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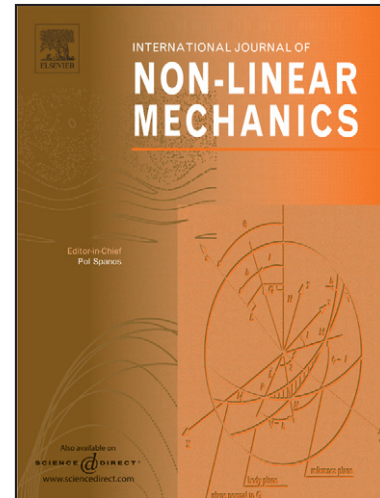
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Non-linear interaction of waves in prestressed material

A. Ravasoo *

*Centre for Nonlinear Studies, Institute of Cybernetics at Tallinn University of
Technology, Akadeemia tee 21, 12618 Tallinn, Estonia*

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Abstract

Interaction of two counterpropagating ultrasonic waves in an inhomogeneously prestressed elastic material (a structural element) with quadratic non-linearity is studied theoretically by using the perturbative formalism. The analytical solution that describes wave-wave, wave-material and wave-prestress non-linear interaction is derived. This rather cumbersome solution is studied in the case of a harmonic boundary condition in the material (specimen) subjected to two-parametric inhomogeneous prestressed state. The model problems are solved and the influence of the prestress on the wave interaction is cleared up. Resulting oscillations on two parallel boundaries of the material are prestress sensitive. It is proposed to use boundary oscillation data for nondestructive characterization of inhomogeneous prestressed state.

Keywords: Ultrasonic waves; Non-linear interaction; Non-linear elastic material; Inhomogeneous prestress; Nondestructive prestress characterization

**E-mail address:* arvi@ioc.ee; Tel.: +372 6204163; Fax: +372 6204151

1. Introduction

Ultrasonic nondestructive evaluation (NDE) of prestress in materials, structural elements, machinery, etc., is intensively studied over the last decade [1-4]. The main tool is the acoustoelastic effect that is defined as the dependence of ultrasonic wave velocity on the value of prestress [5, 6]. Theory and applications for NDE of the homogeneous prestress field on the basis of the acoustoelastic effect have been elaborated in detail [7-10]. The amount of quantities which needs to be determined by the NDE of the inhomogeneous prestress field increases essentially [11-14]. Several methods that combine the acoustoelastic effect with other features of wave motion are proposed for this purpose in order to enhance the accuracy of NDE [15-17].

In this paper the non-linear effects of wave propagation and interaction are proposed to be used as an additional source of information for NDE of inhomogeneous prestress field in the material. For this purpose, counter-propagation of two ultrasonic waves in the non-linear elastic material subjected to inhomogeneous plane strain is studied theoretically. The analytical solution for description of simultaneous propagation of waves with arbitrary smooth initial profiles excited on the parallel boundaries of the specimen is derived making use of the perturbation technique. The obtained solution permits to investigate propagation and interaction effects in detail and to separate linear and non-linear parts of solution.

Analyses of the results of numerical simulations lead to the conclusion that non-linear effects of boundary oscillations caused by the simultaneous propagation of two ultrasonic waves in the material are informative and may be used as addi-

tional information for enhancing the NDE of inhomogeneous prestress field. This is demonstrated by a model problem of simultaneous propagation of two sine waves in the material subjected to two-parametric prestressed state that corresponds to the pure bending with tension or compression. The phenomenon of amplification of the amplitude of boundary oscillations in domains of wave interactions is taken into account. The analysis shows clearly that the non-linear part of boundary oscillations is sensitive to the variation of prestress. The analyses of non-linear boundary oscillation data enable to distinguish qualitatively the special cases of prestress and to estimate the values of the parameters of the prestress field.

2. Problem formulation

The purpose is to excite simultaneously two ultrasonic longitudinal waves in the material (specimen, structural element, etc.) subjected to a prestress state that corresponds to the plane strain. This means a certain geometrical constraint that a specimen must have two parallel surfaces. In the civil engineering and machinery such objects are plates with different types of supports, beams, thin-wall open sections, et al. Ultrasonic NDE of inhomogeneous prestress field in these objects is then possible provided the access to the parallel boundaries of the material is feasible.

Theoretically, it is assumed that the isotropic and homogeneous material with quadratic non-linearity is initially in an undeformed natural state. At some instant material is subjected to the external loading and up to now it is in a prestressed state. Small but finite deformations of the material are described by the non-linear

theory of elasticity [18] in Lagrangian rectangular coordinates $X_J, J = 1, 2, 3$ and the corresponding Kirchhoff pseudostress tensor is $T_{KL}(X_J, t)$, $K, L = 1, 2, 3$. The physical and geometrical non-linearity of the problem is taken into account. The prestressed state is characterized by the displacement $U_K^0(X_J)$. The static (time independent) prestressed state is considered by assumption $U_3^0(X_J) = 0$, i.e., it corresponds to the plane strain.

The ultrasonic waves are excited in the specimen simultaneously on two parallel boundaries $X_1 = 0$ and $X_1 = h$, where h denotes the thickness of the specimen. The problem is treated as a quasi one-dimensional: two one-dimensional longitudinal waves are propagating simultaneously in the material subjected to two-dimensional prestressed state (see Fig. 1). The wave is characterized by displacement $U_1(X_1, t)$ and is governed by the equation of motion [19]

$$\begin{aligned}
 & [1 + k_1 U_{1,1}^0 + k_2 U_{2,2}^0] U_{1,11} + [k_1 U_{1,11}^0 + k_3 U_{1,22}^0 + k_4 U_{2,12}^0] U_{1,1} \\
 & + k_1 U_{1,11} U_{1,1} - c^{-2} U_{1,tt} = 0.
 \end{aligned} \tag{1}$$

Here, indices 1, 2 and t after a comma indicate differentiation with respect to X_1 , X_2 and time t , respectively. The constants

$$\begin{aligned}
 k_1 &= 3 + 6k(\nu_1 + \nu_2 + \nu_3), \quad k_2 = k(\lambda + 6\nu_1 + 2\nu_2), \\
 k_3 &= 1 + k\left(\nu_2 + \frac{3}{2}\nu_3\right), \quad k_4 = k\left[\lambda + \mu + 3\left(2\nu_1 + \nu_2 + \frac{1}{2}\nu_3\right)\right], \\
 k &= (\lambda + 2\mu)^{-1}, \quad c^{-2} = \rho_0 k
 \end{aligned} \tag{2}$$

are functions of the Lamè constants λ and μ , the elastic constants of the third order ν_1, ν_2 and ν_3 and the density ρ_0 .

Equation (1) may be solved provided information about the prestressed state is available. The two-dimensional prestressed state (plane strain) is described analytically by a set of two elliptic PDE

$$\begin{aligned}
& [1 + k_1 U_{I,I}^0 + k_2 U_{J,J}^0] U_{I,II}^0 + [2 k_3 U_{I,J}^0 + 2 k_5 U_{J,I}^0] U_{I,IJ}^0 \\
& + [k_7 + k_3 U_{I,I}^0 + k_3 U_{J,J}^0] U_{I,JJ}^0 + [k_5 U_{I,J}^0 + k_3 U_{J,I}^0] U_{J,II}^0 \\
& + [k_3 U_{I,J}^0 + k_5 U_{J,I}^0] U_{J,JJ}^0 + [k_6 + k_4 U_{I,I}^0 + k_4 U_{J,J}^0] U_{J,JI}^0 = 0,
\end{aligned} \tag{3}$$

where the indices I and J are equal to $I = 1, J = 2$ for the first equation of equilibrium and $I = 2, J = 1$ for the second one. The constants

$$k_5 = k (\mu + \nu_2 + \frac{3}{2} \nu_3), \quad k_6 = k (\lambda + \mu), \quad k_7 = k \mu \tag{4}$$

are dependent on the material properties.

3. Plane strain

The problem (1), (3) is solved by assumption that deformations are small but finite and the plastic deformations are not allowed. This leads to the idea to solve Eqs. (1) and (3) making use of a small parameter $\varepsilon \ll 1$ that has the physical meaning of small strain. The solution to Eq. (1) is sought in the form of series

$$U_1 = \sum_{n=1}^{\infty} \varepsilon^n U_1^{(n)}, \tag{5}$$

and to Eq. (3) by the series

$$U_K^0 = \sum_{m=1}^{\infty} \varepsilon^m U_K^{0(m)}. \tag{6}$$

Here, the small parameters in series (5) and (6) may actually be of the different order. This case was studied in [20]. The conclusion was that the non-linear effects of wave motion contain maximum information about the prestressed state provided the displacements caused by prestress and wave motion are of the same order. This is a reason why here the same small parameter is used in series (5) and (6).

The prestressed state is described by the solution to a set of equations (3). This solution is derived making use of the perturbation technique. The series (6) is introduced into Eq. (3) and following the perturbation procedure, the sets of equations in terms of series (6) are obtained. The first term is the solution to the set of two linear elliptic second order PDE

$$U_{I,II}^{0(1)} + k_7 U_{I,JJ}^{0(1)} + k_6 U_{J,JI}^{0(1)} = 0, \quad (7)$$

where $I = 1, J = 2$ are the indices for the first equation and $I = 2, J = 1$ for the second one.

Equations (7) are solved under eight boundary conditions in terms of Kirchhoff pseudostress tensor

$$\begin{aligned} T_{11}^{0(1)}(0, X_2) = T_{11}^{0(1)}(h, X_2) = T_{12}^{0(1)}(0, X_2) = T_{12}^{0(1)}(h, X_2) = 0, \\ T_{22}^{0(1)}(X_1, \pm L/2) = \sum_{n=0}^5 w_n X_1^n, \quad T_{21}^{0(1)}(X_1, \pm L/2) = 0, \end{aligned} \quad (8)$$

where L denotes the length of the specimen (structural element) and w_n are constants. These boundary conditions can be expressed on the basis of the theory of elasticity [18] in terms of displacement.

The second and the subsequent terms in solution (6) are solutions to the sets of

following equations with known r.h.s.:

$$U_{I,II}^{0(n)} + k_7 U_{I,JJ}^{0(n)} + k_6 U_{J,JI}^{0(n)} = F(U_I^{0(n-1)}, U_J^{0(n-1)}), \quad n = 2, 3, \dots \quad (9)$$

Equations (9) are solved under boundary conditions (8) where the upper index (1) is changed to (n) and the value of $T_{22}^{0(n)}(X_1, \pm L/2)$ is equal to zero.

The aim is to solve the problem of NDE of the parameters of the prestressed state on the basis of wave propagation data. For this purpose, it is convenient to describe the prestressed state by polynomials

$$U_1^0 = \varepsilon P_1^{2,2} + \varepsilon^2 P_2^{5,5}, \quad U_2^0 = \varepsilon P_3^{1,1} + \varepsilon^2 P_4^{5,5}, \quad (10)$$

where $P_l^{m,n} = \sum_{p=0}^m \sum_{s=0}^n e_{p,s} X_1^p X_2^s$, $l = 1, 2, \dots, 4$, and $e_{p,s}$ is a constant.

This leads to a NDE problem where the coefficients of polynomials may be considered as the parameters of the prestressed state that must be evaluated.

Substitution of polynomials $P_1^{2,2}$, $P_3^{1,1}$ into the linear equations of equilibrium (7) by boundary conditions (8) gives the analytical polynomial expressions for the terms $U_1^{0(1)}$ and $U_2^{0(1)}$ in solution (6). The terms $U_1^{0(2)}$ and $U_2^{0(2)}$ are determined making use of the polynomials $P_2^{5,5}$, $P_4^{5,5}$ and corresponding boundary conditions. The result is that the prestressed state of the material is determined by the main domain of the prestress - the component $T_{22}^0 = a + bX_1$ of the Kirchhoff pseudostress tensor. The non-linear correction to the prestressed state is considered of the negligible magnitude.

Consequently, the considered prestressed state corresponds to the plane strain and it may be considered as a two-parametric situation. It is characterized by the parameter a (constant part of prestress) and the parameter b (linearly variable part

of prestress).

4. Counterpropagating waves

These are waves excited simultaneously on the opposite parallel surfaces of the specimen. They propagate into the depth of the material, reach the opposite boundary, reflect and arrive at the boundary of excitation. It is assumed that in this initial stage these waves have arbitrary smooth initial profile, the distortion of wave profile is weak and the shock waves are not generated.

The wave propagation process is described by the solution (5) to the equation of motion (1) with known variable in space coefficients. The solution is derived making use of the perturbation technique. The three first terms in series (5) are solutions to the equations

$O(\varepsilon)$:

$$U_{1,11}^{(1)} - c^{-2} U_{1,tt}^{(1)} = 0, \quad (11)$$

$O(\varepsilon^2)$:

$$\begin{aligned} U_{1,11}^{(2)} - c^{-2} U_{1,tt}^{(2)} = & -k_1 U_{1,11}^{(1)} U_{1,1}^{(1)} - [k_1 U_{1,11}^{0(1)} + k_3 U_{1,22}^{0(1)} + k_5 U_{2,12}^{0(1)}] U_{1,1}^{(1)} \\ & - [k_1 U_{1,1}^{0(1)} + k_2 U_{2,2}^{0(1)}] U_{1,11}^{(1)}, \end{aligned} \quad (12)$$

$O(\varepsilon^3)$:

$$\begin{aligned} U_{1,11}^{(3)} - c^{-2} U_{1,tt}^{(3)} = & -k_1 U_{1,11}^{(1)} U_{1,1}^{(2)} - k_1 U_{1,11}^{(2)} U_{1,1}^{(1)} - [k_1 U_{1,1}^{0(1)} + k_2 U_{2,2}^{0(1)}] U_{1,11}^{(2)} \\ & - [k_1 U_{1,1}^{0(2)} + k_2 U_{2,2}^{0(2)}] U_{1,11}^{(1)} - [k_1 U_{1,11}^{0(1)} + k_3 U_{1,22}^{0(1)} + k_5 U_{2,12}^{0(1)}] U_{1,1}^{(2)} \end{aligned}$$

$$-[k_1 U_{1,11}^{0(2)} + k_3 U_{1,22}^{0(2)} + k_5 U_{2,12}^{0(2)}] U_{1,1}^{(1)}. \quad (13)$$

Counterpropagating waves are excited at the boundaries of the specimen at zero initial conditions

$$U_1(X_1, X_2, 0) = U_{1,t}(X_1, X_2, 0) = 0, \quad (14)$$

and by boundary conditions

$$U_{1,t}(0, X_2, t) = \varepsilon a_0 \varphi(t) H(t), \quad (15)$$

$$U_{1,t}(h, X_2, t) = \varepsilon a_h \psi(t) H(t). \quad (16)$$

Here $H(t)$ is the Heaviside's unit step function, and a_0 and a_h are constants. The smooth arbitrary initial wave profiles are denoted by functions $\varphi(t)$ and $\psi(t)$ with $\max |\varphi(t)| = 1$ and $\max |\psi(t)| = 1$.

The wave propagation in the specimen is studied under the assumption that the properties and the prestressed state of the material are known. From the mathematical point of view equations (11),..., (13) are solved like the second order PDEs with the known constant coefficients and with the known right-hand sides. The coordinate X_2 is regarded as a parameter. The solution (5) consists of the linear part (first term) and the corrections (subsequent terms) that describe the influence of prestress and non-linearity in the process. The advantage of this solution is that the linear part of it is easily separable and the remainder can be used as a source of information in the problems of NDE of material properties and the parameters of its state.

The linear process involving counter-propagation of waves, reflection from the

opposite surfaces and arrival to the boundaries of excitation in the homogeneous prestress free specimen is described by the solution

$$U_1^{(1)}(X_1, X_2, t) = a_0 H(\xi) \int_0^\xi \varphi(\tau) d\tau + a_h H(\eta) \int_0^\eta \psi(\tau) d\tau - a_0 H(\theta) \int_0^\theta \varphi(\tau) d\tau - a_h H(\zeta) \int_0^\zeta \psi(\tau) d\tau, \quad (17)$$

$$\xi = t - X_1/c, \quad \eta = t - h/c + X_1/c,$$

$$\zeta = t - h/c - X_1/c, \quad \theta = t - 2h/c + X_1/c,$$

to Eq. (11) derived under the initial and boundary conditions

$$U_1^{(1)}(X_1, X_2, 0) = U_{1,t}^{(1)}(X_1, X_2, 0) = 0, \quad (18)$$

$$U_{1,t}^{(1)}(0, X_2, t) = a_0 \varphi(t) H(t), \quad (19)$$

$$U_{1,t}^{(1)}(h, X_2, t) = a_h \psi(t) H(t). \quad (20)$$

The wave-wave, wave-material and wave-prestress interaction, i. e., the non-linear affect of wave to wave, physical material properties to wave and prestress to wave is described by the second and the subsequent terms in series (5) as solutions to Eqs. (12), (13). These equations may be expressed in the similar form

$$U_{1,11}^{(n)}(X_1, X_2, t) - c^{-2} U_{1,tt}^{(n)}(X_1, X_2, t) = \sum_{j=1}^m G_j^{(n)}(X_1, X_2) F_j^{(n)}(\vartheta_j^{(n)}), \quad (21)$$

$$\vartheta_j^{(n)} = t - g_j^{(n)}(X_1), \quad g_j^{(n)}(X_1) \geq 0,$$

with known right-hand sides where it is possible to separate the independent variables X_1 and $\vartheta_j^{(n)}$. Equation (21) is solved under the initial and boundary conditions

$$U_1^{(n)}(X_1, X_2, 0) = U_{1,t}^{(n)}(X_1, X_2, 0) = 0, \quad (22)$$

$$U_{1,t}^{(n)}(0, X_2, t) = U_{1,t}^{(n)}(h, X_2, t) = 0, \quad n = 2, 3, \dots, \quad (23)$$

making use of the Laplace integral transformation and it has the solution

$$U_1^{(n)L}(X_1, X_2, p) = \sum_{j=1}^m \left\{ \frac{c}{2p} F_j^{(n)L}(p) [P_j^{(n)}(X_1, X_2, p) - e^{-ph/c} (V_j^{(n)}(X_1, X_2, p) + W_j^{(n)}(X_1, X_2, p))] \right\}, \quad (24)$$

where

$$V_j^{(n)}(X_1, X_2, p) = e^{p(X_1-h)/c} [-e^{-ph/c} P_j^{(n)}(0, X_2, p) + P_j^{(n)}(h, X_2, p)], \quad (25)$$

$$W_j^{(n)}(X_1, X_2, p) = e^{-p(X_1+h)/c} [-e^{ph/c} P_j^{(n)}(0, X_2, p) + P_j^{(n)}(h, X_2, p)]. \quad (26)$$

Here p is the transform parameter, and upper index L denotes the Laplace transforms of corresponding functions.

The initial wave profile determines the function $F_j^{(n)L}(p)$. Functions $G_j^{(n)}(X_1, X_2)$ and $g_j^{(n)}(X_1)$ in

$$P_j^{(n)}(X_1, X_2, p) = e^{-pX_1/c} \left[e^{2pX_1/c} \int e^{-p(X_1/c+g_j^{(n)}(X_1))} G_j^{(n)}(X_1, X_2) dX_1 - \int e^{p(X_1/c-g_j^{(n)}(X_1))} G_j^{(n)}(X_1, X_2) dX_1 \right] \quad (27)$$

are dependent on the inhomogeneity in material properties.

The solution to Eq. (21)

$$U_1^{(n)}(X_1, X_2, t) = \lim_{Y \rightarrow \infty} \frac{1}{2\pi i} \int_{\alpha-iY}^{\alpha+iY} e^{\tau p} U_1^{(n)L}(X_1, X_2, p) dp. \quad (28)$$

valid in time interval

$$0 \leq t < 2h/c. \quad (29)$$

is obtained by applying the inverse Laplace transformation to the expression (24).

The outcome, solutions (17) and (30) determine all the terms in series (5) and this enables us to describe analytically counter-propagation and interaction of one-dimensional longitudinal waves with smooth arbitrary initial profiles $\varphi(t)$ and $\psi(t)$ in the inhomogeneously prestressed elastic medium.

5. Interaction of harmonic waves

Solution (5) may be used for description of the simultaneous propagation and interaction of two counterpropagating longitudinal waves with arbitrary smooth initial profiles in the non-linear elastic material undergoing arbitrary inhomogeneous two-dimensional plain strain. In this paper the possibility to use this solution as a basis for algorithms of NDE techniques of two-parametric prestress field is discussed.

For this purpose, the initial profiles of the excited waves are determined as sine functions with the same amplitude and frequency. Sine waves are excited on opposite surfaces of the specimen in terms of particle velocity

$$U_{1,t}(0, t) = \varepsilon a_0 \sin \omega t H(t), \quad (30)$$

$$U_{1,t}(h, t) = \varepsilon a_h \sin \omega t H(t), \quad (31)$$

where ω denotes the frequency.

The evolution of wave profile is recorded on the same surfaces in terms of stress. Expression for the wave induced stress has the form

$$T_{11} = \varepsilon T_{11}^{(1)} + \varepsilon^2 T_{11}^{(2)} + \dots,$$

$$T_{11}^{(1)} = (\lambda + 2\mu) U_{1,1}^{(1)},$$

$$T_{11}^{(2)} = (\lambda + 2\mu) U_{1,1}^{(2)} + [\lambda/2 + \mu + 3(\nu_1 + \nu_2 + \nu_3)] U_{1,1}^{(1)2}. \quad (32)$$

The linear part of stress $T_{11}^{(1)}$ is a function of displacement $U_{1,1}^{(1)}$ and specifies the stress caused by waves in the prestress free specimen. The term $T_{11}^{(2)}$ depends in addition on displacement $U_{1,1}^{(2)}$ that takes the non-linearity in material properties and the influence of the prestress into account. Consequently, provided the physical properties of the material (functions $U_{1,1}^{(1)}$ and $T_{11}^{(1)}$) are known, the diagnostics problem of the prestressed state turns to extraction of the stress component $T_{11}^{(2)}$ from the recorded data and to the analysis of the function $U_{1,1}^{(2)}$.

The analytical expression for $U_{1,1}^{(2)}$ is derived making use of the symbolic computational software *Maple 9* and it has the form

$$U_{1,1}^{(2)} = A_0^{(2)} + \sum_{j=1}^{m_1} A_{1j}^{(2)} \sin \omega \vartheta_j + \sum_{j=1}^{m_2} A_{2j}^{(2)} \cos \omega \vartheta_j + \sum_{j=1}^{m_3} A_{3j}^{(2)} \sin 2\omega \vartheta_j + \sum_{j=1}^{m_4} A_{4j}^{(2)} \cos 2\omega \vartheta_j, \quad (33)$$

where $\vartheta_j = t + c_{1j} h/c + c_{2j} X_1/c$, and c_{1j} , c_{2j} , m_1, \dots, m_4 are constants. The expression (33) consists of nonperiodic term $A_0^{(2)}$ and periodic terms with arguments $\omega \vartheta_j$ and $2\omega \vartheta_j$. Analytical expressions for nonperiodic term $A_0^{(2)}$ and amplitudes $A_{ij}^{(2)}$, $i = 1, \dots, 4$ are too cumbersome to be presented here.

From the physical point of view, function $U_{1,1}^{(2)}$ describes the main part of non-linear effects including the non-periodic term $A_0^{(2)}$, evolution of the second harmonic (fourth and fifth term in Eq. (33)), influence of the prestress on the evolution of the first harmonic (second and third term in Eq. (33)) and the non-linear interaction

between the first two harmonics (terms with frequencies $\omega = 2\omega - \omega$, $2\omega = \omega + \omega$ in sums of Eq. (33)). Influence of the linear and non-linear physical properties on the wave propagation is involved in all terms of Eq. (33). The evolution of the third and the higher harmonics are neglected here as higher order small quantities.

6. Boundary oscillations versus prestress

There is an analytical solution (5) at our disposal which describes non-linear counter-propagation, reflection and interaction of longitudinal waves with arbitrary smooth initial profiles in the physically non-linear elastic material (specimen) subjected to plane prestrain. For applications it is assumed that the initial wave profiles are determined by sine functions. Harmonic waves are excited in terms of particle velocity (Eqs. (30) and (31)) and recorded in terms of wave induced stress (Eq. (32)). Separation of overlapping elastic waves from measured data is described in [21]. Information about the prestress contains in the higher order terms of stress and the main domain of this information is included into the function $U_{1,1}^{(2)}$ (Eq. (33)), i. e., into non-linear effects of wave motion.

Evolution of non-linear effects in duralumin Al 7475 is illustrated in Fig. 2 on the basis of numerical simulation of Eq. (33). The material has the density $\rho_0 = 2800 \text{ kg/m}^3$, Lamè constants $\lambda = 57 \text{ GPa}$, $\mu = 27 \text{ GPa}$, the third order elastic constants $\nu_1 = -115 \text{ GPa}$, $\nu_2 = -132.5 \text{ GPa}$, $\nu_3 = -9.2 \text{ GPa}$ [22], and the thickness of the sample $h = 0.1 \text{ m}$. Strain is characterized by the dimensionless constant $\varepsilon = 5 * 10^{-4}$ and the excitation frequency equals to $\omega = 1.1868 * 10^{(6)} \text{ rad/s}$.

Initial amplitudes of harmonic waves are chosen by the values of constants a_0 and a_h equal to $a_0 = -a_h = c m/s$. In the discussions below the function $\varepsilon U_{1,1}^{(2)}$ is plotted in the vertical axis of figures. This function characterizes the ratio of the magnitude of the amplitude of non-linear effects with respect to the amplitude of the linear wave motion in the considered special case.

The analytical solution (5) permits to investigate the influence of the value of prestress on the wave motion in the whole $X_1 - t$ plane. In experiments it is more convenient to record the oscillations caused by wave motion on the boundaries of the sample (structural element).

6.1. Boundary oscillation profile distortion

The non-linear part of boundary oscillations evoked by simultaneous propagation of two harmonic waves in duralumin with the physical properties described above is studied. Several special cases of two-parametric prestress state characterized by the component $T_{22}^0 = a + bX_1$ of the Kirchhoff pseudostress tensor are considered.

The main domain of non-linear boundary oscillations in the prestress free specimen ($T_{22}^0 = 0$) is illustrated in Fig. 3. Oscillations on both boundaries $X_1 = 0$ and $X_1 = h$ coincide. It is possible to distinguish two intervals on time axis - the interval of propagation $0 \leq \tau < 1$ and the interval of interaction $1 \leq \tau < 2$. Here τ denotes the dimensionless time ($\tau = t h/c$).

Oscillations in the interval of propagation are plotted in Fig. 4.

Essential is that the non-linear wave interaction amplifies the boundary oscilla-

tions in the interval of interaction in comparison with the oscillations in the interval of propagation about a hundred times. The main physical reason to this effect is the evolution of the second harmonic from the nonentity on the boundary of excitation to the remarkable value on the opposite boundary. This phenomenon facilitates the usage of non-linear effects of wave interaction in NDE of material properties and states.

The non-linear part of boundary oscillations in the prestress free material is characterized by constant amplitudes which are different in the interval of propagation and interaction. Oscillation amplitudes are sensitive to the physical properties of the material (density, linear and non-linear elastic constants) and less sensitive to the value of the excitation frequency.

The homogeneous prestress ($T_{22}^0 = a$) modulates the boundary oscillations (Figs. 5 and 6). The shape and the depth of modulation carry information about the sign and the value of prestress. The oscillation profiles on both boundaries coincide.

The inhomogeneous prestress ($T_{22}^0 = a + bX_1$) modulates oscillations on different boundaries in a different way (see Fig. 7). Interesting is that in the special case of triangular stress distribution the boundary oscillation is not modulated on the boundary where the value of the prestress T_{22}^0 is equal to zero only in the interval $0 \leq \tau < 1$. In the case of pure bending when the stress T_{22}^0 has the equal value but the opposite sign on opposite boundaries the boundary oscillation profiles coincide but have a phase shift.

The analysis of boundary oscillations evoked by non-linear effects of two sine wave simultaneous propagation in the specimen leads to the conclusion that it is

easy to distinguish qualitatively:

- the prestress free specimen. Boundary oscillations have constant amplitudes in both time intervals,
- specimen with homogeneous prestress. Boundary oscillations are modulated and oscillations on both boundaries coincide,
- specimen undergoing pure bending. Modulated boundary oscillations on both boundaries coincide with phase shift,
- specimen with triangular prestress distribution. The boundary oscillation is not modulated on the boundary where the value of the prestress is equal to zero only in the interval $0 \leq \tau < 1$,
- specimen undergoing arbitrary two-parametric prestress. The modulated boundary oscillations on both boundaries do not coincide.

6.2. Boundary oscillation amplitude variation

The depth of modulation of non-linear boundary oscillations caused by counter-propagation of two sine waves in the specimen is sensitive to the value of prestress. This is studied on the basis of the following model problem. The geometry and the physical properties (density, elastic constants of the second and the third order) of the specimen are described above. The specimen is subjected to two-parametric prestressed state that corresponds to the pure bending with tension or compression characterized by the Kirchhoff pseudostress tensor component $T_{22}^0 = a + bX_1$.

Making use the analytical solution (33) the plots boundary oscillation amplitudes

at two instants τ_1 and τ_2 versus prestress parameters a and b for the specimen under investigation are composed (Fig. 8). The values of instants τ_1 and τ_2 are chosen to correspond to two first local maxima of boundary oscillation amplitudes in prestress free material. The values of oscillation amplitudes at the instants τ_1 and τ_2 are fixed on both boundaries.

The remarkable sensitivity of the value of the boundary oscillation amplitude to the prestress parameters leads to the theoretical possibility to evaluate the prestress parameters on the basis of the boundary oscillation amplitude variation data.

It is assumed that the boundary oscillation profiles on opposite boundaries are recorded and plotted in the interval of interaction for the real specimen (Fig. 7) and the Fig. 8 corresponds to the real specimen also. The calculated values of the differences of fixed amplitudes on opposite boundaries at the instants τ_1 and τ_2 in Fig. 8 enable to determine two possible values of the parameter b . The parameter a is determined in Fig. 8 making use of the absolute value of the oscillation amplitude on one of the boundaries at the instant τ_1 or τ_2 . The other free absolute values of the oscillation amplitude may be used for verification of the obtained results.

8. Conclusions

In order to describe inhomogeneity of materials (including also inhomogeneous prestress), more parameters are needed compared to the homogeneous case. Consequently, the ultrasonic NDE of inhomogeneous material properties needs additional information for determining these parameters. One possible way to enhance the

efficiency of NDE techniques is to use the non-linear effects that accompany the multi-wave propagation and interaction in the material (structural element).

In this paper, the counter-propagation of sine waves in physically non-linear elastic material (structural element) subjected to the two-parametric prestress state is studied theoretically. The non-linear effects of boundary oscillations are clarified. They contain additional information which is profitable by composing algorithms for nondestructive characterization of the two-parametric inhomogeneous prestressed state of the specimen.

From the practical point of view the proposed algorithms make use besides the acoustoelastic effect - dependence of the wave phase velocity on the prestress also on the nonlinear effects of wave propagation and interaction that leads in applications to the necessity to modify the existing nondestructive testing equipment.

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Figure captions

Fig. 1. Loading scheme of the specimen.

Fig. 2. Evolution of non-linear effects that accompany counter-propagation of harmonic waves in non-linear elastic material.

Fig. 3. Main domain of non-linear boundary oscillations in prestress free material.

Fig. 4. Non-linear oscillations on the boundaries of the prestress free material in the interval $0 \leq \tau < 1$.

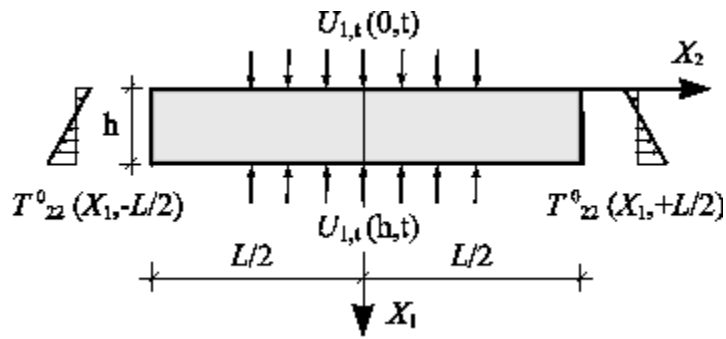
Fig. 5. Effect of homogeneous prestress on non-linear boundary oscillations in the interaction interval.

Fig. 6. Non-linear boundary oscillations in the interval $0 \leq \tau < 1$ of the homogeneously prestressed material.

Fig. 7. Modulation of boundary oscillations by inhomogeneous prestress (solid line - $X_1 = 0$, dashed line - $X_1 = h$).

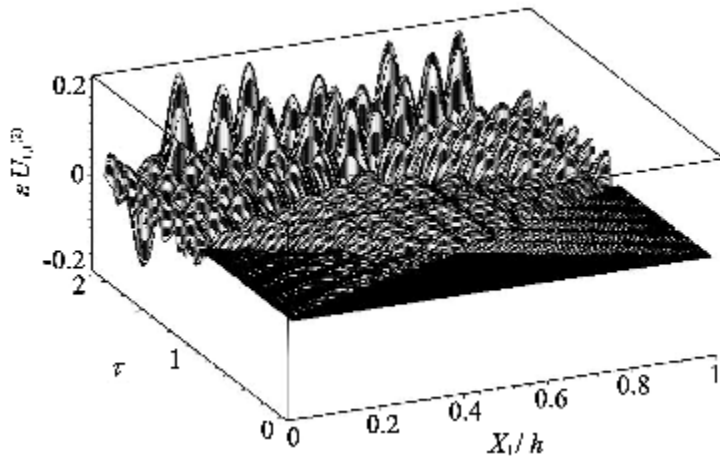
Fig. 8. Boundary oscillation amplitudes at instants τ_1 and τ_2 versus prestress parameters a and b .

Figure 1



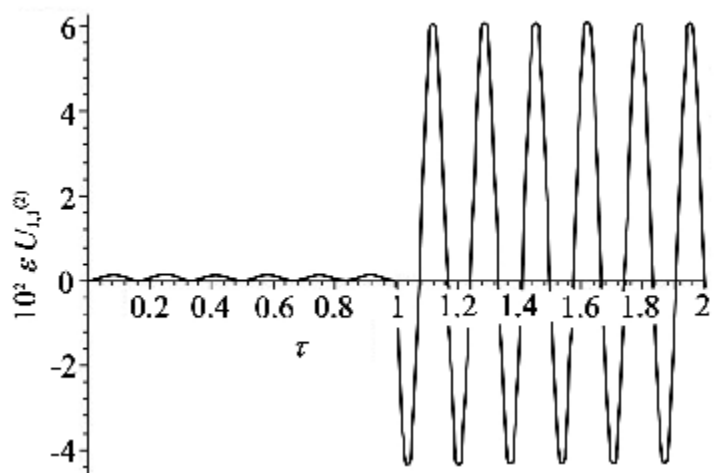
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Figure 2



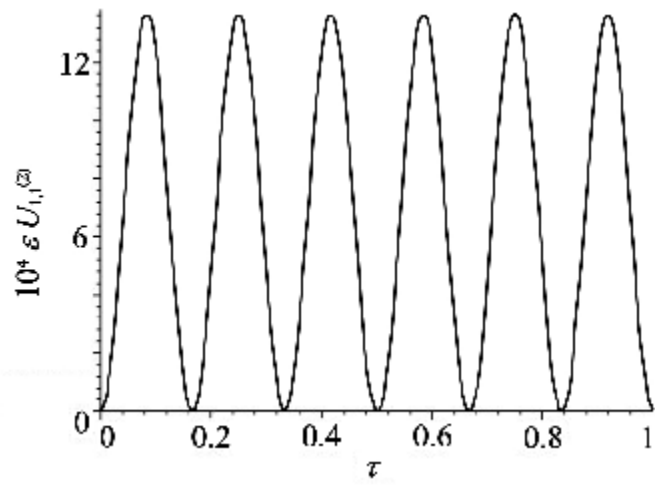
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Figure 3



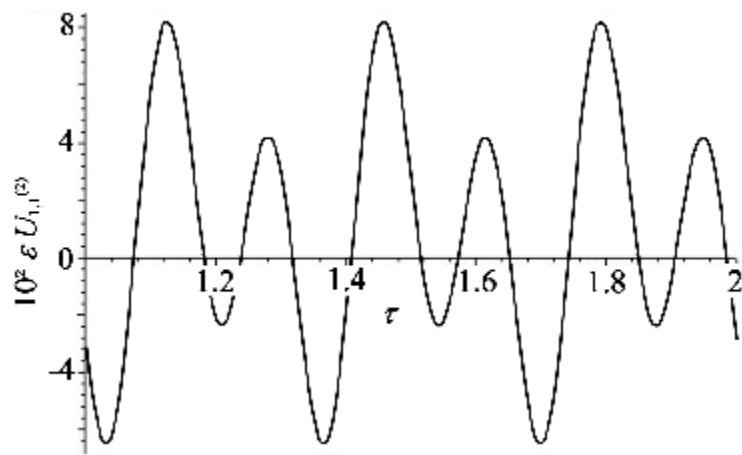
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Figure 4



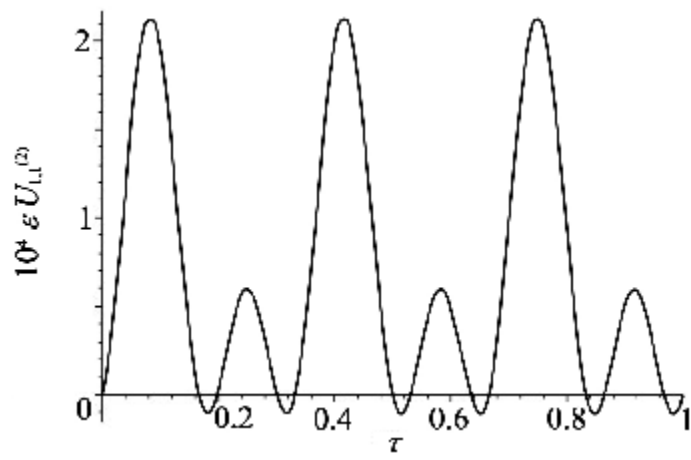
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Figure 5



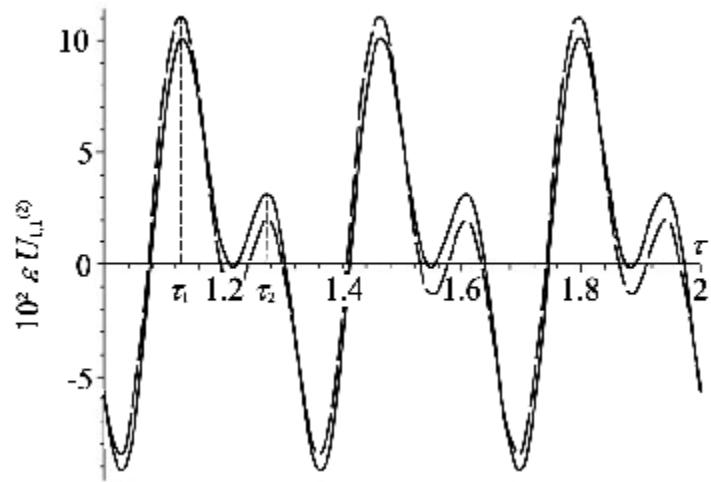
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Figure 6



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Figure 7



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Figure 8

