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Patrice Longère, A. Dragon. Evaluation of the inelastic heat fraction in the context of microstructure supported dynamic plasticity modelling. *International Journal of Impact Engineering*, 2008, 35 (9), pp.992. 10.1016/j.ijimpeng.2007.06.006 . hal-00499103

**HAL Id: hal-00499103**

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Submitted on 9 Jul 2010

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## Author's Accepted Manuscript

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PII: S0734-743X(07)00105-4  
DOI: doi:10.1016/j.ijimpeng.2007.06.006  
Reference: IE 1503

To appear in: *International Journal of Impact*

Received date: 24 November 2006  
Revised date: 21 June 2007  
Accepted date: 22 June 2007

Cite this article as: Patrice Longère and André Dragon, Evaluation of the inelastic heat fraction in the context of microstructure supported dynamic plasticity modelling, *International Journal of Impact* (2007), doi:[10.1016/j.ijimpeng.2007.06.006](https://doi.org/10.1016/j.ijimpeng.2007.06.006)

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# Evaluation of the inelastic heat fraction in the context of microstructure supported dynamic plasticity modelling

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## Abstract

Under dynamic adiabatic conditions the plastic work is known to dissipate into heat and inducing thermal softening. From both theoretical and numerical viewpoints the proportion of effectively dissipated plastic work is commonly evaluated using the so-called TAYLOR-QUINNEY coefficient usually assumed to be a constant empirical value. On the other hand experimental investigations have shown its dependence on strain, strain rate and temperature. A methodology combining dislocation theory in the domain of thermally activated inelastic deformation mechanisms and internal variable approach applied to thermo-elastic/viscoplastic behaviour is developed allowing for obtaining a physically based inelastic heat fraction expression. The latter involves explicitly the combined influence of the parameters mentioned above and highly evolving nature of the inelastic heat fraction.

*Keywords* : Dislocation theory – Irreversible thermodynamics – Inelastic heat fraction – Non-linear modelling – Viscoplasticity - Dynamics.

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## 1. Introduction

As a consequence of the inelastic deformation under adiabatic condition, the most part of the plastic work is dissipated into heat. The subsequent temperature rise induced in the material provokes a softening effect which favours potential instability as encountered notably in adiabatic shear banding phenomenon (see RECHT [1], BAI AND DODD [2], and LONGÈRE ET AL. [3,4]). For applications involving high strain rate – speed machining, crash, impact problems f.ex. – an accurate evaluation of the plastic work induced heating is consequently of major interest. The latter is usually evaluated using the inelastic heat fraction also known as TAYLOR AND QUINNEY coefficient in reference to their original work [5]. Starting from the definition of the stored energy of cold work (see WILLIAMS [6] and BEVER ET AL., [7]), the inelastic heat fraction is defined as the ratio of the generated heat by the plastic work. Depending on the authors, its value is assumed to be a constant comprised between 80 and 100%. Nevertheless many experimental investigations devoted to plastic deformation related temperature rise during dynamic loading have shown the dependence of the inelastic heat fraction on strain,

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strain rate and temperature (see CHRYSOCHOOS ET AL. [8], MASON ET AL. [9], KAPOOR AND NEMAT-NASSER [10], ROSAKIS ET AL. [11], LERCH AT AL. [12] and OLIFERUK ET AL., [13]) thanks to the improvement of the temperature measurement devices. Thus the inelastic heat fraction is strongly dependent on the loading path and neglecting this feature constitutes for most materials a coarse simplification which may lead to an erroneous estimation of the temperature rate. Following a thermodynamics based approach the present study attempts to provide some complementary insight for a more realistic evaluation of dynamic heat generation in the context of strain, strain rate and temperature dependent material behaviour.

The modelling methodology used herein is referred to dislocation theory at strain rate high enough to consider the deformation mechanisms as thermally activated but low enough to exclude phonon-drag phenomenon, i.e. at intermediate strain rate and temperature. The flow stress of the polycrystalline aggregate is assumed to be split in an athermal contribution characterizing the rate independent resistance to dislocation motion due to long-range barriers, such as grain boundary, and a viscous-thermal contribution characterizing the rate dependent resistance to dislocation motion due to short-range obstacles, such as forest dislocations (see ZERILLI AND ARMSTRONG [14], FOLLANSBEE AND KOCKS [15], KLEPACZKO AND RESAIG [16] and NEMAT-NASSER AND LI [17] f.ex.). The former increases with increasing accumulated dislocations but saturates according to the assumption of bounded dislocation density. The latter increases with increasing strain rate but decreases with increasing temperature, including different effects regarding the geometry of the crystalline pattern (fcc or bcc).

The dislocation kinetics and related formalism is further integrated into internal variable procedure in the framework of irreversible thermodynamics (see also VOYIADJIS AND ABED [18] for a similar approach). Free energy and dissipative potential are proposed preserving the aforementioned basic concepts. The yield function involves the athermal processes of plasticity as described in dislocation theory while the viscous overstress is included in PERZYNA's type dissipative potential [19]. Starting from the first and second principles of thermodynamics, the heat equation is deduced leading to a rigorous expression of the inelastic heat fraction. In the latter form, the combined influence of strain, strain rate and temperature appears clearly. From this thermodynamic viewpoint, the inelastic heat fraction rate is shown to be negative for a strain hardening material, and positive for a strain softening material.

In Sect.2 of the present paper, the basic concepts are introduced. They include firstly the notion of flow stress according to dislocation theory in the regime of thermally controlled inelastic deformation mechanisms. Secondly they detail the irreversible thermodynamics based context applied to thermo-elastic/viscoplastic modelling. The heat equation leading to the expression of the inelastic heat fraction is also deduced. Finally a unified approach coupling both aforementioned methodologies is proposed.

Sect.3 is devoted to numerical evaluation of the inelastic heat fraction of two very different materials, a Copper type one (fcc structure) and a Tantalum type one (bcc structure) under simple shear loading at various strain rates and initial temperatures. The tendency observed agrees experimental conclusions.

## **2. Basic concepts and unified approach**

This part aims at reconciling both main methodologies of modelling, namely physically based one and phenomenological one. In a first sub section the former is briefly applied to plastic deformation mechanisms controlled by thermal activation in the cases of fcc and bcc materials. Afterwards the irreversible thermodynamics related internal variable procedure is

considered regarding thermo-elastic/viscoplastic materials. In the last sub section a unified approach is employed in which the dislocation interaction mechanisms based modelling is incorporated in the formalism of standard generalized materials.

## 2.1. Dislocation mechanics based modelling

The following modelling is devoted to metallic materials which deform plastically under dislocation motion and accumulation/annihilation mechanisms. It refers explicitly to the concept of thermally controlled mechanisms. In the range of strain rate and temperature considered, the resistance to dislocation motion is supposed to be due to two kinds of obstacles : long-range barriers typically formed by grain boundaries and other far-field influent microstructural elements relative to a rate and temperature independent stress (athermal stress), and short-range barriers formed by disoriented dislocations and other point defects relative to a rate and temperature dependent stress (thermal/viscous stress).

According to these microstructural concepts, the flow stress  $\tau$  may be decomposed into an athermal contribution  $\tau_a$  and a thermal/viscous contribution  $\tau_{th}$  as follows :

$$\tau = \tau_a(\gamma^p) + \tau_{th}(\dot{\gamma}^p, T) \quad (1)$$

where  $\gamma^p$  represents the plastic strain,  $\dot{\gamma}^p$  the plastic strain rate and T absolute temperature. The athermal stress  $\tau_a$  reflects the influence of the presence of solute, original dislocation density and grain size (the material state considered here is not the virgin one if the material was submitted to thermo-mechanical treatments) through a constant contribution  $\tau_0$  and the accumulation of dislocation through a hardening contribution  $\bar{\tau}$  :

$$\tau_a(\gamma^p) = \tau_0 + \bar{\tau}(\gamma^p) \quad (2)$$

Assuming a bounded dislocation density at large deformation, the hardening stress is supposed to saturate, following VOCE's form :

$$\bar{\tau}(\gamma^p) = \tau_\infty [1 - \exp(-b\gamma^p)] \quad (3)$$

where  $\tau_\infty$  represents the maximum hardening stress and b a material constant characterizing the hardening kinetics.

According to OROWAN's law in the context of thermally activated inelastic mechanisms, the plastic strain is assumed in the following ARRHENIUS form :

$$\dot{\gamma}^p = \dot{\gamma}_0 \exp\left(-\frac{\Delta G}{kT}\right) \quad (4)$$

where the constant pre-exponential term  $\dot{\gamma}_0$  is notably related to mobile dislocation density and obstacle overcoming frequency, k represents the BOLTZMANN constant and  $\Delta G$  the activation energy or energetic barrier needed for dislocation to overcome. The latter is written in the following form :

$$\Delta G = G_0 \left[ 1 - \left\langle \frac{\tau_{th}}{\hat{\tau}} \right\rangle^p \right]^q \quad (5)$$

with  $\langle x \rangle = \max(x, 0)$  and where the total energy  $G_0$  is related to the material strain-rate sensitivity via the activation volume,  $\hat{\tau}$  the maximum glide resistance, and  $p$  and  $q$  express the statistical shape of the obstacle profile. Injecting (5) into (4) yields :

$$\dot{\gamma}^p = \dot{\gamma}_0 \exp \left\{ -\frac{G_0}{kT} \left[ 1 - \left\langle \frac{\tau_{th}}{\hat{\tau}} \right\rangle^p \right]^q \right\} \quad (6)$$

Inverting the relation (6) above leads to following expression for the thermal/viscous stress contribution  $\tau_{th}$  in (1) :

$$\tau_{th} = \hat{\tau} \left\{ 1 - \left[ -\frac{kT}{G_0} \ln \left\langle \frac{\dot{\gamma}^p}{\dot{\gamma}_0} \right\rangle \right]^{1/q} \right\}^{1/p} \quad (7)$$

In the case of bcc materials, the maximum glide resistance  $\hat{\tau}$  is assumed to be a constant :

$$\hat{\tau} = \hat{\tau}_0 \quad (8)$$

According to (7) and (8) the thermal/viscous stress contribution  $\tau_{th}$  becomes for bcc materials:

$$\tau_{th} = \hat{\tau}_0 \left\{ 1 - \left[ -\frac{kT}{G_0} \ln \left\langle \frac{\dot{\gamma}^p}{\dot{\gamma}_0} \right\rangle \right]^{1/q} \right\}^{1/p} \quad (9)$$

Injecting (3) into (2) and considering (9) the total flow stress in (1) is written as :

$$\tau = \tau_0 + \tau_\infty [1 - \exp(-b\gamma^p)] + \hat{\tau}_0 \left\{ 1 - \left[ -\frac{kT}{G_0} \ln \left\langle \frac{\dot{\gamma}^p}{\dot{\gamma}_0} \right\rangle \right]^{1/q} \right\}^{1/p} \quad (10)$$

This expression for the bcc material flow stress reflects experimental observations showing that, for isothermal processes, the apparent strain hardening  $d\tau/d\gamma^p$  is neither affected by strain rate at a given initial temperature nor by initial temperature at a given strain rate. This explains the additive decomposition of the flow stress into separated strain hardening contribution and thermal/viscous contribution (see ZERILLI AND ARMSTRONG [14] and VOYIADJIS AND ABED [18]).

In the case of fcc materials, the maximum glide resistance is assumed to involve the former athermal stress contribution affected by temperature. This corresponds to a simplification of expressions available in literature (see f.ex. NEMAT-NASSER AND LI [17] for reference form). Let consider the following expression :

$$\hat{\tau} = [\tau_0 + \bar{\tau}(\gamma)]a(T) ; a(T) = 1 - \left( \frac{T}{T_m} \right)^2 \quad (11)$$

According to (3) and (11) the thermal/viscous stress contribution in (7) is expressed as :

$$\tau_{th} = \left\{ \tau_0 + \tau_\infty [1 - \exp(-b\gamma^p)] \right\} \left[ 1 - \left( \frac{T}{T_m} \right)^2 \right] \left\{ 1 - \left[ -\frac{kT}{G_0} \ln \left\langle \frac{\dot{\gamma}^p}{\dot{\gamma}_0} \right\rangle \right]^{1/q} \right\}^{1/p} \quad (12)$$

Injecting (12) into (1) yields :

$$\tau = \left\{ \tau_0 + \tau_\infty [1 - \exp(-b\gamma^p)] \right\} \left[ 1 + \left[ 1 - \left( \frac{T}{T_m} \right)^2 \right] \left\{ 1 - \left[ -\frac{kT}{G_0} \ln \left\langle \frac{\dot{\gamma}^p}{\dot{\gamma}_0} \right\rangle \right]^{1/q} \right\}^{1/p} \right] \quad (13)$$

Contrarily to bcc materials, the flow stress for fcc materials is known to combine multiplicatively the influence of both strain hardening and thermal/viscous contributions (see ZERILLI AND ARMSTRONG [14] and VOYIADJIS AND ABED [20]). This feature is respected in expression (13).

## 2.2. Thermodynamic framework

Irreversible thermodynamics framework (COLEMAN AND GURTIN [21], MEIXNER, [22] and BATAILLE AND KESTIN, [23]) is now applied to thermo-elastic/viscoplastic behaviour in the case of linear thermo-elasticity, pressure insensitive plasticity and isotropic strain hardening. The instantaneous state of the material is supposed to be described via the free energy  $\tilde{\psi} = \rho\psi$ . The set  $y$  of state variables includes  $\underline{e}^e$  representing a measure of moderate elastic strain ( $\underline{e}^e = \ln \underline{V}^e$ ;  $\underline{F}^e = \underline{V}^e \underline{Q}$  where  $\underline{F}^e$  proceeds from the multiplicative split of the deformation gradient  $\underline{F} = \underline{F}^e \underline{F}^p$  and where  $\underline{V}^e$  and  $\underline{Q}$  represent the pure elastic stretching and the frame rotation tensor respectively) and  $\kappa$  representing the isotropic strain hardening variable ( $\kappa = \int_t \sqrt{\frac{2}{3} \underline{d}^p : \underline{d}^p} dt$ ), such that  $y = (\underline{e}^e, \kappa)$ . The rate of deformation  $\underline{d}$  is further decomposed into a reversible part  $\underline{d}^e$  and an irreversible part  $\underline{d}^p$  such that  $\underline{d} = [\underline{l}]^s = \underline{d}^e + \underline{d}^p$ , where  $[\underline{l}]^s$  represents the symmetric part of the velocity gradient  $\underline{l}$ . Derivating free energy  $\tilde{\psi}(T; \underline{e}^e, \kappa)$  yields :

$$\dot{\tilde{\psi}} = \frac{\partial \tilde{\psi}}{\partial T} \dot{T} + \frac{\partial \tilde{\psi}}{\partial \underline{e}^e} : \underline{d}^e + \frac{\partial \tilde{\psi}}{\partial \kappa} \dot{\kappa} = -\tilde{s} \dot{T} + \underline{\underline{\sigma}} : \underline{d}^e + R \dot{\kappa}; \quad \tilde{s} = -\frac{\partial \tilde{\psi}}{\partial T}; \quad \underline{\underline{\sigma}} = \frac{\partial \tilde{\psi}}{\partial \underline{e}^e}; \quad R = \frac{\partial \tilde{\psi}}{\partial \kappa} \quad (14)$$

where  $\tilde{s}$  represents entropy,  $\underline{\underline{\sigma}}$  the thermo-elastic CAUCHY stress tensor,  $R$  the isotropic hardening conjugate force and  $\underline{d}^e = \overset{\nabla}{\underline{e}^e} = \dot{\underline{e}^e} - \underline{\underline{\omega}} \underline{e}^e + \underline{e}^e \underline{\underline{\omega}}$ ,  $\nabla$  representing the objective JAUMANN derivative of a 2<sup>nd</sup> order tensor (according to the assumption of small elastic strain as stated here) and  $\underline{\underline{\omega}} = [\underline{l}]^{AS}$  the spin defined as the antisymmetric part of the velocity gradient. According to the second principle of thermodynamics intrinsic mechanical dissipation is written as :

$$D_{mech} = \underline{\underline{\sigma}} : \underline{d}^p - R \dot{\kappa} \geq 0 \quad (15)$$

where  $\underline{\underline{\sigma}}: \underline{\underline{d}}^p$  represents the plastic part of the mechanical work rate and  $R\dot{\kappa}$  the stored energy rate. Assuming normality rule, evolution laws are deduced from:

$$\underline{\underline{d}}^p = \Lambda \frac{\partial F}{\partial \underline{\underline{\sigma}}} ; -\dot{\kappa} = \Lambda \frac{\partial F}{\partial R} \quad (16)$$

where  $\Lambda \geq 0$  represents here the viscous multiplier and  $F$  designates the yield function.

Combining (14) with the first law of thermodynamics and assuming that  $R$  is independent on temperature (see (20) below) lead to the following form for the heat equation :

$$\tilde{c}_y \dot{T} + \text{div} \underline{\underline{q}} - r = T \frac{\partial \underline{\underline{\sigma}}}{\partial T} : \underline{\underline{d}}^e + \underline{\underline{\sigma}} : \underline{\underline{d}}^p - R\dot{\kappa} \quad (17)$$

where  $\tilde{c}_y$  represents the heat capacity per unit mass at fixed  $y$   $\left( \tilde{c}_y = -T \frac{\partial^2 \tilde{\psi}}{\partial T^2} \Big|_y \right)$ ,  $\underline{\underline{q}}$  heat flux vector per unit surface and  $r$  heat supply per unit volume. The context considered herein concerns loading at high strain rate excluding heat supply and for which conditions can be assumed as adiabatic. The relation (17) above is thus reduced to :

$$\tilde{c}_y \dot{T} = T \frac{\partial \underline{\underline{\sigma}}}{\partial T} : \underline{\underline{d}}^e + \underline{\underline{\sigma}} : \underline{\underline{d}}^p - R\dot{\kappa} = D_{\text{mech}} + T \frac{\partial \underline{\underline{\sigma}}}{\partial T} : \underline{\underline{d}}^e \quad (18)$$

where  $T \frac{\partial \underline{\underline{\sigma}}}{\partial T} : \underline{\underline{d}}^e$  represents the thermo-elastic coupling contribution. Considering that  $D_{\text{mech}} \geq 0$  and  $T \frac{\partial \underline{\underline{\sigma}}}{\partial T} : \underline{\underline{d}}^e \leq 0$  for tensile loading (implying cooling) or  $T \frac{\partial \underline{\underline{\sigma}}}{\partial T} : \underline{\underline{d}}^e \geq 0$  for compressive loading (implying heating), thermo-elastic and thermo-viscoplastic mechanisms act inversely or together regarding temperature rise.

According to the aforementioned assumptions, the free energy  $\tilde{\psi}(T; \underline{\underline{e}}^e, \kappa)$  is now expressed in the following form:

$$\tilde{\psi}(T; \underline{\underline{e}}^e, \kappa) = \frac{\lambda}{2} (\text{Tr} \underline{\underline{e}}^e)^2 + \mu \underline{\underline{e}}^e : \underline{\underline{e}}^e - \alpha K \text{Tr} \underline{\underline{e}}^e \vartheta - \tilde{c}_0 \left[ T \ln \left( \frac{T}{T_0} \right) - \vartheta \right] + h(\kappa) - h(0) \quad (19)$$

where  $\lambda$  and  $\mu$  represent LAMÉ elasticity constants,  $K = \lambda + \frac{2}{3}\mu$  the bulk modulus,  $\alpha$  the thermal dilatation coefficient,  $\tilde{c}_0$  heat capacity which is supposed to have a constant value (i.e.  $\tilde{c}_y = \tilde{c}_0$ ),  $\vartheta = T - T_0$  the temperature rise,  $h(\kappa)$  the stored energy of cold work as a function of strain hardening variable. After partial derivation of (19) with respect to state variables, the thermodynamic forces are detailed as follows :

$$\underline{\underline{\sigma}} = (\lambda \text{Tr} \underline{\underline{e}}^e - \alpha K \vartheta) \underline{\underline{\delta}} + 2\mu \underline{\underline{e}}^e ; R = h'(\kappa) ; \tilde{s} = \alpha K \text{Tr} \underline{\underline{e}}^e + \tilde{c}_0 \ln \left( \frac{T}{T_0} \right) \quad (20)$$

The dissipative potential is assumed of the PERZYNA's type [19]  $\phi(\underline{\underline{\sigma}}, R) = \phi(\langle F(\underline{\underline{\sigma}}, R) \rangle)$ . The viscoplastic multiplier and the yield function in (16) are defined as :

$$\Lambda = \frac{\partial \phi}{\partial F} = H(\langle F(\underline{\underline{\sigma}}, R) \rangle) ; F(\underline{\underline{\sigma}}, R) = J_2(\underline{\underline{\sigma}}) - g(R) ; J_2(\underline{\underline{\sigma}}) = \sqrt{\frac{3}{2} \underline{\underline{s}} : \underline{\underline{s}}} ; \underline{\underline{s}} = \underline{\underline{\sigma}} - \frac{\text{Tr} \underline{\underline{\sigma}}}{3} \underline{\underline{\delta}} \quad (21)$$

where H is a function of the yield surface F. The strain hardening function g(R) in (21) represents the VON MISES surface radius. It is assumed in the form :

$$g(R) = R_0 + R(\kappa) \quad (22)$$

It includes the first term  $R_0$  independent of strain hardening and the isotropic hardening force R as the second term. The quantity  $R_0$  accounts for residual stresses potentially induced by the previous thermo-mechanical history of the material. Evolution laws (16) are thus detailed as :

$$\underline{\underline{d}}^p = \frac{3}{2} \Lambda \frac{\underline{\underline{s}}}{J_2(\underline{\underline{\sigma}})} ; \dot{\kappa} = \Lambda \quad (23)$$

Inverting (21)<sub>1</sub> and using (21)<sub>2</sub> and (22) yield :

$$J_2 = R_0 + R(\kappa) + \Phi(\kappa, \dot{\kappa}, T) ; \Phi = H^{-1} \quad (24)$$

According to (23)<sub>1</sub> and (23)<sub>2</sub>, the rate of plastic work  $\underline{\underline{\sigma}} : \underline{\underline{d}}^p$  in (15) is thus given by :

$$\underline{\underline{\sigma}} : \underline{\underline{d}}^p = J_2 \dot{\kappa} = [R_0 + R(\kappa) + \Phi(\kappa, \dot{\kappa}, T)] \dot{\kappa} \quad (26)$$

Finally injecting (26) into (15) yields :

$$D_{\text{mech}} = [R_0 + \Phi] \dot{\kappa} \geq 0 \quad (27)$$

One can note that all terms in the above equation are positive.

### 2.3. Dislocation mechanics-irreversible thermodynamics unified approach

The concepts of thermally activated processes developed previously are now incorporated in the internal variable procedure formalism (see also VOYIADJIS AND ABED [18]). The first step consists in unifying the notations. The corresponding terms are reported in Table 1.

The following yield function describing athermal processes is also assumed (see (1)(2) and (21)<sub>2</sub>(22)) :

$$F(\tau, \bar{\tau}) = \tau - (\tau_0 + \bar{\tau}) = F(\underline{\underline{\sigma}}, R) = J_2(\underline{\underline{\sigma}}) - (R_0 + R) \quad (28)$$

Noting that  $\tau_{\text{th}}(\gamma, \dot{\gamma}, T) = \tau - \tau_a(\gamma) = F(\tau, \bar{\tau}) \geq 0$  stands for viscoplastic yielding, expression (6) is converted into :

$$\dot{\kappa} = \dot{\kappa}_0 \exp \left\{ -\frac{G_0}{kT} \left[ 1 - \left\langle \frac{F(\underline{\sigma}, R)}{\hat{\tau}} \right\rangle^p \right]^q \right\} = H \langle F(\underline{\sigma}, R) \rangle \quad (29)$$

**Table 1.** Corresponding terms for dislocation mechanics-irreversible thermodynamics unified approach

Dislocation mechanics	Internal variable procedure
$\gamma^p$	$\kappa$
$\dot{\gamma}^p$	$\dot{\kappa}$
$\tau$	$J_2$
$\tau_a(\gamma^p) = \tau_0 + \bar{\tau}(\gamma^p)$	$g(\kappa) = R_0 + R(\kappa)$
$\tau_0$	$R_0$
$\bar{\tau}(\gamma^p) = \tau_\infty [1 - \exp(-b\gamma^p)]$	$R(\kappa) = R_\infty [1 - \exp(-b\kappa)]$
$\tau_{th}(\gamma, \dot{\gamma}, T)$	$\Phi(\kappa, \dot{\kappa}, T)$
$\tau_\infty$	$R_\infty$

Inverting (29) yields :

$$F = H^{-1} = \Phi \quad (30)$$

Using (28) and (30) leads to the following form for the flow stress :

$$J_2(\underline{\sigma}) = R_0 + R(\kappa) + \Phi(\kappa, \dot{\kappa}, T) \quad (31)$$

The expression (31) above has to be compared to the following one obtained from (1) and (2):

$$\tau = \tau_0 + \bar{\tau}(\gamma^p) + \tau_{th}(\gamma^p, \dot{\gamma}^p, T) \quad (32)$$

### 3. Evaluation of the inelastic heat fraction for fcc and bcc materials

According to the dislocation mechanics-irreversible thermodynamics unified viscoplasticity approach developed previously, this section aims at showing the influence of the modelling regarding the evolution of the inelastic heat fraction and related temperature rise. Actually it is shown that, from the thermodynamic viewpoint, the inelastic heat fraction rate is strongly dependent on the strain hardening/softening rate. Fcc and bcc materials are modelled and the inelastic heat fraction is deduced in both cases. Its evolution is analysed considering a simple shear loading.

#### 3.1. General expression for the inelastic heat fraction

In the following the effects of strain hardening, thermal softening and viscosity on stress/strain behaviour, temperature rise and inelastic heat fraction are studied. Thermo-elastic coupling contribution to temperature rise is actually particularly significant in problems involving very high velocity impact and/or high pressure shock loading. In the context of this

work, velocity and pressure are considered moderately high and thermo-elastic coupling is neglected. Heat equation in (18) is thus reduced to :

$$\tilde{c}_0 \dot{T} = D_{\text{mech}} \quad (33)$$

Starting from the definition of the inelastic heat fraction  $\beta$  as  $\beta = \frac{\tilde{c}_0 \dot{T}}{\underline{\underline{\sigma}} : \underline{\underline{d}}^p}$ , the following expression is deduced :

$$\beta = 1 - \frac{R\dot{\kappa}}{\underline{\underline{\sigma}} : \underline{\underline{d}}^p} \quad (34)$$

Accounting for (26), relations (33) and (34) are thus reduced to:

$$\dot{T} = \frac{1}{\tilde{c}_0} [J_2 - R] \dot{\kappa} ; \beta = 1 - \frac{R}{J_2} \quad (35)$$

Injecting the expression (26) of the stress invariant  $J_2$  yields:

$$\dot{T} = \frac{R_0 + \Phi(\kappa, \dot{\kappa}, T)}{\tilde{c}_0} \dot{\kappa} ; \beta = 1 - \frac{h'(\kappa)}{R_0 + h'(\kappa) + \Phi(\kappa, \dot{\kappa}, T)} \quad (36)$$

Consequently, the inelastic heat fraction  $\beta$  appears explicitly as a function of thermal/athermal hardening/softening and viscosity mechanisms via  $\Phi(\kappa, \dot{\kappa}, T)$  and  $h'(\kappa)$  and of the prior plastic deformation history via  $R_0$ . The form (36)<sub>2</sub> highlights the evolving nature of  $\beta$  with temperature, strain and strain rate evolution. On this basis further remarks can be made as follows.

#### Remark 1

Under the modelling assumption, for plastic strain  $\kappa_2 > \kappa_1$  close enough to consider that  $T_1 \approx T_2 \approx T$ , one can write from (36) :

$$\beta(\kappa_2, \dot{\kappa}, T) - \beta(\kappa_1, \dot{\kappa}, T) \approx -[h'(\kappa_2) - h'(\kappa_1)] \frac{1}{R_0 + h'(\kappa) + \Phi(\kappa, \dot{\kappa}, T)} \quad (37)$$

Using the notation

$$\chi = \frac{1}{R_0 + h'(\kappa) + \Phi(\kappa, \dot{\kappa}, T)} ; \chi \geq 0 \quad (38)$$

relation (37) is reduced to:

$$\frac{\partial \beta}{\partial \kappa} \approx -\chi h''(\kappa) \quad (39)$$

According to (39), it is possible to conclude that for material exhibiting strain hardening ( $h''(\kappa) > 0$ ), the inelastic heat fraction is decreasing with increasing strain, i.e.  $\frac{\partial\beta}{\partial\kappa} < 0$ . On the contrary, for material exhibiting strain softening ( $h''(\kappa) < 0$ ), the inelastic heat fraction is increasing with increasing strain, i.e.  $\frac{\partial\beta}{\partial\kappa} > 0$ .

### Remark 2

According to (36)<sub>2</sub>,  $\beta(\kappa, \dot{\kappa}, T)$  is equal to unity when  $h'(\kappa) = 0$  which is satisfied for a perfectly plastic material (no strain hardening).

### 3.2. Application to fcc and bcc materials

The unified approach is now applied to fcc and bcc materials in the case of simple shearing such that  $l_{ij} = \partial v_i / \partial x_j = 0$  except  $l_{12} = \partial v_1 / \partial x_2 = \dot{\Gamma} \neq 0$ . Material constants used for numerical simulations have been identified to reproduced Copper type material (see VOYIADJIS AND ABED [20] for experimental results) and Tantalum type material (see VOYIADJIS AND ABED [18] for experimental results) and are reported in Table 2.

**Table 2.** Material constants for simple shearing simulation

	Copper (fcc)	Tantalum (bcc)
E (GPa)	120	170
$\nu$	0.33	0.34
$\rho_0$ (kg/m <sup>3</sup> )	8930	16600
$c_0$ (J/kg.K)	380	140
$T_m$ (K)	1350	3300
$p$	1/2	1/2
$q$	3/2	3/2
$k/G_0$ (K <sup>-1</sup> )	4.9E-5	8E-5
$\dot{p}_0$ (s <sup>-1</sup> )	1E10	1E9
$R_0$ (MPa)	100	100
$R_\infty$ (MPa)	300	100
$b$	10	5
$\hat{\tau}_0$ (MPa)	X	1700

Numerical evolution of stress invariant  $J_2$ , temperature  $T$  and inelastic heat fraction  $\beta$  is given versus shear strain  $e_{12} = \gamma_{12}/2$  (the strain tensor  $\underline{e}$  is obtained by time integration of the non objective strain rate tensor  $\underline{\dot{e}}$ , with  $\underline{\dot{e}} = \underline{d} + \underline{\omega e} - \underline{e \omega}$ ) for various strain rates and initial temperatures considering Copper behaviour model in Fig.1 and Tantalum behaviour model in Fig.2. Adiabatic conditions are assumed for strain rates higher than 100 s<sup>-1</sup>.

Figs.1a-2a show the increase of the flow stress with the increase of the strain rate whereas Figs.1d-2d show the increase of the flow stress with the decrease of the initial temperature. Fig.2a shows also the thermal softening induced in the flow stress of Tantalum while thermal softening is not significant in Fig.1a concerning Copper.

Values of numerical temperature in Fig.2b are very similar to those measured by (KAPOOR AND NEMAT-NASSER [10]) on Ta-2.5%W alloy under dynamic compression.

According to Figs.1c-1f and 2c-2f initial value of  $\beta$  is equal to 1 whatever the strain rate and the initial temperature. At large strain  $\beta$  converges to a value which depends on strain rate and initial temperature with a rate (negative according to remark 1) whose absolute value increases with decreasing strain rate and increasing initial temperature. The value for  $\beta$  at convergence is much smaller for Copper than for Tantalum.

Recent experimental investigations using fast response infrared optical device devoted to the measurement of heating during dynamic loading on Aluminium alloy (fcc) and steel (bcc) have shown that the inelastic heat fraction decreases with increasing strain (see respectively LERCH AT AL. [12] and JOVIC ET AL. [24]). Unfortunately, time resolved data obtaining with this type of reliable device are missing concerning Copper and Tantalum.

### 3. Concluding remarks

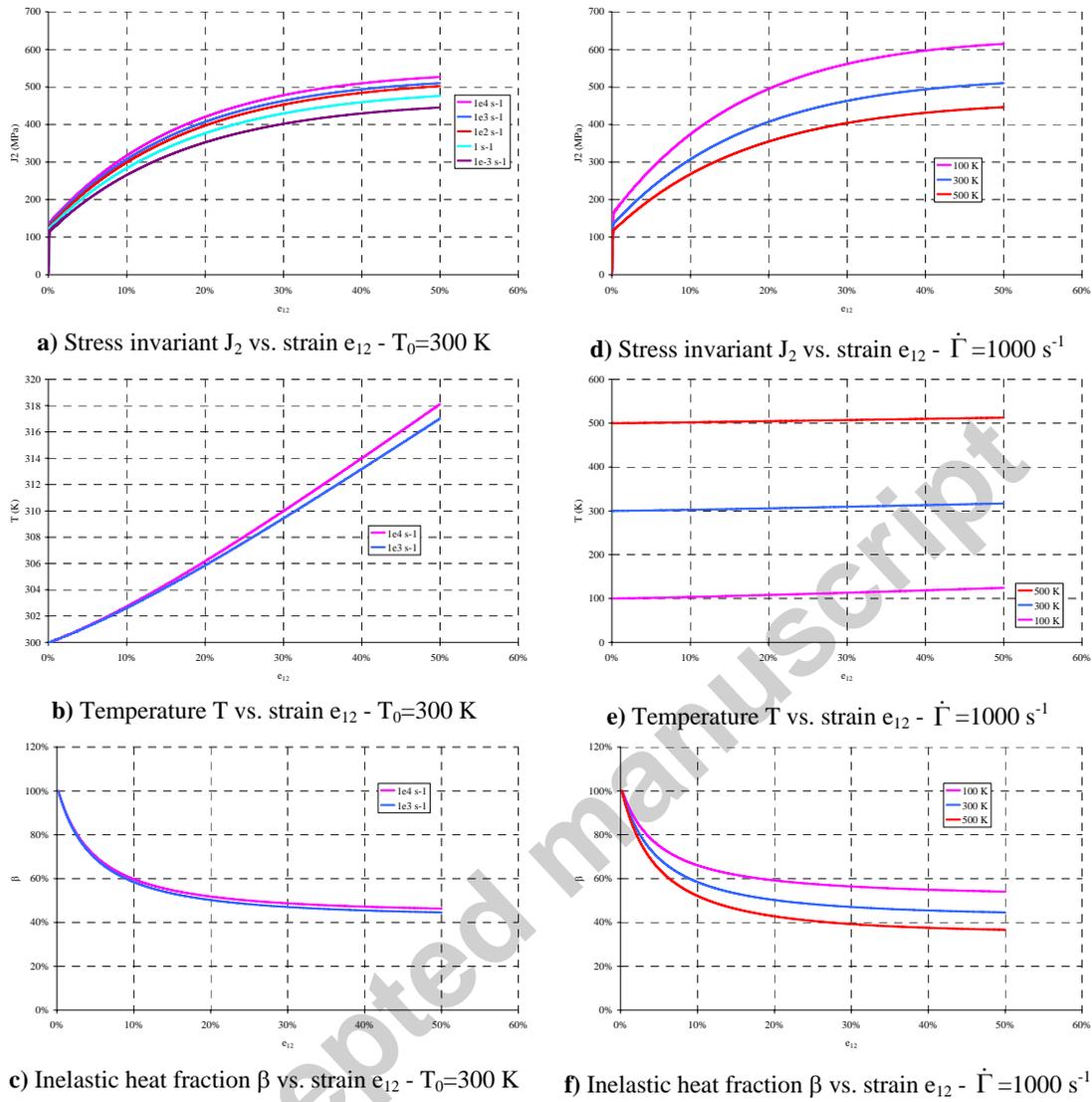
A unified approach combining both concepts of dislocation mechanisms controlled by thermal activation and internal variable viscoplasticity for macroscopic modelling is introduced in the present paper. It is applied to strain, strain rate and temperature dependent metallic materials behaviour in a range covering low velocity to moderately dynamic loading. Following the internal variable procedure and assuming the existence of thermodynamic potentials (free energy and dissipation potential), a consistent expression for the inelastic heat fraction has been obtained. The latter form involves explicitly the influence of strain, strain rate and temperature as observed experimentally and allows for concluding that for a strain hardening model the inelastic heat fraction is decreasing for increasing strain.

These theoretical results show the influence of the modelling in terms of strain hardening/softening, thermal softening and strain rate dependence on the inelastic heat fraction and its highly evolving nature, notably for larger strain. Some models are actually intrinsically able to reproduce observed phenomena, notably the temperature rise induced by plastic deformation under adiabatic conditions, while others are not.

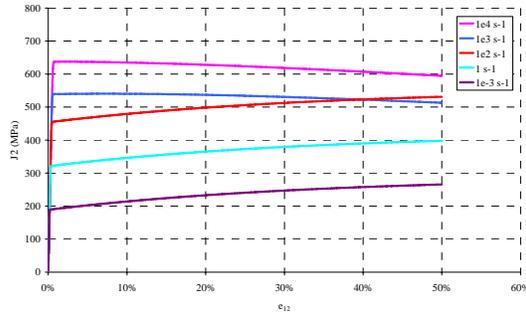
The interest of satisfactory quantification of temperature rise in dynamic plasticity is evident. As shown f.ex. by KLEPACZKO AND RESAIG [16] for adiabatic shear banding involving strain rates of about  $10^5 \text{ s}^{-1}$  the increase of temperature for a class of bcc metals is close to the melting point.

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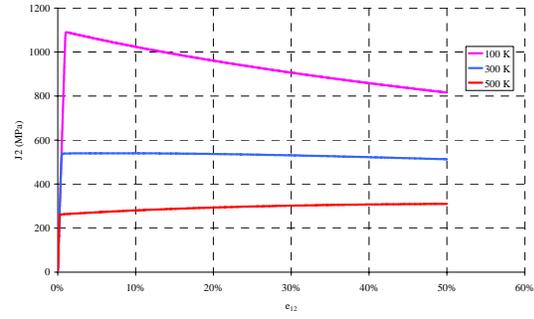
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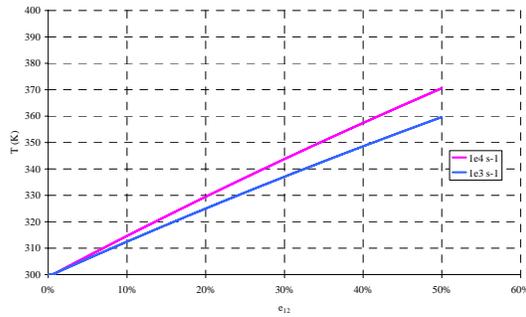
**Fig. 1 :** Influence of shear strain rate and initial temperature on stress invariant and inelastic heat fraction. Adiabatic conditions are assumed for strain rates higher than  $100$  s $^{-1}$ . Copper (fcc material)



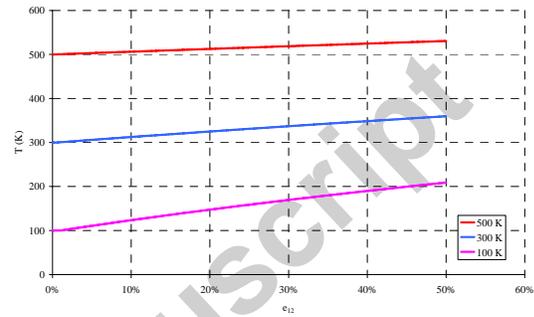
a) Stress invariant  $J_2$  vs. strain  $e_{12}$  -  $T_0=300$  K



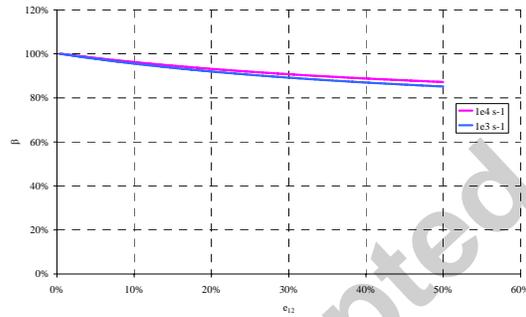
d) Stress invariant  $J_2$  vs. strain  $e_{12}$  -  $\dot{\Gamma}=1000$  s<sup>-1</sup>



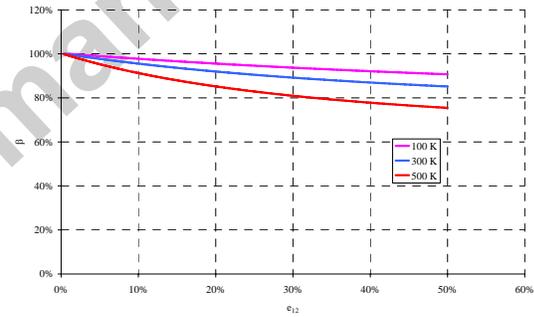
b) Temperature  $T$  vs. strain  $e_{12}$  -  $T_0=300$  K



e) Temperature  $T$  vs. strain  $e_{12}$  -  $\dot{\Gamma}=1000$  s<sup>-1</sup>



c) Inelastic heat fraction  $\beta$  vs. strain  $e_{12}$  -  $T_0=300$  K



f) Inelastic heat fraction  $\beta$  vs. strain  $e_{12}$  -  $\dot{\Gamma}=1000$  s<sup>-1</sup>

**Fig. 2 :** Influence of shear strain rate and initial temperature on stress invariant and inelastic heat fraction. Adiabatic conditions are assumed for strain rates higher than 100 s<sup>-1</sup>. Tantalum (bcc material)