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► **To cite this version:**

F. Gori. Mass and energy-capital conservation equations to study price evolution of non-renewable energy resources Part III – Energy supply curve. *Applied Thermal Engineering*, 2009, 29 (11-12), pp.2172. 10.1016/j.applthermaleng.2008.10.013 . hal-00505528

HAL Id: hal-00505528

<https://hal.science/hal-00505528>

Submitted on 24 Jul 2010

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Accepted Manuscript

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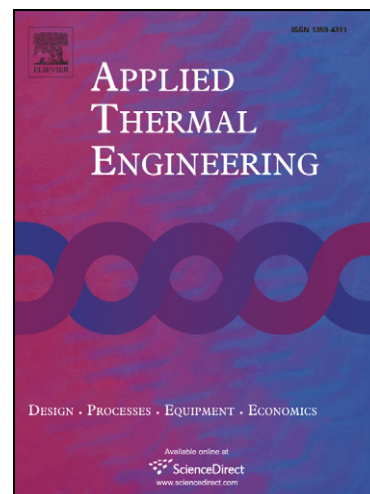
PII: S1359-4311(08)00438-9
DOI: [10.1016/j.applthermaleng.2008.10.013](https://doi.org/10.1016/j.applthermaleng.2008.10.013)
Reference: ATE 2651

To appear in: *Applied Thermal Engineering*

Received Date: 16 April 2008
Revised Date: 19 September 2008
Accepted Date: 30 October 2008

Please cite this article as: F. Gori, Mass and energy-capital conservation equations to study price evolution of non-renewable energy resources Part III – Energy supply curve, *Applied Thermal Engineering* (2008), doi: [10.1016/j.applthermaleng.2008.10.013](https://doi.org/10.1016/j.applthermaleng.2008.10.013)

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**Mass and energy-capital conservation equations to study
price evolution of non-renewable energy resources**

Part III – Energy supply curve

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Keywords: Extracted and Sold resources; Extraction rate; Inflation rate; Discount rate; Prime rate; Ratio of interest rate of non-extracted resources on extraction rate; Ratio of interest rate of sold resources on extraction rate; Dimensionless critical initial price; Dimensionless critical initial extreme price.

Abstract

The price evolution of non-renewable resources versus the consumption rate is investigated with the aim of constructing the energy supply curve. The case studied is without accumulation nor depletion of the resources and the mass and energy-capital conservation equations are solved under the condition of the same mass flow rate of extraction and sale. The energy supply curve of extracted resource is dependent on the newly defined parameter, RINE, Rate of Interest of Non-extracted resources on the Extraction rate. The energy supply curve of sold resource is dependent on the newly defined parameter, RISE, Rate of Interest of Sold resources on the Extraction rate, in case the rate of interest of non-extracted resources, r_N , is nil. In general, the energy supply curve of sold resource is dependent also on two dimensionless parameters, Dimensionless Critical Initial Price of Sold resources, i.e. DCIPS, and Dimensionless Critical Initial Price Extreme of Sold resources, i.e. DCIPES. The energy supply curve of sold resources is investigated under different relations between three parameters, i.e. extraction rate and interest rates of non-extracted and extracted/sold resources. New trends are observed in the economic market of non-renewable energy resources. The energy supply curve of the difference between sold and extracted resource is also obtained and is dependent on two dimensionless parameters, Critical Initial Price Difference, i.e. CIPD, and Critical Extreme of the Initial Price Difference, i.e. CEIPD. Finally, the predictions obtained with the present approach are compared to the real evolution of the world price of oil and the European price of gas versus the world consumption during the last three decades, i.e. from 1980 until 2005 for oil and from 1984 until 2005 for gas, taking into account inflation, discount and prime rates of the economic market. The agreement is acceptable but, more important, the trend is correctly predicted. The price difference between sold and extracted resources is also investigated versus the dimensionless mass flow rate of extraction. The evolution is dependent on four parameters: RINE, RISE, DCIPS, and DCIPES.

1. Introduction

The price evolution of non-renewable resources is a very important problem in our economy. The economic issues related to an economy based on exhaustible resources have been investigated in [1] and later on revised in [2]. The approach of using mass and energy-capital conservation equations to investigate the price evolution with time throughout the use of economic parameters has been proposed in [3-5] by the present author who is here extending his analysis to the price evolution versus the rate of consumption with the aim of constructing the energy supply curve of non-renewable resources.

2. Supply curve of extracted resources

The mass conservation equations of extracted and sold resources, [3], under the hypothesis of no accumulation nor depletion of the resources, can be written in dimensionless form as

$$G'_E = G_E / G_{E0} = G'_S = G_S / G_{S0} = \exp(\alpha t) \quad (1)$$

The price evolution with time of extracted resources, [3], is, in dimensionless form,

$$p'_E = p_E / p_{E0} = \exp(\beta t) \quad (2)$$

The extraction rate, α , can be obtained from Eq. (1) on the base of the mass flow rates of extraction in two successive years,

$$\alpha = 1 / t \ln G'_S \quad (3)$$

The elimination of the time variable, t , between Eqns. (1) and (2) allows to obtain the relation, called supply curve of extracted resources, between the dimensionless price, p'_E , and the dimensionless mass flow rate, G'_E , of extracted resources,

$$p'_E = G'_E^{(\beta/\alpha)} = G'_E^{(y-1)} \quad (4)$$

where the new variable, y ,

$$y = r_N / \alpha \quad (5)$$

is called Rate of Interest of Non-extracted resources on the Extraction rate, RINE.

The variation of p'_E with G'_E is presented in Fig. 1 where the only variable affecting the evolution is y .

- For $y = 0$ (i.e. $r_N = 0$ or $\alpha = \pm \infty$), Eq. 4 is an equilateral hyperbole: p'_E decreases with the increase of G'_E or increases with the decrease of G'_E .
- For $0 < y < 1$, p'_E decreases with the increase of G'_E .
- For $y = 1$ (i.e. $\alpha = r_N$), p'_E is constant with the increase of G'_E .
- For $y > 1$, p'_E increases with the increase of G'_E .

- For $y = \pm \infty$ (or $\alpha = 0$), Eq. 4 is a vertical line: p'_E increases at constant $G'_E = 1$.
- For $y < 0$, p'_E increases with the decrease of G'_E .

3. Supply curve of sold resources

The price evolution with time of sold resources, [4], becomes, in dimensionless form,

$$p'_s = p_s / p_{s0} = \exp(\beta' t) - p'^*_{s0} [\exp(\beta' t) - \exp(\beta t)] \quad (6)$$

where

$$p'^*_{s0} = p^*_{s0} / p_{s0} = r_N p_{E0} / p_{s0} / (\beta' - \beta) = p_{E0} / p_{s0} y / (x - y) \quad (7)$$

is the Dimensionless Critical Initial Price of Sold resources, DCIPS, and the new variable, x ,

$$x = r_E / \alpha \quad (8)$$

is the Rate of Interest of Sold resources on the Extraction rate, RISE.

For $x \neq y$ (i.e. $r_N \neq r_E$) Eq. (6), combined with Eq. (1), becomes

$$p'_s = p_s / p_{s0} = G'_s (\beta' / \alpha) - p'^*_{s0} (G'_s (\beta' / \alpha) - G'_s (\beta / \alpha)) = G'_s (x-1) - p'^*_{s0} (G'_s (x-1) - G'_s (y-1)) = G'_s (x-1) (1 - p'^*_{s0}) + p'^*_{s0} G'_s (y-1) \quad (9)$$

An extreme value of p'_s is present for

$$G'_s = [y(1-y)/(x-1)/(x-2y)]^{1/(x-y)} \quad (10)$$

when $p_{E0} = p_{s0}$, and

$$G'_s = [p'^*_{s0} (1-y)/(x-1)/(1-p'^*_{s0})]^{1/(x-y)} \quad (11)$$

when $p_{E0} \neq p_{s0}$.

The maximum of p'_s is present at $G'_s = 1$ for p'_{s0}

$$p'_{s0} = p'^*_{s0} = p^*_{s0} / p_{s0} = p'^*_{s0} [(\beta' - \beta) / \beta'] = p_{E0} / p_{s0} r_N / \beta' = p_{E0} / p_{s0} y / (x-1) \quad (12)$$

which is the Dimensionless Critical Initial Price Extreme of Sold resources, DCIPES.

3.1 Supply curve of sold resources for $r_N = 0$, i.e. $p'^*_{s0} = 0$

For $r_N = 0$, i.e. $p'^*_{s0} = 0$, Eq. (9) becomes

$$p'_s = p_s / p_{s0} = \exp(\beta' t) \quad (13)$$

which, combined with Eq. (1), gives

$$p'_s = G'_s (\beta' / \alpha) = G'_s (x-1) \quad (14)$$

The variation of p'_s with G'_s is presented in Fig. 2 for $y = 0$ (i.e. $r_N = 0$). The only variable affecting the evolution is x .

- For $x = 0$ (i.e. $r_E = 0$ or $\alpha = \pm \infty$), Eq. (14) is an equilateral hyperbole: p'_s decreases with the increase of G'_s or increases with the decrease of G'_s .
- For $0 < x < 1$, p'_s decreases with the increase of G'_s .
- For $x = 1$ (i.e. $\alpha = r_E$), p'_s is constant with the increase of G'_s .

- For $x > 1$, p'_s increases with the increase of G'_s .
- For $x = \pm \infty$ (or $\alpha = 0$), Eq. (14) is a vertical line: p'_s increases at constant $G'_s = 1$.
- For $x < 0$, p'_s increases with the decrease of G'_s .

3.2 Supply curve of sold resources for $p'^*_{s0} = 1$.

For $p'^*_{s0} = 1$, Eq. (9) reduces to

$$p'_s = p_s / p_{s0} = G'_s{}^{(y-1)} \quad (15)$$

and the variation of p'_s is only dependent on y , *RINE*.

The variation of p'_s with G'_s for $p'^*_{s0} = 1$ (i.e. $x = 2$ y if $p_{E0} = p_{s0} = 1$) is presented in Fig. 3. The only variable involved in the evolution is y .

- For $y = 0$ (i.e. $r_E = 0$ or $\alpha = \pm \infty$), Eq. (15) is an equilateral hyperbole: p'_s decreases with the increase of G'_s or increases with the decrease of G'_s .
- For $0 < y < 1$, p'_s decreases with the increase of G'_s .
- For $y = 1$, p'_s is constant with the increase of G'_s .
- For $y > 1$, p'_s increases with the increase of G'_s . A special case is $y = 2$ when the variation of p'_s is a linear increase with the increase of G'_s .
- For $y = \pm \infty$ (or $\alpha = 0$), Eq. (15) is a vertical line: p'_s increases at constant $G'_s = 1$.
- For $y < 0$, p'_s increases with the decrease of G'_s .

3.3 Supply curve of sold resources for $|x| > |y| > 1$.

The variation of p'_s with G'_s is presented in Fig. 4 for $|x| > |y| > 1$ (i.e. $r_E > r_N > \alpha$). The variables involved in the evolution are here two: i.e. p'^*_{s0} and p''^*_{s0} . The main role is played by p'^*_{s0} when $p'^*_{s0} \leq 1$, but, when $p'^*_{s0} > 1$ it is determinant the second variable p''^*_{s0} .

- For $p'^*_{s0} \leq 1$, p'_s increases with the increase or the decrease of G'_s .
- For $p'^*_{s0} > 1 > p''^*_{s0}$, p'_s increases up to a maximum and then decreases with the increase or the decrease of G'_s .
- For $p''^*_{s0} = 1$, p'_s decreases with the increase or the decrease of G'_s and the maximum is at $G'_s = 1$.
- For $p'^*_{s0} > p''^*_{s0} > 1$, p'_s decreases with the increase or the decrease of G'_s .

3.4 Supply curve of sold resources for $|y| > |x| > 1$ or $y > 1 > x$.

The variation of p'_s with G'_s is presented in Fig. 5 for $|y| > |x| > 1$ (i.e. $r_N > r_E > \alpha$ and $p'^*_{s0} < 0 < p''^*_{s0}$). The only variable influencing the evolution is p''^*_{s0} because $p'^*_{s0} < 0$.

- For $p'^{**}_{s0} < 1$, p'_s increases up to a maximum and then decreases with the decrease or the increase of G'_s . The maximum value of p'_s corresponds to the values of G'_s given by Eqs. (10) or (11).

- For $p'^{**}_{s0} = 1$, p'_s decreases with the decrease or the increase of G'_s and the maximum is at $G'_s = 1$.

- For $p'^{**}_{s0} > 1$, p'_s decreases with the increase or the decrease of G'_s .

The variation of p'_s with G'_s is presented in Fig. 5 also for $y > 1 > x$ (i.e. $r_N > \alpha > r_E$ and $0 > p'^{**}_{s0} > p'^{**}_{s0}$). The price p'_s decreases with the increase of G'_s .

The conclusion of Fig. 5 is that p'_s cannot increase indefinitely for $|y| > |x| > 1$ or $y > 1 > x$.

3.5 Supply curve of sold resources for $y=1$ or $x=1$.

The variation of p'_s with G'_s is presented in Fig. 6 for $y=1$ (i.e. $\alpha = r_N > 0$ and $p'^{**}_{s0} = p'^{**}_{s0}$). The main variable influencing the evolution is $p'^{**}_{s0} = p'^{**}_{s0}$.

- For $x > y = 1$ (i.e. $\alpha = r_N < r_E$ and $p'^{**}_{s0} = p'^{**}_{s0} > 0$), p'_s increases with the increase of G'_s if $p'^{**}_{s0} < 1$; p'_s remains constant with G'_s if $p'^{**}_{s0} = 1$; p'_s decreases with G'_s if $p'^{**}_{s0} > 1$.

- For $x < y = 1$ (i.e. $\alpha = r_N > r_E$ and $p'^{**}_{s0} = p'^{**}_{s0} < 0$), p'_s decreases asymptotically to p'^{**}_{s0} with the increase of G'_s .

The evolution of p'_s with G'_s is presented in Fig. 6 also for $x=1 < y$ (i.e. $r_N > r_E = \alpha$ and $p'^{**}_{s0} < 0$, $p'^{**}_{s0} = +\infty$). The price p'_s decreases with the increase of G'_s .

3.6 Supply curve of sold resources for $y < 1$.

The variation of p'_s with G'_s is presented in Fig. 7 for $y < 1$ (i.e. $\alpha > r_N$). The main variables influencing the evolution are p'^{**}_{s0} and p'^{**}_{s0} .

- For $x > 1 > y$ (i.e. $r_E > \alpha > r_N$ and $p'^{**}_{s0} > p'^{**}_{s0} > 0$) five cases are possible: p'_s increases with G'_s if $p'^{**}_{s0} \leq 1$; p'_s increases with G'_s , after a minimum, if $p'^{**}_{s0} > 1 > p'^{**}_{s0}$; p'_s decreases asymptotically to $+0$ with G'_s if $p'^{**}_{s0} > p'^{**}_{s0} = 1$; p'_s decreases with G'_s if $p'^{**}_{s0} > p'^{**}_{s0} > 1$.

- For $x = 1 > y$ (i.e. $r_E = \alpha > r_N$ and $p'^{**}_{s0} = +\infty$, $p'^{**}_{s0} > 0$) p'_s decreases asymptotically to $(1 - p'^{**}_{s0})$ with the increase of G'_s .

- For $1 > x > y$ (i.e. $\alpha > r_E > r_N$ and $p'^{**}_{s0} > 0 > p'^{**}_{s0}$) two cases are possible: p'_s decreases asymptotically to $+0$ with G'_s if $p'^{**}_{s0} \leq 1$; p'_s decreases asymptotically to -0 with G'_s , after a minimum, if $p'^{**}_{s0} > 1$.

- For $1 > y > x$ (i.e. $\alpha > r_N > r_E$ and $0 > p'^{**}_{s0} > p'^{**}_{s0}$) p'_s decreases asymptotically to -0 with G'_s after a minimum.

3.7 Supply curve of sold resources for $x=y$ (i.e. $r_N = r_E$)

The price evolution of sold resources for $x=y$ (i.e. $r_N=r_E$), [4], can be written in dimensionless form, as

$$p'_s = p_s / p_{s0} = (1 - p^{**}_{s0} \beta t) \exp(\beta t) \quad (16)$$

Using Eq. (1), Eq. (16) becomes finally

$$p'_s = p_s / p_{s0} = [1 - p^{**}_{s0} \beta / \alpha \ln(G'_s)] G'_s^{(\beta/\alpha)} = G'_s^{(y-1)} [1 - p^{**}_{s0} (y-1) \ln(G'_s)] \quad (17)$$

The maximum value of p'_s corresponds to

$$G'_s = \exp[(x-y-1)/y/(y-1)] \quad (18)$$

for $p_{E0}=p_{s0}$, and to

$$G'_s = \exp[(1/p^{**}_{s0}-1)/(y-1)] \quad (19)$$

for $p_{E0} \neq p_{s0}$.

The maximum of p'_s is present at $G'_s=1$ for

$$p'_{s0} = p^{**}_{s0} = p^{**}_{s0} / p_{s0} = p^{**}_{s0} [(\beta' - \beta) / \beta'] = p_{E0} / p_{s0} r_N / \beta' = p_{E0} / p_{s0} y / (x-1) \quad (20)$$

which is the Dimensionless Critical Initial Price Extreme of Sold resources, DCIPES.

The price evolution of sold resources for $x=y=1$ (i.e. $r_N=r_E=\alpha$) becomes, in dimensionless form,

$$p'_s = p_s / p_{s0} = 1 - p_{E0} / p_{s0} \ln G'_s \quad (21)$$

The evolution of p'_s with G'_s is presented in Fig. 8 for $x=y$ (i.e. $r_N=r_E$).

- For $|x|=|y|>1$ (i.e. $r_E=r_N>\alpha$ and $p^{**}_{s0}>0$, $p^{**}_{s0} = +\infty$) the only variable affecting the evolution is $p^{**}_{s0}>0$. The price p'_s decreases with the increase or the decrease of G'_s for $p^{**}_{s0} \geq 1$. The price p'_s increases up to a maximum and then decreases with the decrease or the increase of G'_s for $p^{**}_{s0} < 1$.

- For $x=y<1$ (i.e. $r_E=r_N<\alpha$ and $p^{**}_{s0}<0$, $p^{**}_{s0} = +\infty$) the price p'_s decreases asymptotically to -0 with the increase of G'_s after a negative minimum.

- For $x=y=1$ (i.e. $r_E=r_N=\alpha$ and $p^{**}_{s0}=p^{**}_{s0}=+\infty$) the price p'_s decreases with the increase of G'_s .

4. Supply curve of the difference between the price of sold and extracted resources.

The difference Δp between the price of sold and extracted resources is given by

$$\Delta p = p_s - p_E = (\Delta p_0 - \Delta^* p_0) G'_s^{(x-1)} + \Delta^* p_0 G'_s^{(y-1)} \quad (22)$$

where

$$\Delta^* p_0 = p^*_{s0} - p_{E0} = p_{E0} (2 r_N - r_E) / (r_E - r_N) \quad (23)$$

is the Critical Initial Price Difference, CIPD.

The price difference Δp has an extreme for

$$G'_s = [\Delta p^*_{s0} (1-y)/(x-1)/(1 - \Delta p^*_{s0})]^{1/(x-y)} \quad (24)$$

which is equal to 1 for Δp_0 equal to

$$\Delta^{**}p_0 = \Delta^*p_0 (x-y)/(x-1) = \Delta^*p_0 [(\beta' - \beta)/\beta'] = p_{E0} (2 r_N - r_E) / \beta' \quad (25)$$

which is the Critical Extreme of the Initial Price Difference, CEIPD.

The relative evolutions of Δp with G'_s are presented in Figs. 9-12.

4.1 Supply curve of Δp for $y=0$ and $|x|>1$.

Fig. 9 presents the price difference Δp for $y=0$ (i.e. $r_N=0$) and $|x|>1$ (i.e. $\alpha < r_E$, $\Delta^*p_0 = -p_{E0} < 0$ and $\Delta^*p_0 < \Delta^{**}p_0 < 0$).

- For $\Delta p_0 > 0$ the price difference Δp increases with the increase or the decrease of G'_s , i.e. for $G'_s < 1$ or $G'_s > 1$. For $G'_s < 1$ the difference Δp increases exponentially while the difference Δp increases at a lower rate for $G'_s > 1$.

- For Δp_0 in the range from 0 to $\Delta^{**}p_0$, i.e. $\Delta^{**}p_0 < \Delta p_0 < 0$, the price difference Δp increases with the increase or the decrease of G'_s . The increase is exponential for $G'_s < 1$ while has a lower rate of increase for $G'_s > 1$.

- For $\Delta p_0 = \Delta^{**}p_0$, the price difference Δp increases with the increase or the decrease of G'_s . For $G'_s < 1$ the difference Δp increases exponentially with the minimum at $G'_s = 1$ while the increase has a lower rate for $G'_s > 1$.

- For Δp_0 in the range from Δ^*p_0 to $\Delta^{**}p_0$, i.e. $\Delta^*p_0 < \Delta p_0 < \Delta^{**}p_0$, the price difference Δp increases exponentially, after a minimum, with the decrease of G'_s . For $G'_s > 1$ the difference Δp increases with a lower rate.

- For $\Delta p_0 = \Delta^*p_0$, the price difference Δp increases asymptotically towards - 0 for $G'_s > 1$, while Δp decreases exponentially for $G'_s < 1$.

4.2 Supply curve of Δp for $y=0$ and $0 < x \leq 1$.

The price difference Δp for $y=0$ (i.e. $r_N=0$, $\Delta^*p_0 = -p_{E0} < 0$) and $0 < x \leq 1$ (i.e. $\alpha \geq r_E > 0$) has the following trends.

- For $x=1$ (i.e. $\alpha=r_E$ and $\Delta^{**}p_0=-\infty$) the price difference Δp increases asymptotically to p_{S0} for any value of Δp_0 (i.e. $\Delta p_0>0$, $0>\Delta p_0>\Delta^*p_0$ or $\Delta p_0=\Delta^*p_0$).

- For $x<1$ (i.e. $\alpha>r_E>0$ and $\Delta^{**}p_0>0$) the price difference Δp tends to 0 for any value of Δp_0 .

4.3 Supply curve of Δp for $|x|>2|y|>=1$, or $y>x=1$, or $y>1>x$.

Fig. 10 presents the price difference Δp for: $|x|>2|y|>=1$ (i.e. $r_E > 2 r_N \geq \alpha$ and $\Delta^*p_0<\Delta^{**}p_0<0$); $y>x=1$ (i.e. $r_N>\alpha=r_E$ and $\Delta^*p_0<0$); $y>1>x$ (i.e. $r_N>\alpha>r_E$ and $\Delta^{**}p_0<\Delta^*p_0<0$).

- For $|x|>2|y|>=1$ and $\Delta p_0 > 0$ or $\Delta p_0 > \Delta^{**}p_0$ the price difference Δp increases exponentially with the increase or the decrease of G'_s . For $\Delta p_0=\Delta^{**}p_0$ the price difference Δp increases exponentially with a minimum at $G'_s=1$. For $\Delta^{**}p_0>\Delta p_0>\Delta^*p_0$ the price difference Δp increases exponentially, after a minimum, with the decrease or the increase of G'_s . For $\Delta p_0\leq\Delta^*p_0$ the price difference Δp decreases exponentially with the decrease or the increase of G'_s .

- For $y > x = 1$ (i.e. $r_N>\alpha=r_E$ and $\Delta^*p_0<0$), and for $y>1>x$ (i.e. $r_N>\alpha>r_E$ and $\Delta^{**}p_0<\Delta^*p_0<0$). The price difference Δp decreases exponentially with G'_s .

4.4 Supply curve of Δp for $|y|>1$ and $|x|=2 y$.

The price difference Δp for $|y|>1$ and $|x|=2|y|$ (i.e. $\alpha<r_N$, $r_E=2 r_N$, and $\Delta^*p_0=\Delta^{**}p_0=0$) has the following trends. For $\Delta p_0>0$ the price difference Δp increases exponentially with the increase or the decrease of G'_s . For $\Delta p_0<0$ the price difference Δp decreases exponentially with the increase or the decrease of G'_s .

4.5 Supply curve of Δp for $|y|>1$ and $|x|<2 y$.

Fig. 11 presents the price difference Δp for $|y|>1$ and $|x|<2|y|$ (i.e. $\alpha<r_N$, $r_E<2 r_N$ and $\Delta^*p_0>\Delta^{**}p_0>0$). For $\Delta p_0 \geq \Delta^*p_0$ the price difference Δp increases exponentially with the increase or the decrease of G'_s . For $\Delta^*p_0>\Delta p_0>\Delta^{**}p_0$ the price difference Δp increases up to a maximum and then decreases exponentially with the increase or the decrease of G'_s . For $\Delta p_0=\Delta^{**}p_0$ the price difference Δp decreases exponentially with the increase or the decrease

of G'_s and the maximum is at $G'_s=1$. For $\Delta p_0 \leq \Delta^{**}p_0$ the price difference Δp decreases exponentially with the increase or the decrease of G'_s .

4.6 Supply curve of Δp for $y=1$.

The price difference Δp for $y=1$ (i.e. $\alpha = r_N$) has the following trends.

- For $x>y=1$ (i.e. $0 < \alpha = r_N < r_E$ and $\Delta^*p_0 = \Delta^{**}p_0$), three cases are possible. For $\Delta p_0 > \Delta^*p_0$ the price difference Δp increases with the increase of G'_s ; for $\Delta p_0 = \Delta^*p_0$ the price difference Δp remains constant with the increase of G'_s ; for $\Delta p_0 < \Delta^*p_0$ the price difference Δp decreases with the increase of G'_s .

- For $x<y=1$ (i.e. $\alpha = r_N > r_E$ and $\Delta^*p_0 = \Delta^{**}p_0$), the price difference Δp decreases with the increase of G'_s .

4.7 Supply curve of Δp for $y < 1$.

The price difference Δp for $y < 1$ (i.e. $\alpha > r_N$) and $\Delta^{**}p_0 > \Delta^*p_0 > 0$ has the following trends. For $\Delta p_0 > \Delta^{**}p_0$ the price difference Δp increases with the increase of G'_s . For $\Delta p_0 = \Delta^{**}p_0$ the price difference Δp increases with the increase of G'_s and the minimum is at $G'_s=1$. For $\Delta^{**}p_0 > \Delta p_0 > \Delta^*p_0$ the price difference Δp increases with the increase of G'_s after a minimum. For $\Delta p_0 = \Delta^*p_0$ the price difference Δp decreases asymptotically to $+0$ with the increase of G'_s . For $\Delta p_0 < \Delta^*p_0$ the price difference Δp decreases with the increase of G'_s .

The price difference Δp for $y < 1$ (i.e. $\alpha > r_N > 0$) and $\Delta^{**}p_0 < \Delta^*p_0 < 0$ has the following trends. For $\Delta p_0 > 0 > \Delta^*p_0$ the price difference Δp increases with the increase of G'_s . For $\Delta p_0 = \Delta^*p_0$ the price difference Δp increases asymptotically to -0 with the increase of G'_s . For $\Delta^*p_0 > \Delta p_0 > \Delta^{**}p_0$ the price difference Δp increases up to a maximum and then decreases with the increase of G'_s . For $\Delta p_0 \leq \Delta^{**}p_0$ the price difference Δp decreases with the increase of G'_s .

The price difference Δp for the other three cases which are possible according to the relation between x and y (i.e. α and r_E) has the following trends.

- For $1=>x>y$ (i.e. $\alpha=>r_E>r_N$) the price difference Δp decreases asymptotically to $+0$ with the increase of G'_s .

- For $l > y > x$ (i.e. $\alpha > r_N > r_E$) the price difference Δp decreases asymptotically to -0 after a negative minimum with the increase of G'_s .

4.8 Supply curve of Δp for $x = y$.

The price difference Δp , [4], is, in dimensionless form,

$$\Delta p = p_S - p_E = (\Delta p_0 - p^{**}_{s0} \beta' t) \exp(\beta' t) \quad (26)$$

For $x=y$ the critical initial price difference $\Delta^* p_0$ and the critical initial extreme price difference $\Delta^{**} p_0$ are not defined and the only critical price defined is the critical initial price extreme of the sold resources p^{**}_{s0} .

The price difference Δp , as a function of G'_s , is then

$$\Delta p = [\Delta p_0 - p^{**}_{s0} (y-1) \ln(G'_s)] G'_s^{(y-1)} \quad (27)$$

The price difference Δp has an extreme for

$$G'_s = \exp\{[(\Delta p_0 - p^{**}_{s0}) / (1-y) / p^{**}_{s0}]\} \quad (28)$$

which is equal to 1 if

$$\Delta p_0 = p^{**}_{s0} \quad (29)$$

Fig. 12 presents the price difference Δp for $x=y$ (i.e. $r_N=r_E$).

- For $x < 0$ (i.e. $\alpha < 0 < r_E$ and $p^{**}_0 > 0$), three cases are possible with the decrease of G'_s . For $\Delta p_0 > p^{**}_0$ the price difference Δp decreases after a maximum; for $\Delta p_0 = p^{**}_0$ the price difference Δp decreases with the maximum at $G'_s = 1$; for $\Delta p_0 < p^{**}_0$ the price difference Δp decreases exponentially.

- For $x < 1$ (i.e. $\alpha > r_E$ and $p^{**}_0 < 0$), three cases are possible with the increase of G'_s . For $\Delta p_0 > 0 > p^{**}_0$ the price difference Δp decreases asymptotically to -0 after a negative minimum; for $0 > \Delta p_0 > = < p^{**}_0$ the price difference Δp increases asymptotically to -0 .

- For $x=y=1$ (i.e. $\alpha=r_E=r_N$) the price difference Δp , which has the following dependence on G'_s ,

$$\Delta p = \Delta p_0 - y p_{E0} \ln(G'_s) \quad (30)$$

decreases with the increase of G'_s .

5. Real evolution and predictions of the price of non-renewable resources.

5.1 Oil price evolution from 1980 to 2005.

The real evolution of world oil price versus consumption from 1980 to 2005 [6] is reported in Table 1 and Figure 13.

Table 1 reports the current year on the first column, the oil price, p ($US\$/GJ$), on the second one, the consumption rate, G (GT/y), on the third one and the extraction rate of oil, α ($1/annum$), on the fourth one. The extraction rate is evaluated from the consumption rates of the previous and current year. Table 1 reports also the economic data employed in the present model, i.e. the inflation rate on the fifth column, the discount rate on the sixth one and the prime rate on the last one. The inflation rates from 1980 to 2005 have been assumed according to [7] while the discount and prime rates from [8].

Figure 13 presents the evolution of world price versus the consumption rate. The oil price is the average world price registered during the corresponding year [6]. The first oil price reported on the left part of the curve is relative to 1980 ($6.52 US\$/GJ$) with a consumption of $3.15 GT/y$. From 1980 to 1983 the consumption has decreased down to $2.94 GT/y$ (and then the curve is going towards the left) along to the price which has decreased down to $5.3 US\$/GJ$ in 1983. From 1983 the consumption rate has continuously increased (and then the curve is always going towards the right) until 2005 when a small decrease has been registered. The world oil price registered oscillations with a decrease from 1983 to 1986, an increase in 1987, decrease in 1988, sensible increase in 1989 and 1990, decrease from 1991 to 1994, increase from 1995 to 1996, decrease from 1997 to 1998, sensible increase from 1999 to 2000, decrease in 2001, and a final continuous increase from 2002 to 2005. The curve from 2004 to 2005 is almost vertical, i.e. the price has increased while the consumption rate has been almost constant (a slight decrease).

The price evolution from 1980 to 1983 (i.e. a price decrease along to a consumption decrease) cannot be predicted by Eq. 4 for the price of extracted resources, p'_E , and is not present among the curves reported in Fig. 1. Similar considerations can be done for the price evolution of sold resources, p'_S , which cannot be predicted by Eq. 14, under the assumption of $r_N=0$, i.e. zero interest rate of the non extracted resources, or by Eq. 15 for $p'^{*}_{SO}=1$. This evolution is not present, among the others, in Fig. 2 and Fig. 3

The decrease of the price p'_s and the consumption G'_s (i.e. $x < 0$) can be seen in Fig. 4 for $p'^*_{s0} > p''^*_{s0} \geq 1$ and after the maximum for $p'^*_{s0} > 1 > p''^*_{s0}$, in Fig 5 for $p''^*_{s0} \geq 1$ and after the maximum for $p''^*_{s0} < 1$, in Fig. 8 for $p''^*_{s0} \geq 1$ and after the maximum for $p''^*_{s0} < 1$.

Table 3 reports the current year on the first column, the oil price, p ($US\$/GJ$), on the second one, the consumption rate, G (GT/y), on the third one, the extraction rate, α (1/annum), on the fourth one, the economic data employed in the best prediction, i.e. r_N on the fifth column and r_E on the sixth one. Table 3 reports also the resulting values of y , x , the Figure of reference, p''^*_{s0} , p'^*_{s0} and the ratio p_{s0}/p_{E0} used in the best prediction.

The price decrease from 1981 to 1986 can be predicted by the present approach under the assumption that the inflation rate is equal to $r_E = r_N$ and the guideline of Fig. 8 for $x = y$, i.e. $r_E = r_N$. On the base of the data of Table 1 it is possible to forecast numerically:

- a decrease of p'_s in 1981 when $x = y < 0$ and $p''^*_{s0} = 0.994$ under the assumption $p_{s0} = 0.75 p_{E0}$. The decrease of the consumption is $G'_s(1981) = G_s(1981)/G_s(1980) = 0.968$, i.e. $G'_s(1981)$ is smaller than the value of $G'_s(p'_{max}) = 0.998$ at the maximum p'_{max} and the value of $G'_s(p'_s = 1) = 0.996$ at $p'_s = 1$.
- a decrease of p'_s in 1982 when $x = y < 0$ and $p''^*_{s0} = 0.97$ under the assumption $p_{s0} = 0.75 p_{E0}$. The decrease of the consumption is $G'_s(1982) = G_s(1982)/G_s(1981) = 0.977$, i.e. $G'_s(1982)$ is lower than $G'_s(p'_{max}) = 0.991$ and $G'_s(p'_s = 1) = 0.983$.
- a decrease of p'_s in 1983 when $x = y < 0$ and $p''^*_{s0} = 0.992$ under the assumption $p_{s0} = 0.72 p_{E0}$. The decrease of the consumption is $G'_s(1983) = G_s(1983)/G_s(1982) = 0.987$, i.e. $G'_s(1983)$ is lower than $G'_s(p'_{max}) = 0.998$ and $G'_s(p'_s = 1) = 0.995$.
- a decrease of p'_s in 1984 when $x = y > 1$ and $p''^*_{s0} = 1.76$ under the assumption $p_{s0} = p_{E0} = 1$.
- a decrease of p'_s in 1985 when $x = y > 1$ and $p''^*_{s0} = 1.14$ under the assumption $p_{s0} = p_{E0} = 1$.
- a decrease of p'_s in 1986 when $x = y < 1$ and $p''^*_{s0} = -2.35$ under the assumption $p_{s0} = p_{E0} = 1$.

The price evolution from 1987 to 1996 can be predicted by the present approach under the assumption that the inflation rate is equal to r_N , the discount rate is equal to r_E and the guideline of Fig. 4 for $x > y > 1$, i.e. $r_E > r_N > \alpha > 0$. On the base of the data of Table 1 it is possible to forecast numerically:

- an increase of p'_s in 1987 when $p'^*_{s0} = 1.59 > 1 > p''^*_{s0} = 0.87$ under the assumption $p_{s0} = 1.15 p_{E0}$. The increase of the consumption, before the maximum, is $G'_s(1987) = G_s(1987)/G_s(1986) = 1.019$, i.e. $G'_s(1987)$ is smaller than $G'_s(p'_{max}) = 1.22$.

- a decrease of p'_s in 1988 when $p'^*_{s0}=1.925>p'^**_{s0}=1.22>1$ under the assumption $p_{s0}=p_{e0}=1$.
- an increase of p'_s in 1989, when $p'^*_{s0}=2.3>1>p'^**_{s0}=0.92$ under the assumption $p_{s0}=p_{e0}=1$. The increase of the consumption, before the maximum, is $G'_s(1989) = G(1989)/G(1988) = 1.018$, i.e. $G'_s(1989)$ is smaller than $G'_s(p_{max})=1.048$.
- an increase of p'_s in 1990 when $p'^*_{s0}=3.39>1>p'^**_{s0}=0.78$ under the assumption $p_{s0}=p_{e0}=1$. The increase of the consumption, before the maximum, is $G'_s(1990) = G_s(1990)/G_s(1989) = 1.0009$, i.e. $G'_s(1990)$ is smaller than $G'_s_{max}=1.005$.
- a decrease of p'_s in 1991, when $p'^*_{s0}=3.94>p'^**_{s0}=1.04>1$ under the assumption $p_{s0} = 0.9 p_{e0}$.
- a decrease of p'_s in 1992, when $p'^*_{s0}=13.77>p'^**_{s0}=1.01>1$ under the assumption $p_{s0}=p_{e0}=1$.
- a decrease of p'_s in 1993 when $p'^*_{s0}=74>p'^**_{s0}=1.13>1$ under the assumption $p_{s0}=p_{e0}=1$.
- a decrease of p'_s in 1994 when $p'^*_{s0}=2.64>p'^**_{s0}=1.55>1$ under the assumption $p_{s0}=p_{e0}=1$.
- an increase of p'_s in 1995 when $p'^*_{s0}=1.18>1>p'^**_{s0}=0.97$ under the assumption $p_{s0}=p_{e0}=1$. The increase of the consumption, before the maximum, is $G'_s(1995) = G_s(1995)/G_s(1994) = 1.023$, i.e. $G'_s(1995)$ is smaller than $G'_s_{max}=1.13$.
- an increase of p'_s in 1996 when $p'^*_{s0}=1.40>1>p'^**_{s0}=0.97$ under the assumption $p_{s0}=p_{e0}=1$. The increase of the consumption, before the maximum, is $G'_s(1996) = G_s(1996)/G_s(1995) = 1.02$, i.e. $G'_s(1996)$ is smaller than $G'_s_{max}=1.064$.

The decrease of p'_s in 1997 and 1998 can be predicted by the present approach under the assumption that the inflation rate is $r_E=r_N$, and the guideline of Fig. 8 for $x=y$, i.e. $r_E=r_N$. On the base of the data of Table 1 and the assumption $p_{s0}=p_{e0}=1$ it is possible to forecast numerically:

- a decrease of p'_s in 1997 when $x=y<1$ and $p'^**_{s0}=-146<0$.
- a decrease of p'_s in 1998 when $x=y>1$ and $p'^**_{s0}=1.39>1$.

The price increase in 1999 and 2000 can be predicted by the present approach under the assumption that the inflation rate is equal to r_N , the prime rate is equal to r_E and the guidelines of Fig. 7 in 1999 for $x>1>y$, i.e. $r_E>\alpha>r_N$, and Fig. 4 in 2000 for $x>y>1$, i.e. $r_E>r_N>\alpha>0$. On the base of the data of Table 1 and the assumption $p_{s0}=p_{e0}=1$ it is possible to forecast numerically:

- an increase of p'_s in 1999 when $p'^{**}_{s0}=0.44<1$ and $p'^{*}_{s0}=0.38$.
- an increase of p'_s in 2000 when $p'^{*}_{s0}=0.57<1$ and $p'^{**}_{s0}=0.43$.

The price decrease in 2001 can be predicted by the present approach under the assumption that the inflation rate is equal to r_N , the discount rate is equal to r_E and the guideline of Fig. 4 for $x>y>1$, i.e. $r_E>r_N>\alpha>0$. On the base of the data of Table 1 and the assumption $p_{s0}=p_{e0}=1$ it is possible to forecast numerically a decrease because $p'^{*}_{s0}=4.88>p'^{**}_{s0}=1.19>1$.

The price evolution from 2002 until 2005 can be predicted by the present approach under the assumption that the inflation rate is equal to r_N , the prime rate is equal to r_E and the guidelines of Fig. 4 in 2002-2003 for $x>y>1$, i.e. $r_E>r_N>\alpha>0$, Fig. 7 in 2004 for $x>1>y$, i.e. $r_E>\alpha>r_N$, and Fig. 4 in 2005 for $x<y<0$, i.e. $r_E>r_N>0>\alpha$. On the base of the data of Table 1 it is possible to forecast numerically:

- an increase of p'_s in 2002 when $p'^{*}_{s0}=0.52<1$ and $p'^{**}_{s0}=0.44$ under the assumption $p_{s0}=p_{e0}=1$.
- an increase of p'_s in 2003 when $p'^{*}_{s0}=0.99<1$ and $p'^{**}_{s0}=0.88$ under the assumption $p_{s0}=1.25 p_{e0}$.
- an increase of p'_s in 2004 when $p'^{*}_{s0}=0.996<1$ and $p'^{**}_{s0}=0.55$ under the assumption that $p_{s0}=2.89 p_{e0}$.
- an increase of p'_s in 2005 when $p'^{*}_{s0}=0.99<1$ and $p'^{**}_{s0}=0.44$ under the assumption $p_{s0}=1.2 p_{e0}$.

5.2 Gas price evolution from 1984 to 2005.

The evolution of the gas price in Europe versus the consumption in the world from 1984 to 2005, [6], is reported in Table 2 and Fig. 14.

Table 2 reports the current year on the first column, the gas price in Europe, p ($US\$/GJ$), on the second one, the consumption rate in the world, G ($10^9 m^3/y$), on the third one, the extraction rate, α ($1/annum$), on the fourth one. Table 2 reports also the economic data employed in the present model, i.e. the inflation rate on the fifth column, the discount rate on the sixth one and the prime rate on the last one. The inflation rates from 1984 to 2005 have been assumed according to [7] while the discount and the prime rates from [8].

Figure 14 presents the evolution of the gas price in Europe versus the consumption in the world. The price of the gas is the average price registered during the corresponding year [6]. The first gas price reported on the left of the curve is relative to 1984 ($3.56 US\$/GJ$) with

a consumption of $1621.1 \text{ Gm}^3/\text{y}$. From 1984 to 2005 the consumption has usually increased, except during 1997. The gas price has oscillated up and down with a first increase in 1985, decrease from 1986 to 1989, increase from 1990 to 1991, decrease from 1992 to 1994, increase from 1995 to 1997, decrease from 1998 to 1999, increase in 2000 and 2001, decrease in 2002, and final increase from 2003 to 2005.

Table 4 reports the current year on the first column, the gas price, p ($US\$/GJ$), on the second one, the consumption rate, G ($10^9 \text{ m}^3/\text{y}$), on the third one, the extraction rate, α (1/annum), on the fourth one, the economic data employed in the best prediction, i.e. r_N on the fifth column and r_E on the sixth one. Table 4 reports also the resulting values of y , x , the Figure of reference, p^{**}_{S0} , p^*_{S0} and the ratio p_{S0}/p_{E0} used in the best prediction.

The price increase in 1985 can be predicted by the present approach under the assumption that the inflation rate is equal to r_N , the discount or the prime rate is equal to r_E and the guideline of Fig. 4 for $x>y>1$ (i.e. $r_E>r_N>\alpha$). On the base of the data of Table 2 and the assumption $p_{S0}=p_{E0}=1$ it is possible to forecast the increase in 1985 when $p^*_{S0}=0.86<1$ if the discount rate is equal to r_E or $p^{**}_{S0}=0.55<1$ if the prime rate is equal to r_E .

The price decrease from 1986 to 1989 can be predicted by the present approach under the assumption that the inflation rate is equal to $r_E=r_N$ and the guideline of Fig. 8 for $x=y$, i.e. $r_E=r_N$. On the base of the data of Table 2 and the assumption $p_{S0}=p_{E0}=1$ it is possible to forecast numerically:

- a decrease of p'_S in 1986 when $x=y>1$ (i.e. $r_E=r_N>\alpha$) and $p^{**}_{S0}=2.49>1$.
- a decrease of p'_S in 1987 when $x=y<1$ (i.e. $r_E=r_N<\alpha$) and $p^{**}_{S0}=-2.99<1$.
- a decrease of p'_S in 1988 when $x=y<1$ (i.e. $r_E=r_N<\alpha$) and $p^{**}_{S0}=-8.35<1$.
- a decrease of p'_S in 1989 when $x=y>1$ (i.e. $r_E=r_N>\alpha$) and $p^{**}_{S0}=26.37>1$.

The price increase from 1990 to 1991 can be predicted by the present approach under the assumption that the inflation rate is equal to r_N , the prime rate is equal to r_E and the guideline of Fig. 4, $x>y>1$ (i.e. $r_E>r_N>\alpha$). On the base of the data of Table 2 and the assumption $p_{S0}=p_{E0}=1$ it is possible to forecast numerically:

- an increase of p'_S in 1990 when $p^*_{S0}=1.17>1>p^{**}_{S0}=0.70$. The increase of the consumption, before the maximum, is $G'_S(1990)=G_S(1990)/G_S(1989)=1.024$, i.e. $G'_S(1990)$ is smaller than the maximum $G'_S(p'_{max})=1.67$.

- an increase of p'_s in 1991 when $p'^*_{s0}=1.82>1>p'^**_{s0}=0.9$. The increase of the consumption, before the maximum, is $G'_s(1991)=G_s(1991)/G_s(1990)=1.019$, i.e. $G'_s(1991)$ is smaller than the maximum $G'_s(p_{max})=1.097$.

The price decrease from 1992 to 1994 can be predicted by the present approach:

- under the assumption that the inflation rate is equal to r_N , the discount rate is equal to r_E and the guidelines of Fig. 4 when $x>y>1$, i.e. $r_E>r_N>\alpha>0$. On the base of the data of Table 2 and the assumption $p_{s0}=p_{E0}=1$ it is possible to forecast:

- a decrease of p'_s in 1992 when $p'^*_{s0}=13.77>p'^**_{s0}=1.12>1$;
- a decrease of p'_s in 1993 when $p'^*_{s0}=74>p'^**_{s0}=2.3>1$;
- a decrease of p'_s in 1994 when $p'^*_{s0}=3.3>p'^**_{s0}=1.01>1$ under the assumption $p_{s0}=0.8 p_{E0}$.

- under the assumption that the inflation rate is $r_E=r_N$ and the guideline of Fig. 8 for $x=y$, i.e. $r_E=r_N$. On the base of the data of Table 2 and the assumption $p_{s0}=p_{E0}=1$ it is possible to forecast numerically:

- a decrease of p'_s in 1992 when $p'^**_{s0}=1.22>1$;
- a decrease of p'_s in 1993 when $p'^**_{s0}=2.38>1$;
- a decrease of p'_s in 1994 when $p'^**_{s0}=1.17>1$.

The price increase from 1995 to 1997 can be predicted by the present approach under the assumption that the inflation rate is equal to r_N , the prime rate is equal to r_E and the guideline of Fig. 7 in 1995-1996 when $x>1>y$, i.e. $r_E>\alpha>r_N$ and Fig. 4 in 1997 when $x<y<0$, i.e. $r_E>r_N>0>\alpha$. On the base of the data of Table 2 and the assumption $p_{s0}=p_{E0}=1$ it is possible to forecast numerically:

- an increase of p'_s in 1995 when $p'^**_{s0}=0.51<1$;
- an increase of p'_s in 1996 when $p'^**_{s0}=0.83<1$;
- an increase of p'_s in 1997 when $p'^*_{s0}=0.38<1$.

The price decrease from 1998 to 1999 can be predicted by the present approach under the assumption that the inflation rate is $r_E=r_N$, and the guideline of Fig. 8 for $x=y$, i.e. $r_E=r_N$. On the base of the data of Table 1 and the assumption $p_{s0}=p_{E0}=1$ it is possible to forecast numerically:

- a decrease of p'_s in 1998 when $x=y<1$ (i.e. $r_E=r_N<\alpha$) and $p'^**_{s0}=-14.42<0$;
- a decrease of p'_s in 1999 when $x=y>1$ (i.e. $r_E=r_N>\alpha$) and $p'^**_{s0}=35.44>1$.

The price evolution from 2000 to 2005 can be predicted by the present approach under the assumption that the inflation rate is equal to r_N , the prime rate is equal to r_E and the

guidelines of Fig. 7 in 2000, 2002-2004, when $x > 1 > y$, i.e. $r_E > \alpha > r_N$, and Fig. 4 in 2001, 2005 when $x > y > 1$, i.e. $r_E > r_N > \alpha$. On the base of the data of Table 2 it is possible to forecast:

- an increase of p'_s in 2000 when $p'^{**}_{s0} = 0.66 < 1$ under the assumption $p_{s0} = p_{e0} = 1$.
- an increase of p'_s in 2001 when $p'^{**}_{s0} = 0.72 < 1$ under the assumption $p_{s0} = p_{e0} = 1$.
- a decrease of p'_s in 2002 when $p'^{**}_{s0} = 1.06 > 1 > p'^{**}_{s0} = 0.52$ under the assumption $p_{s0} = p_{e0} = 1$. The increase of the consumption is $G'_s(2002) = G_s(2002)/G_s(2001) = 1.032$, i.e. $G'_s(2002)$ is smaller than consumption of the minimum $G'_s(p'_{min}) = 1.13$;
- an increase of p'_s in 2003 when $p'^{**}_{s0} = 0.995 < 1$ under the assumption $p_{s0} = 1.35 p_{e0}$;
- an increase of p'_s in 2004 when $p'^{**}_{s0} = 0.982 < 1$ under the assumption $p_{s0} = 3.1 p_{e0}$;
- an increase of p'_s in 2005 when $p'^{**}_{s0} = 0.99 < 1$ under the assumption $p_{s0} = 1.2 p_{e0}$.

6. Conclusions

6.1 Oil price evolution from 1980 to 2005.

The conclusions of the best predictions obtained by the present method in the entire period from 1980 to 2005 are the following:

- the interest rate of non extracted resources, r_N , is equal to the inflation rate;
- the interest rate of extracted resources, r_E , varies from 1981 to 2005. It is equal to the inflation rate from 1981 until 1986; to the discount rate from 1987 until 1996; to the inflation rate from 1997 until 1998; to the prime rate from 1999 until 2000; to the discount rate in 2001; to the prime rate from 2002 until 2005.

The best predictions of the oil price obtained with the present approach are reported in Fig. 15 according to the abovementioned assumptions about the interest rates of non extracted, r_N , and extracted resources, r_E . The conclusion is that the agreement can be considered as acceptable.

6.2 Gas price evolution from 1984 to 2005.

The conclusions of the best predictions obtained with the present method in the period from 1984 to 2005 are the following:

- the interest rate of non extracted resources, r_N , is equal to the inflation rate;
- the interest rate of extracted resources, r_E , varies from 1984 to 2005. It is equal to the discount or the prime rate in 1985; to the inflation rate from 1986 until 1989; to the prime rate from 1990 to 1991; to the discount rate from 1992 until 1994; to the prime rate from 1995 to 1997; to the inflation rate from 1998 until 1999; to the prime rate from 2000 until 2005.

The best predictions of the gas price obtained with the present approach are reported in Fig. 16 according to the abovementioned assumptions about the interest rate of non extracted, r_N , and extracted resources, r_E . The conclusion is that the agreement can be considered as acceptable.

Nomenclature

Latin

G	mass flow rate of resources
p	price
p'^{*}_{s0}	Dimensionless Critical Initial Price of Sold resources, DCIPS
p''^{*}_{s0}	Dimensionless Critical Initial Price Extreme of Sold resources, DCIPES
r	interest rate
t	time
$x=r_E/\alpha$	Rate of Interest of Sold resources on Extracted rate, RISE
$y=r_N/\alpha$	Rate of Interest of Non-extracted resources on Extracted rate, RINE

Greek

α	extraction rate
$\beta=r_N-\alpha$	price-increase factor of non-extracted resources
$\beta'=r_E-\alpha$	price-increase factor of extracted resources
$\Delta p=p_S-p_E$	difference between price of sold and extracted resources
Δ^*p_0	Critical Initial Price Difference, CIPD
$\Delta^{**}p_0$	Critical Extreme Initial Price Difference, CEIPD

Subscript

0	initial
E	extracted resources
max	maximum
N	non extracted resources
S	sold resources

Superscript

'	dimensionless
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Captions to Tables

Table 1 – World oil price, world consumption, extraction, inflation, discount and prime rates from 1980 to 2005.

Table 2 – Europe gas price, world consumption, extraction, inflation, discount and prime rates from 1984 to 2005.

Table 3 – Best predictions for world oil price.

Table 4 – Best predictions for Europe gas price.

Captions to figures

Figure 1 - Dimensionless price of extracted resources, p'_E , versus dimensionless mass flow rate of extraction, G'_E .

Figure 2 - Dimensionless price of sold resources, p'_S , versus dimensionless mass flow rate of extraction, G'_S , for $y=0$ (i.e. $r_N=0$).

*Figure 3 - Dimensionless price of sold resources, p'_S , versus dimensionless mass flow rate of extraction, G'_S , for $p'^*_{S0} = 1$, (i.e. $x = 2y$ if $p_{S0} = p_{E0} = 1$).*

Figure 4- Dimensionless price of sold resources, p'_S , versus dimensionless mass flow rate of extraction, G'_S , for $|x| > |y| > 1$.

Figure 5- Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $|y| > |x| > 1$, or $y > 1 > x$.

Figure 6- Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $y = 1$ and $x = 1$.

Figure 7- Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $y < 1$.

Figure 8- Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $x = y$.

Figure 9- Price difference, $\Delta p = p_s - p_E$, versus dimensionless mass flow rate of extraction, G'_s , for $y = 0$ and $|x| > 1$.

Figure 10- Price difference, $\Delta p = p_s - p_E$, versus dimensionless mass flow rate of extraction, G'_s , for $|x| > 2|y| \geq 1$, or $y > x = 1$, or $y > 1 > x$.

Figure 11 - Price difference, $\Delta p = p_s - p_E$, versus dimensionless mass flow rate of extraction, G'_s , for $|y| > 1$ and $|x| < 2|y|$.

Figure 12- Price difference, $\Delta p = p_s - p_E$, versus dimensionless mass flow rate of extraction, G'_s , for $x = y$.

Figure 13 – World oil price, p (US\$/GJ), versus world consumption, G (GT/y), from 1980 (initial point of the curve close to $G = 3.2$ GT/y) until 2005 (last point on the right).

Figure 14 – Europe gas price, p (US\$/GJ) versus world consumption, G ($G \text{ m}^3/\text{y}$) from 1984 (initial point on the left) until 2005 (last point on the right).

Figure 15 – World oil price (black square), p (US\$/GJ), versus world consumption, G (GT/y), from 1980 (initial point close to $G = 3.2$ GT/y) until 2005 (last point on the right) compared to the best predictions (white circle).

Figure 16 – Europe gas price (US\$/GJ) versus world consumption ($G \text{ m}^3/\text{y}$) in the world from 1984 (initial point of the left) until 2005 (last point on the right) compared to the best predictions (white circle).

Table 1 – World oil price, world consumption, extraction, inflation, discount and prime rates from 1980 to 2005.

Year	p (US\$/GJ)	G (GT/y)	α (1/annum)	Inflation rate (1/annum)	Discount rate (1/annum)	Prime rate (1/annum)
1980	6.52	3.154		13.50	11.77	15.85
1981	6.39	3.045	-3.517	10.30	13.42	18.47
1982	5.86	2.975	-2.326	6.20	11.02	14.58
1983	5.30	2.937	-1.286	3.22	8.50	10.83
1984	5.10	2.992	1.855	4.30	8.80	12.00
1985	4.88	3.005	0.434	3.55	7.70	10.00
1986	2.62	3.088	2.725	1.91	6.33	8.25
1987	3.34	3.15	1.988	3.66	5.66	8.53
1988	2.78	3.241	2.848	4.08	6.20	9.50
1989	3.42	3.296	1.683	4.83	6.92	11.00
1990	4.27	3.299	0.091	5.39	6.98	10.00
1991	3.74	3.329	0.905	4.25	5.45	6.58
1992	3.58	3.337	0.240	3.03	3.25	6.00
1993	3.22	3.35	0.389	2.96	3.00	6.00
1994	3.00	3.415	1.922	2.61	3.60	7.30
1995	3.21	3.495	2.316	2.81	5.20	8.75
1996	3.86	3.566	2.011	2.93	5.02	8.25
1997	3.59	3.651	2.356	2.34	5.00	8.50
1998	2.508	3.667	0.437	1.55	4.92	8.00
1999	3.36	3.78	3.035	2.19	4.62	8.02
2000	5.28	3.83	1.314	3.38	5.73	9.27
2001	4.52	3.87	1.039	2.83	3.41	6.77
2002	4.55	3.91	1.028	1.59	1.17	4.67
2003	5.43	3.99	2.025	2.27	0.63	4.10
2004	7.22	4.13	3.449	2.68	0.85	4.38
2005	9.84	4.12	-0.242	3.39	2.71	6.25

Table 2 – Europe gas price, world consumption, extraction, inflation, discount and prime rates from 1984 to 2005.

Year	p (US\$/GJ)	G (GT/y)	α (1/annum)	Inflation rate (1/annum)	Discount rate (1/annum)	Prime rate (1/annum)
1980				13.50	11.77	15.85
1981				10.30	13.42	18.47
1982				6.20	11.02	14.58
1983				3.22	8.50	10.83
1984	3.565	1621.10		4.30	8.80	12.00
1985	3.628	1671.70	3.074	3.55	7.70	10.00
1986	3.462	1690.90	1.142	1.91	6.33	8.25
1987	2.451	1775.50	4.882	3.66	5.66	8.53
1988	2.241	1858.50	4.569	4.08	6.20	9.50
1989	1.983	1946.90	4.647	4.83	6.92	11.00
1990	2.670	1992.80	2.330	5.39	6.98	10.00
1991	3.014	2030.10	1.854	4.25	5.45	6.58
1992	2.613	2041.40	0.555	3.03	3.25	6.00
1993	2.400	2076.70	1.714	2.96	3.00	6.00
1994	2.123	2084.40	0.370	2.61	3.60	7.30
1995	2.246	2153.00	3.238	2.81	5.20	8.75
1996	2.303	2257.50	4.740	2.93	5.02	8.25
1997	2.512	2249.70	-0.346	2.34	5.00	8.50
1998	2.142	2287.30	1.658	1.55	4.92	8.00
1999	1.706	2336.50	2.128	2.19	4.62	8.02
2000	3.080	2435.40	4.146	3.38	5.73	9.27
2001	3.933	2460.80	1.038	2.83	3.41	6.77
2002	3.279	2540.00	3.168	1.59	1.17	4.67
2003	4.170	2601.90	2.408	2.27	0.63	4.10
2004	4.322	2694.70	3.504	2.68	0.85	4.38
2005	5.952	2749.60	2.017	3.39	2.71	6.25

Table 3 – Best predictions for world oil price.

Year	p (US\$/GJ)	G (GT/y)	α (1/annum)	r_N (1/annum) (inflation)	r_E (1/annum) (i, inflation) (d, discount) (p, prime)	y	x	Fig.	p^{***so}	p^{*so}	P_{SO}/P_{E0}
1980	6.52	3.154									
1981	6.39	3.045	-3.517	10.30	10.30 (i)	-2.93	-2.93	8	0.994	∞	0.75
1982	5.86	2.975	-2.326	6.20	6.20 (i)	-2.67	-2.67	8	0.97	∞	0.75
1983	5.3	2.937	-1.286	3.22	3.22 (i)	-2.50	-2.50	8	0.992	∞	0.72
1984	5.1	2.992	1.855	4.30	4.30 (i)	2.32	2.32	8	1.76	∞	1.00
1985	4.88	3.005	0.434	3.55	3.55 (i)	8.19	8.19	8	1.14	∞	1.00
1986	2.62	3.088	2.725	1.91	1.91 (i)	0.70	0.70	8	-2.35	∞	1.00
1987	3.34	3.15	1.988	3.66	5.66(d)	1.84	2.85	4	1.59	0.87	1.15
1988	2.78	3.241	2.848	4.08	6.20 (d)	1.43	2.18	4	1.925	1.22	1.00
1989	3.42	3.296	1.683	4.83	6.92 (d)	2.87	4.11	4	2.3	0.92	1.00
1990	4.27	3.299	0.091	5.39	6.98 (d)	59.25	76.72	4	3.39	0.78	1.00
1991	3.74	3.329	0.905	4.25	5.45 (d)	4.69	6.02	4	3.94	1.04	0.90
1992	3.58	3.337	0.240	3.03	3.25 (d)	12.62	13.54	4	13.77	1.01	1.00
1993	3.22	3.35	0.389	2.96	3.00 (d)	7.61	7.72	4	74.00	1.13	1.00
1994	3	3.415	1.922	2.61	3.60 (d)	1.36	1.87	4	2.64	1.55	1.00
1995	3.21	3.495	2.316	2.81	5.20 (d)	1.21	2.25	4	1.18	0.97	1.00
1996	3.86	3.566	2.011	2.93	5.02 (d)	1.46	2.50	4	1.40	0.97	1.00
1997	3.59	3.651	2.356	2.34	2.34 (i)	0.99	0.99	8	-146	∞	1.00
1998	2.508	3.667	0.437	1.55	1.55 (i)	3.54	3.54	8	1.39	∞	1.00
1999	3.36	3.78	3.035	2.19	8.02 (p)	0.72	2.64	7	0.44	0.38	1.00
2000	5.28	3.83	1.314	3.38	9.27 (p)	2.57	7.05	4	0.43	0.57	1.00
2001	4.52	3.87	1.039	2.83	3.41 (d)	2.72	3.28	4	1.19	4.88	1.00
2002	4.55	3.91	1.028	1.59	4.67 (p)	1.55	4.54	4	0.44	0.52	1.00
2003	5.43	3.99	2.025	2.27	4.10 (p)	1.12	2.02	4	0.88	0.99	1.25
2004	7.22	4.13	3.449	2.68	4.38 (p)	0.78	1.27	7	0.996	0.55	2.89
2005	9.84	4.12	-0.242	3.39	6.25 (p)	-13.98	-25.78	4	0.44	0.99	1.20

Table 4 – Best predictions for Europe gas price.

Year	p (US\$/GJ)	G (Gm ³ /y)	α (1/annum)	r_N (1/annum) (inflation)	r_E (1/annum) (i, inflation) (d, discount) (p, prime)	y	x	Fig.	p ^{***so}	p ^{*so}	p _{SO} /p _{E0}
1980											
1981											
1982											
1983											
1984	3.565	1621.10	3.074								
1985	3.628	1671.70	1.142	3.55	7.70 (d)	1.16	2.51	4	0.77	0.86	1.00
1986	3.462	1690.90	4.882	1.91	1.91 (i)	1.67	1.67	8	2.49	∞	1.00
1987	2.451	1775.50	4.569	3.66	3.66 (i)	0.75	0.75	8	-2.99	∞	1.00
1988	2.241	1858.50	4.647	4.08	4.08 (i)	0.89	0.89	8	-8.35	∞	1.00
1989	1.983	1946.90	2.330	4.83	4.83 (i)	1.04	1.04	8	26.37	∞	1.00
1990	2.670	1992.80	1.854	5.39	10.00 (p)	2.31	4.29	4	0.70	1.17	1.00
1991	3.014	2030.10	0.555	4.25	6.58 (p)	2.3	3.56	4	0.90	1.82	1.00
1992	2.613	2041.40	1.714	3.03	3.25 (d)	5.46	5.86	4	1.12	13.77	1.00
1993	2.400	2076.70	0.370	2.96	3.00 (d)	1.73	1.75	4	74	2.3	1.00
1994	2.123	2084.40	3.238	2.61	3.60 (d)	7.05	9.73	4	3.3	1.01	0.80
1995	2.246	2153.00	4.740	2.81	8.75 (p)	0.87	2.70	7	0.51	0.47	1.00
1996	2.303	2257.50	-0.346	2.93	8.25 (p)	0.62	1.74	7	0.83	0.55	1.00
1997	2.512	2249.70	1.658	2.34	8.50 (p)	-6.76	-24.57	4	0.265	0.38	1.00
1998	2.142	2287.30	2.128	1.55	1.55 (i)	0.94	0.94	8	-14.42	∞	1.00
1999	1.706	2336.50	4.146	2.19	2.19 (i)	1.03	1.03	8	35.44	∞	1.00
2000	3.080	2435.40	1.038	3.38	9.27 (p)	0.82	2.23	7	0.66	0.57	1.00
2001	3.933	2460.80	3.168	2.83	6.77 (p)	2.72	6.52	4	0.49	0.72	1.00
2002	3.279	2540.00	2.408	1.59	4.67 (p)	0.50	1.47	7	1.06	0.52	1.00
2003	4.170	2601.90	3.504	2.27	4.10 (p)	0.94	1.7	7	0.995	0.92	1.35
2004	4.322	2694.70	2.017	2.68	4.38 (p)	0.77	1.25	7	0.982	0.51	3.10
2005	5.952	2749.60	3.074	3.39	6.25 (p)	1.68	3.1	4	0.67	0.99	1.20

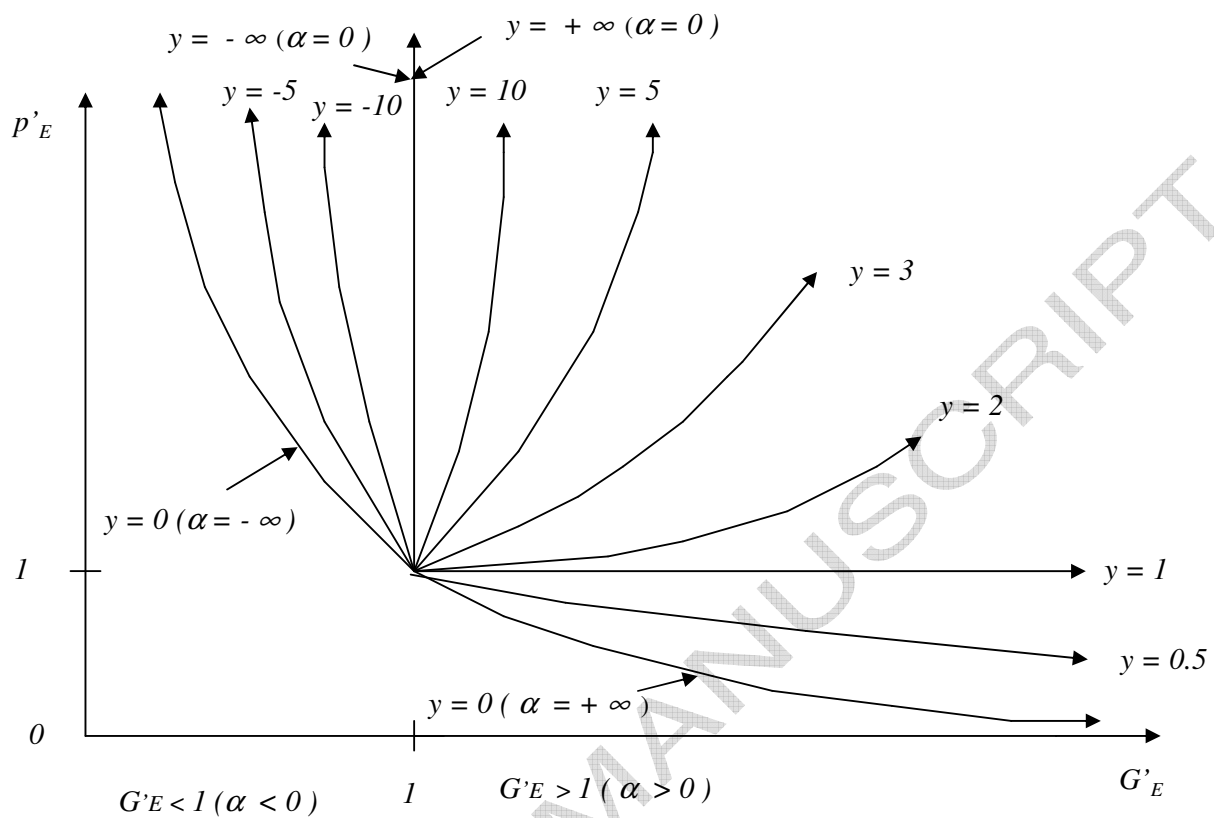


Figure 1 - Dimensionless price of extracted resources, p'_E , versus dimensionless mass flow rate of extraction, G'_E .

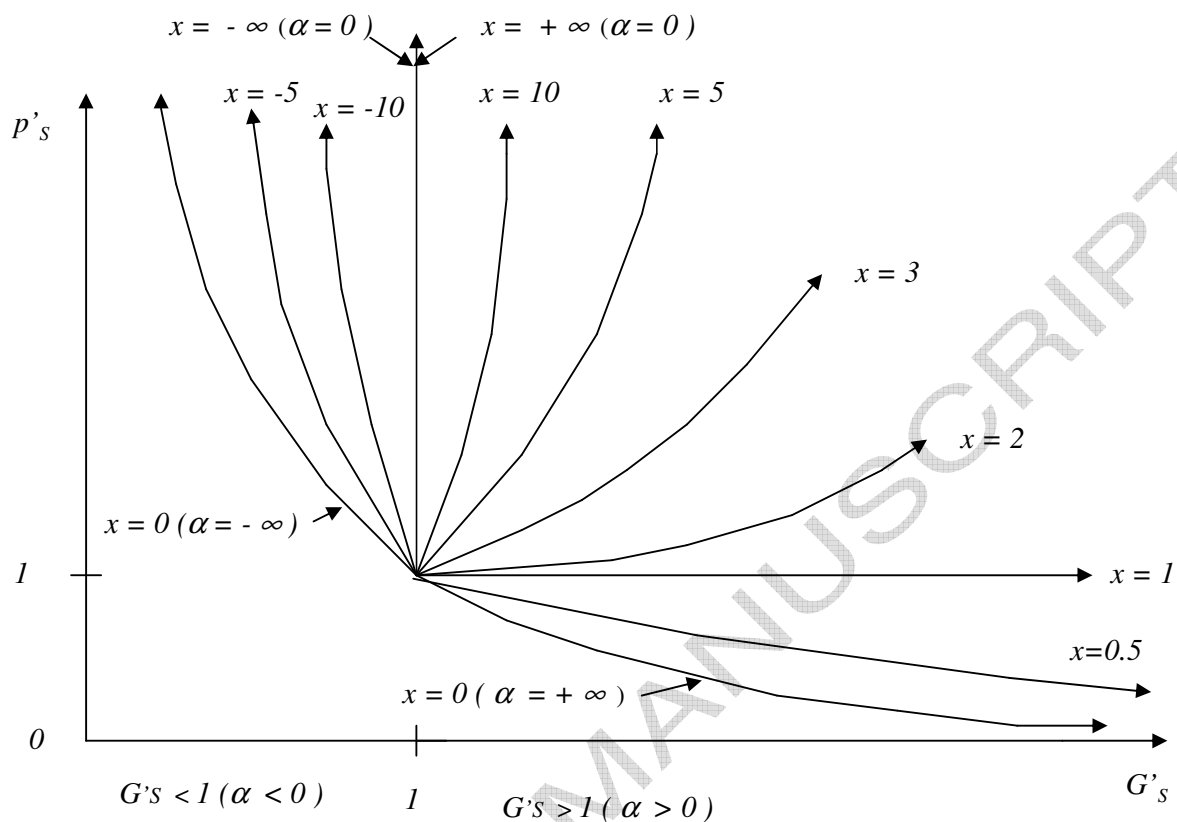


Figure 2 - Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $y=0$ (i.e. $r_N=0$).

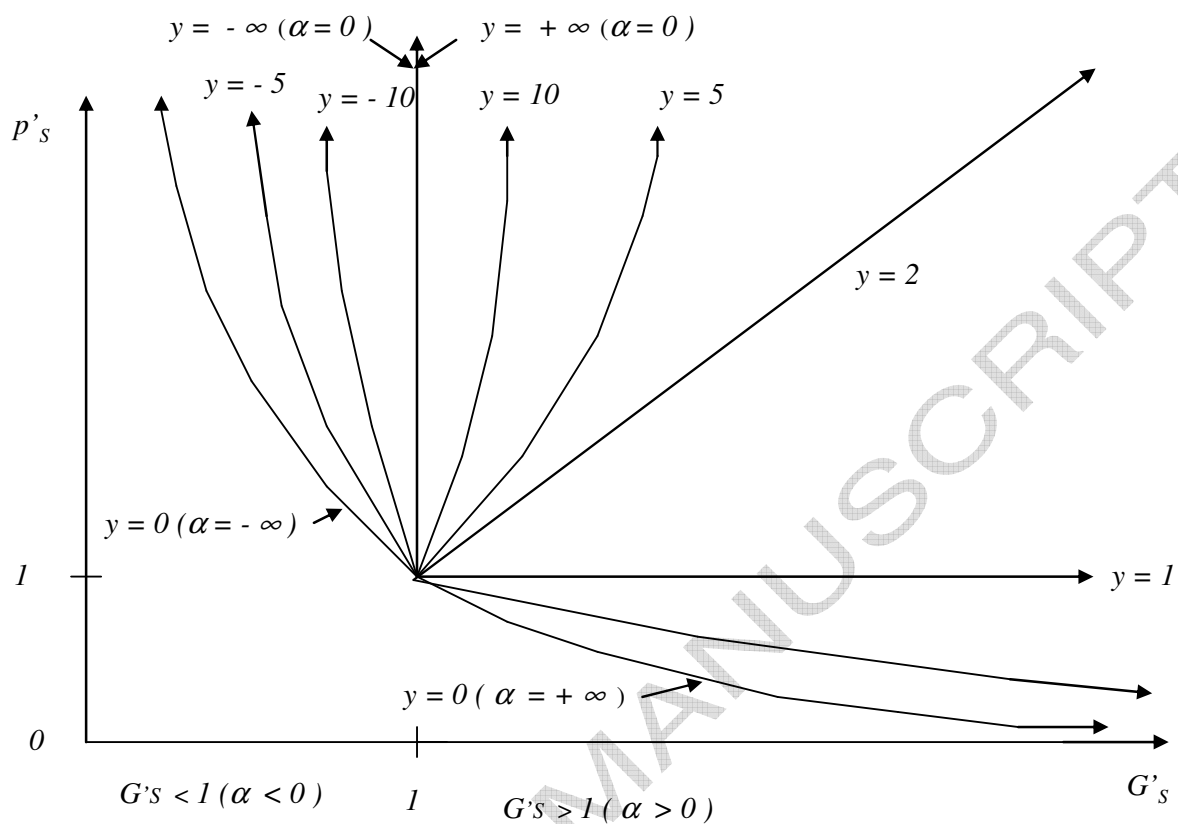


Figure 3 - Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $p'^*_{s0} = 1$, (i.e. $x = 2$ y if $p_{s0} = p_{E0} = 1$).

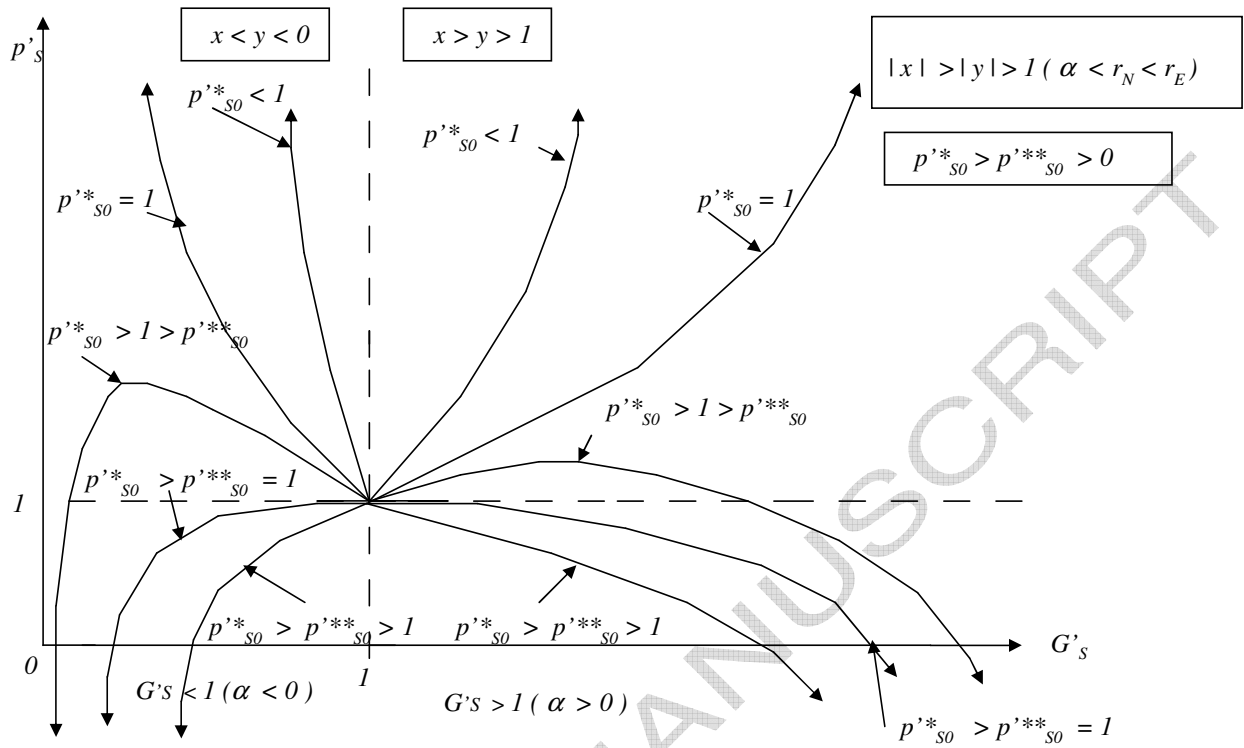


Figure 4- Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $|x| > |y| > 1$.

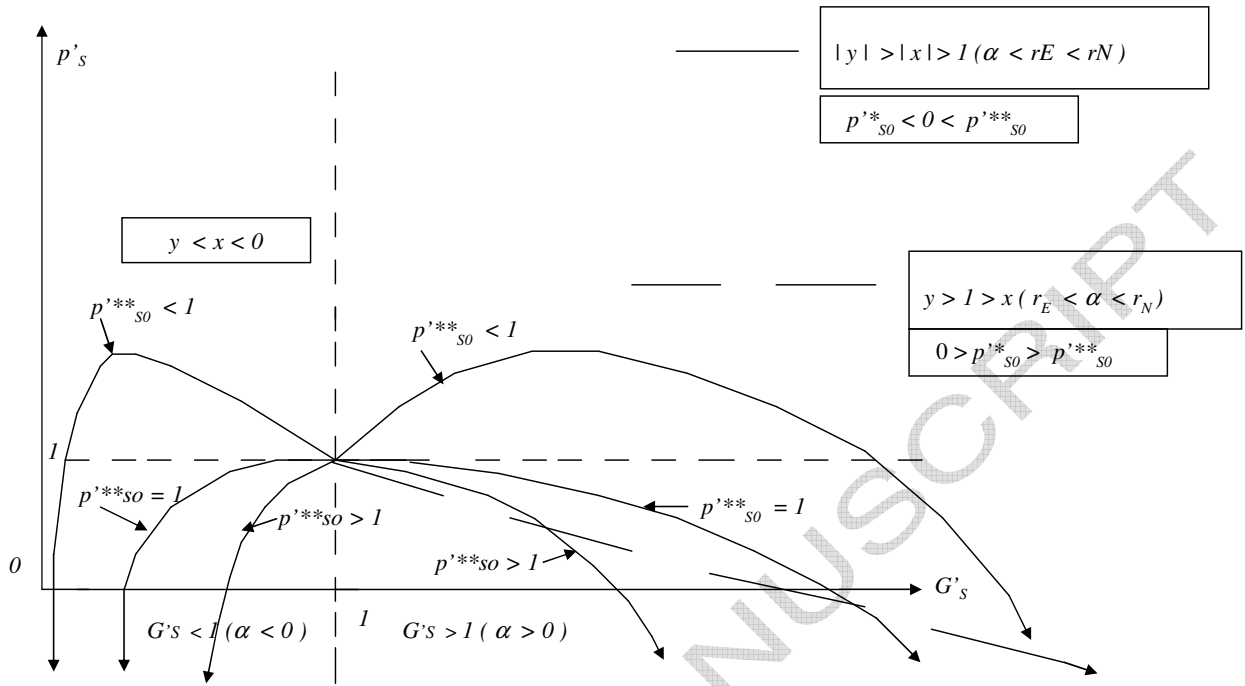


Figure 5- Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $|y| > |x| > 1$ or $y > 1 > x$.

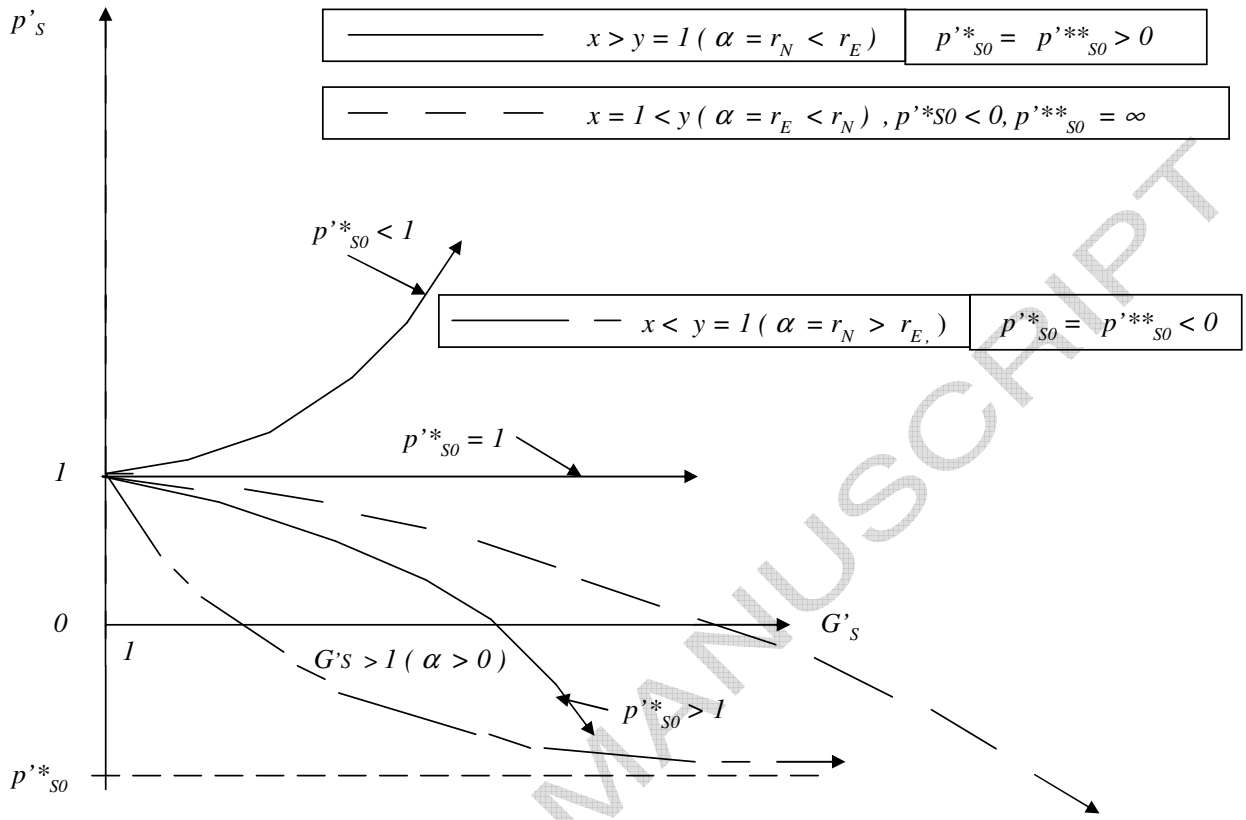


Figure 6- Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $y = 1$ or $x=1$.

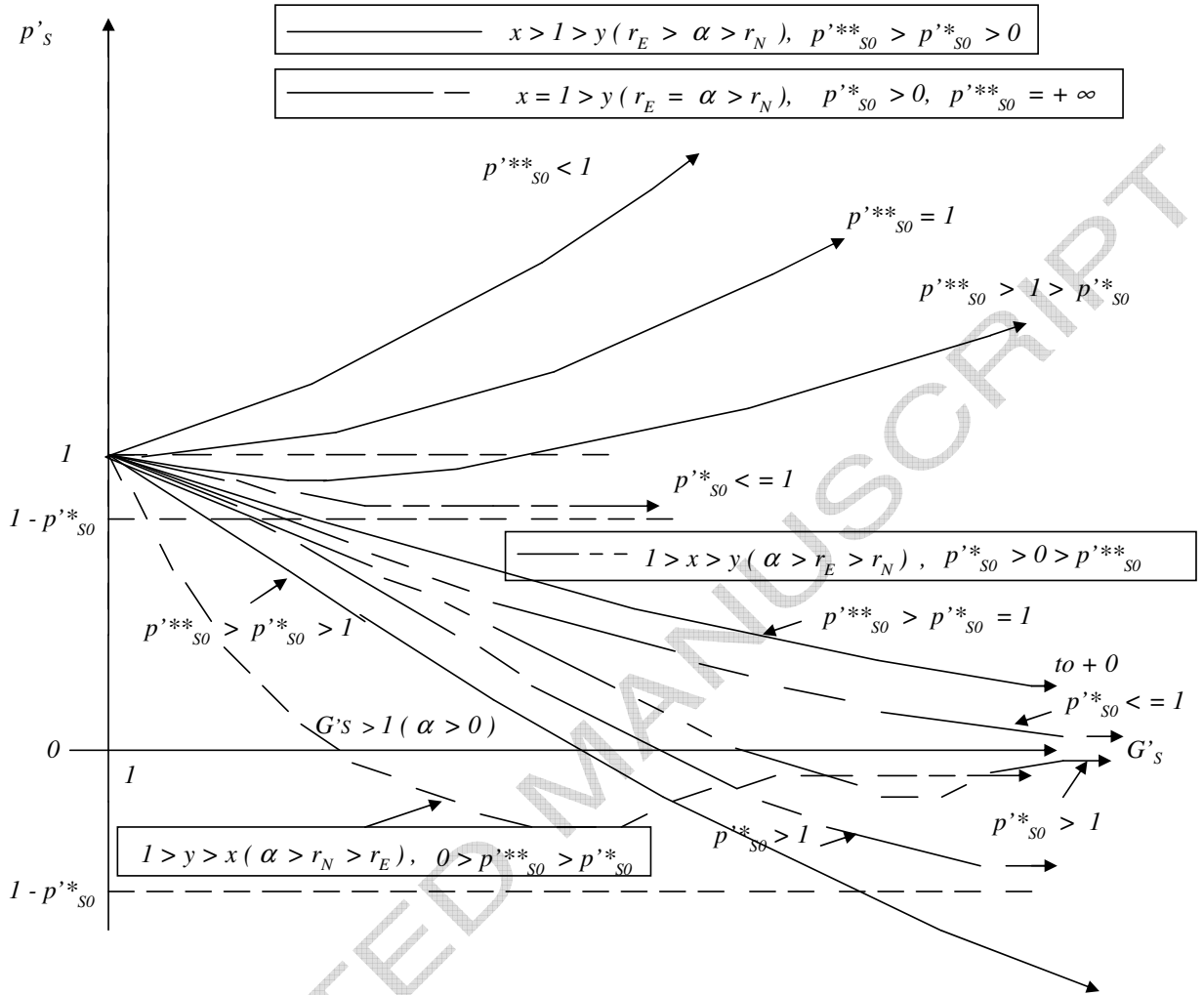


Figure 7- Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $y < 1$.

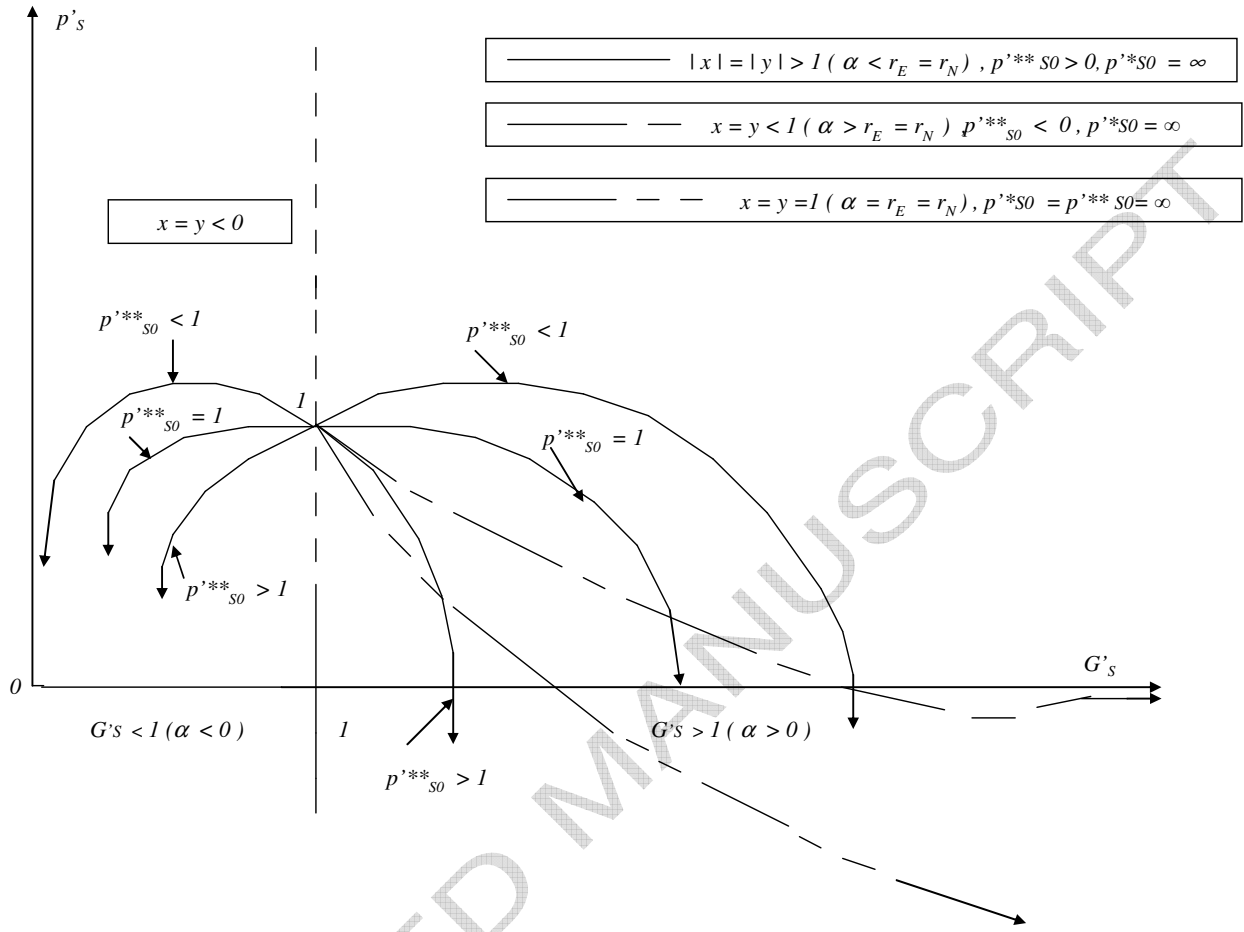


Figure 8- Dimensionless price of sold resources, p'_s , versus dimensionless mass flow rate of extraction, G'_s , for $x = y$.

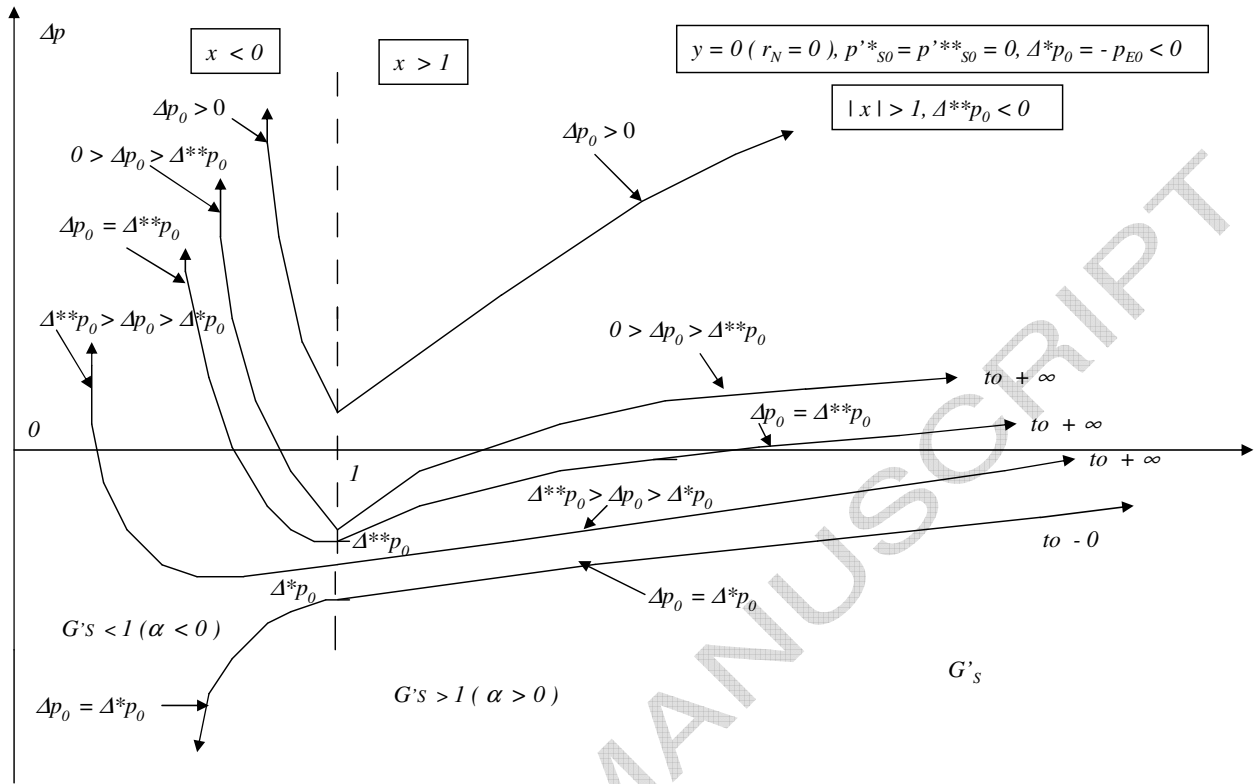


Figure 9- Price difference, $\Delta p = p_s - p_E$, versus dimensionless mass flow rate of extraction, G'_s , for $y = 0$ and $|x| > 1$.

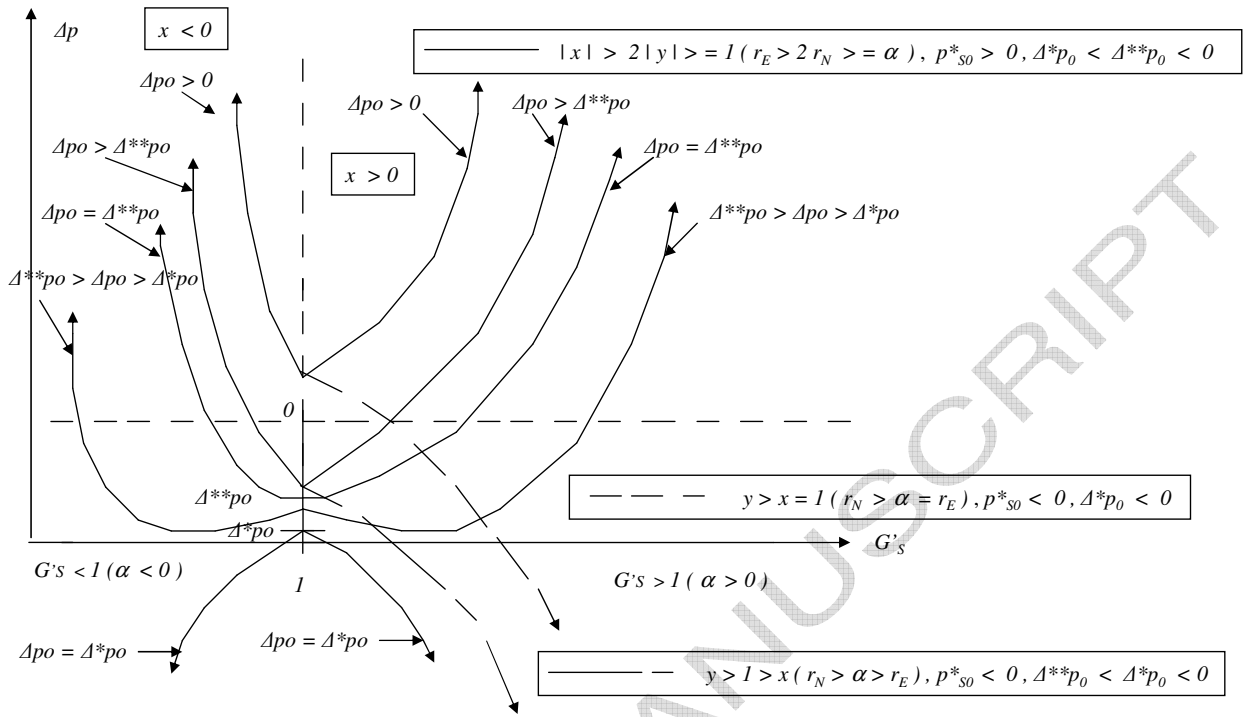


Figure 10- Price difference, $\Delta p = p_S - p_E$, versus dimensionless mass flow rate of extraction, G'_s , for $|x| > 2|y| \geq 1$, or $y > x = 1$, or $y > 1 > x$.

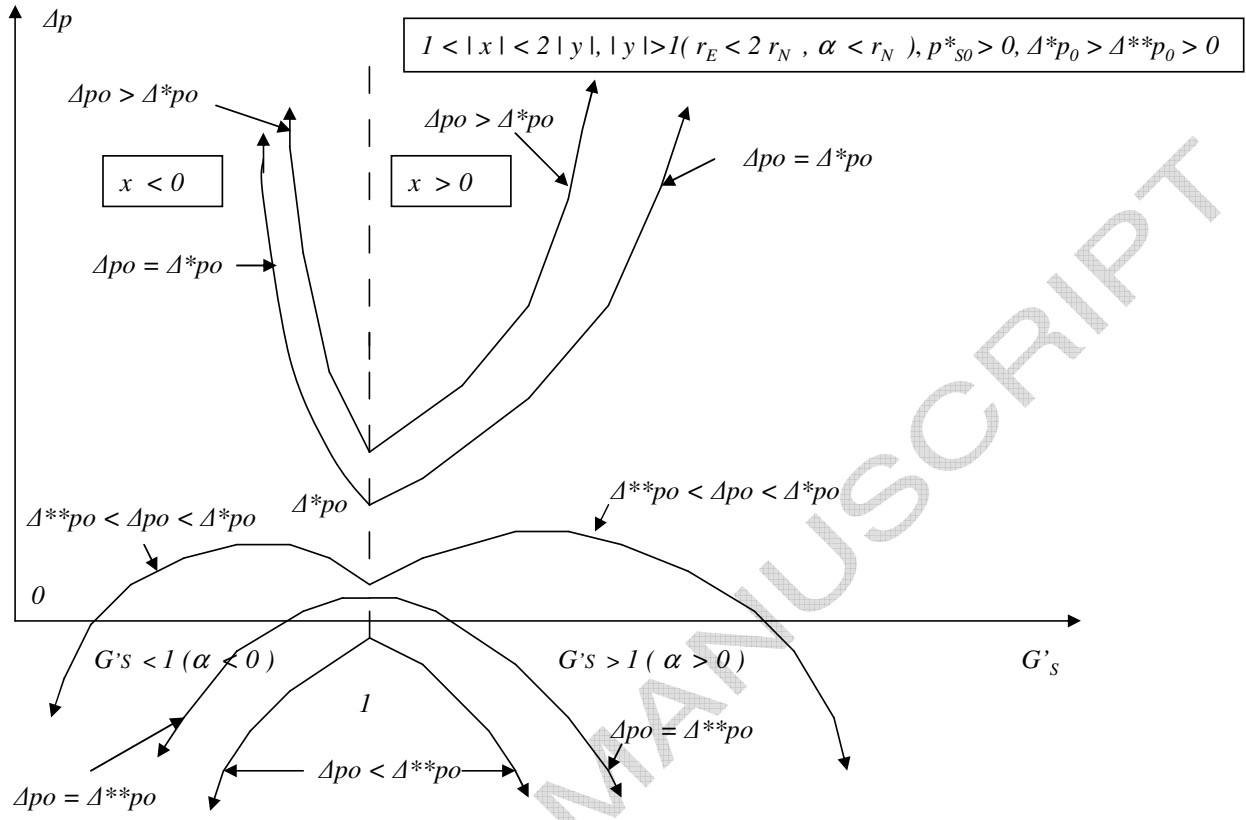


Figure 11 - Price difference, $\Delta p = p_S - p_E$, versus dimensionless mass flow rate of extraction, G'_s , for $|y| > 1$ and $|x| < 2|y|$.

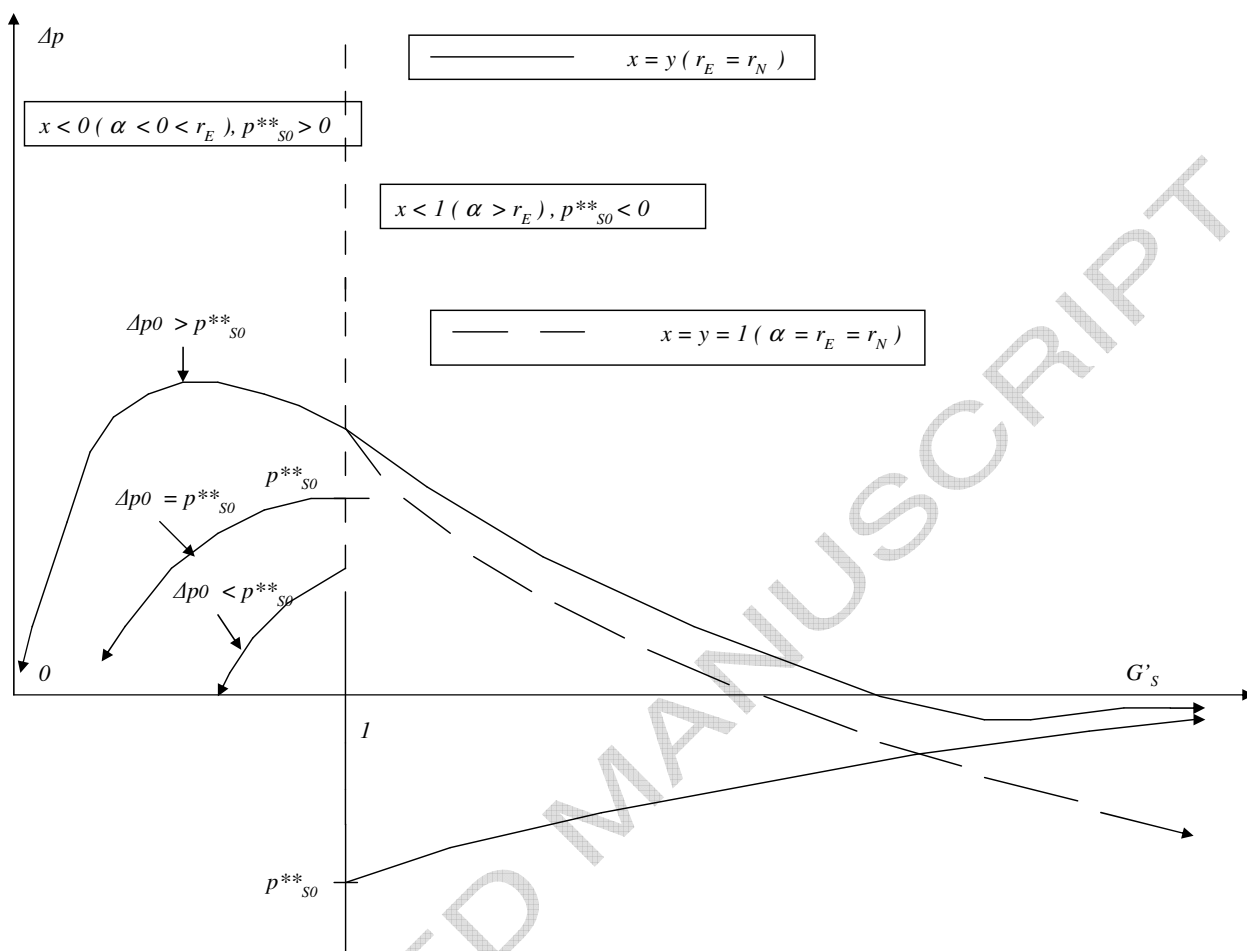


Figure 12- Price difference, $\Delta p = p_S - p_E$, versus dimensionless mass flow rate of extraction, G'_s , for $x = y$.

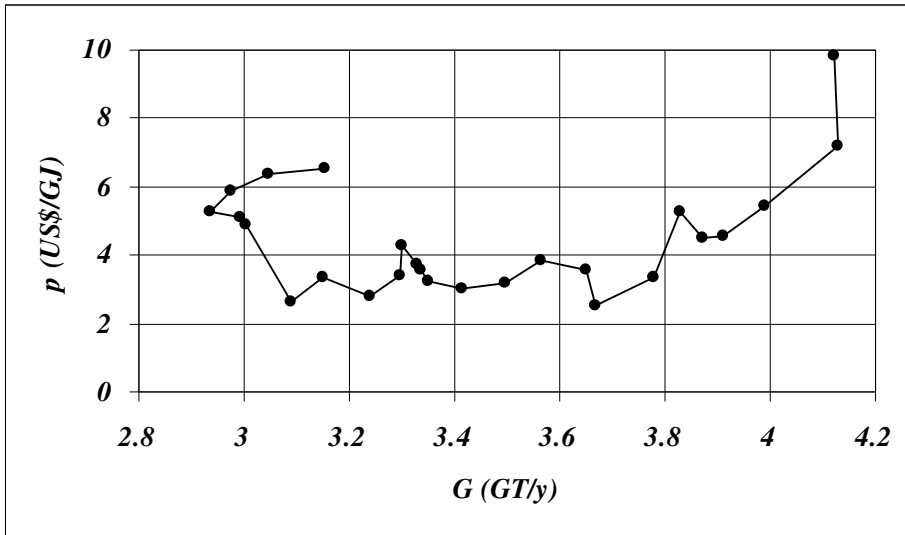


Figure 13 – World oil price, p (US\$/GJ), versus consumption, G (GT/y), from 1980 (initial point of the curve close to $G=3.2$ GT/y) until 2005 (last point on the right).

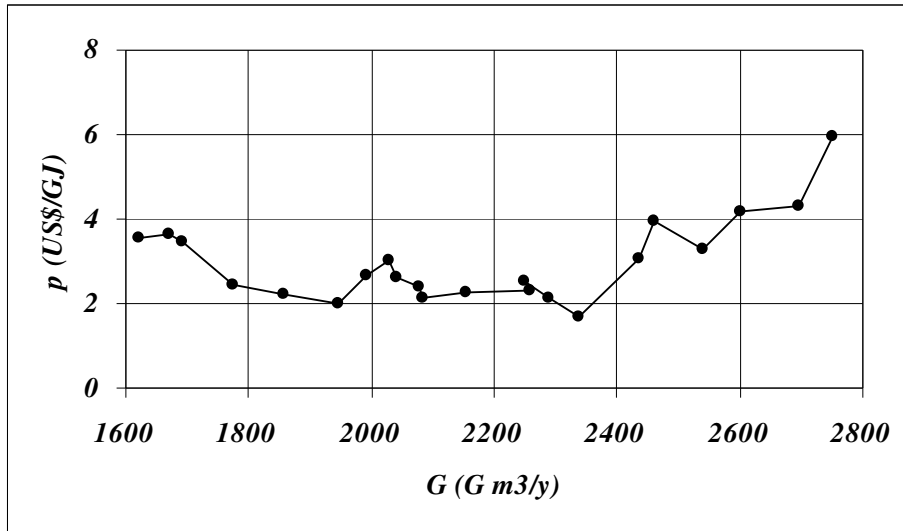


Figure 14 – World gas price, p (US\$/GJ) versus consumption, G (G m³/y) in the world from 1984 (initial point on the left) until 2005 (last point on the right).

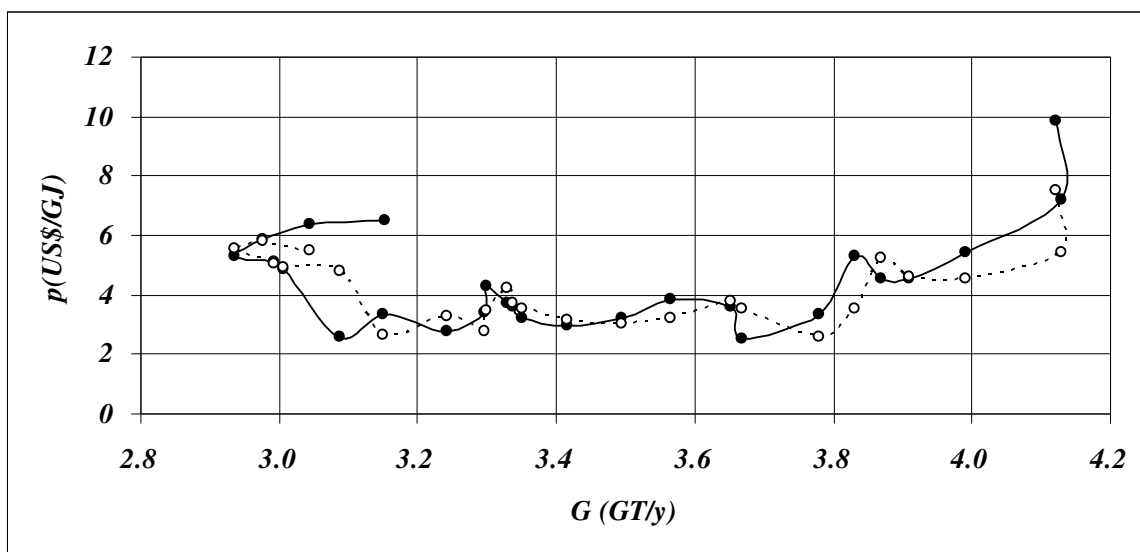


Figure 15 – World oil price (black square), p (US\$/GJ), versus consumption, G (GT/y), from 1980 (initial point close to $G=3.2$ GT/y) until 2005 (last point on the right) compared to the best predictions (white circle) as reported in Table 3.

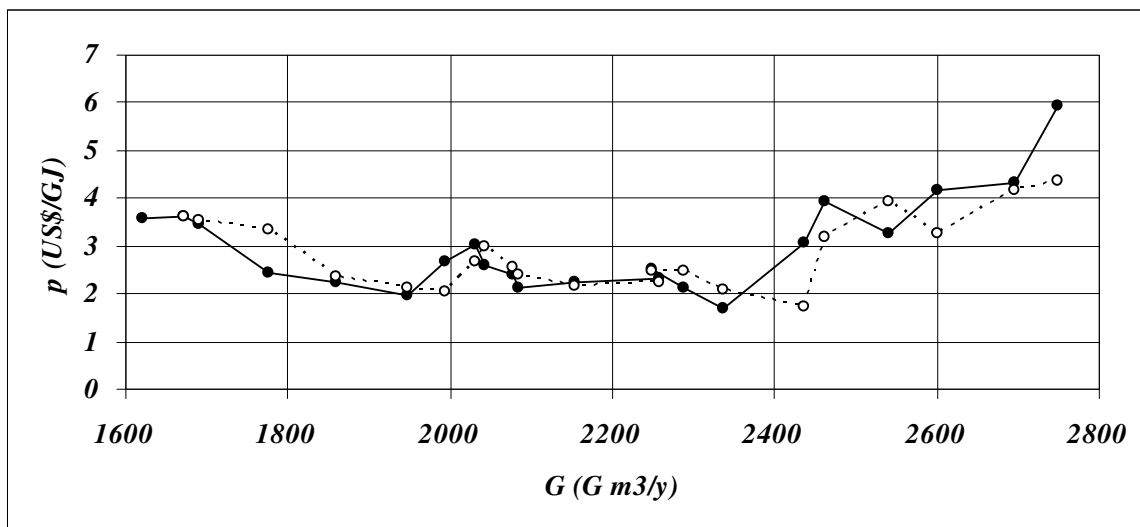


Figure 16 – World gas price (US\$/GJ) versus consumption (G m³/y) in the world from 1984 (initial point of the left) until 2005 (last point on the right) compared to the best predictions (white circle) as reported in Table 4.

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