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# Multimodal vibration control by using piezoelectric transducers and passive circuits

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## Abstract

In this paper we solve the problem of finding an optimal passive electrical controller of beam vibrations exploiting distributed piezoelectric transducers. To this aim an electrical circuit analog to Euler beam is synthesized using a Lagrangian method and by paralleling capacitive flux linkages to rotations and transverse displacements. Therefore a piezo-electromechanical (PEM) beam is conceived by uniformly distributing piezoelectric transducers on a beam and interconnecting their electric terminals via the found analog circuit, completed with suitable resistors. The high performance features of the synthesized novel circuit include: i) the circuit topology is extremely reduced, the used components are all but one two terminal passive elements, and the only two port network needed is an ideal transformer; ii) one and the same dissipative circuit assures a multi resonance coupling with the vibrating beam and the optimal electrical dissipation of mechanical vibrations energy.

## 1 Introduction

### 1.1 The state of the art

Among the first applications conceived for piezoelectric transducers in the field of active vibrations control there was the so called "electronic damping" (see e.g. Forward [1], Hanagud et al [2]): a set of piezoceramic devices is placed on a host

structure to sense and control the mechanical vibrations. The deformation of the sensing transducers results in electrical signals, which are conditioned by suitably designed feedback electronics and then, once amplified, applied to actuating transducers; the actuators convert the applied electrical energy into mechanical energy, transmitting mechanical control actions to the host structure. When active control systems are used, the piezoelectric transducers driving requires complex power-amplifiers and associated precise sensing electronics, resulting into the consumption of a significant amount of electrical power.

In Hagood and von Flotow [3] the possibility of damping mechanical vibrations by means of a piezoelectric transducer positioned on a structural element and shunted with completely passive electrical circuits is investigated. In particular two different shunting circuits are considered: a resistive (R) and a resonant (LR) one. The method proposed in Hagood and von Flotow [3] allows for an efficient single-mode control of structural vibrations, whenever the resonant circuit is tuned to the mechanical mode to be suppressed; nevertheless the efficiency of the electromechanical coupling strongly depends on the position of the transducer over the host structure. Moreover the technical feasibility of the passive piezoelectric controller proposed in Hagood and von Flotow [3] is limited, since too large inductors are required to produce low-frequency electrical resonance standing the small inherent capacitance of the available piezoelectric transducer.

In (dell'Isola and Vidoli [4], Alessandroni et al. [5]) it is proposed to control beam vibrations interconnecting an array of uniformly distributed piezoelectric transducers to an electrical waveguide. Such an electrical propagation medium must be designed in order to support an electrical signal the evolution of which is governed by a differential operator with a spectrum as close as possible to the spectrum of the operator governing the mechanical vibrations. Indeed when this condition on spectral properties of the electromechanical evolution operators is fulfilled, the energy transduction is the most efficient (see Vidoli and dell'Isola [6]). However all the electric circuits employed as vibrations controllers must be fed, i.e. are constituted by active elements.

## 1.2 Problem statement and objectives

In the present paper we develop the ideas exposed in (Alessandroni et al. [7], dell'Isola et al. [8]) in order to find a truly passive controller constituted by inductors, resistors and elementary two port transformers. Indeed there are relevant technological feasibility problems when one wants to exploit piezoelectric transduction to efficiently electrically dissipate the mechanical vibration energy: we prove in the following (by using the results in Andreus et al. [9]) that by using commercially available transducers and low-cost, low-weight electric components it is possible to achieve the multimodal damping of a specific aluminum beam (for the properties of which we refer to section 3).

Therefore in section 2 we address the problem of synthesizing an electrical circuit which is the analog of the Euler beam. To this aim as the theory of circuits supplies synthesis methods suitable to find lumped circuits governed by finite dimensional Lagrangian equations, we formulate in a Lagrangian form the evolution

equations of an Euler beam and spatially discretize its Lagrangian functional. We chose to do it via a standard Euler finite differences method (see Richards [10]).

In section 3 a piezo-electromechanical beam is designed interconnecting an array of piezoelectric bender transducers bonded to a host beam via the found completely passive analog circuit. Assuming that a large number of transducers is positioned over the structural element a continuous model for the PEM beam is adopted, and governing evolution equations in terms of the involved kinematical electromechanical descriptors are derived and discussed.

In section 4 the analysis of a simply supported PEM beam is performed via a standard modal analysis. An optimality criterion for the damping of vibrations is proposed and a proper dissipative networks is added to the analog circuit to accomplish the optimality condition. The achieved electrical dissipation will be shown to be independent of the beam vibration mode, confirming the efficiency of PEM beams in the damping and control of multimodal vibrations.

## 2 Synthesis of the analog circuit of the Euler beam

In this section an analog circuit of the vibrating Euler beam is synthesized. The method proposed exploits the basic version of the Euler finite difference method and is articulated as follows:

- i. we recall the variational principle governing the free vibrations of Euler beams,
- ii. we define a grid over the beam reference configuration and introduce- as a set of Lagrange coordinates- the sampled values of the fields describing the beam kinematics at the grid nodes,
- iii. we approximate the infinite dimensional Lagrangian for Euler beam via a finite dimensional Lagrangian in terms of introduced Lagrange coordinates,
- iv. with a well-known procedure (Crandall et al. [11]) we synthesize a lumped electrical circuit, the evolution of which is governed by the so found finite dimensional Lagrangian.

Every material particle of the considered bending Euler beam is labelled by an abscissa (varying in the interval  $[0, l]$  where  $l$  is the length) in its reference configuration and its state is characterized by a transverse displacement  $u$  and a change of attitude  $u'$  (unshearable beam hypothesis).

Let us subdivide the interval from 0 to  $l$  on the  $x$  axes putting equally spaced points  $\Delta x = l/N$  units apart, where  $N$  is the number of intervals (see figure 1), and label

$$x_j = j\Delta x, \quad (1)$$

for  $j = 0, \dots, N$ , so  $x_0 = 0$  and  $x_N = l$ .

The Hamilton's principle is given as:

$$\delta \int_{t_0}^{t_1} L dt = 0, \quad (2)$$

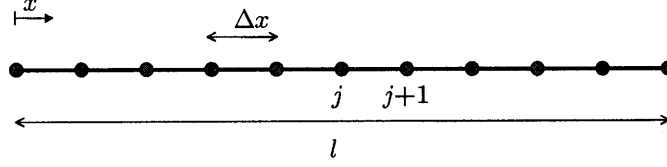


Figure 1: Discretization grid on the beam.

where the Lagrangian is given by

$$L = K - E - W_{in},$$

where  $K$  is the total kinetic energy,  $E$  is the total strain energy,  $W_{in}$  is the total input of external energy and  $t_0$  and  $t_1$  are two arbitrary instants in time. For an Euler beam excited only at the edges, the Lagrangian reads:

$$L = \int_0^l k(\dot{u}) dx - \int_0^l e(u'') dx - T_0 u(0) - T_l u(l) - M_0 u'(0) - M_l u'(l), \quad (3)$$

where the density of kinetic energy  $k$  and the density of elastic strain energy  $e$  are given by:

$$\begin{cases} k(\dot{u}) = \frac{1}{2} \rho \dot{u}^2 \\ e(u'') = \frac{1}{2} K_M (u'')^2 \end{cases}, \quad (4)$$

and  $T_0, T_l, M_0, M_l$  represent the transversal loads and bending moments applied at the edges of the beam,  $\rho$  the mass density per unit length,  $K_M$  the bending stiffness. In order to synthesize the electrical analog of the Euler beam, we exploit an Euler finite differences approximation of the Lagrangian in (3), and parallel the kinematical descriptors rotation and deflection sampled at the grid nodes with the flux linkages of a  $2N + 2$  nodes electrical circuit (measured with respect to a common reference ground). Following Andreaus et al. [9] the electric analog of the Euler beam is found to be that one depicted in figure 2, where the electrical elements are determined by the relation:

$$\frac{Ln^2 C}{(\Delta x)^4} = \frac{\rho}{K_M}. \quad (5)$$

Let us observe that the synthesis of the boundary circuital elements is essentially done by paralleling the applied edge loads with applied current sources, see Andreaus et al. [9]. The governing equations for the internal nodes become:

$$C\ddot{\psi}_j + \frac{1}{Ln^2} (\psi_{j+2} - 4\psi_{j+1} + 6\psi_j - 4\psi_{j-1} + \psi_{j-2}) = 0, \quad j = 2, \dots, N - 2, \quad (6)$$

which is immediately recognized as a central finite difference approximation of the governing equation of the Euler beam.

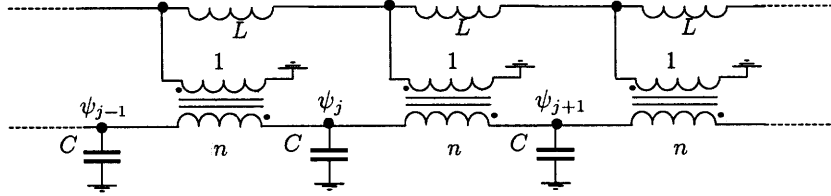


Figure 2: Electrical analog of the Euler beam.

### 3 PEM beam

In the following section we will define a novel smart structure which we have called (see dell'Isola and Vidoli [4], Alessandroni et al. [5], Alessandroni et al. [7], dell'Isola et al. [8], Andreus et al. [9]) piezo-electromechanical beam, making use of the analog circuit designed in the previous section interconnecting (see figure 3) an array of uniformly distributed bender piezoelectric transducers.

The piezoelectric layer positioned over the structural element, drastically modify the constitutive equation for the bending moment in

$$M(x, t) = K_M u''(x, t) + k_{me} \dot{\psi}(x, t) \quad (7)$$

where  $\psi$ , assuming the number of transducers to be sufficiently large, denotes the flux-linkage field, defined over the entire beam span. From an electrical point of view the following electrical constitutive equation for the charge stored per unit length arises:

$$q(x, t) = \frac{k_{ee}}{\Delta x} \dot{\psi}(x, t) - k_{me} u''(x, t) \quad (8)$$

The constants  $k_{me}$ ,  $K_M$  and  $k_{ee}$  are determined by the geometrical and electromechanical properties of the beam and the transducers (see Andreus et al. [9]): they respectively represent the piezoelectric coupling coefficient, bending stiffness, and piezoelectric capacitance.

#### 3.1 Non dissipative PEM beam

##### 3.1.1 Deduction of evolution equations

Interconnecting the bender transducers as the capacitors in figure 2 (which specifies the meaning of the symbols when  $C = k_{ee}$ ), the homogenized governing equation for the mechanically fed analog circuit is:

$$\frac{(\Delta x)^3}{Ln^2} \psi^{IV} + \frac{k_{ee}}{(\Delta x)} \ddot{\psi} - k_{me} \dot{u}'' = 0. \quad (9)$$

Considering the constitutive equation (7), the equation for the beam free vibrations in terms of the deflection becomes:

$$K_M u^{IV} + \rho \ddot{u} + k_{me} \dot{\psi}'' = 0, \quad (10)$$

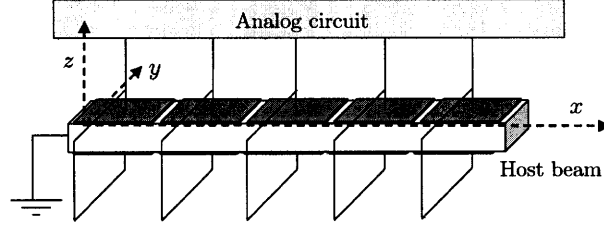


Figure 3: PEM beam.

where  $\rho$  is the density of mass per unit length of the PEM beam.

In order to guarantee an efficient multimodal control based on internal resonance it is necessary, as proved in Vidoli and dell'Isola [6], to tune the electrical circuit so as to satisfy eqn (5) with  $C = k_{ee}$ . Furthermore it is very useful to non dimensionalize the previous coupled equations in order to estimate the efficiency of the gyroscopic coupling assured by the coefficient  $k_{me}$ . Thus introducing as usual the scaling parameters  $\tilde{u}$ ,  $\tilde{\psi}$ ,  $l$ ,  $\varpi$  for the deflection, the flux linkage, length and frequency, the dimensionless coupled equations become:

$$\begin{cases} u^{IV} + \alpha^4 \ddot{u} + \beta^2 \dot{\psi}'' = 0 \\ \psi^{IV} + \alpha^4 \ddot{\psi} - \beta^2 \dot{u}'' = 0 \end{cases}, \quad \begin{cases} \alpha^4 = \frac{\varpi^2 \rho l^4}{K_M} \\ \beta^2 = \frac{\varpi l^2 k_{me}}{K_M} \sqrt{\frac{\rho}{\gamma}} \\ \frac{\tilde{\psi}}{\tilde{u}} = \sqrt{\frac{\rho}{\gamma}} \end{cases}, \quad (11)$$

where  $\gamma$  denotes the capacitance per unit length and with an abuse of notation the dimensional and dimensionless variables are denoted with the same symbols.

### 3.1.2 Design of a prototype of PEM beam

In order to assess the physical realizability and the efficiency of the proposed device, let us consider an aluminum beam, the geometry of which is presented in table 1.

Let us position 9 bender transducers constituted by piezoceramic patches (T110-H4E-602 (PSI-5H4), (see Piezo Systems [12])) made of lead zirconate titanate [PZT]. The characteristics of these piezoceramic transducers are presented in table 2.

The step for the finite difference grid is given by:

$$\Delta x = l/9 \simeq 56.667 \times 10^{-3} m. \quad (12)$$

Using transformers the turns ratio of which is equal to 10 the inductances to be used should be:

$$L \simeq 6.587 mH. \quad (13)$$

Let us explicitly remark that the extremely reduced value of the used inductances and of the transformers turns ratio lead to a completely passive realization

Table 1: Properties and dimensions of the host beam.

Coefficient	Value	Units
$l$	$510 \cdot 10^{-3}$	$m$
$w$	$40 \cdot 10^{-3}$	$m$
$h$	$4 \cdot 10^{-3}$	$m$
$E$	$70 \cdot 10^9$	$Pa$
$\rho$	2700	$kg/m^3$

of the whole electrical analog network.

### 3.2 Dissipative PEM beam

Let us introduce in the coupled set of dimensionless homogenized evolution equations an electrical dissipation of the following differential form:

$$D[\dot{\psi}] = \sum_{k=0}^M y_{2k} \frac{\partial^{2k} \dot{\psi}}{\partial x^{2k}}, \quad (14)$$

where the constant  $M$  is a prescribed number and  $y_{2k}$  are real constants; thus the governing equations for the dissipative PEM beam are:

$$\begin{cases} u^{IV} + \alpha^4 \ddot{u} + \beta^2 \dot{\psi}'' = 0 \\ \psi^{IV} + \alpha^4 \ddot{\psi} - \beta^2 \dot{u}'' - D[\dot{\psi}] = 0 \end{cases} \quad (15)$$

The class of dissipation operators (14) has been chosen for the following reasons:

- i. as shown in Andreaus et al. [9] it is easy to synthesize a circuit governed by the equation (15)<sub>2</sub> with  $\beta = 0$ ;
- ii. in the case of a simply supported PEM beam it represents a proportional damping;
- iii. as it will be shown in section 4, in it one can find a dissipation operator which is optimal simultaneously for all vibration modes.

In the following section we will find (according to a well defined optimality criterion) the optimal form of the electrical dissipation operator  $D[\dot{\psi}]$ ; once such an expression is determined the synthesis problem of a dissipative network governed by the so found optimal form of eqn (15)<sub>2</sub> will be addressed.

## 4 Application sample: simply supported PEM beam

In the present section we address the problem of damped free vibrations of a simply supported PEM beam via a modal analysis method (subsection 4.1). Subsequently (subsection 4.2) we consider purely electrically dissipative PEM beams and determine the optimal dissipation operator in the specified class (14), the associated circuit topology and consequently the required optimal resistances.



Table 2: Properties and dimensions of the T110-H4E-602 (PSI-5H4).

Coefficient	Value	Units
$\rho_p$	7800	$kg/m^3$
$(s_{11}^E)^{-1}$	$62 \cdot 10^9$	$Pa$
$d_{31}$	$320 \cdot 10^{-12}$	$m/V$
$\epsilon_3^T$	$3800 \cdot 8.85 \cdot 10^{-12}$	$F/m$
$\delta$	$0.267 \cdot 10^{-3}$	$m$

#### 4.1 Modal analysis

In this subsection a modal analysis for a simply-supported P<sub>1</sub>EM beam, based upon the Galerkin method, is performed. The set of basis functions chosen to represent the deflection  $u(x, t)$  and the flux linkage  $\psi(x, t)$  of the P<sub>1</sub>EM beam is constituted by the eigenfunctions  $\varphi_m(x)$  of the spatial differential operator determining the modal forms for both the uncoupled motions:

$$\varphi_m(x) = \sqrt{2} \sin(m\pi x), \quad m = 1, 2, \dots \quad (16)$$

Consequently a Fourier expansion for the solution of eqns (15) is considered:

$$\begin{bmatrix} u(x, t) \\ \psi(x, t) \end{bmatrix} = \sum_{m=1}^{\infty} \begin{bmatrix} p_m(t) \\ q_m(t) \end{bmatrix} \varphi_m(x), \quad (17)$$

where  $p_m(t)$  and  $q_m(t)$  are respectively the mechanical and electrical Fourier coefficients of the expansion. With simple calculation the following system for the ODEs governing the time evolution of a generic pair of electrical and mechanical Fourier coefficients is obtained:

$$\frac{d}{dt} \begin{bmatrix} p_m(t) \\ q_m(t) \\ \dot{p}_m(t) \\ \dot{q}_m(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_m & 0 & 0 & c_m \\ 0 & -a_m & -c_m & -d_m \end{bmatrix} \begin{bmatrix} p_m(t) \\ q_m(t) \\ \dot{p}_m(t) \\ \dot{q}_m(t) \end{bmatrix}, \quad (18)$$

with:

$$a_m = \frac{(m\pi)^4}{\alpha^4}, \quad c_m = \frac{\beta^2 (m\pi)^2}{\alpha^4}, \quad d_m = \frac{P_D(m)}{\alpha^4}, \quad (19)$$

where the polynomial  $P_D(m)$  is defined by:

$$P_D(m) = \sum_{k=0}^M (-1)^{k+1} y_{2k} (m\pi)^{2k}. \quad (20)$$

Let us explicitly remark that because of the performed choices we got a set of uncoupled evolution problems for each electromechanical mode. Thus it is possible to easily derive the analytical solution for the Fourier coefficients: otherwise cumbersome numerical analysis should be performed. Let us remark that the dissipative parameter  $d_m$  depends linearly on the set of dimensionless damping parameters  $y_i$ .

## 4.2 Resistance optimization by pole placement technique

A reasonable possibility for choosing the coefficient  $y_{2k}$  in the dissipation operator (14) to be employed in the electrical analog is provided by a different optimality criterion that is the pole placement technique widely discussed in Alessandrini et al. [7]. Once it is defined the  $m$ -th modal time rate decay

$$\tau_m(y_0, \dots, y_M) = \min_{i=1, \dots, 4} [|\operatorname{Re}[\lambda_i^m]|], \quad (21)$$

where  $\lambda_i^m$  are the eigenvalues of the state space matrix  $A_m$  in eqn (18), it has to be required that the chosen values  $\{y_i\}$  must maximize  $\tau_m$ .

Considering the eigenvalues of the generic state matrix  $A_m$  it is easy to show that  $\tau_m$  is maximized when

$$d_m = 2c_m, \quad (22)$$

and in this case there are only two complex conjugate eigenvalues of multiplicity two (see e.g. Miller and Crawley [13]). The maximum time rate decay  $\tau_m^{opt}$  can be easily evaluated to be equal to  $c_m/2$ . The imaginary parts of the eigenvalues  $\lambda_i^m$  evaluated at  $d_m = 2c_m$  are recognized to be equal to  $\pm\sqrt{a_m - (c_m/2)^2}$ . Hence, the damping ratio evaluated when condition (22) hold, becomes:

$$\zeta_m = \frac{c_m}{2\sqrt{a_m}} = \frac{k_{me}}{2} \sqrt{\frac{1}{K_M \gamma}} := \zeta; \quad (23)$$

which does not depend on the mode number  $m$ .

It is possible to accomplish this optimality condition for each mode by means of a unique choice of the resistances. In fact choosing  $d_m = 2c_m$  we get:

$$\frac{P_D(m)}{\alpha^4} = 2(m\pi)^2 \frac{\beta^2}{\alpha^4}, \quad (24)$$

which can be satisfied, according to (20), for every mode choosing:

$$\begin{cases} y_{2k}^{opt} = 0, & k = 0, 2, 3, \dots, M \\ y_2^{opt} = 2\beta^2 \end{cases} \quad (25)$$

The interconnection to be used is simply constituted by a set of resistors interconnecting the PZT capacitors, the value of these resistors is determined by:

$$R_2^{opt} = \frac{L_1 n^2}{y_2^{opt} (\Delta x)^2} \omega l^2 = \frac{1}{2} \frac{(\Delta x)}{k_{me}} \sqrt{\frac{\rho}{\gamma}}. \quad (26)$$

For the PEM beam designed in the previous section the pole placement optimization criterion yields:

$$R_2^{opt} \simeq 2.415 k\Omega. \quad (27)$$

In figure 4 we show a detail of the first mode root locus in the neighborhood of the bifurcation (critical) point, which correspond to the optimal value for the resistance, as the dissipative parameter  $d_1$  varies from 0 to infinity.

The optimal damping ratio achieved by the pole placement criterion is equal to

$$\zeta \simeq 12.34\%. \quad (28)$$

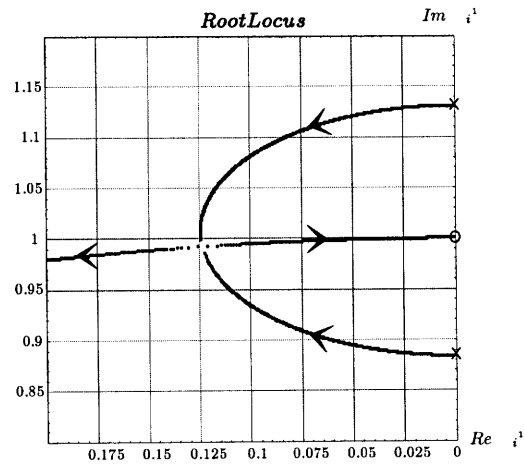


Figure 4: First mode root locus for increasing value of the damping parameter  $\gamma_2$ .

## 5 Conclusions

A novel completely passive device for multimodal structural vibration damping based on distributed piezoelectric effect has been conceived and the design of a specific structural element has been performed. The novel passive smart structure consists of a host beam, an array of bender transducers uniformly distributed on it and an analog passive circuit completed by dissipative elements interconnecting the terminals of the piezo transducers. Such an analog passive circuit can be regarded as a distributed electric controller for mechanical vibrations. The introduced PEM beam allows for an efficient multimodal vibration damping and control in despite of inductances of extremely reduced value in the analog circuit and low-cost piezoelectric transducers. The design of an experiment to validate the novel concepts and methodology is addressed for an aluminum beam, considering available PZT transducers and practically interesting results are exhibited via analytical and numerical techniques. With a view towards the construction of a demonstrative prototype it has to be remarked that:

- i. the topology of the analog circuit is extremely simple;
- ii. the analog circuit can be entirely realized by means of light, low-cost, passive elements, dispensing the use of any external feeding;
- iii. an efficient multimodal control of mechanical vibrations is guaranteed;
- iv. mechanical vibrations are rapidly suppressed by means of suitable resistors properly inserted in the analog circuit.

## References

- [1] Forward, R. L. and Swigert, G. J., Electronic Damping of Orthogonal Bending Modes in a Cylindrical Mast—Theory. *Journal of Spacecraft and Rockets*, **18**, pp. 5-10, 1981.
- [2] Hanagud, S., Obal, M. W., and Meyyappa, M., Electronic Damping Techniques and Active Vibration Control. *Proceedings of the 26th AIAA/ASME/ASCE/AHS 26th Structures, Structural Dynamics and Materials Conference*, Orlando, Florida, April (AIAA Paper 85-0752), 1985.
- [3] Hagood, N. W. and von Flotow, A., Damping of structural vibrations with piezoelectric materials and passive electrical networks. *Journal of Sound and Vibrations*, **146**, pp. 243-268, 1991.
- [4] dell'Isola, F., Vidoli, S., Continuum modelling of piezoelectromechanical truss beams: an application to vibration damping. *Archive of Applied Mechanics*, **68**, pp. 1-19, 1998.
- [5] Alessandrini, S., dell'Isola, F., Frezza, F., Optimal Piezo-Mechanical Coupling to Control Plate Vibrations. To appear in *International Journal of Applied Electromagnetics and Mechanics*.
- [6] Vidoli, S., dell'Isola, F., Modal coupling in one-dimensional electromechanical structured continua. *Acta Mechanica*, **141**, pp. 37-50, 2000.
- [7] Alessandrini, S., dell'Isola, F., Porfiri, M., A revival of electric analogs for vibrating mechanical systems aimed to their efficient control by PZT actuators. *International Journal of Solids and Structures*, **39**(20), pp.5295-5324, 2002.
- [8] dell'Isola, F., Henneke, E. G., Porfiri, M., Synthesis of electrical networks interconnecting PZT actuators to damp mechanical vibrations. To appear in *International Journal of Applied Electromagnetics and Mechanics*.
- [9] Andreaus, U., dell'Isola, F., Porfiri, M., Piezoelectric passive distributed controllers for beam flexural vibrations. To appear in *Journal of Vibration and Control*.
- [10] Richards, T. H., *Energy methods in stress analysis: with an introduction to finite element techniques*, Halsted Press: New York, pp. 219-220, 1977.
- [11] Crandall, S. H., Karnopp, D. C., Kurtz, E. F. Jr., and Pridmore-Brown, F. C. *Dynamics of Mechanical and Electromechanical systems*, McGraw-Hill, New York, 1968.
- [12] Piezo Systems, Inc., Cambridge, Massachusetts, web site:  
<http://www.piezo.com>.
- [13] Miller, D. W., and Crawley, E. F., Theoretical and experimental investigation of space-realizable inertial actuation for passive and active structural control. *Journal of Guidance, Control and Dynamics*, **11**(5), pp. 449-458, 1988.