



HAL
open science

Time discontinuous Galerkin method for dynamic crack growth using X-FEM

Julien Réthoré, Anthony Gravouil, Alain Combescure

► **To cite this version:**

Julien Réthoré, Anthony Gravouil, Alain Combescure. Time discontinuous Galerkin method for dynamic crack growth using X-FEM. 11th International Conference on Fracture, Mar 2005, Rome, Italy. hal-00507160

HAL Id: hal-00507160

<https://hal.science/hal-00507160>

Submitted on 26 Mar 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

TIME DISCONTINUOUS GALERKIN METHOD FOR DYNAMIC CRACK GROWTH USING X-FEM

J. Réthoré¹, A. Gravouil¹ and A. Combescure¹

¹ LaMCoS, Laboratoire de Mécanique des Contacts et des Solides
UMR 5514, INSA Lyon
Bat. Jean d'Alembert, 18,20 rue des Sciences 69621 Villeurbanne France

ABSTRACT

In the past few years, lots of techniques were developed for modeling dynamic crack growth. One of the main difficulties is that the discretization of the problem is time dependent. This can produce numerical instabilities, uncontrolled energy tranfers and high frequency oscillations in the solution due to time discontinuities in the numerical model. This paper proposes, in the framework of the eXtended Finite Element Method (X-FEM), a study of Time Discontinuous Galerkin Method (T-DGM). Combining efficient tools like X-FEM and T-DGM , the obtained results are well accurate and will allow to check efficiency for crack initiation, growth and arrest criteria.

1. INTRODUCTION

In the past few years, lots of techniques were developed for modeling dynamic crack growth : EFG (see Krysl [1], Organ [2]), GEM (see Duarte [3]) and various PUM (see Remmers [4], Belytschko [5]). One of the main difficulties is that the discretization of the problem is time dependent. This can produce numerical instabilities, uncontrolled energy tranfers and high frequency oscillations in the solution due to time discontinuities in the numerical model. Authors have proposed an enrichment strategy for X-FEM (Réthoré [6]) that allows stable numerical simulation and energy preservation. The results they obtained let think that the limits of classical Newmark time scheme are reached. It means that high frequency oscillations appears in the solution because such time integrator are not well appropriate for time discontinuities. This paper proposes, in the framework of the eXtended Finite Element Method (X-FEM), a study of Time Discontinuous Galerkin Method (T-DGM) (see for e.g Michler [7] or Li [8]). We first describe the space discretization of the model and then the time integration. The results are compared with those obtained using a Newmark type scheme and seem promising.

2. SPACE DISCRETIZATION

In the method presented, we use the eXtended Finite Element Method first introduced in Black [9]. In this method, an enrichment is added to the classical finite element approximation using the PUM developed in Babuska [10]. For static problem, the displacement field can be written with enriched basis of shape functions :

$$U = \sum_{i \in \mathcal{N}} N_i(x) U_i + \sum_{i \in \mathcal{N}_{cut}} N_i(x) H(x) a_i + \sum_{i \in \mathcal{N}_{branch}} \sum_{\alpha} N_i(x) B_{\alpha}(x) b_{i,\alpha} \quad (1)$$

where \mathcal{N} is the set of all nodes in the mesh, \mathcal{N}_{cut} the set of nodes which belong to elements completely cut by the crack and \mathcal{N}_{branch} the set of nodes which belong to elements partially cut by the crack. N_i are the classical shape functions, H is a discontinuous function which value is

1 if x is above the crack surface and -1 if x is below. $[B_\alpha]$ are branch functions ((r, θ) are local cylindrical crack tip coordinates):

$$[B_\alpha] = \left[\sqrt{r} \sin\left(\frac{\theta}{2}\right), \sqrt{r} \cos\left(\frac{\theta}{2}\right), \sqrt{r} \sin\left(\frac{\theta}{2}\right) \sin(\theta), \sqrt{r} \cos\left(\frac{\theta}{2}\right) \sin(\theta) \right] \quad (2)$$

In our approach, all fields (displacement, velocity and acceleration) are discretized with Equation (1). Consequently, the discrete enriched problem can be written as a classical dynamic problem in the scope of the Finite Element Method : we define the mass matrix and the stiffness matrix in the usual manner. New space shape functions are added to simulate the crack growth. As shown by Figure 1, new singular enrichment is added on the new set $\mathcal{N}_{branch}^{n+1}$. For discontinuous enrichment, new shape functions are only added on the set $\mathcal{N}_{cut}^{new} = \mathcal{N}_{cut}^{n+1} \setminus \mathcal{N}_{cut}^n$. This strategy is stable and energy preserving (see Réthoré [6]) referring to theoretical studies of Newmark type schemes for time dependent discretization (see Réthoré [11]).

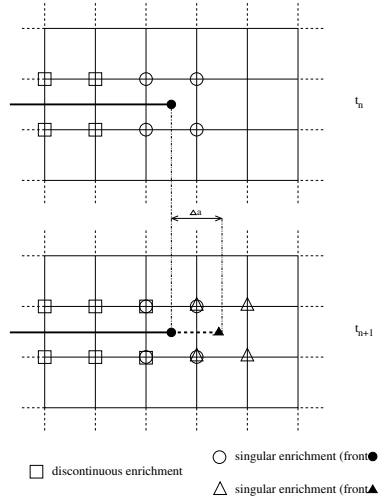


Figure 1. Strategy for enrichment when the crack propagates

3. TIME INTEGRATION

The formulation we choose is from Michler [7] and generalized for X-FEM space discretization. As soon as the formulation is velocity-based, the velocity is interpolated using a piecewise-linear approximation. Inside a time-slab $I_n = [t_n^+; t_{n+1}^-]$ we have :

$$V(t) = V_n \lambda_n(t) + V_{n+1} \lambda_{n+1}(t) \quad (3)$$

where λ_i are linear functions. The kinematic relation $\dot{U} = V$ and displacement continuity at t_n are strongly enforced. Velocity continuity at t_n is weakly enforced and the variational statement for I_n is written: find a linear function V such that for all linear test function δV we have:

$$\int_{I_n} \delta V(t) \left[M \dot{V}(t) + K \left(U(t_n^-) + \int_{t_n^+}^t V(\tau) d\tau \right) - F_{ext}(t) \right] dt + \delta V(t_n^+) (V(t_n^+) - V(t_n^-)) = 0 \quad (4)$$

This formulation is quasi-equivalent to the classical P3-P1 presented in Li[8] : computed values of velocity and displacement at the discrete instant are the same and the displacement continuity

is ensured. The difference concerns the displacement interpolation and the kinematic relation : here the displacement approximation is P2 because the kinematic relation is strongly enforced, P3-P1 approximation allowed high order approximation but the kinematic relation is only weakly enforced.

4. EXAMPLE

The example we choose is the infinite plate with a semi-infinite crack because the theoretical solution is known (see Freund [12]). Several authors have already treated this example so we can also compare our results to their works. Under those assumptions (infinite plate with a semi-infinite crack), for the geometry described Fig. (2), the analytical solution is only valid for time $t \leq 3t_c = 3H/c_d$ when the reflected stress wave arrives on the crack tip. As the wave reaches the crack, the mode I stress intensity factor can be written for a moving crack :

$$K_1^{dyn}(\dot{a}, t) = \frac{2\sigma_0}{1-\mu} \sqrt{\frac{c_d t(1-2\mu)}{\pi}} \frac{1 - \frac{\dot{a}}{c_r}}{1 - \frac{\dot{a}}{2c_r}} \quad (5)$$

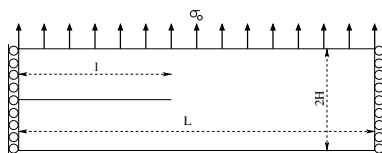


Figure 2. Geometry and loading for the example of the infinite plate

The numerical results are compared with this theoretical one. We will be interested in the case where the crack propagates at a prescribed constant speed v_0 after time $t = 1.5t_c$. The following numerical results are obtained with $H = 2m$, $L = 10m$ and $l = 5m$ for plate dimensions and $E = 210GPa$, $\nu = 0.3$ and $\rho = 8000kgm^{-3}$ for material properties. The tensile stress σ_0 is $500MPa$. v_0 is $1500m.s^{-1}$ and stress intensity factors are normalized by $\sigma_0\sqrt{H}$. Solutions are computed using a 40×80 quadrangle elements with linear approximation.

Results are presented on Figure 3. As shown in Michler [7], the convergence of the T-DGM formulation allowed us to use an approximately four time larger time step for the same accuracy. Using a Newmark type scheme (average acceleration method, $\gamma = \frac{1}{2}$, $\beta = \frac{1}{4}$), the slope is well captured but oscillations appear when the crack is growing. Such oscillations are not observed with the T-DGM and the solution remains satisfying.

5. CONCLUSION

This paper presents a combination of X-FEM and T-DGM for dynamic crack propagation problem. We propose an enrichment strategy for time dependent problems with X-FEM which is stable and energy preserving (see Réthoré [6]). The time integration is treated with T-DGM using a velocity-based formulation which ensures displacement continuity at the discrete instant. The results are compared with Newmark time integrator which seems to reach its limits for such time discontinuous problems. Combining efficient tools like X-FEM and T-DGM, the obtained results are accurate and will allow to check efficiency for crack initiation, growth and arrest criteria.

REFERENCES

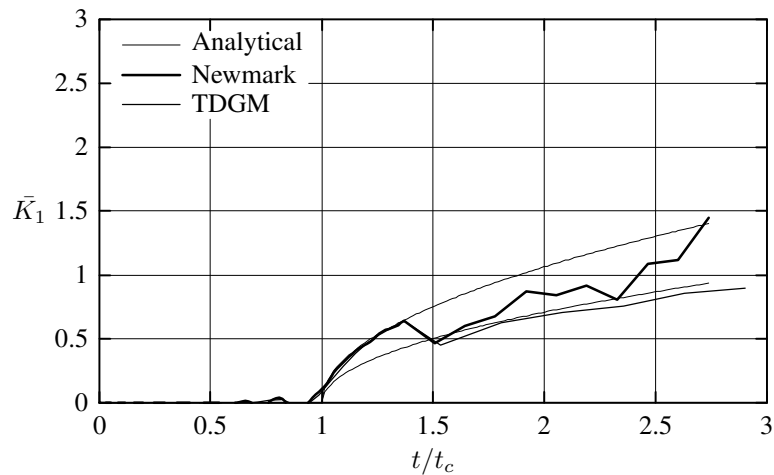


Figure 3. Numerical and analytical solutions \bar{K}_I for a stationary then moving crack

1. P. Krysl and T. Belytschko. Dynamic propagation of arbitrary 3-d cracks. *International Journal for Numerical Methods in Engineering*, 44(6):767–800, 1999.
2. D. Organ. *Numerical Solutions to Dynamic Fracture Problems Using the Element-Free Galerkin Methods*. PhD thesis, Northwestern University, 1996.
3. Duarte CA, Hamzeh ON, Liska TJ, and Tworzydło WW. A generalized finite element method for the simulation of three-dimensional dynamic crack propagation. *Computer Methods in Applied Mechanics and Engineering*, 190:2227–2262, 2001.
4. Remmers JJC, de Borst R, and Needleman A. A cohesive segments method for the simulation of crack growth. *Computational Mechanics*, 31:69–77, 2003.
5. Belytschko T, Chen H, Jingxiao X, and Goangseup Z. Dynamic crack propagation based on loss of hyperbolicity and a new discontinuous enrichment. *International Journal for Numerical Methods in Engineering*, 58:1873–1905, 2003.
6. J. Réthoré, A. Gravouil, and A. Combescure. An energy conserving scheme for dynamic crack growth with the extended finite element method. *International Journal for Numerical Methods in Engineering*, submitted.
7. C. Michler, E.H. van Brummelen, S.J. Hulshoff, and R. de Borst. The relevance of conservation for stability and accuracy of numerical methods for fluid-structure interaction. *Computer Methods in Applied Mechanics and Engineering*, 192:4195–4215, 2003.
8. Xikui Li, Dongmei Yao, and R.W. Lewis. A discontinuous galerkin finite element method for dynamic and wave propagation problems in non-linear solids and saturated porous media. *International Journal for Numerical Methods in Engineering*, 57:1775–1800, 2003.
9. Black T and Belytschko T. Elastic crack growth in finite elements with minimal remeshing. *International Journal for Numerical Methods in Engineering*, 45:601–620, 1999.
10. I. Babuska and JM. Melenk. The partition of unity method. *International Journal for Numerical Methods in Engineering*, 40:727–758, 1997.
11. J. Réthoré, A. Gravouil, and A. Combescure. A stable numerical scheme for the finite element simulation of dynamic crack propagation with remeshing. *Computer Methods in Applied Mechanics and Engineering*, to appear.
12. L. B. Freund. *Dynamic fracture mechanics*. Cambridge Monographs on Mechanics and Applied Mathematics, 1990.