small number of zones. Thus, correcting for the quantum efficiency, the intrinsic squeezing appears to be

at most  $-10$  dB.

Relative number squeezing is also related to the violation of a classical Cauchy-Schwarz inequality [24, 25],

$$\langle N_i N_j \rangle \leq \sqrt{\langle N_i^2 \rangle \langle N_j^2 \rangle}, \quad (2)$$

relating the count rates in two correlated zones  $i$  and  $j$ . For equal count rates in the two zones, relative number squeezing is strictly equivalent to the violation of the inequality (2). The situation in our experiment is more complex because the average count rates are not exactly equal, in which case squeezing and Cauchy-Schwarz violation are not equivalent [26]. Nevertheless, we do indeed observe a violation of the inequality (2). Since the squeezing we describe here is multimode, more sophisticated inequalities can be invoked involving correlation functions of atoms with opposite and parallel momenta. These will be studied in future work.

In order to do interferometry with the production mechanism we have described, a pair of modes must be recombined using a technique such as Bragg diffraction of the atoms by a laser standing wave. Because of the angular selectivity of Bragg reflection, such “mirrors” and beam splitters would also serve the function of the slits in an optical experiment which select a single pair of correlated photon modes [18]. To increase count rates and the number of atoms per mode, one can also use an optical lattice to modify the dispersion relations of the atoms so as to populate a single pair of modes [27–29]. Such well defined twin atom beams would permit the realization of experiments such as the celebrated HOM experiment [30], or the realization of interferometry in the spirit of [8–10]. Even more ambitious would be the demonstration of entanglement of the pairs by making Bell-type measurements of the well separated neutral atoms, in analogy with the measurement made in Ref. [18] using photons.

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