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Distributed electric absorbers of beam vibrations

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ABSTRACT

Several electric vibration absorbers based on distributed piezoelectric control of beam vibrations are studied. The damping devices are conceived by interconnecting with different modular electric networks an array of piezoelectric transducers uniformly distributed on a beam. Five different vibration absorbers made of five different network interconnecting topologies are considered and their damping performances are analyzed and compared. The analysis is based on homogenized models of modular piezo-electromechanical systems. The optimal parameters of these absorbers are found by adopting the criterion of critical damping of waves with a single wave number. We show that: i) there is an interconnecting network providing an optimal multimodal damping; ii) the performances required to the electr(on)ic components can be significantly decreased by increasing the number (and decreasing the dimensions) of the piezoelectric transducers.

Keywords: piezoelectric transducers, distributed vibration absorbers, electric damping, passive vibration damping, electromechanical coupling, piezoelectric shunting, electromechanical waves.

1. INTRODUCTION

Passive vibration control can be achieved by coupling a mechanical structure S to an auxiliary dissipative system S' by means of an appropriate coupling device T . Once the properties of T are given, a proper design of the auxiliary system S' allows to enhance the energy exchange between S and S' and the energy dissipation in S' by exploiting an internal resonance phenomenon in (S, T, S') . The electromechanical coupling provided by piezoelectric transducers allows for using *electric* additional systems to damp vibrations of mechanical structures. For example, a given vibration mode of a beam can be damped by positioning on it a piezoelectric transducer shunted to a resistor and an inductor (resonant shunted piezoelectric transducer), as shown by Hagood and Von Flotow¹ and Hollkamp². In this paper, we analyze *distributed* vibration absorbers for beam vibrations in which the additional system S' is a distributed electric network and the coupling is attained through distributed piezoelectric transducers. These systems can be designed by interconnecting an array of piezoelectric transducers bonded on a beam by suitable RL electric networks, aiming at synthesizing a smart structure consisting of an electromechanical continuum the damping properties of which are controlled by electric parameters (see Vidoli and dell'Isola^{3,4} and Alessandroni et al.⁵). We analyze and compare the optimal damping performances achievable by means of five different interconnecting networks which are synthesized in order to realize, together with the piezoelectric transducers, electric auxiliary systems with assigned equations of motion.

If an Euler model is accepted, the beam flexural vibrations are governed by a fourth order differential equation of the type (here and henceforth $\square' = \partial/\partial x$, $\square = \partial/\partial t$)

$$U^{IV}(X, t) + \frac{1}{c_b^2} \ddot{U}(X, t) = 0 \quad (1)$$

where $U(X, t)$ is the beam axis transversal displacement at time t at the point labelled by the abscissa X and c_b is the bending waves phase velocity. We consider the damping performances which can be achieved by

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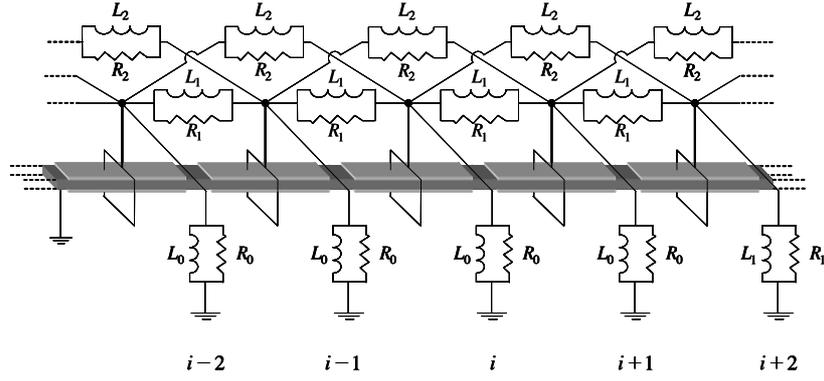


Figure 1: Generic modular PiezoElectroMechanical (*PEM*) beam

piezoelectrically coupling a beam to the electric networks characterized by the following homogenized evolution equations for the electric flux-linkage Ψ (the electric flux-linkage is defined as the time primitive of the electric potential).

$$(Z, Z) \text{ - network: } \ddot{\Psi}(X, t) + \delta_0 \dot{\Psi}(X, t) - \beta_0 \Psi(X, t) = 0 \quad (2a)$$

$$(S, Z) \text{ - network: } \ddot{\Psi}(X, t) + \delta_0 \dot{\Psi}(X, t) - \beta_2 \Psi''(X, t) = 0 \quad (2b)$$

$$(S, S) \text{ - network: } \ddot{\Psi}(X, t) - \delta_2 \dot{\Psi}''(X, t) - \beta_2 \Psi''(X, t) = 0 \quad (2c)$$

$$(F, Z) \text{ - network: } \ddot{\Psi}(X, t) + \delta_0 \dot{\Psi}(X, t) + \beta_4 \Psi^{IV}(X, t) = 0 \quad (2d)$$

$$(F, S) \text{ - network: } \ddot{\Psi}(X, t) - \delta_2 \dot{\Psi}''(X, t) + \beta_4 \Psi^{IV}(X, t) = 0 \quad (2e)$$

The (\cdot, \cdot) - **network** nomenclature above refers to the order of spatial derivatives appearing respectively in the third and second term of the LHS of the homogenized equations of motion (2). For example, the (F, S) - **network** is characterized by a fourth order spatial derivative on Ψ and a second order spatial derivative on $\dot{\Psi}$ ($Z \equiv 0^{th}$, $S \equiv 2^{nd}$, $F \equiv 4^{th}$). In order to synthesize *PiezoElectroMechanical (PEM)* beams in which the electric networks (2) are coupled to the beam vibrations (1) through distributed piezoelectric transducers, it is necessary to find suitable topologies for the electric interconnection of the piezoelectric transducers. To this end, we consider as interconnecting network a one-dimensional *electric lattice with next-to-nearest-neighbour interactions* as defined by Brillouin⁶ (see Fig.1). In section 2, we establish a continuous homogenized model for such a system and we show that the desired *PEM* beams can be obtained by particular choices of the inductors and resistors of the assumed interconnecting network. In section 3, the dynamics of the so obtained electromechanical systems is analyzed, the electric parameters of the different electric networks are optimized and their damping performances are compared. The analysis is based on the study of the properties of wave-form solutions for the homogenized equations of motion.

2. MATHEMATICAL MODELLING

In this section homogenized equations of motion for the modular *PEM* beam in Fig.1 are derived. We introduce two mathematical descriptions of such a system: a *micro model*, which regards the system as an axially non homogeneous layered piezoelectric beam coupled to a lumped electric network; a *macro model*, which regards the system as an electromechanical microstructured continuum the constitutive properties of which are determined by those of the basic module (see Eringen⁷). In the micro model, the system dynamics is expressed by a partial differential equation coupled to a set of ordinary differential equations; in the macro model by two coupled partial differential equations. In the following sections, the dynamic analysis is based on the homogenized model. However, the description at micro level is necessary to rationally deduce the macro constitutive properties. To this aim, we adopt the identification method in virtual powers presented by Woźniac.⁸ This method starts

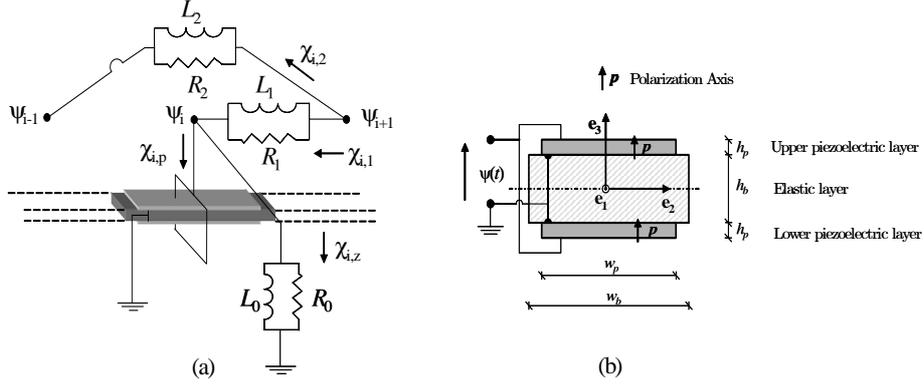


Figure 2. Generic modular *PEM* beam: (a) basic module; (b) beam cross section: piezoelectric layers in bending bimorph configuration.

from the weak formulation of evolutionary problems based on D'Alembert principle of virtual powers (for its modern presentation see Maugin⁹).

2.1. Kinematics

The *PEM* beam can be regarded as a modular system composed by the assembly of basic modules like the one in Fig.2. In each module a pair of piezoelectric transducer is bonded in the bimorph bending configuration. In the following, the beam axis is denoted by \mathcal{A} and the part of the beam axis in the i -th module by \mathcal{A}_i . The basic module length is denoted by l_b .

In the micro model the kinematical state of the *PEM* beam is specified by the transverse displacement field $u : \mathcal{A} \times \mathbb{R} \rightarrow \mathbb{R}$ and by a set of nodal flux-linkages $\{\psi_h : \mathbb{R} \rightarrow \mathbb{R}\}$. In the homogenized model the kinematical state is specified by the macro scalar fields

$$U : \mathcal{A} \times \mathbb{R} \rightarrow \mathbb{R} \quad \Psi : \mathcal{A} \times \mathbb{R} \rightarrow \mathbb{R} \quad (3a)$$

representing the homogenized beam axis transversal displacement and flux-linkage, respectively. At macro level, the system is an electromechanical beam which, at each axial point X , has the microstructure in Fig.2: the part \mathcal{A}_i of the beam axis and the corresponding circuitry, should be regarded as an infinitesimal neighborhood of the generic point X . Locating the generic point in \mathcal{A}_i by a micro coordinate $\xi \in [-l_b/2, l_b/2]$, we assume that the micro state of the neighborhood \mathcal{A}_i of the point X is given as a function of the macro kinematical fields through the following expansions

$$u(\xi, t) = U(X, t) + U'(X, t)\xi + U''(X, t)\frac{\xi^2}{2} \quad (4a)$$

$$\psi_i(t) = \Psi(X, t) \quad (4b)$$

$$\psi_{i\pm 1}(t) = \Psi(X, t) \pm \Psi'(X, t)l_b + \Psi''(X, t)\frac{l_b^2}{2} \quad (4c)$$

The relations above can be regarded as a local (around the point X) kinematical map between the micro and macro models. This map assumes that $l_b/\lambda \rightarrow 0$, where λ is the characteristic wavelength of the considered phenomena. Here and henceforth we denote by the upper-case letters the macro quantities, by lower-case the micro ones.

2.2. Micro model

Once the micro kinematical descriptors are introduced, the micro model is specified by defining the internal and external virtual power functionals per module. By denoting with $\dot{u}^* : \mathcal{A}_i \rightarrow \mathbb{R}$ and $\{\dot{\psi}_h^* \in \mathbb{R}\}$ the generalized

virtual velocities acting as test functions, we assume

$$\mathcal{P}_{int}(t) = \chi_{i,0}(t)\dot{\psi}_i^* + \chi_{i,1}(t)\left(\dot{\psi}_{i+1}^* - \dot{\psi}_i^*\right) + \chi_{i,2}(t)\left(\dot{\psi}_{i+1}^* - \dot{\psi}_{i-1}^*\right) + \int_{\mathcal{A}_i} m(\xi, t)\dot{u}^{*''}(\xi)d\xi \quad (5a)$$

$$\mathcal{P}_{ext}(t) = \int_{\mathcal{A}_i} q(\xi, t)\dot{u}^*(\xi)d\xi \quad (5b)$$

where the bending moment $m(\xi, t)$ and the electric current $\chi_{i,0}(t)$, $\chi_{i,1}(t)$, $\chi_{i,2}(t)$ (refer to Fig.2 for naming) are determined by the following micro constitutive relations

$$m(\xi, t) = g_{mm}u''(\xi, t) + \dot{\psi}_i(t) \int_{\mathcal{A}_i} g_{me}(\xi)d\xi \quad (6)$$

$$\chi_{i,0}(t) = \mathcal{T}_0(\psi_i(t)) - \int_{\mathcal{A}_i} g_{me}(\xi)\dot{u}''(\xi, t)d\xi + \ddot{\psi}_i(t) \int_{\mathcal{A}_i} g_{ee}(\xi)d\xi \quad (7)$$

$$\chi_{i,1}(t) = \mathcal{T}_1(\psi_{i+1}(t) - \psi_i(t)) \quad (8)$$

$$\chi_{i,2}(t) = \mathcal{T}_2(\psi_{i+1}(t) - \psi_{i-1}(t)) \quad (9)$$

where

$$\mathcal{T}_h(\cdot) := \frac{1}{R_h} \frac{\partial(\cdot)}{\partial t} + \frac{1}{L_h}(\cdot) \quad (10)$$

When dynamic phenomena are considered, the basic inertia effects are modelled by the constitutive equation

$$q(\xi, t) = -\sigma(\xi)\ddot{u}(\xi, t) \quad (11)$$

The power functionals (5) and the constitutive equations (6) and (11) assume an *equivalent single layer Euler model* for the layered piezoelectric beam and a constant distribution of the electric field inside each piezoelectric transducer. If one assumes also an uniaxial stress state both in the piezoelectric and elastic layers, a perfect bonding conditions between the different layers and a purely extensional membranal behavior of the piezoelectric layers, the following expressions for the constitutive coefficients g_{mm}, g_{ee}, g_{me} are found as a function of geometric and material properties of the layered beam as a three dimensional continuum (see¹⁰ for further details)

$$g_{mm}(\xi) = Y_b I_b + Y_p^E h_c^2 S_p \Pi(\xi) \quad g_{me}(\xi) = -d_{31} Y_p^E \frac{h_c S_p}{h_p} \Pi(\xi) \quad g_{ee}(\xi) = \epsilon_{33}^S \frac{S_p}{h_p^2} \Pi(\xi) \quad \sigma(\xi) = \rho_b S_b + \rho_p S_p \Pi(\xi) \quad (12)$$

In the expressions above, Y_b is the Young modulus of the elastic layer, Y_p^E the Young modulus of the piezoelectric layer for null electric field, d_{31} the charge piezoelectric coupling coefficient, ϵ_{33}^S the dielectric constant of the piezoelectric material for null mechanical deformation, ρ_b and ρ_p the mass densities of the elastic and piezoelectric layers, h_b and h_p their thicknesses, w_b and $w_p = \vartheta_w w_b$ the respective widths, with the so defined transversal covering factor ϑ_w . The function $\Pi(\xi)$ is introduced in order to describe the axial discontinuities caused by the piezoelectric transducers: its value is 1 in the axial region where the piezoelectric transducers are present, 0 everywhere else. Moreover, the following geometrical quantities have been introduced

$$I_b = \frac{w_b h_b^3}{12} \quad S_p = 2\vartheta_w w_b h_p \quad S_b = w_b h_b \quad h_c = \frac{h_b + h_p}{2} \quad (13)$$

The interested reader can refer to Saravanos¹¹ for further details and references about models of laminated piezoelectric beams.

2.3. Macro model

We impose that the power densities per unit length of the macro model are given by the average of the corresponding micro quantities over a module, once the macro-micro kinematical map (4) is prescribed. In this way, the balance and constitutive equations of the macro model are obtained. For sake of brevity, we omit all the details and we simply present the homogenized equations of motion in the following dimensionless

matrix form by introducing the dimensionless electromechanical state vector $\mathbf{s}(X, t) = \{U(X, t), \Psi(X, t)\}^t$ (the dimensionless variables are denoted by the same letters of the dimensional ones)

$$\ddot{\mathbf{s}}(X, t) + \mathcal{D}^{(1)}\dot{\mathbf{s}}(X, t) + \mathcal{D}^{(0)}\mathbf{s}(X, t) = 0 \quad (14)$$

In the equations above \mathcal{D}_1 and \mathcal{D}_0 are space-differential operators. It is possible to show that for the PEM beam in Fig.1

$$\mathcal{D}^{(1)} = \begin{bmatrix} 0 & \gamma \frac{\partial^2}{\partial X^2} \\ -\gamma \frac{\partial^2}{\partial X^2} & \delta_0 - \delta_2 \frac{\partial^2}{\partial X^2} + \delta_4 \frac{\partial^4}{\partial X^4} \end{bmatrix} \quad \mathcal{D}^{(0)} = \begin{bmatrix} \frac{\partial^4}{\partial X^4} & 0 \\ 0 & \beta_0 - \beta_2 \frac{\partial^2}{\partial X^2} + \beta_4 \frac{\partial^4}{\partial X^4} \end{bmatrix} \quad (15)$$

where γ is a coupling parameter given by

$$\gamma = \frac{\bar{g}_{me}}{\sqrt{\bar{g}_{mm}\bar{g}_{ee}}}, \quad (16)$$

$(\beta_0, \beta_2, \beta_4)$ and $(\delta_0, \delta_2, \delta_4)$ are sets of tuning and damping parameters, respectively. They depend on the micro parameters as follows

$$\beta_0 = \frac{1}{L_0}nc_\beta \quad \beta_2 = \left(\frac{1}{L_1} + \frac{4}{L_2}\right)\frac{c_\beta}{n} \quad \beta_4 = \frac{1}{4L_1}\frac{c_\beta}{n^3} \quad \delta_0 = \frac{1}{R_0}nc_\delta \quad \delta_2 = \left(\frac{1}{R_1} + \frac{4}{R_2}\right)\frac{c_\delta}{n} \quad \delta_4 = \frac{1}{4R_1}\frac{c_\delta}{n^3} \quad (17)$$

where

$$c_\beta = \frac{\bar{\sigma}}{\bar{g}_{ee}\bar{g}_{mm}}X_0^3 \quad c_\delta = \frac{1}{\bar{g}_{ee}}\sqrt{\frac{\bar{\sigma}}{\bar{g}_{mm}}}X_0 \quad (18)$$

The dimensionless parameters $n := X_0/l_b$, which represents the number of modules in the characteristic length X_0 , has been introduced. Moreover, the following expressions for the homogenized constitutive coefficients $(\bar{g}_{mm}, \bar{g}_{ee}, \bar{g}_{me}, \bar{\sigma})$ are found ($\vartheta_l := l_p/l_b$ is a longitudinal covering factor)

$$\begin{aligned} \bar{g}_{mm} &= \frac{Y_b w_b h_p^3}{12} + 2\vartheta_l \vartheta_w Y_p^E h_c^2 h_p w_b & \bar{g}_{me} &= -2\vartheta_l \vartheta_w d_{31} Y_p^E h_c w_b \\ \bar{g}_{ee} &= 2\vartheta_l \vartheta_w \epsilon_{33}^S \frac{w_b}{h_p} & \bar{\sigma} &= \rho_b w_b h_b + 2\vartheta_l \vartheta_w \rho_p w_b h_p \end{aligned} \quad (19)$$

The dimensionless form (14-15) implies the choice of the scaling time $t_0 = X_0^2 \sqrt{\bar{\rho}/\bar{g}_{mm}}$ and the scaling flux-linkage $\Psi_0 = \sqrt{\bar{\rho}/\bar{g}_{ee}}U_0$. For further details refer to the forthcoming paper.¹²

The equations of motions above have been deduced by considering the circuital topology in Fig.1. An opportune choice of the electrical parameters appearing in $\mathcal{T}_0, \mathcal{T}_1, \mathcal{T}_2$ allows to design the impedances of each circuital branch in order to obtain a desired system of differential equations. In particular we report in Table 1 how the circuital parameters $(R_0, R_1, R_2, L_0, L_1, L_2)$ can be chosen in order to synthesize PEM beams in which the electric evolution is governed (as far as a homogenized model is concerned) by each one of the differential equations (2). When the beam is coupled with one of the distributed electric vibration absorbers (2) only one between the three tuning parameters $(\beta_0, \beta_2, \beta_4)$ and one between the three damping parameters $(\delta_0, \delta_2, \delta_4)$ is not vanishing, as it is ruled by Table 2. We underline that the adopted circuital topology requires the use of active components as negative inductors in order to synthesize the forth order networks (\mathbf{F}, \mathbf{Z}) and (\mathbf{F}, \mathbf{S}) . Negative inductors can be electronically simulated by active filters. However, one can aim at avoiding the use of active components to synthesize passive systems like the forth order networks. To this end, different synthesis solutions can be developed as discussed in Karplus¹³ and Andreaus et al.¹⁴

3. ELECTROMECHANICAL DYNAMICS

In the previous section, the evolution equations for homogenized models of the *PEM* beams were derived. In the present section, the dependence of their dynamic properties on the electric parameters is studied and the damping performances of the different network configurations are compared. To this end, the temporal decay of *k*-waves (i.e. solutions in the form of electromechanical waves characterized by a single real wave number *k*) evolving in infinite PEM beams is studied. The optimal electric parameters are found as function of *k* for the five vibration absorbers and the corresponding damping performances are compared. An application to the case of a simply-supported beam is presented.

Table 1. Electric parameters in order to obtain the five distributed vibration absorbers from the generic network topology.

Network	(Z,Z)	(S,Z)	(S,S)	(F,Z)	(F,S)
L_0	L	∞	∞	∞	∞
L_1	∞	∞	∞	L	L
L_2	∞	L	L	$-4L$	$-4L$
R_0	R	R	∞	R	∞
R_1	∞	∞	∞	∞	∞
R_2	∞	∞	R	∞	R

Table 2: Non-vanishing tuning and damping parameter for the five distributed vibration absorbers.

Network	(Z,Z)	(S,Z)	(S,S)	(F,Z)	(F,S)
Tuning	β_0	β_2	β_2	β_4	β_4
Damping	δ_0	δ_0	δ_0	δ_2	δ_2

3.1. K-waves solutions

Let us look for solutions $\mathbf{s}_k(X, t)$ of the system (14) in the form

$$\mathbf{s}_k(X, t) = \mathbf{s}(t)e^{ikX} \quad (20)$$

We call a solution in this form a k -wave, which is a particular standing wave as defined by Courant and Hilbert.¹⁵ The analysis of k -waves is relevant because, by means of a Fourier Transform (or Series), the general solution of a dynamic problem for a infinite (or finite) beam can be written as the superposition of k -waves, each one with a fixed wave number. The substitution of the wave form solution (20) into the equations of motion (14) leads to the following system of two second-order ordinary differential equations for the temporal evolution $\mathbf{s}(t)$

$$\ddot{\mathbf{s}}(t) + D_k^{(1)}\dot{\mathbf{s}}(t) + D_k^{(0)}\mathbf{s}(t) = \mathbf{0} \quad (21)$$

where $D_k^{(1)}, D_k^{(0)}$ are *real algebraic operators* since only *even* order spatial derivatives appear in $\mathcal{D}^{(1)}$ and $\mathcal{D}^{(0)}$. The system (21) can be conveniently rewritten as a system of four first-order ordinary differential equations with constant coefficients in terms of the state vector $\mathbf{y}(t) = \{\mathbf{s}(t), \dot{\mathbf{s}}(t)\}^t$, as follows

$$\dot{\mathbf{y}}(t) = \mathbf{A}_k \mathbf{y}(t) \quad (22)$$

For the general system in Fig.1

$$\mathbf{A}_k = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k^4 & 0 & 0 & \gamma k^2 \\ 0 & -\beta(k) & -\gamma k^2 & -\delta(k) \end{bmatrix} \quad (23)$$

where subscript k on \mathbf{A}_k underlines that the system (22) is written for the temporal evolution of k -waves. The following parameters have been introduced

$$\beta(k) := \beta_0 + k^2\beta_2 + k^4\beta_4 \quad \delta(k) := \delta_0 + k^2\delta_2 + k^4\delta_4 \quad (24)$$

They can be regarded as equivalent tuning and damping parameters as a function of wave number. The characteristic polynomial associated to (23) is

$$\mu^4 + \delta(k)\mu^3 + (\beta(k) + k^4(1 + \gamma^2))\mu^2 + k^4\delta(k)\mu + k^4\beta(k) = 0 \quad (25)$$

We denote by $\{\mu_i\}_{i=1\dots 4}$ the corresponding roots. The solution of (22) starting from a generic initial condition $\mathbf{y}_0 = \mathbf{y}(0)$ is given by

$$\mathbf{y}(t) = \mathbf{e}^{\mathbf{A}_k t} \mathbf{y}_0 \quad (26)$$

We characterize the damping properties of the $\mathbf{y}(t)$ (and consequently of $\mathbf{s}(t)$) by the following exponential *decay rate* ϱ and *damping time* τ

$$\varrho := \min_{j=1,\dots,4} (-\operatorname{Re}(\mu_j)) \quad \tau := 1/\varrho \quad (27)$$

Moreover, we define a *system damping ratio* ζ as

$$\zeta := \min_{j=1,\dots,4} \left(\frac{-\operatorname{Re}(\mu_j)}{\sqrt{\operatorname{Re}(\mu_j)^2 + \operatorname{Im}(\mu_j)^2}} \right) \quad (28)$$

All the quantities above are referred to a fixed wave number k and, when necessary, the functional dependence on k is explicitly underlined. We remark that the proposed characterization of the temporal evolution $\mathbf{s}(t)$ is based only on the system eigenvalues and it is completely independent on the initial data \mathbf{y}_0 .

3.2. K-waves optimal damping

The system performances are optimized by minimizing the temporal *decay* obtained for a \bar{k} -wave (i.e. an electromechanical wave in the form (20) with a *fixed wave number* \bar{k}). Since the temporal evolution of a k -wave is determined by means of the system of the two second order differential equations (21), the optimal design of the electric part of that system is equivalent to the optimization of a dissipative one-degree-of-freedom dynamic vibration absorber gyroscopically coupled to a harmonic oscillator. Hence, the optimization techniques which were developed for that application can be fruitfully revisited. In particular, we follow a *pole-placement* method analog to the one that was adopted by Hagood and Von Flotow¹. We choose as performance index *PI* the system decay rate (27). Let us underline its functional dependence on the wave number k , the tuning parameter β and damping parameter δ by writing

$$PI(k, \beta, \delta) := \varrho(k, \beta, \delta) = \min_{j=1\dots 4} (-\operatorname{Re}(\mu_j(k, \beta, \delta))) \quad (29)$$

We assume that $\delta, \beta \geq 0$. The definition (29) implies the assumption of a *pole-placement* criterion for the system optimization. The optimization of a given network implies the maximization of (29) with respect to (β, δ) when the wave number k is fixed and equal to \bar{k} . To this end, we proceed by employing an useful properties of (29): *it is maximum when the four eigenvalues of \mathbf{A}_k appear in the form of two coincident complex conjugate pairs*. This property is widely exploited in the literature (see e.g. Hagood and Von Flotow¹ and Alessandroni et al.⁵). With this method the following optimal values for the equivalent tuning and damping parameters $\beta(k)$ and $\delta(k)$ defined by (24) are found

$$\beta^{opt}(\bar{k}) = \bar{k}^4 \quad \delta^{opt}(\bar{k}) = 2\gamma\bar{k}^2 \quad (30)$$

The corresponding eigenvalues of (23) are

$$\mu_{1,2} = \mu_{3,4} = -\frac{1}{2}\bar{k}^2\gamma \pm \bar{k}^2\sqrt{1 - \frac{\gamma^2}{4}} \quad (31)$$

The associated system damping ratio is

$$\zeta = \frac{\gamma}{2} \quad (32)$$

The optimal damping ratio depends only on the dimensionless coupling parameter γ and it is the same for the five network configurations. For each network a single pair (β_i, δ_j) , with proper i and j , must be considered, as it is ruled by Table 2. Hence, relations (30) and definitions (24) allow for deriving the following optimal parameters.

$$\beta_0^{opt}(\bar{k}) = \bar{k}^4 \quad \beta_2^{opt}(\bar{k}) = \bar{k}^2 \quad \beta_4^{opt}(\bar{k}) = 1 \quad \delta_0^{opt}(\bar{k}) = 2\gamma\bar{k}^2 \quad \delta_2^{opt}(\bar{k}) = 2\gamma \quad \delta_4^{opt}(\bar{k}) = 2\gamma/\bar{k}^2 \quad (33)$$

The corresponding optimal inductors and resistors as a function of \bar{k} and of the number of modules in a characteristic length n are immediately found through the definitions (17). They are reported in Table 3. By comparing the expressions for the optimal parameters of the different circuitual topologies, the following facts can be noted.

Table 3: Optimal inductors and resistors in each module for the five distributed vibration absorbers

Network	Optimal Inductors	Optimal Resistors
(Z,Z)	$L_0^{opt}(\bar{k}) = \frac{c_\beta}{k^4} n$	$R_0^{opt}(\bar{k}) = \frac{c_\delta}{2\gamma k^2} n$
(S,Z)	$L_2^{opt}(\bar{k}) = \frac{4c_\beta}{k^2} \frac{1}{n}$	$R_0^{opt}(\bar{k}) = \frac{c_\delta}{2\gamma k^2} n$
(S,S)	$L_2^{opt}(\bar{k}) = \frac{4c_\beta}{k^2} \frac{1}{n}$	$R_2^{opt}(\bar{k}) = \frac{2c_\delta}{\gamma} \frac{1}{n}$
(F,Z)	$L_1^{opt}(\bar{k}) = \frac{c_\beta}{4} \frac{1}{n^3}$ $L_2^{opt}(\bar{k}) = -c_\beta \frac{1}{n^3}$	$R_0^{opt}(\bar{k}) = \frac{c_\delta}{2\gamma k^2} n$
(F,S)	$L_1^{opt}(\bar{k}) = \frac{c_\beta}{4} \frac{1}{n^3}$ $L_2^{opt}(\bar{k}) = -c_\beta \frac{1}{n^3}$	$R_2^{opt}(\bar{k}) = \frac{2c_\delta}{\gamma} \frac{1}{n}$

1. **Dependence on location.** The optimal values of the resistors and the inductors in each module are the same if they are located in the same circuital branch (e.g. the optimal value of the ground resistor $R_0(\bar{k})$ is the same in the (Z,Z), (S,Z) and (F,Z) networks, as well as the optimal value of the line inductor $L_2(\bar{k})$ is the same in the (S,Z) and (S,S) networks);
2. **Dependence on wave number.** If the resistors and the inductors are properly located, their optimal values are independent of the wave number \bar{k} . Indeed, the optimal resistors are independent of the wave number if the corresponding dissipation term appears in the homogenized equations with a second order spatial derivative ((S,S) and (F,S) networks); the optimal inductors are independent of the wave number if they are located so as to lead to fourth order networks ((F,Z) and (F,S)). The (F,S) network satisfies both the conditions, therefore we expect that such a system *can be optimized for all the wave numbers at the same time* (this important result is confirmed by the plots in Fig.3).
3. **Dependence on number of modules.** The optimal inductance in each module is proportional to n in the zeroth order network, proportional to $1/n$ in the second order networks, proportional to $1/n^3$ in the fourth order networks. Hence, in second order and fourth order networks the optimal inductances can be significantly decreased by increasing the number of modules per unit of length. This fact can be relevant because the construction of high-value inductors implies significant technological problems. Moreover, in the networks with a second order dissipation, the optimal value of the parallel-connected resistances R_2 decreases for increasing n . Also this effect is desirable because the unavoidable parasite resistances of the electric components imply a technological upper bound on the value of R_2 .

Once a system is optimized for a fixed wave number \bar{k} , it is interesting to study the corresponding damping properties for k -waves which are characterized by a wave number k different from \bar{k} . Let us denote with $\beta_i^{opt}(\bar{k}), \delta_j^{opt}(\bar{k})$ the optimal damping and tuning parameters relative to a generic i -th order electric network with a j -th order dissipation (refer to equations (2a-2e)). As it is evident from equation (32) the corresponding *optimal* damping ratio is independent of the electric network. In order to point out the different damping performances shown by different distributed vibration absorbers it is useful to define a function, named **characteristic decay time** function, as follows

$$T_{opt}(k, \bar{k}) := \frac{1}{PI(k, \beta_i^{opt}(\bar{k}), \delta_j^{opt}(\bar{k}))} \quad (34)$$

It furnishes the value of the characteristic (dimensionless) decay time for a wave number k in a structure the electric circuitry of which is optimized for the wave number \bar{k} . The **optimal decay time** function obtained for $\bar{k} = k$, i.e. the characteristic decay time obtained by optimizing the system for the current wave number k , can be evaluated explicitly from relations (31)

$$T_{opt}(k, k) = \frac{2}{\gamma k^2} \quad (35)$$

Expression (35) depends only on the considered wave number and coupling coefficient and, as one could expect, the bigger is the wave number the smaller is the decay time. Hence, it shows that *the same optimal decay time function may be attained regardless of the circuital topologies*. On the contrary, we anticipate that, when $k \neq \bar{k}$, the performances of the different PEM system depend significantly on the chosen circuital topology (see

Table 4: Aluminum beam and piezoelectric sheets cross section dimensions and constitutive parameters

	Beam	PZT
Width (w_b, w_p)	40×10^{-3} m	36.2×10^{-3} m
Thickness (h_b, h_p)	4.0×10^{-3} m	0.267×10^{-3} m
Covering factor θ_l	-	0.9
Young Modulus (Y_b, Y_p^E)	70×10^9 N/m ²	62×10^9 N/m ²
Mass Density (ρ_b, ρ_p)	2700 kg/m ³	7800 kg/m ³
Dielectric Constant (ϵ_{33}^T)	-	$3800 \times \epsilon_0 = 3.36 \times 10^{-8}$ F m ⁻¹
Charge Constant (d_{31})	-	-320×10^{-12} m/V

e.g. the plots in the following Fig.3). Since the optimal decay time function depends on k as ruled by (35), it is convenient to define a **relative decay time** function as follows

$$\tilde{T}_{opt}(k, \bar{k}) := \frac{T_{opt}(k, \bar{k})}{T_{opt}(k, k)} = \frac{\gamma}{2} k^2 T_{opt}(k, \bar{k}) \quad (36)$$

It will be interesting also to analyze the behavior of the system damping ratio $\zeta(k, \bar{k})$ as function of k in a system optimized for the wave number \bar{k} .

3.3. Design of PEM beams

In the present subsection, considering a numerical example, the optimal values of the electric parameters are found for a generic wave number \bar{k} and the corresponding damping performances of the five network configurations are analyzed and compared. Since the systems described by the dimensionless differential equations (21) are studied, it is always possible to consider $\bar{k} = 1$ (if \bar{k}_d is the dimensional value of the wave number \bar{k} , the scaling length $X_0 = 1/\bar{k}_d$ can be fixed).

Let us consider as a numerical case study a rectangular cross section aluminum beam on which piezoelectric sheets made by the piezoelectric material PSI-5H-S4-ENH are bonded. In Table 4, the cross section and material properties of the beam and of the piezoelectric sheets are reported. The corresponding numerical values of the homogenized material properties, the dimensionless coupling coefficient γ and the constants (c_β, c_δ) are

$$\begin{aligned} \bar{g}_{mm} &= 19.8 \text{ N m}^2 & \bar{g}_{ee} &= 6.83 \times 10^{-6} \frac{\text{F}}{\text{m}} & \bar{g}_{me} &= 2.76 \times 10^{-3} \frac{\text{N m}}{\text{V}} \\ \bar{\sigma} &= 0.568 \frac{\text{kg}}{\text{m}} & c_\beta &= 4.19 \times 10^3 X_0^3 \text{ H} & c_\delta &= 24.8 \times 10^3 X_0 \Omega \end{aligned}$$

The associated dimensionless coupling coefficient is $\gamma = 0.237$ and the following optimal damping ratio (32) is found (it is the same for the five vibration absorbers)

$$\zeta = 11.78\% \quad (37)$$

In Fig.3 the relative decay time function (36) and the system damping ratio (28) are plotted as a function of k for the numerical values above. These plots show some important results. The (F,S)-Network *attains the optimal performances for all the wave numbers*: this means that $\tilde{T}_{opt}(k, 1) := T_{opt}(k, \bar{k})/T_{opt}(k, k) = 1$ for each k . This fact could be established also from Table 3, which shows that in this case the optimal resistances and inductances are independent of k . Furthermore, the (S,S)-Network can appreciably damp free oscillations with $k > 1$, although the corresponding damping ratio is much lower than the optimal one which is reached only for $k = \bar{k} = 1$. Also this behavior is in accordance with the relations for the optimal resistances and inductances in Table 3. Indeed, in the (S,S)-Network the dissipation is always optimal, while the electromechanical resonant energy exchange is effective only for a narrow band of wave numbers.

It is important to recall that the proposed dynamic analysis is based on *homogenized beam model* of the modular systems. As a consequence the presented results are *meaningful only for wave numbers which are small enough*. Roughly speaking, the dimensionless wave number k must be smaller than i) the wave number $k_w = 2\pi/(w_b/X_0)$ corresponding to a wave length equal to the beam width (we assume $h_b < w_b$), ii) the wave

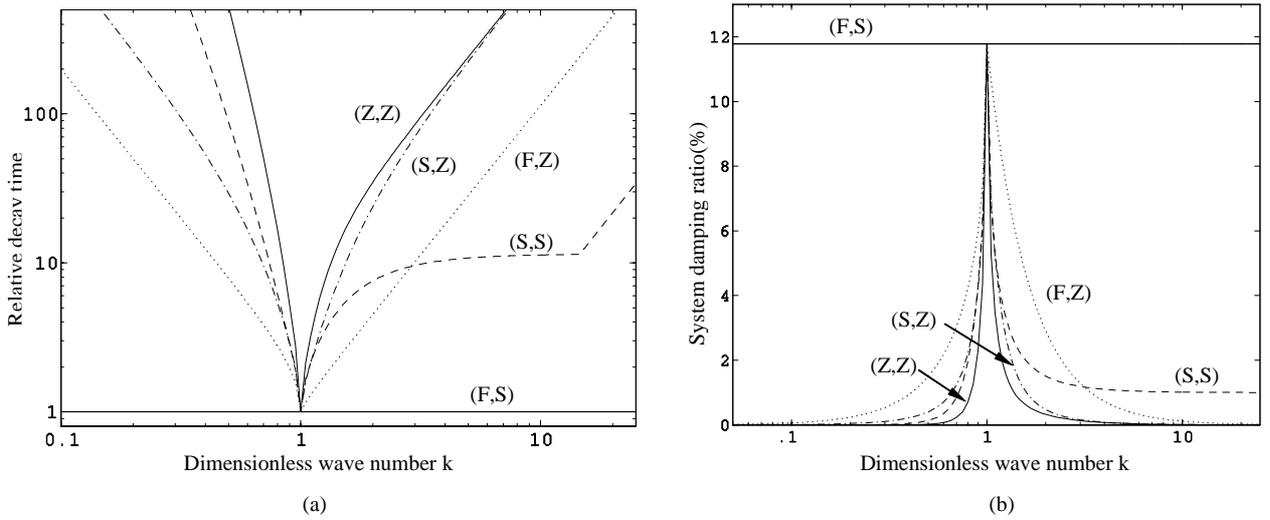


Figure 3. Damping performances for the five network configurations when optimized for a dimensionless wavenumber $\bar{k} = 1$. (a) relative decay time $\bar{T}_{opt}(k, 1)$, (b) system damping ratio $\zeta(k, 1)$.

Table 5: Optimal inductors and resistors in each module for the simply supported PEM beam

Network	Optimal Inductors	Optimal Resistors
(Z,Z)	$L_0^{opt}(\bar{k}) = 66.5 \text{ H}$	$R_0^{opt}(\bar{k}) = 32.1 \text{ k}\Omega$
(S,Z)	$L_2^{opt}(\bar{k}) = 18.2 \text{ H}$	$R_0^{opt}(\bar{k}) = 32.1 \text{ k}\Omega$
(S,S)	$L_2^{opt}(\bar{k}) = 18.2 \text{ H}$	$R_2^{opt}(\bar{k}) = 8.80 \text{ k}\Omega$
(F,Z)	$L_1^{opt}(\bar{k}) = 781 \text{ mH}$ $L_2(\bar{k}) = -312 \text{ mH}$	$R_0^{opt}(\bar{k}) = 32.1 \text{ k}\Omega$
(F,S)	$L_1^{opt}(\bar{k}) = 781 \text{ mH}$ $L_2(\bar{k}) = -312 \text{ mH}$	$R_2^{opt}(\bar{k}) = 8.80 \text{ k}\Omega$

number $k_l = \pi/(l_b/X_0) = n\pi$ corresponding to a wavelength equal twice the longitudinal dimension of a single module. The first limit is an intrinsic drawback of the assumption of a one-dimensional beam model. The second restriction is related to the approximation implied by the homogenized model which was based on a second order Taylor expansion of the state variables in each module.

The performance of the five different distributed vibration absorbers for the forced response are compared in Fig.4. The plot refers to the case of a simply supported beam of length $l_d = 0.5 \text{ m}$, equipped with twelve circuital modules. The boundary modules of the electric networks were chosen in order to realize the boundary conditions $\Psi(0, t) = \Psi(l, t) = \Psi''(0, t) = \Psi''(l, t) = 0$ for the fourth order networks, and the boundary conditions $\Psi(0, t) = \Psi(l, t) = 0$ for the second order networks. All the electric networks are optimized in order to damp a wave with a wavelength corresponding to the first vibration mode of the beam. The numerical values of the optimal inductors and resistor in each module are reported in Table 5.

4. CONCLUSIONS

In the present paper the concept of distributed piezoelectric vibration control was proved to be interesting for technological applications. We considered several electromechanical systems made of distributed piezoelectric transducers bonded on a beam and interconnected by properly designed electric networks. These systems can be regarded as waveguides supporting electromechanical oscillations. The electric network interconnecting the piezoelectric transducers are designed in order to enhance, by taking advantage of internal resonance effects, the transformation of mechanical energy into the electric form. In these networks, the mechanical energy is dissipated in dedicated resistors, which are designed to obtain optimal vibration decay times. The performances of different circuital topologies interconnecting the piezoelectric transducers were compared and it was shown

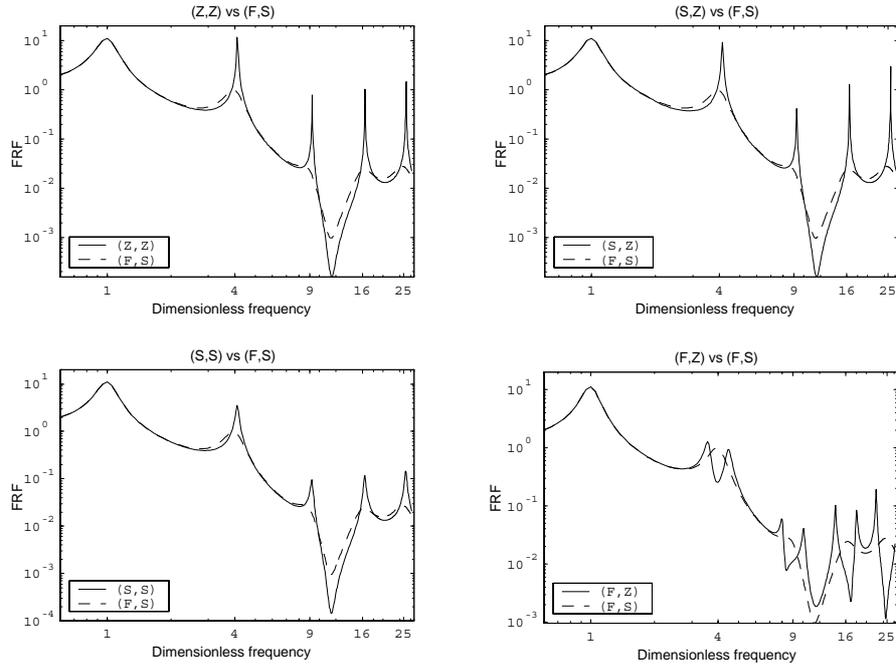


Figure 4. Mechanical frequency response function for the vibration absorbers optimized to damp the free oscillations of the first mechanical mode. The response obtained by means of the fourth order line (F,S) is compared with those related to the other networks. The plot corresponds to the transverse displacement of the beam axis point $x_u = 0.3l$ for an excitation in the form of a transverse force applied at the point $x_f = 0.7l$.

that a passive controller able to optimally damp multiple vibration modes at the same time can be obtained. The main results obtained for the control of the transversal vibrations of an Euler beam are resumed in the following list.

- i) Every considered electric vibration absorber may be optimized to achieve the optimal damping performances for a single wave number. This optimal damping is the same for every circuital topology and depends only on the electromechanical coupling coefficient.
- ii) The performances of the different absorbers optimized for a single wave number, vary significantly when waves having different wave numbers are considered. There is a distributed vibration absorber with proper dynamic properties for which the optimal electric parameters are independent of the wave number. To this purpose, the "dissipation" term associated to the resistors must appear in the homogenized equations with a second order spatial derivative, the "electric stiffness" term due to the inductors with a fourth order spatial derivative. This system is the fourth order transmission line with second order dissipation ((F,S) – network). It supplies the optimal decay time for every wave number.
- iv) For different electric networks the nominal value of the inductances in each module depends on the number n of modules (i.e. transducers) per unit length in different ways. The most convenient circuit is again the fourth order network: in this case the inductances in each module are proportional to $1/n^3$. This fact is interesting because the lower the inductances are the easier is their technological construction with electr(on)ic components.
- v) The piezoelectric controller obtained with the (F,S) network is robust under forcing frequency shifts, accomplishing a multimodal optimal damping. This point was illustrated by means of a numerical example consisting of a simply-supported PEM beam (see Fig.4).

The proposed damping devices can be actually constructed. Although technological aspects have not been detailed in this paper, it should be noted that the required high or negative inductances are feasible only using electronic active filters. The technological and theoretical problems to be solved in this context are related to the synthesis of equivalent electric networks which are completely passive.

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