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NON-LINEAR MAGNETIC MODEL REFINEMENT VIA A FINITE ELEMENT SUBPROBLEM METHOD

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***Abstract*— Model refinements of non-linear magnetic circuits are performed via a finite element subproblem method. A complete problem is split into subproblems to allow a progression from 1-D to 3-D including linear to non-linear model corrections. Its solution is then expressed as the sum of the subproblem solutions supported by different meshes. A convenient and robust correction procedure is proposed allowing independent overlapping meshes for both source and reaction fields. The procedure simplifies both meshing and solving processes, and quantifies the gain given by each model refinement on both local fields and global quantities.**

I. INTRODUCTION

The perturbation of finite element (FE) solutions provides clear advantages in repetitive analyses and helps improving the solution accuracy [1]-[6]. It allows to benefit from previous computations instead of starting a new complete FE solution for any variation of geometrical or physical data. It also allows different problem-adapted meshes and computational efficiency due to the reduced size of each subproblem.

A FE subproblem method (SPM) is herein developed for coupling solutions of various dimensions, starting from simplified models, based on ideal flux tubes defining 1-D models, that evolve towards 2-D and 3-D accurate models, allowing leakage flux and end effects. Progressions from linear to non-linear models are aimed to be performed at any step, which extends the method proposed in [3]-[6]. A convenient and robust correction procedure is proposed here. It combines both types of changes, via volume sources (VSs) and surfaces sources (SSs), in single correction steps. This allows independent overlapping meshes for both source and reaction fields, which simplifies the meshing procedure.

The developments are performed for the magnetic vector potential FE magnetostatic formulation, paying special attention to the proper discretization of the constraints involved in each SP. The method will be illustrated and validated on test problems.

II. PROGRESSIVE MAGNETIC SUBPROBLEMS

A. Sequence of Subproblems

Instead of solving a complete problem, generally with a 3-D model, it is proposed to split it into a sequence of SPs, some of lower dimensions, i.e. 1-D and 2-D models, and others performing adequate corrections. Linear to non-linear corrections can be involved at any level of this sequence. The complete solution is then to be expressed as the sum of the SP solutions. This offers a way to perform model refinements, with a direct access to each correction, usually of useful physical meaning.

Each SP is defined in its own domain, generally distinct from the complete one. At the discrete level, this aims to decrease the problem complexity and to allow distinct meshes with suitable refinements. Each SP has to approximate at best its contribution to the complete solution. The domains of the SPs usually overlap.

B. Canonical magnetic problem

A canonical magnetostatic problem p is defined in a domain Ω_p, with boundary $\partial\Omega_p = \Gamma_p = \Gamma_{h,p} \cup \Gamma_{b,p}$. Subscript p refers to the associated problem p. The equations, material relation, boundary conditions (BCs) and interface conditions (ICs) of problem p are

$$\operatorname{curl} \boldsymbol{h}_p = \boldsymbol{j}_p \,, \quad \operatorname{div} \boldsymbol{b}_p = 0 \,, \quad \boldsymbol{h}_p = \mu_p^{-1}\, \boldsymbol{b}_p + \boldsymbol{h}_{s,p} \,, \qquad \text{(1a-b-c)}$$

$$\boldsymbol{n} \times \boldsymbol{h}_p|_{\Gamma_{h,p}} = 0 \,, \quad \boldsymbol{n} \cdot \boldsymbol{b}_p|_{\Gamma_{b,p}} = 0 \,, \qquad \text{(1d-e)}$$

$$[\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_p} = \boldsymbol{j}_{f,p}, \quad [\boldsymbol{n} \cdot \boldsymbol{b}_p]_{\gamma_p} = \boldsymbol{b}_{f,p}, \qquad \text{(1f-g)}$$

where $\boldsymbol{h}_p$ is the magnetic field, $\boldsymbol{b}_p$ is the magnetic flux density, $\boldsymbol{j}_p$ is the prescribed current density, μ_p is the magnetic permeability and $\boldsymbol{n}$ is the unit normal exterior to Ω_p. The notation $[\,\cdot\,]_\gamma = \cdot|_{\gamma^+} - \cdot|_{\gamma^-}$ expresses the discontinuity of a quantity through any interface γ (with sides γ^+ and γ^-) in Ω_p, which is allowed to be non-zero.

The field $\boldsymbol{h}_{s,p}$ in (1) is a VS, usually used for fixing a remnant induction. With the SPM, $\boldsymbol{h}_{s,p}$ is also used for expressing changes of permeability. For a change of permeability of a region, from μ_q for problem q to μ_p for problem p, the VS $\boldsymbol{h}_{s,p}$ in this region is

$$\boldsymbol{h}_{s,p} = (\mu_p^{-1} - \mu_q^{-1})\, \boldsymbol{b}_q \,, \qquad (2)$$

for the total fields to be related by $\boldsymbol{h}_q + \boldsymbol{h}_p = \mu_p^{-1} (\boldsymbol{b}_q + \boldsymbol{b}_p)$.

The surface fields $\boldsymbol{j}_{su,p}$ and $\boldsymbol{b}_{su,p}$ in (1f-g) are generally zero to define classical ICs for the fields. If nonzero, they define possible SSs. This is the case when some field traces in a previous problem q have been forced to be discontinuous, e.g. for neglecting leakage fluxes and reducing the problem to a lower dimension [2]-[6]. The continuity has to be recovered after a correction via a problem p. The SSs in problem p are thus to be fixed as the opposite of the trace solution of problem q.

Each problem p is constrained via the so defined VSs and

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SSs from parts of the solutions of other problems. This is a key element of the developed method, offering a wide variety of possible corrections [2]-[6], that should welcome linear to non-linear corrections as well.

III. Various Possible Problem Splittings

For a typical magnetic circuit, e.g. an electromagnet, the SP procedure splits the problem in a minimum of 3 SPs (Fig. 1): (1) the magnetic region and the air gaps considered as an ideal flux tube (with possible start from 1-D models [4]-[5]), (2) the stranded inductor alone, and (3) the consideration of the leakage flux via a SS $\boldsymbol{j}_{f,3}$ on the flux tube boundary, simultaneously with the change of permeability due to the addition of the magnetic region in the inductor source field [6]. In this way, step 2 and step 3 are based on totally independent meshes; step 1 uses a portion of mesh 3. All the resulting total solutions have been successfully validated.

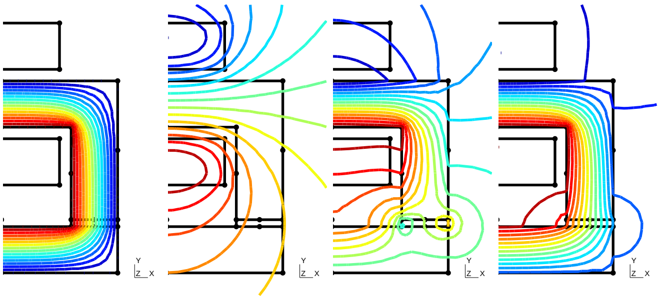

Fig. 1. Field lines in the ideal flux tube ($\boldsymbol{b}_1$, $\mu_{r,core}$ = 100), for the inductor alone ($\boldsymbol{b}_2$), for the leakage flux ($\boldsymbol{b}_3$) and for the total field ($\boldsymbol{b}$) (*left to right*).

It is herein proposed to couple changes from linear to non-linear material characteristics with the already developed correction SP method. An initially linear μ_q can change to a non-linear μ_p to be expressed as a function of the total magnetic flux density. The resulting VS (2) supported by the non-linear region is

$$\boldsymbol{h}_{s,p} = (\mu_p^{-1}(\boldsymbol{b}_q + \boldsymbol{b}_p) - \mu_q^{-1})\,\boldsymbol{b}_q\,, \qquad (3)$$

At the discrete level, the source quantity $\boldsymbol{b}_q = \text{curl}\,\boldsymbol{a}_q$, initially given in mesh q, is projected in the mesh p [6], limited to the non-linear region. A classical non-linear iterative process has then to lead to the convergence of $\boldsymbol{b}_p = \text{curl}\,\boldsymbol{a}_p$. This solution corrects the flux linkages of the inductors, and consequently their reluctances.

The non-linear SP can be split in several portions, each of them involved at a certain level of the whole SPM. Various combinations of problem splitting will be studied and discussed in the extended paper. The results of a two-step SPM are shown in Figs. 2, 3 and 4 for a low reluctance circuit, illustrating the way the correction fields behave.

IV. Conclusions

The developed FE subproblem method allows to split magnetic models into subproblems of lower complexity with regard to meshing operations and computational aspects. A natural progression from simple to more elaborate models, from 1-D to 3-D geometries, including linear to non-linear corrections, is thus possible, while quantifying the gain given by each model refinement. From the so calculated field corrections, the associate corrections of global quantities inherent to magnetic models, i.e. fluxes, magnetomotive forces, can be evaluated.

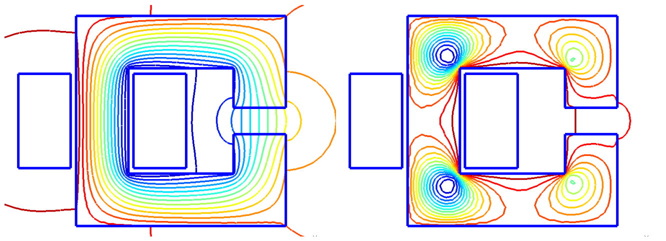

Fig. 2. Field lines for the linear model ($\boldsymbol{b}_1$, *left*) and for the non-linear correction ($\boldsymbol{b}_2$, *right*).

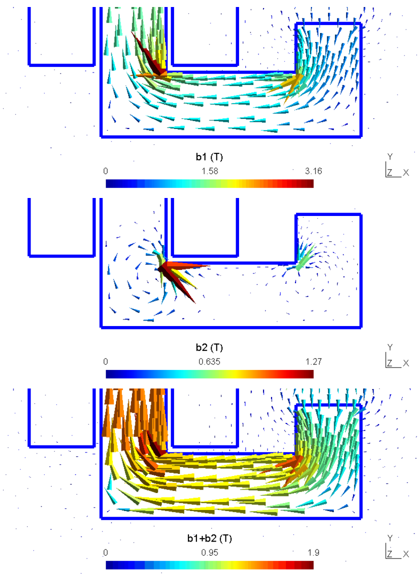

Fig. 3. Magnetic flux density for the linear model ($\boldsymbol{b}_1$, *top left*), for the non-linear correction ($\boldsymbol{b}_2$, *top right*) and the total solution ($\boldsymbol{b}_1$+$\boldsymbol{b}_2$, *bottom left*).

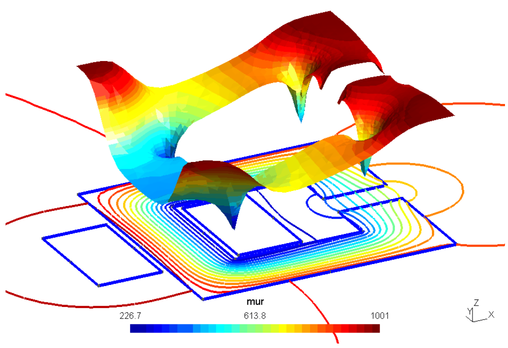

Fig. 4. Field lines and relative permeability (elevated color map) for the total solution ($\boldsymbol{b}_1$+$\boldsymbol{b}_2$, $\mu_{r,2}$).

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