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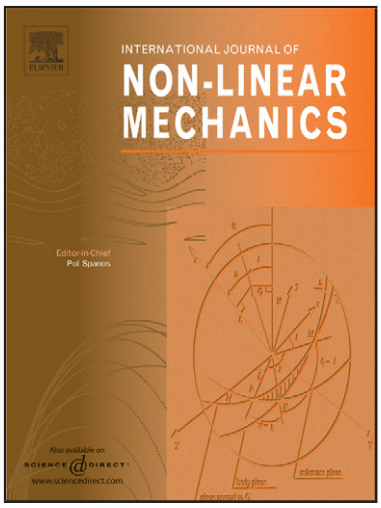

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A note on the second problem of Stokes for Newtonian fluids

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Abstract

New and simpler exact solutions corresponding to the second problem of Stokes for Newtonian fluids are established by the Laplace transform method. These solutions, presented as a sum of the steady-state and transient solutions are in accordance with the previous results (see Figs. 1, 2, 3 and 4). The amplitudes of the wall shear stresses corresponding to the cosine and sine oscillations are almost identical, except for a small initial time interval. The time required to attain the steady-state for the cosine oscillations of the boundary is smaller than that for the sine oscillations of the boundary. This time decreases if the frequency of the velocity of the boundary increases.

Keywords: Stokes' second problem; Newtonian fluids; Exact solutions.

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1. Introduction

The study on the flow of a viscous fluid over an oscillating plate is not only of fundamental theoretical interest but it also occurs in many applied problems. In the literature this motion is termed as Stokes' second problem or Rayleigh problem [1]. The first exact solutions corresponding to this problem for non-Newtonian fluids seem to be those of Rajagopal [2]. Recently, Erdogan [3] established exact transient solutions for the motion of a Newtonian fluid due to the cosine and sine oscillations of the plate. For large times these solutions tend to the steady-state solutions. However, the Erdogan's solutions are complicated enough and they are not presented as sum of the steady-state and transient solutions.

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The purpose of this note is to present new exact solutions for the Stokes' second problem describing the flow for small and large times. In order to obtain these solutions the Laplace transform method is again used, but our solutions are directly presented as sum of the steady-state and transient solutions and are simpler than the previous ones. Direct computations show that the steady-state solutions can be reduced to the classical forms [2, 3], while the diagrams of the transient solutions, as it was proved by graphical illustrations, are in close proximity of those obtained in [4] by a different technique. The required time to reach the steady-state for cosine oscillations of the wall is smaller than that for the sine oscillations of the wall. This time decreases if the frequency of the velocity of the boundary increases.

2. Statement of the problem

Consider a Newtonian fluid at rest over an infinitely extended plate which is situated in the (x, z) – plane. At time $t = 0^+$ the plate begins to oscillate in its plane and transmits the motion into the fluid. Its velocity is of the form

$$\mathbf{v} = \mathbf{v}(y,t) = u(y,t)\mathbf{i}, \tag{1}$$

where $\mathbf{i}$ denotes the unit vector in the x – direction.

The governing equation, in the absence of a pressure gradient along the x – axis and neglecting body forces, is [2, 3]

$$\frac{\partial u(y,t)}{\partial t} = \nu \frac{\partial^2 u(y,t)}{\partial y^2}; \quad y,t > 0, \tag{2}$$

where ν is the kinematic viscosity of the fluid. The boundary and initial conditions are

$$u(0,t) = U\cos(\omega t) \quad \text{for all } t > 0, \tag{3a}$$

$$u(y,0) = 0 \quad \text{for } y > 0, \tag{3b}$$

$$u(y,t) \to 0 \quad \text{as } y \to \infty, \tag{3c}$$

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$$u(0,t) = U\sin(\omega t) \quad \text{for all} \quad t > 0, \tag{4a}$$

$$u(y,0) = 0 \quad \text{for} \quad y > 0, \tag{4b}$$

$$u(y,t) \to 0 \quad \text{as} \quad y \to \infty, \tag{4c}$$

where ω is the frequency of the velocity of the wall.

3. Solution of the problem

In order to determine exact solutions to the previous problems, describing the motion of the fluid at small and large times after the start of the boundary, the Laplace transform method is used. Consequently, applying the Laplace transform to Eq. (2) and using the initial and boundary conditions (3) and (4), we find that

$$\bar{u}(y,q) = \frac{Uq}{q^2+\omega^2}\exp\left(-\frac{y}{\sqrt{\nu}}\sqrt{q}\right), \quad \text{respectively,} \quad \bar{u}(y,q) = \frac{U\omega}{q^2+\omega^2}\exp\left(-\frac{y}{\sqrt{\nu}}\sqrt{q}\right). \tag{5}$$

Regarding the right parts of Eqs. (5) as a product of two functions $\bar{U}_1(q)$ and $\bar{U}_2(y,q)$ and using the known results [5]

$$L^{-1}\left\{\frac{q}{q^2+\omega^2};\ \frac{\omega}{q^2+\omega^2}\right\} = \{\cos(\omega t), \sin(\omega t)\};\ L^{-1}\left\{\exp\left(-\frac{y}{\sqrt{\nu}}\sqrt{q}\right)\right\} = \frac{y}{2t\sqrt{\pi\nu t}}\exp\left(-\frac{y^2}{4\nu t}\right) \tag{6}$$

and

$$L^{-1}\left\{\bar{U}_1(q)\cdot\bar{U}_2(y,q)\right\} = U_1(t) * U_2(y,t) = \int_0^t U_1(t-s)U_2(y,s)\,ds = \int_0^t U_1(s)U_2(y,t-s)\,ds, \tag{7}$$

where $*$ denotes the convolution product of two functions, $U_1(t) = L^{-1}\left\{\bar{U}_1(q)\right\}$ and $U_2(y,t) = L^{-1}\left\{\bar{U}_2(y,q)\right\}$, we find for the velocity field $u(y,t)$ the simple expressions

$$u(y,t) = \frac{Uy}{2\sqrt{\pi\nu}}\int_0^t \frac{\cos[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds, \tag{8}$$

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respectively,

$$u(y,t)=\frac{Uy}{2\sqrt{\pi\nu}}\int_0^t\frac{\sin[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds. \tag{9}$$

The solutions (8) and (9) can be also written into equivalent forms

$$u(y,t)=\frac{Uy}{2\sqrt{\pi\nu}}\int_0^\infty\frac{\cos[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds-\frac{Uy}{2\sqrt{\pi\nu}}\int_t^\infty\frac{\cos[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds, \tag{10}$$

respectively,

$$u(y,t)=\frac{Uy}{2\sqrt{\pi\nu}}\int_0^\infty\frac{\sin[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds-\frac{Uy}{2\sqrt{\pi\nu}}\int_t^\infty\frac{\sin[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds, \tag{11}$$

in which the second terms are going to zero for $t\to\infty$. Of course, our previous solutions clearly satisfy the initial condition $u(y,0)=0$. However, under these forms, the boundary conditions (3a) and (4a) seem not to be satisfied. In order to do away with this inconvenience we shall present them into more suitable forms.

Making the change of variable $s=1/\sigma$ into the first integrals from (10) and (11) and using the fact that $\cos x=\cosh(ix)$, $\sin(x)=-i\sinh(ix)$ and [6]

$$\int_0^\infty\frac{\exp\left(-a^2s-\frac{b^2}{4s}\right)}{2\sqrt{s}}ds=\frac{\sqrt{\pi}}{2a}e^{-ab},$$

we find that

$$u(y,t)=Ue^{-y\sqrt{\frac{\omega}{2\nu}}}\cos\left(\omega t-y\sqrt{\frac{\omega}{2\nu}}\right)-\frac{Uy}{2\sqrt{\pi\nu}}\int_t^\infty\frac{\cos[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds \tag{12}$$

and

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$$u(y,t)=Ue^{-y\sqrt{\frac{\omega}{2\nu}}}\sin\left(\omega t-y\sqrt{\frac{\omega}{2\nu}}\right)-\frac{Uy}{2\sqrt{\pi\nu}}\int_t^{\infty}\frac{\sin[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds. \tag{13}$$

As soon as the velocity fields have been determined, the corresponding shear stresses can be immediately obtained by means of the known relation

$$\tau(y,t)=\mu\frac{\partial u(y,t)}{\partial y}. \tag{14}$$

Consequently, introducing (12) and (13) into (14), we find that

$$\begin{aligned}\tau(y,t)=\mu U\sqrt{\frac{\omega}{\nu}}e^{-y\sqrt{\frac{\omega}{2\nu}}}\sin\left(\omega t-y\sqrt{\frac{\omega}{2\nu}}-\frac{\pi}{4}\right)-\frac{\mu U}{2\sqrt{\pi\nu}}\int_t^{\infty}\frac{\cos[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds+\\+\frac{\mu U y^2}{4\nu\sqrt{\pi\nu}}\int_t^{\infty}\frac{\cos[\omega(t-s)]}{s^2\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds,\end{aligned} \tag{15}$$

respectively,

$$\begin{aligned}\tau(y,t)=-\mu U\sqrt{\frac{\omega}{\nu}}e^{-y\sqrt{\frac{\omega}{2\nu}}}\cos\left(\omega t-y\sqrt{\frac{\omega}{2\nu}}-\frac{\pi}{4}\right)-\frac{\mu U}{2\sqrt{\pi\nu}}\int_t^{\infty}\frac{\sin[\omega(t-s)]}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds+\\+\frac{\mu U y^2}{4\nu\sqrt{\pi\nu}}\int_t^{\infty}\frac{\sin[\omega(t-s)]}{s^2\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds.\end{aligned} \tag{16}$$

By letting now $t\to\infty$ into these last four relations, we attain to the classical steady-state solutions (cf. with [2] or [3], Eqs. (12) and (17))

$$u_{sc}(y,t)=Ue^{-y\sqrt{\frac{\omega}{2\nu}}}\cos\left(\omega t-y\sqrt{\frac{\omega}{2\nu}}\right)\text{ and }u_{ss}(y,t)=Ue^{-y\sqrt{\frac{\omega}{2\nu}}}\sin\left(\omega t-y\sqrt{\frac{\omega}{2\nu}}\right) \tag{17}$$

and

$$\tau_{sc}(y,t)=\mu U\sqrt{\frac{\omega}{\nu}}e^{-y\sqrt{\frac{\omega}{2\nu}}}\sin\left(\omega t-y\sqrt{\frac{\omega}{2\nu}}-\frac{\pi}{4}\right),\ \tau_{ss}(y,t)=-\mu U\sqrt{\frac{\omega}{\nu}}e^{-y\sqrt{\frac{\omega}{2\nu}}}\cos\left(\omega t-y\sqrt{\frac{\omega}{2\nu}}-\frac{\pi}{4}\right), \tag{18}$$

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which are periodic in time and independent of the initial conditions. As regards the corresponding transient solutions

$$u_{tc}(y,t) = -\frac{Uy}{2\sqrt{\pi\nu}} \int_t^\infty \frac{\cos[\omega(t-s)]}{s\sqrt{s}} \exp\left(-\frac{y^2}{4\nu s}\right) ds, \tag{19}$$

$$u_{ts}(y,t) = -\frac{Uy}{2\sqrt{\pi\nu}} \int_t^\infty \frac{\sin[\omega(t-s)]}{s\sqrt{s}} \exp\left(-\frac{y^2}{4\nu s}\right) ds, \tag{20}$$

$$\tau_{tc}(y,t) = -\frac{\mu U}{2\sqrt{\pi\nu}} \int_t^\infty \frac{\cos[\omega(t-s)]}{s\sqrt{s}} \exp\left(-\frac{y^2}{4\nu s}\right) ds + \frac{\mu U y^2}{4\nu\sqrt{\pi\nu}} \int_t^\infty \frac{\cos[\omega(t-s)]}{s^2\sqrt{s}} \exp\left(-\frac{y^2}{4\nu s}\right) ds \tag{21}$$

and

$$\tau_{ts}(y,t) = -\frac{\mu U}{2\sqrt{\pi\nu}} \int_t^\infty \frac{\sin[\omega(t-s)]}{s\sqrt{s}} \exp\left(-\frac{y^2}{4\nu s}\right) ds + \frac{\mu U y^2}{4\nu\sqrt{\pi\nu}} \sin \int_t^\infty \frac{\sin[\omega(t-s)]}{s^2\sqrt{s}} \exp\left(-\frac{y^2}{4\nu s}\right) ds, \tag{22}$$

as it results from Figs. 1, 2, 3 and 4, they are identical to those obtained in [7] by a different technique and given by

$$u_{tc}(y,t) = -\frac{2U}{\pi} \int_0^\infty \frac{\xi^3}{\xi^4 + (\omega/\nu)^2} e^{-\nu\xi^2 t} \sin(y\xi)\, d\xi, \tag{23}$$

$$u_{ts}(y,t) = \frac{2U}{\pi}\frac{\omega}{\nu} \int_0^\infty \frac{\xi}{\xi^4 + (\omega/\nu)^2} e^{-\nu\xi^2 t} \sin(y\xi)\, d\xi, \tag{24}$$

$$\tau_{tc}(y,t) = -\frac{2\mu U}{\pi} \int_0^\infty \frac{\xi^4}{\xi^4 + (\omega/\nu)^2} e^{-\nu\xi^2 t} \cos(y\xi)\, d\xi, \tag{25}$$

respectively,

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$$\tau_{ts}(y,t) = \frac{2\mu U}{\pi}\frac{\omega}{\nu}\int_0^\infty \frac{\xi^2}{\xi^4 + (\omega/\nu)^2} e^{-\nu\xi^2 t}\cos(y\xi)\,d\xi. \tag{26}$$

From (15) and (16) we can immediately determine the shear stress at the wall

$$\tau_{wc}(t) = \mu U\sqrt{\frac{\omega}{\nu}}\ \sin\left(\omega t - \frac{\pi}{4}\right) - \frac{\mu U}{2\sqrt{\pi\nu}}\int_t^\infty \frac{\cos[\omega(t-s)]}{s\sqrt{s}}\,ds, \tag{27}$$

for the cosine oscillations of the boundary, and

$$\tau_{ws}(t) = -\mu U\sqrt{\frac{\omega}{\nu}}\ \cos\left(\omega t - \frac{\pi}{4}\right) - \frac{\mu U}{2\sqrt{\pi\nu}}\int_t^\infty \frac{\sin[\omega(t-s)]}{s\sqrt{s}}\,ds, \tag{28}$$

for the sine oscillations of the boundary. The graphs of the wall shear stresses, corresponding to the cosine and sine oscillations of the boundary, are presented in Figs. 5 and 6. For a better comparison their plots are presented together as well as separately. The two oscillations have similar amplitudes and a phase shift that persists for all times. Note that the improper integral in Eq. (27) is divergent at $t = 0$. The some behavior is observed by analyzing Eq. (25) for $y = 0$ which strengthens the excellent agreement (as demonstrated by figure 3) between formulas (21) and (25). On the other hand formula (28), for the sine oscillations of the boundary, contains an improper integral which is convergent at $t = 0$ in agreement with the expression (26) obtained by the Fourier transform technique.

Finally, as it was to be expected, it is worthy pointing out that making $\omega \to 0$ into Eqs. (12) and (15) we obtain the classical solutions

$$u(y,t) = U\left[1 - \mathrm{erf}\left(\frac{y}{2\sqrt{\nu t}}\right)\right] = U\,\mathrm{erfc}\left(\frac{y}{2\sqrt{\nu t}}\right),\ \tau(y,t) = -\frac{\mu U}{\sqrt{\pi\nu t}}\exp\left(-\frac{y^2}{4\nu t}\right), \tag{29}$$

corresponding to the first problem of Stokes [8]. In order to obtain these results, we have proved (using the change of variable $y/(2\sqrt{\nu s}) = u$ and an integrating by parts) that

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$$\int_t^\infty \frac{1}{s\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds = \frac{2\sqrt{\nu\pi}}{y}\operatorname{erf}\left(\frac{y}{2\sqrt{\nu t}}\right),$$
$$\int_t^\infty \frac{1}{s^2\sqrt{s}}\exp\left(-\frac{y^2}{4\nu s}\right)ds = -\frac{4\nu\sqrt{\nu}}{y^2\sqrt{\nu t}}\exp\left(-\frac{y^2}{4\nu t}\right)+\frac{4\nu\sqrt{\nu\pi}}{y^3}\operatorname{erf}\left(\frac{y}{2\sqrt{\nu t}}\right). \tag{30}$$

In the above relations, $\operatorname{erf}(\cdot)$ and $\operatorname{erfc}(\cdot)$ are the error function and the complementary error function of Gauss.

4. Conclusions

In this note we have established new exact solutions corresponding to the second problem of Stokes for Newtonian fluids. These solutions, unlike those obtained by Erdogan [3], are simpler and directly presented as a sum of steady-state and transient solutions. The steady-state solutions can be brought to the classical forms, while the graphs of the transient solutions, as depicted in Figs.1, 2, 3 and 4, are in close proximity to those corresponding to the similar solutions obtained in [7] by a different technique. For large values of t, when the transients disappear, the motion of the fluid is described by the steady-state solutions which are periodic in time and independent of the initial conditions. However, they satisfy the governing equation and the associated boundary conditions.

The variations of the wall shear stresses for the cosine and sine oscillations of the boundary are presented in Figs. 5 and 6 for several time intervals. The two oscillations have similar amplitudes and a phase shift that persists for all times. The two sets of formulas (one obtained by Laplace transform, the other by Fourier sine transform) are in perfect agreement.

Finally, we would like to know the required time to attain the steady-state for the cosine and sine oscillations of the boundary. The variations with the distance from the wall of the corresponding starting and steady-state velocities for the oscillations of the boundary by $V\cos(\omega t)$ and $V\sin(\omega t)$ are presented in Figs. 7 and 8. It is clearly seen from these figures that the required time to reach the steady-state for sine oscillations of the wall is greater than that for the cosine oscillations of the boundary. Furthermore, this time decreases if the frequency ω of the velocity increases.

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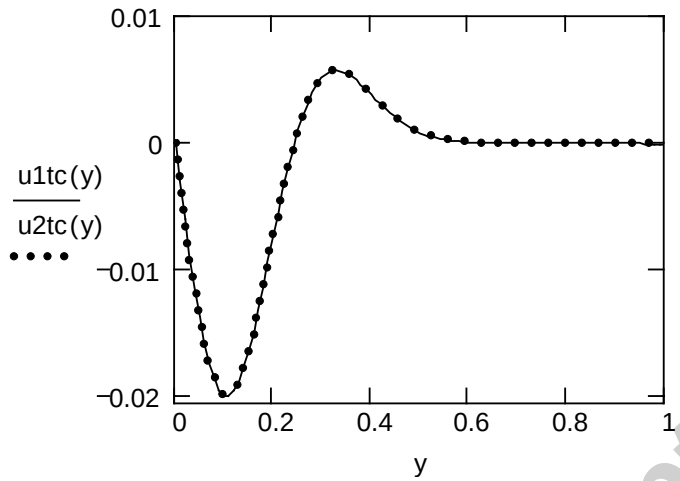
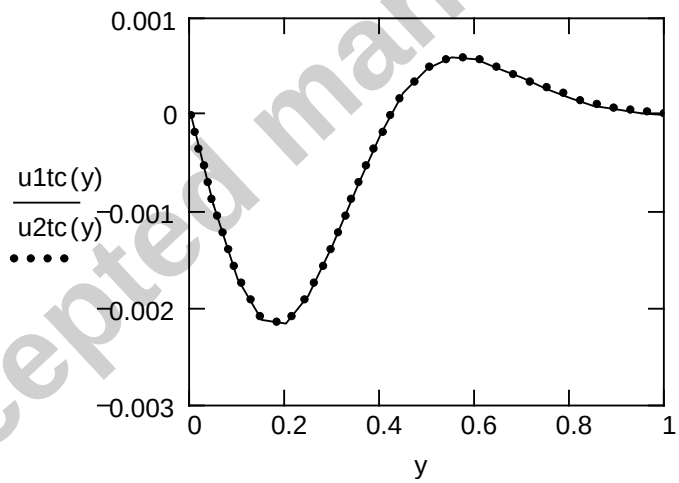


a. $t = 8s$



b. $t = 25s$

Fig.1. Profiles of the transient velocities, given by Eqs. (15) - curve u1(y) and (17) - curve u2(y), for $\omega = 1$, $U = 5$, $\nu = 0.0011746$.

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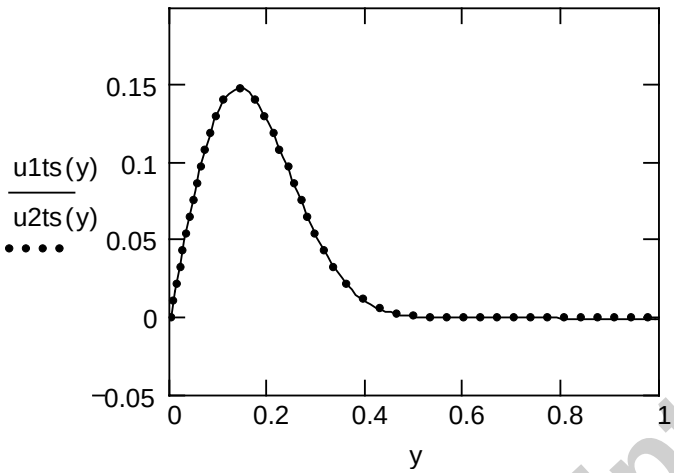

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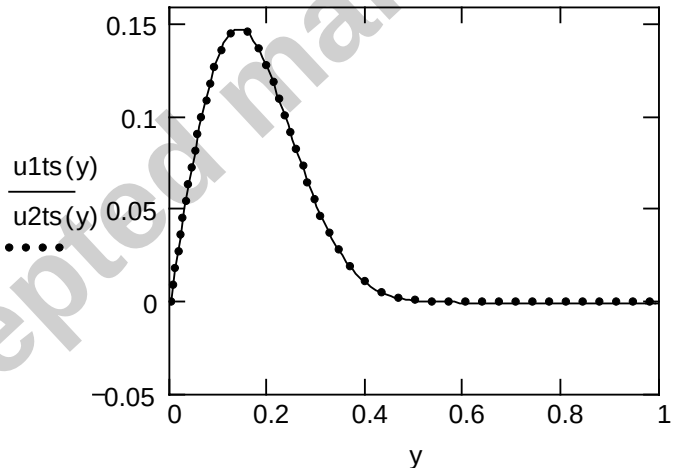

b. $t = 25s$

Fig. 2. Profiles of the transient velocities, given by Eqs. (16) - curve u1(y) and (18) – curve u2(y), for $\omega = 1$, $U = 5$, $\nu = 0.0011746$.

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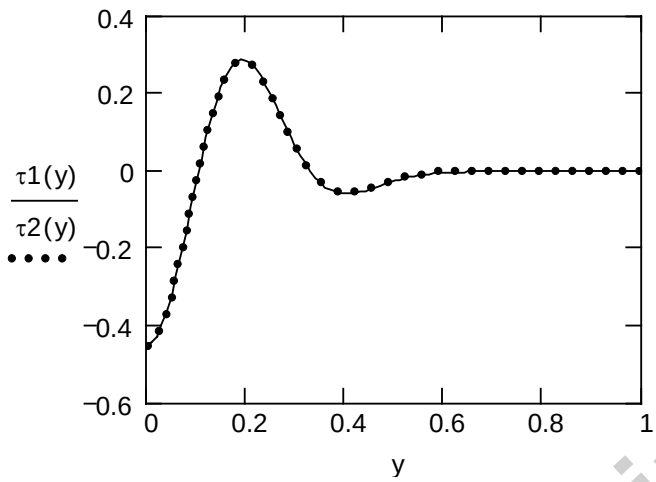

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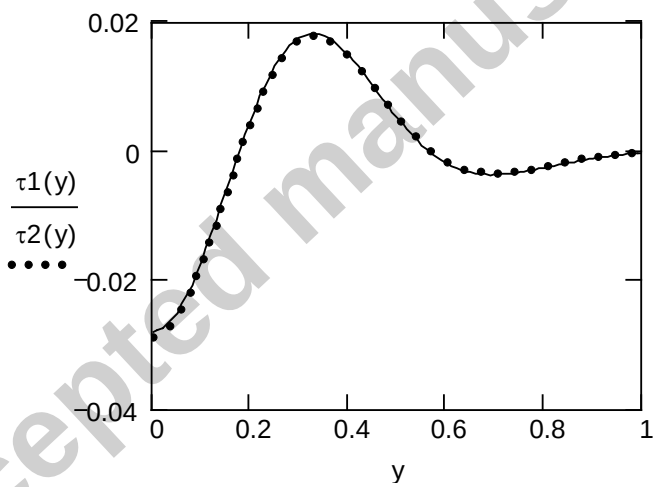

b. $t = 25s$

Fig. 3. Profiles of the transient shear stresses for the cosine oscillations of the boundary, given by Eqs. (21) – curve $\tau 1(y)$ and (25) – curve $\tau 2(y)$, for $\omega = 1$, $U = 5$ and $\nu = 0.0011746$ (glycerin).

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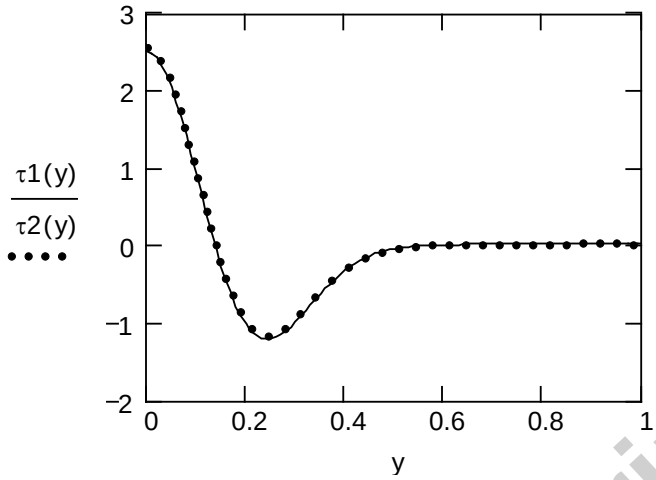
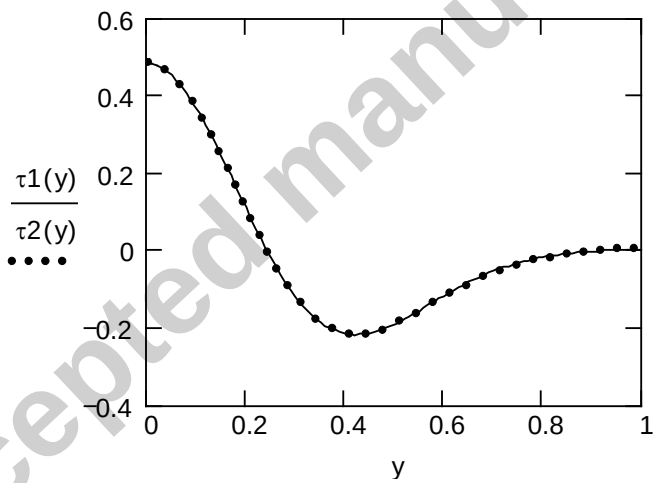


a. $t = 8s$



b. $t = 25s$

Fig. 4. Profiles of the transient shear stresses for the sine oscillations of the boundary, given by Eqs. (22) – curve $\tau 1(y)$ and (26) – curve $\tau 2(y)$, for $\omega = 1$, $U = 5$ and $\nu = 0.0011746$ (glycerin).

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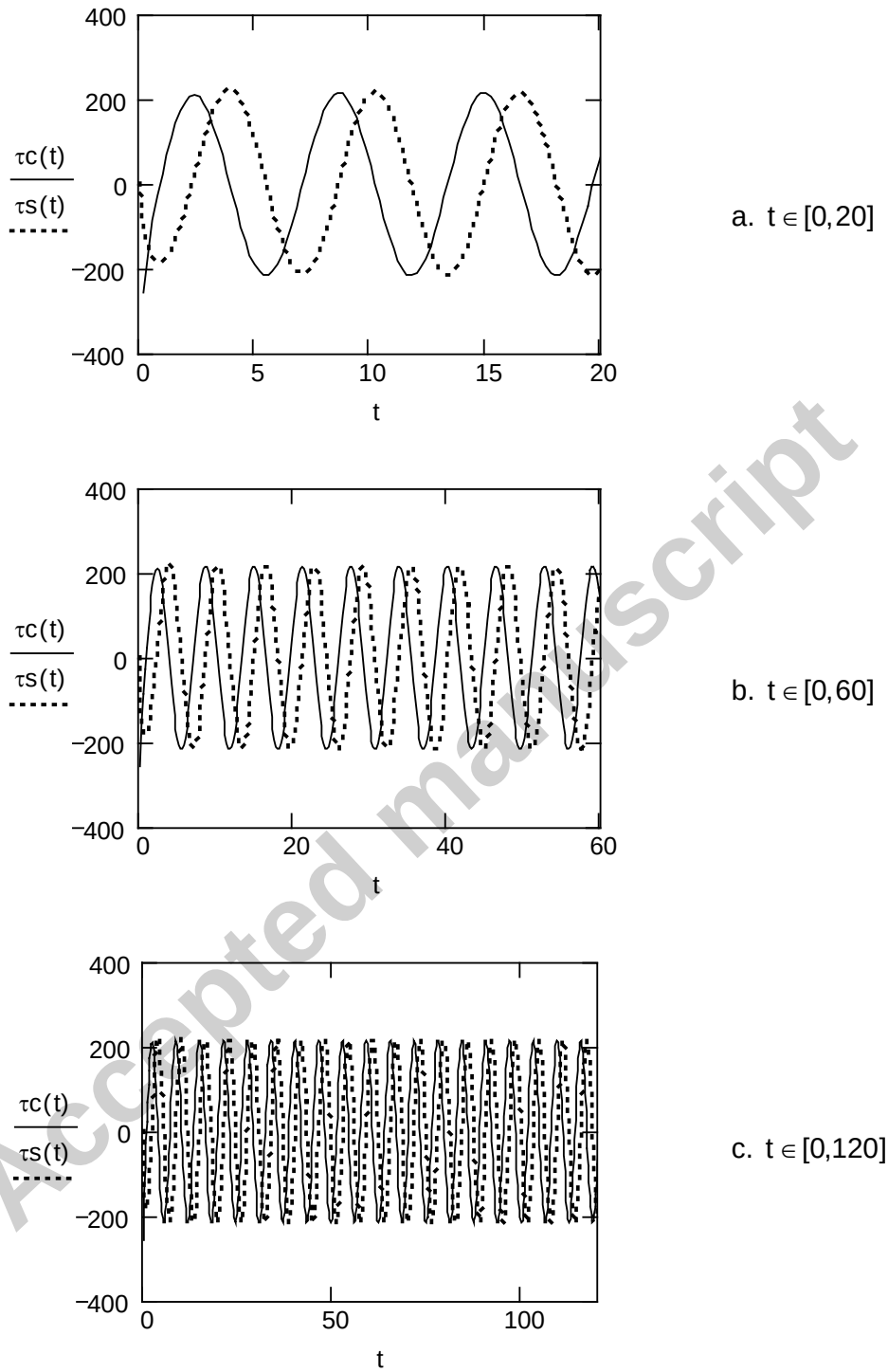

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Fig. 5. Profiles of the wall shear stresses corresponding to the cosine, and sine oscillations of the boundary, $\tau c(t)$ and $\tau s(t)$, for a relatively short time interval, for $\omega = 1, U = 5$ and $\nu = 0.0011746$ (glycerin).

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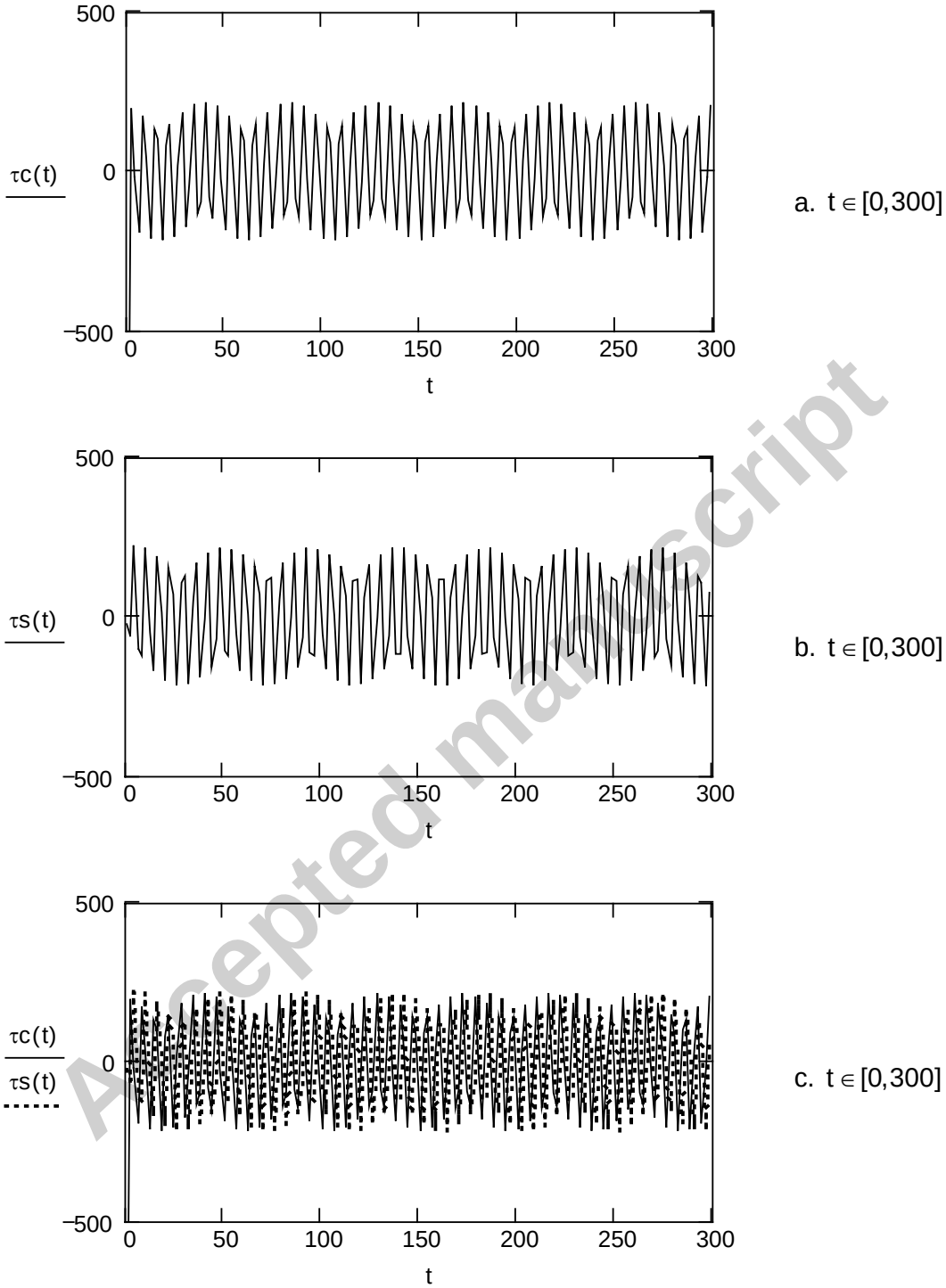

Fig. 6. Long time profiles of the wall shear stresses corresponding to the cosine, and sine oscillations of the boundary, $\tau c(t)$ and $\tau s(t)$, for $\omega = 1$, $U = 5$ and $\nu = 0.0011746$ (glycerin).

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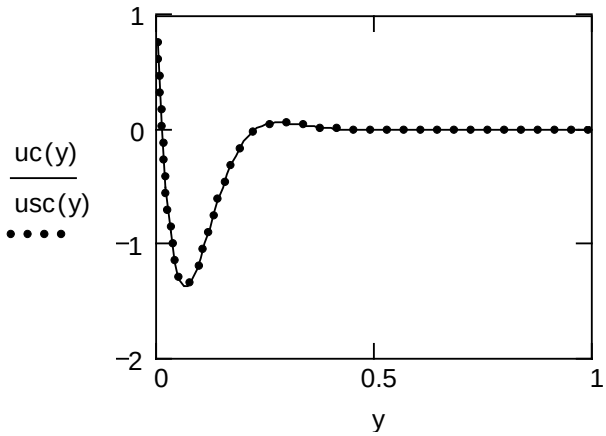
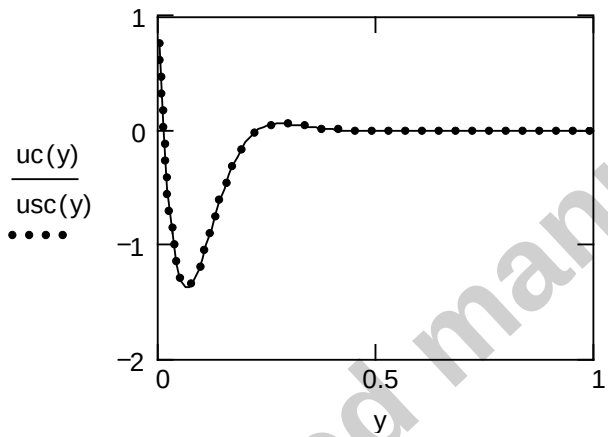
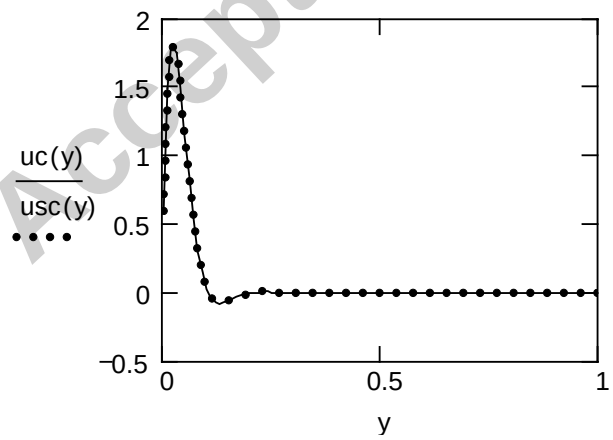


a. $\omega = 0.5$, $t = 60s$



b. $\omega = 1$, $t = 45s$



c. $\omega = 2$, $t = 29s$

Fig. 7. The required times to reach the steady-state for cosine oscillations, with an error of 10^{-4}, for $U = 5$, $\nu = 0.0011746$ and different values of ω.

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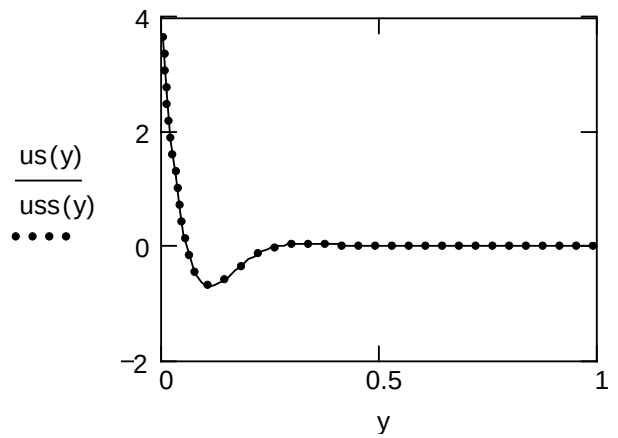
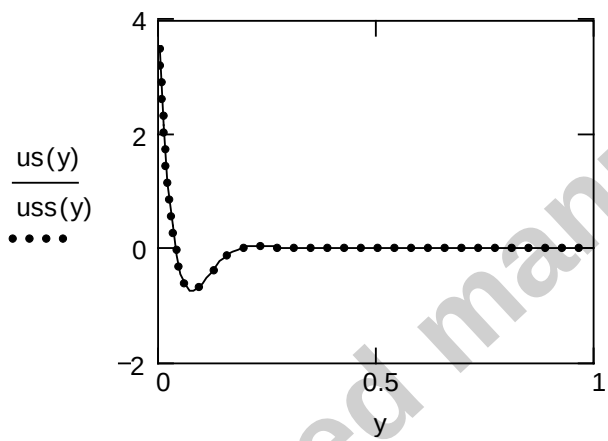
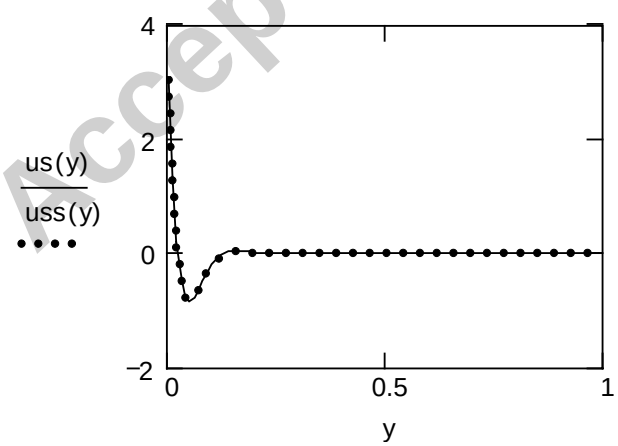


a. $\omega = 0.5$, $t = 165s$



b. $\omega = 1$, $t = 139s$



c. $\omega = 2$, $t = 104s$

Fig. 8. The required times to reach the steady-state for sine oscillations, with an error of 10^{-4}, for $U = 5$, $\nu = 0.0011746$ and different values of ω.

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