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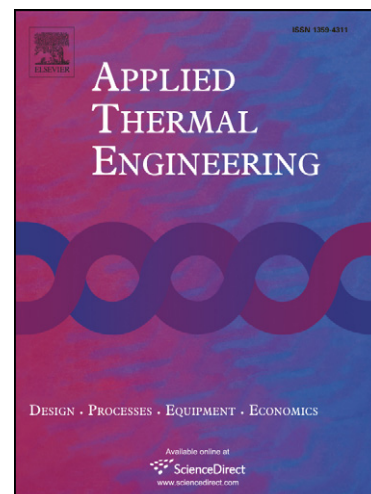
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# Estimation of the transient surface temperature- and heat flux of a steel slab using an inverse method

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## Abstract

In the Steel industry it is of great importance to be able to control the surface temperature and heating- or cooling rates during heat treatment processes. An experiment was performed in which a steel slab was heated up to 1250°C in a fuel fired test furnace. The transient surface temperature and heat flux of a steel slab is calculated using a model for inverse heat conduction. That is, the time dependent local surface temperature and heat flux of a slab is calculated on the basis of temperature measurements in selected points of its interior by using a model of inverse heat conduction. Time- and temperature histories were measured at three points inside a steel slab. Measured temperature histories at the

two lower locations of the slab were used as input to calculate the temperature at the position of the third location. A comparison of the experimentally measured and the calculated temperature histories was made to verify the model. The results showed very good agreement and suggest that this model can be applied to similar applications in the Steel industry or in other areas where the target of investigation for some reason is inaccessible to direct measurements.

## Nomenclature

$c$	specific heat capacity	J/(kg·K)
$f, f_m$	exact and measured temperature histories	Celcius (°C)
$h, h_m$	exact and measured heat flux histories	W/m <sup>2</sup>
$L_i$	distance between thermocouples or thermocouples/surface	meters (m)
$t$	time	seconds (s)
$T$	temperature	Celcius (°C)
$T_0$	duration in time of experiment	seconds (s)
Greek alphabet		
$\alpha$	thermal diffusivity	m <sup>2</sup> /s
$\lambda$	thermal conductivity	W/(m·K)
$\rho$	density	kg/m <sup>3</sup>

## Subscripts

$i$	integer
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## Keywords

Inverse heat conduction, heat treatment, heat transfer, steel processing, slab

## 1. Introduction

The motivations for using an inverse method may be manifold. Sometimes, especially in the field of thermal physics, one wants to calculate the transient temperature or heat flux on the surface of a body. This body may be a slab, or billet in metallurgical applications. However, it may be the case that the surface for some reason is inaccessible to exterior measurements with the aid of some measurement device. Such a device could be a thermocouple if contact with the surface in question is possible or a pyrometer if a non-invasive method is preferred. Sometimes though, these kinds of devices may be an inappropriate choice. It could be the case that the installation of any such device may disturb the experiment in some way or that the environment is chemically destructive or just that such instruments might give incorrect results. In the Steel industry, one large problem is oxide scale formed on the billets or slabs during reheating. The formation of oxide scale causes severe mass losses and affects product quality and economy. One of the main parameters that govern this growth is the surface temperature of the steel surface. However, the surface temperature is difficult to calculate using conventional calculation methods since many parameters inside the furnace chamber must be known for a complete model. The heat transfer mechanism inside the furnace to heat up the slab includes radiation from refractory which often includes a complex geometry and the

convective contribution from burners which often includes a complex flow field. A heat transfer coefficient must thus be known. In most existing models, the growth of oxide scale formation is omitted which makes the heat transferred to the slab erroneous. These kinds of studies are becoming increasingly more important as new combustion technologies are developed with requirements of being environmentally friendly and effective energy wise. In such situations one is directed to using an inverse method based on interior measurements in the body, and in which the desired temperature is calculated by a numerical procedure. The only parameters needed are the temperature histories at some selected locations inside the steel slab and the material properties of the slab.

In the literature [1, 2], this family of problems is most often referred to as inverse heat conduction problems (IHCP). This paper is focused on the application of an inverse method developed in [3].

The aim of this paper is to demonstrate that, in practice, it is possible to determine the transient surface temperature, and heat flux, using a numerical method for solving the IHCP. That is, we present an experiment where the time dependent surface temperature of a slab is calculated on the basis of temperature measurements collected at selected points in the interior of the body. In our experiment the temperature data is recorded by thermocouples inserted into the material at appropriate locations beneath the surface we are interested in studying.

A common family of methods for solving the inverse heat conduction problems transforms the problem into an integral equation of first kind [4, 5]. The drawback of these methods is that often the kernel in the corresponding integral equation is not known explicitly. This is the case, for instance, if the properties of the material, e.g. thermal conductivity and density, are dependent on the temperature, i.e. the problem is non-linear. In metallurgical applications, such as the experiments described in this paper, such methods cannot easily be used since the material properties of steel change considerably in the large temperature range present in our experiment.

The methods developed in [3] allow for problems in which the material properties depend on the temperature, i.e. the heat equation is:

$$\frac{\partial}{\partial x} \left( \lambda(T) \frac{\partial T}{\partial x} \right) = \rho(T) \cdot c(T) \cdot \frac{\partial T}{\partial t} \quad (1)$$

The dependence of the material properties  $\lambda$ ,  $c$  and  $\rho$ , on  $T$  makes the problem non-linear. This approach is most suitable for the application described in this paper. A method for solving the IHCP using wavelets were proposed in [6], as was a method based on using the Fourier Transform. Both of these methods are based on the idea of rewriting the original partial differential equation as a system of ordinary differential equations. The ill-posedness of the problem is dealt with by approximating the time derivative by a bounded approximation. For the purposes of analyzing the data presented in this paper the method based on the Fourier Transform was selected. The Fourier method is simpler than the Wavelet based method, and both methods can be expected to provide similar



results. An application to aluminum was done [7] in which a cooling problem was considered. This paper sets out to use the method for a heating problem in a temperature range relevant to reheating furnaces in the Steel industry [8].

This text is structured in the following way. A mathematical formulation of the IHCP is given in Section 2. In Section 3 we note that the problem is ill-posed and discuss briefly how the problem is turned into solving a system of ordinary differential equations. Section 4 contains details on how the experiment was performed and we follow up with presentation and analysis of the computational results in Section 5. A simplified error estimate for the calculations is given in Section 6 and we close this text by a discussion and some concluding remarks that are entailed in Section 7.

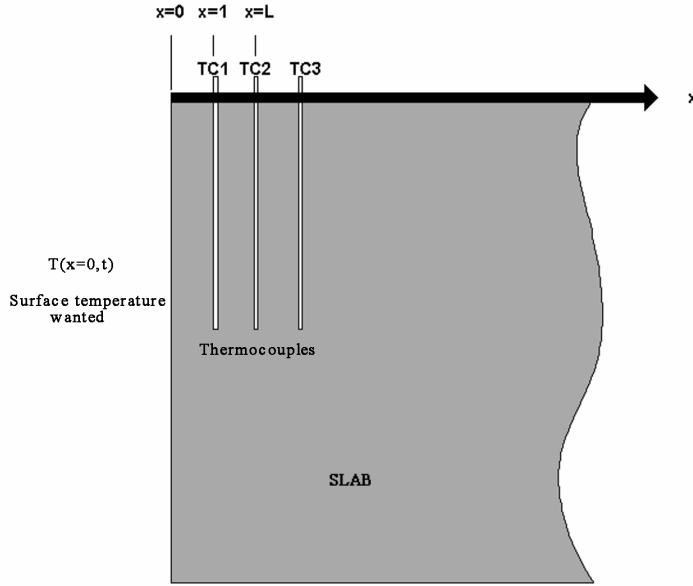
## 2. Mathematical modelling

In this section we present the theoretical model used for analyzing our experimental data. The mathematical problem is as follows: determine the temperature distribution  $T(x, t)$  for  $0 \leq x < 1$  from measurements of temperature  $f(t)$  and heat flux  $h(t)$  along the line  $x = 1$  where  $T(x, t)$  satisfies:

$$\begin{cases} \frac{\partial}{\partial x} \left( \lambda(T) \frac{\partial T}{\partial x} \right) = \rho(T) \cdot c(T) \frac{\partial T}{\partial t}, & 0 < x < 1, \quad t \geq 0, \\ T(1, t) = f(t), & t \geq 0, \\ \lambda(T) \cdot \frac{\partial T}{\partial x}(1, t) = h(t), & t \geq 0, \\ T(x, 0) = 0, & 0 < x < 1. \end{cases} \quad (2)$$

In this text  $\lambda$  is the thermal conductivity,  $\rho$  is the density and  $c$  is the specific heat capacity of the material. These material properties are considered to be temperature dependent. We set out to determine the transient surface temperature of the slab, that is  $T(0, t)$ .

In practice the heat flux cannot be measured in the interior of the material. Instead we measure the temperature at a second location,  $x = L$ , and compute the desired heat flux at  $x = 1$  by solving a well-posed problem in the interval  $1 < x < L$ , using the measured temperatures along  $x = 1$  and  $x = L$  as boundary data. The experimental setup is illustrated in Figure 1.



**Figure 1.** The positions of the thermocouples inside the material are shown.

### 3. Ill-posedness and regularization

The specific feature that characterizes the family of inverse problems is that they are ill-posed [8]. By this we mean that the solution to the problem does not depend continuously on the data. Since the temperature history inside the slab is recorded by means of a thermocouple, measurement errors will be present in the data. In this section we briefly investigate the nature of the ill-posedness and demonstrate that, unless special methods are used, even small perturbations in the data may completely destroy the solution. The complete description of the Fourier analysis for solving this problem is given in [3].

In this section we emphasize the difference between the exact temperature data  $f(t)$  at the location of the measurement device, which by the nature of the heat equation will be a perfectly smooth and continuous function, and the actual measured data  $f_m \in L^2$ . Since, any temperature data recorded by a thermocouple will contain random measurement errors, the actual measured data  $f_m(t)$  will not be a smooth function. However the random noise component is mostly related to high frequency phenomena, and in our application we expect the signal, i.e. the exact temperature history inside the slab, to be slowly varying and thus mainly contain low frequency components. Similarly, as with the temperature data, we also introduce a noisy heat flux data  $h_m \in L^2$ . For the analysis in this section we assume that the measurement errors are bounded by,

$$\|f_m - f\| = \|f_m - T(1, t)\| \leq \varepsilon_1, \text{ and } \|h_m - h\| = \|h_m - \lambda \cdot \frac{\partial T}{\partial x}(1, t)\| \leq \varepsilon_2 \quad (3)$$

where the constants  $\varepsilon_1, \varepsilon_2 > 0$  are assumed to be known.

The fact that the IHCP is severely ill-posed is a consequence of the fact that the time derivative is an unbounded operator while the spatial derivative does not cause any problems. By replacing the time derivative  $\partial/\partial t$  by a bounded approximation a well-posed problem is obtained. By separating the space and time discretizations we can thus focus on treating the ill-posedness of the problem by constructing an appropriately bounded approximation of the time derivative. Since no difficulties are associated with the spatial derivative, this part of the problem can easily be solved using any standard numerical method for integrating ordinary differential equations.

Our numerical problem is based on rewriting the original problem as a set of ordinary differential equations. The time interval is discretized using an equidistant grid, and in this paragraph the unknown temperature  $T(x, :)$ , and the data  $f_m$  and  $g_m$ , are vectors containing samples from the corresponding temperatures on the grid. We obtain the following system of ordinary differential equations:

$$\frac{\partial}{\partial x} \left( \alpha \frac{\partial T}{\partial x} \right) = \begin{pmatrix} 0 & \alpha^{-1} \\ D & 0 \end{pmatrix} \begin{pmatrix} T \\ \alpha \frac{\partial T}{\partial x} \end{pmatrix}, \quad 0 < x < L_1, \quad t > 0, \quad (4)$$

with the initial- and boundary conditions:

$$T(x, 0) = 0, \quad 0 < x < L_1, \quad T(L_1, :) = f_m \text{ and } \lambda \cdot \frac{\partial T}{\partial x}(L_1, :) = h_m. \quad (5)$$

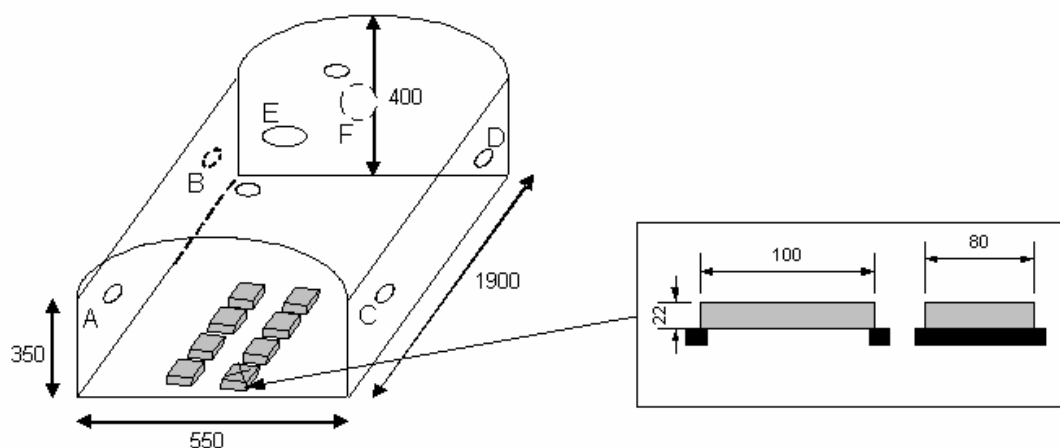
where  $D$  is a matrix representing a discrete approximation of the derivative on the time grid. In our numerical computations discrete derivatives are calculated using the Fast Fourier transform (FFT) and the corresponding differential operator is bounded by explicitly removing frequency components above a certain threshold from the computations. The problem (4) is an initial value problem for a system of ordinary differential equations that can be solved using standard methods, e.g. Runge Kutta Methods. In our computations we use the MATLAB routine *ode45* for solving the initial value problem.

#### 4. Experimental set up

A fuel fired test furnace, see Figure 2, with the dimensions shown in Figure 3 was equipped with a burner at position F and thermocouples at positions A, B, C, D and E respectively. Gas analyzers were also positioned at suitable positions, allowing for measurements of the flue gas composition. The maximum allowed heat density in the furnace is  $200kW/m^3$  which allows for a burner capacity of  $78kW$ . The total inner volume of the furnace was  $0.39m^3$ . A total of eight steel slabs with dimensions  $100\times 80\times 22$  mm as shown were heated in the test furnace. The eight slabs were arranged in two rows, with four slabs each, as seen in Figure 3. All slabs were elevated from the floor by means of ceramic clutches in order to allow for flue gases to circulate under the slabs.



**Figure 2.** A photograph of the test furnace.



**Figure 3.** The dimensions of the test furnace and the position of the slabs are shown. The slabs were elevated a small distance to allow for circulation of flue gases.

The composition of the material of the steel slabs is shown in Table 1.

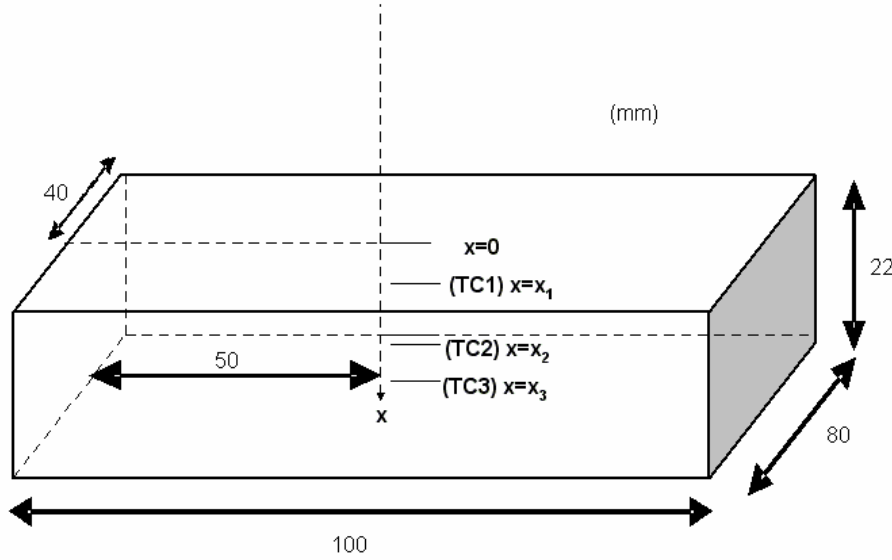
**Table 1.** The composition of the steel used for these investigations.

Element	C	Si	Mn	S	P	Cr	Ni	Mo	Cu	Al
Wt-%	0.06	0.01	0.38	0.035	0.017	0.022	0.055	0.030	0.08	0.001

The first slab in the right row of slabs (the slab pointed on and marked with a cross, see Figure 3), was equipped with thermocouples for temperature readings that were recorded

by a data acquisition system. Three thermocouples were installed beneath the surface at positions  $x_1$ ,  $x_2$  and  $x_3$  respectively in the center of the slab and along the x-axis orthogonal to the top surface, as is shown in Figure 4. The thermocouples were positioned at distances  $x_1 = 5 \text{ mm}$ ,  $x_2 = 11 \text{ mm}$  and  $x_3 = 17 \text{ mm}$  respectively from the surface at  $x = 0 \text{ mm}$ . For later convenience, we refer to Figure 4 and define the distance between the surface at  $x = 0$  and the thermocouple  $TC1$  positioned at  $x = x_1$  as  $L_1$ . Similarly, the distance between the thermocouple  $TC1$  at  $x = x_1$  and thermocouple  $TC2$  at  $x = x_2$  is denoted  $L_2$ . The dimensions of the slab and its position in the test furnace allows for essentially one dimensional heat flux approximation through the slab, with exception or near the corners where end effects may occur. The temperature in the furnace was approximately  $1250^\circ\text{C}$  and the slabs were initially at  $25^\circ\text{C}$ .





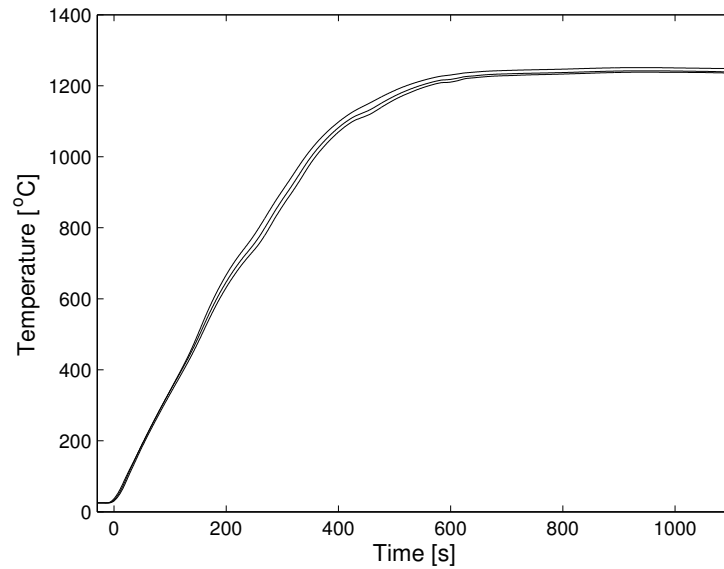
**Figure 4.** The dimensions of the test slab equipped with thermocouples. The surface of the slab is positioned at  $x = 0$ .

Since the intention of this paper is to investigate the appliance of an inverse method to calculate the transient surface temperature in a high temperature heating application, a comparison between the measured and the calculated temperature histories at position  $x_1$  is made. In order to obtain the heat flux at location  $x_2$  needed as boundary data for the inverse problem, a well-posed boundary problem, in the interval  $[x_2, x_3]$ , is solved.

## 5. Computational results

In this section we present the computational results obtained by solving the ill-posed problem (2) numerically. The original data vectors were sampled only at  $0.25\text{Hz}$ . This is insufficient for our purposes, and therefore the data vectors, originally of length 358, were

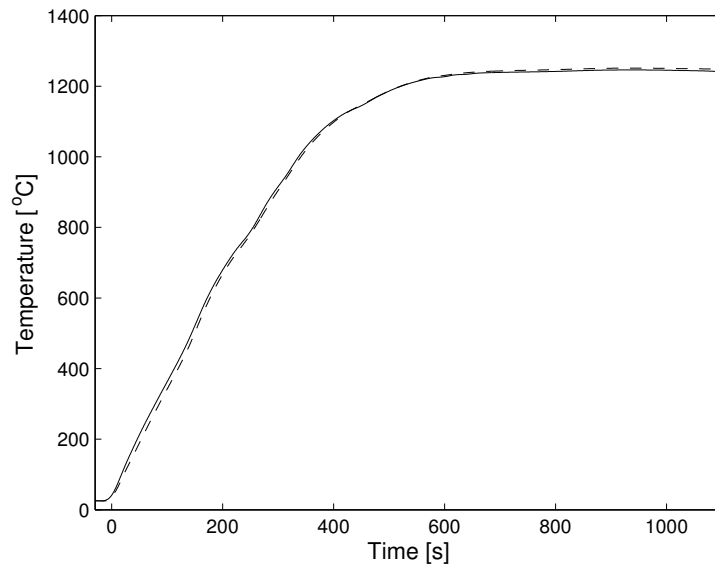
resampled to a larger grid using a smoothing cubic spline. The data vectors used in the actual computations were of length 2048. The data vectors at  $x_1$ ,  $x_2$  and  $x_3$  are shown in Figure 5. For solving the resulting system of ordinary differential equations (4), the standard routine *ode45* was used. The initial step of computing the heat flux, needed as initial data for the ordinary differential equations solver, was done using the Crank-Nicholson Implicit Scheme. All computations were performed using MATLAB 7.



**Figure 5.** The measured temperature histories inside the slab are shown at  $5\text{ mm}$ ,  $11\text{ mm}$  and  $17\text{ mm}$  respectively from the top of the slab surface as shown in Figure 4.

Although we are interested in computing the temperature history at the surface of the slab, we initially used the data recorded by the thermocouple located at  $x = x_2$  and  $x = x_3$  for calculating the temperature history at the location  $x = x_1$ . The calculated

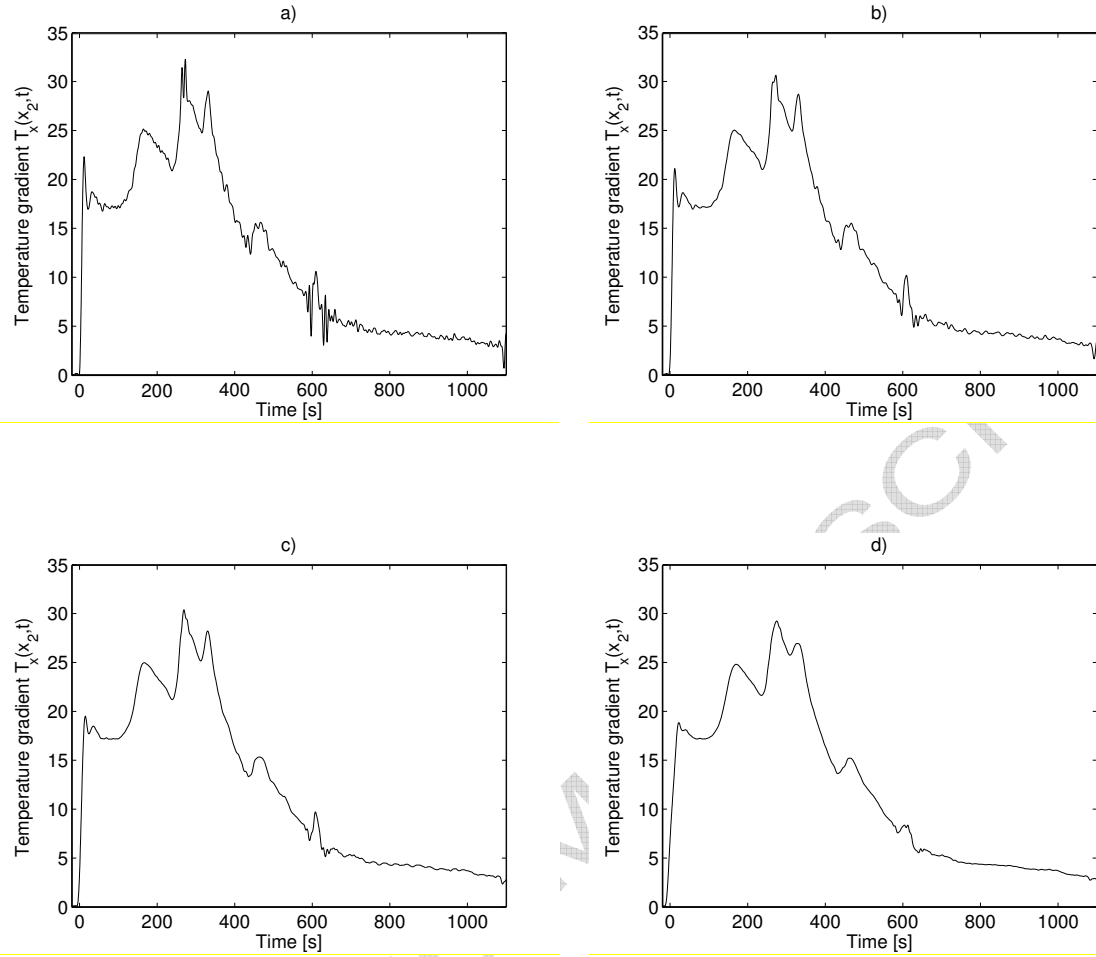
temperature at point  $x_1$  is shown, together with the actual measured temperature at this point, in Figure 6. The cut-off level was  $\xi_c = 100$  and a data averaging of 20 points was used. These values are justified below. The average difference between the measured and the calculated value was  $6.8^\circ\text{C}$ . Since the temperature ranged from  $25 - 1250^\circ\text{C}$  we conclude that this is a very good result.



**Figure 6.** The calculated temperature (solid) and the measured temperature history (dashed) at  $x_1 = 5\text{ mm}$  below the surface.

Although the inverse solution method contains regularization, some denoising smoothing of the data is necessary for the initial calculation of the heat flux to provide sufficiently accurate results. The heat flux data calculated at the thermocouple at  $x = x_2$  is used as input data for the inverse method. The degree of smoothing of the data will affect the calculations. In Figure 7 we demonstrate how the calculated temperature gradients at

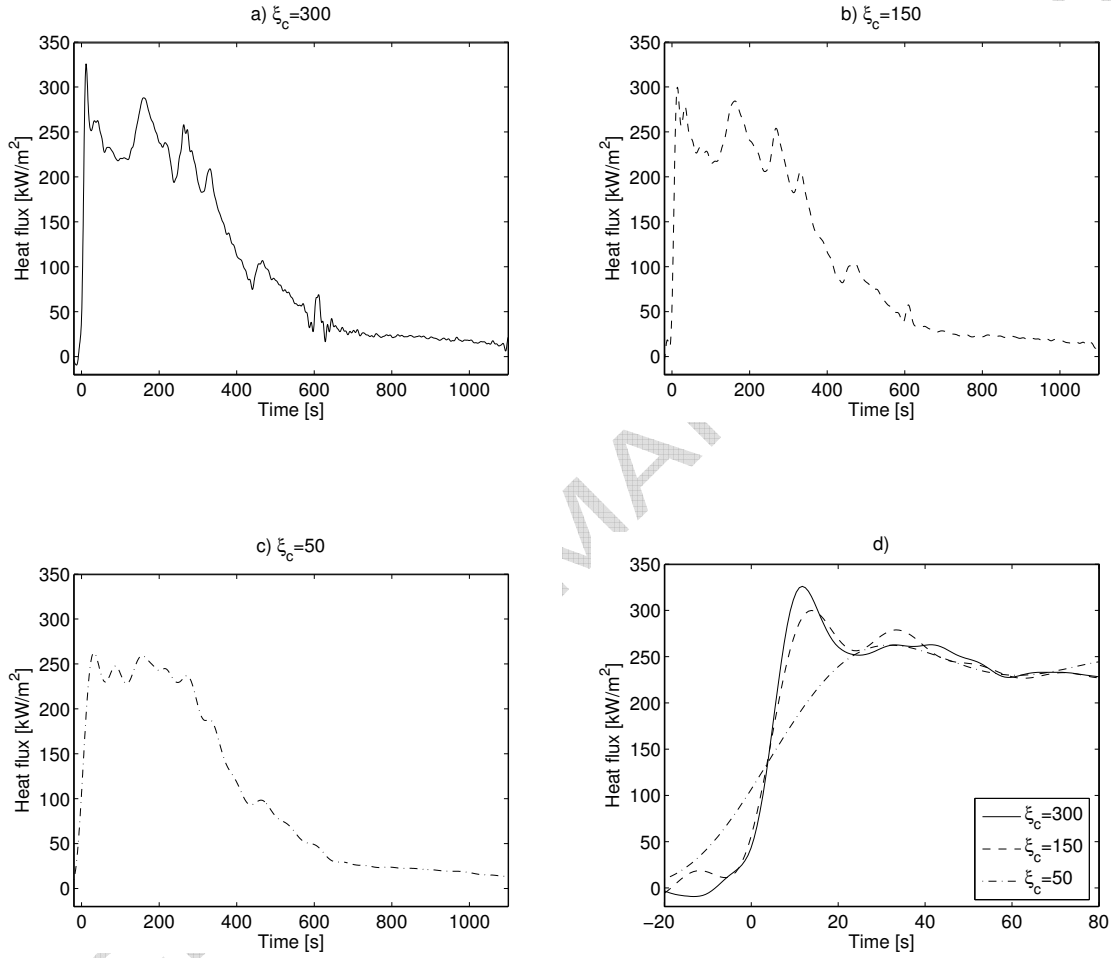
$x = x_2$  is affected by the degree of smoothing being used. In our computations the smoothing is performed by computing a moving average of data points. Note that in Figure 7, we use the more compact notation  $T_x$  instead of  $\partial T / \partial x$ . The temperature gradient is calculated for four cases: no data averaging, averaging of 10 points, 20 points and 40 points, respectively.



**Figure 7.** The effects of the data averaging are demonstrated. We display the calculated heat fluxes at  $x = x_2$ . No smoothing of the data results in a curve with sharp edges **a)**. Using 10 point averaging gives an improvement **b)**. If the 20 points **c)** or 40 points **d)** of data averaging are used the resulting curves are rather smooth.

As can be seen from Figure 7 the curves are improved by increasing the averaging data from zero to 40 points. A value of 20 should be sufficient.

The degree of regularization used when solving the inverse problem is controlled by the *cut-off frequency*,  $\xi_c$ , being used. This parameter has a significant influence on the computational results. Theoretically, a too high cut-off level will result in a solution in which a significant part of the high-frequency noise remains, making the resulting curves very cusp, and unphysical. If, on the other hand, a low cut-off frequency is used then relevant parts of the data are removed along with the noise, with loss of actual information, and resulting a solution that is very smooth, but false. We now investigate this dependence in more detail. We calculate the heat flux at  $x_1$  using different cut-off frequencies  $\xi_c$  and using different levels of data smoothing. The cut-off levels used ranges from as high as 300 to as low as 50. The results obtained without any data averaging is presented in Figure 8.



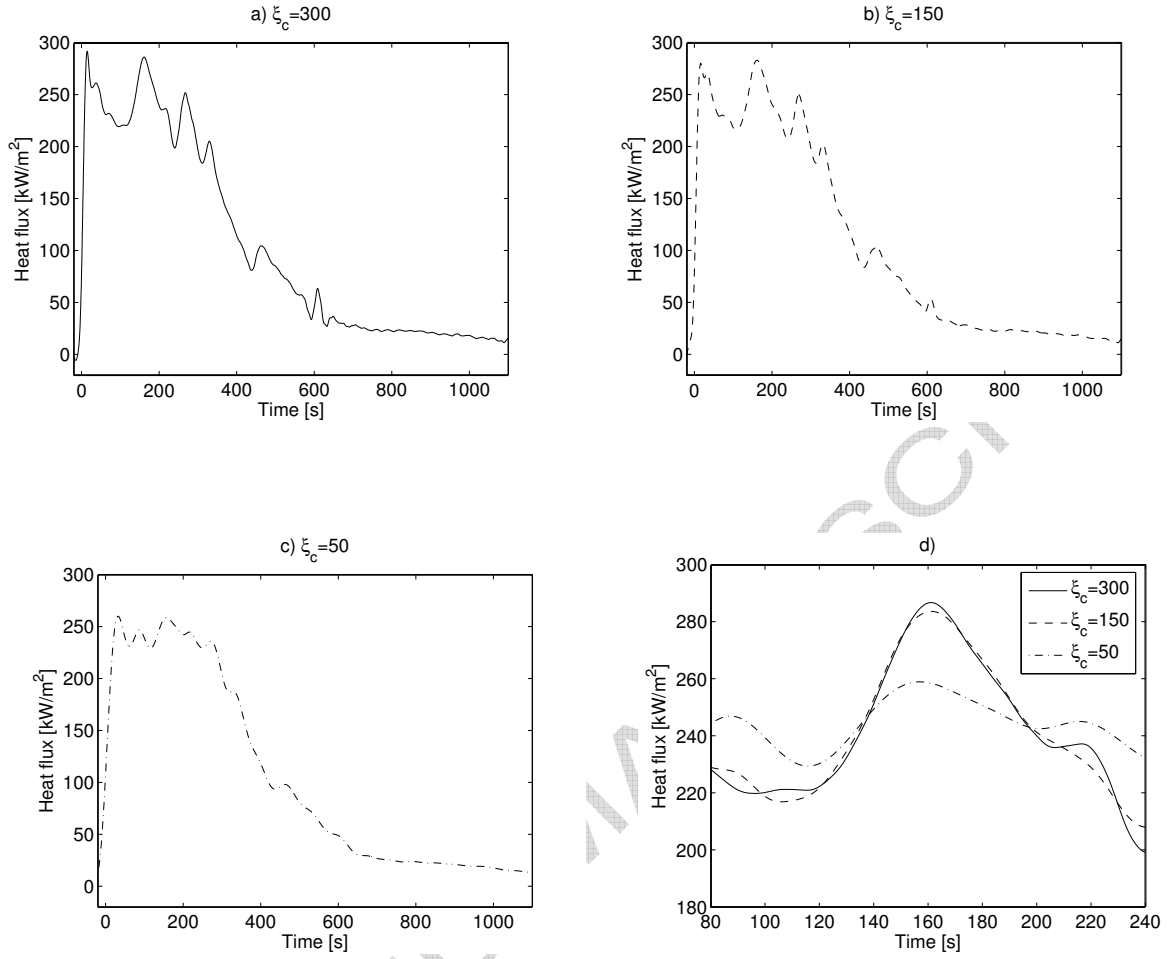
**Figure 8.** **a-c)** The calculated heat fluxes at  $x = x_1$  with no data smoothing and for cut-off frequencies 300, 150 and 50. **d)** The calculated heat fluxes for the three cut-off frequencies are shown in the interval  $-20 < t < 80$  s.

The calculated curves, in particular Figure 8a with  $\xi_c = 300$ , display some sharp, and unphysical features. Note in particular the very sharp “over-shots” close to  $t = 0$  s, these are the typical Gibbs-phenomena you expect when trying to approximate a function that has a discontinuity by a truncated Fourier series. This is the moment when the heating process starts and hence we expect the heat flux to behave almost as a step-function at this point. In Figure 8d we see how this phenomenon depends on the cut-off frequency in more detail. The peaks at  $t = 0$  s shown in Figures 8a and b are more similar than that in Figure 8c and this behavior is underlined in Figure 8d.

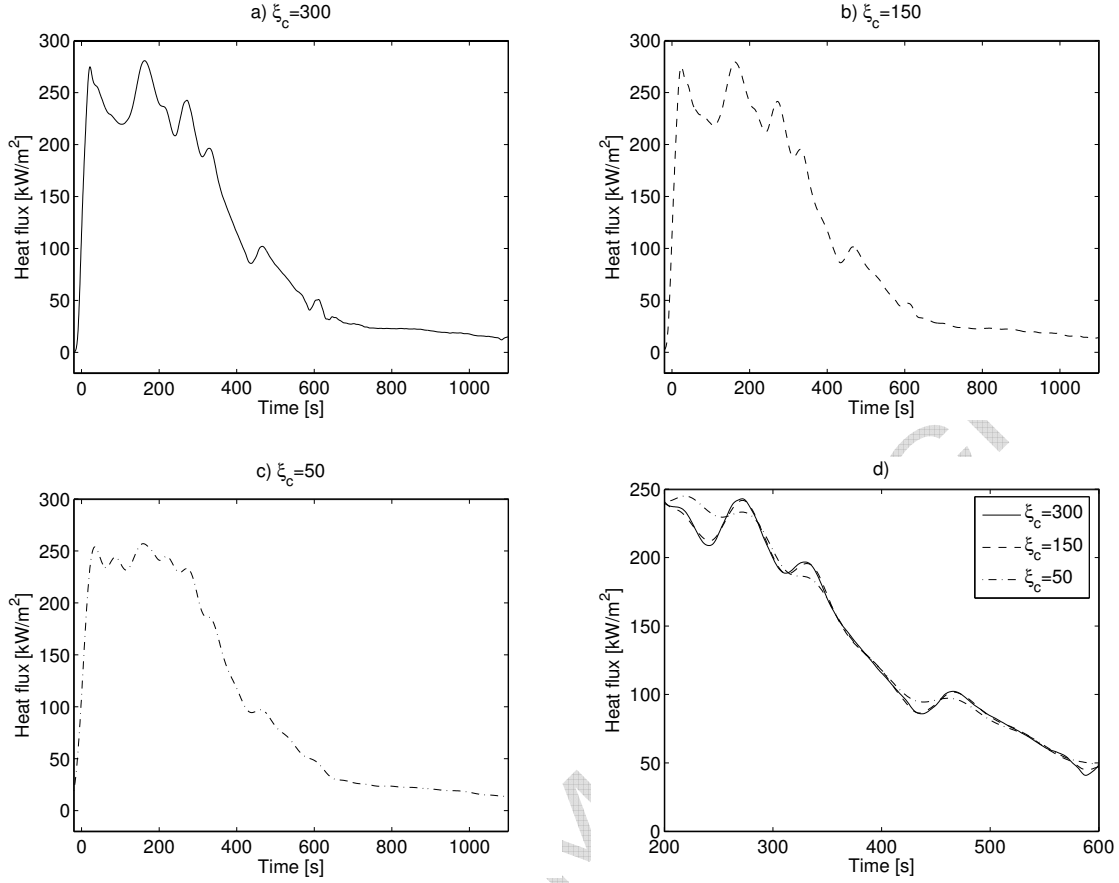
In Figure 9 we again display different computed heat flux curves this time using 20 points of data averaging. In Figure 9d, we conclude that a value of  $\xi_c = 50$  deviates clearly from the shapes of the higher cut-off frequencies. The curve corresponding to  $\xi_c = 150$  is more similar to that of  $\xi_c = 300$ .

Curves computed from 40 points of data averaging are displayed in Figure 10. Figure 10d shows that the discrepancy between the cut-off frequencies being used is not as highly accentuated in the later part of the time interval as is in Figures 8d and 9d, which is due to the transient heating reaching steady state.





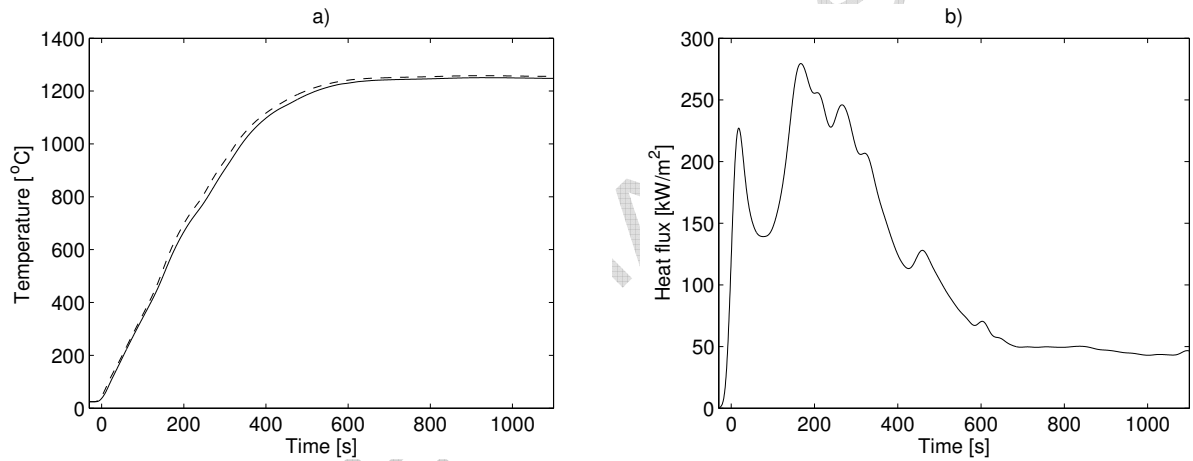
**Figure 9.** a-c) The calculated heat fluxes at  $x = x_1$  for 20 data points of averaging and for cut-off frequencies 300,150 and 50 . d) The calculated heat fluxes for the three cut-off frequencies are shown in the interval  $80 < t < 240$  s.



**Figure 10.** a-c) The calculated heat fluxes at  $x = x_1$  for 40 data points of averaging and for cut-off frequencies 300,150 and 50. d) The calculated heat fluxes for the three cut-off frequencies are shown in the interval  $200 < t < 600$  s.

From these results we conclude that an appropriate *cut-off* level would be around  $100 < \xi_c < 300$ . Further investigations show that a value of  $\xi_c = 100$  seems sufficient to capture the essential behavior of the actual heat flux curve, and without the solution being too cluttered by the noise. It is also seen that data smoothing is important. Averaging over 20 points in the data vectors seems to be sufficient.

Our original purpose was to calculate the surface temperature of the slab, i.e.  $T(0,t)$ . The calculations are the same as the ones described for calculating  $T(x_1,t)$  only this time we use the thermocouples at  $x_1$  and  $x_2$  to calculate the heat flux needed as boundary condition. In Figure 11a, the estimated surface temperature and the measured data at the point  $x_1$  are shown and the corresponding surface heat flux is shown in 11b. A cut-off frequency of 100 and an averaging of 20 data points were used.



**Figure 11.** **a)** The calculated surface temperature (dashed) and the measured temperature history (solid) 5 mm below the surface are shown. **b)** the calculated heat flux at the surface ( $x = 0$ ).

## 6. Influence of measurement errors

We note that the analysis in this section is carried out for the case of constant coefficients. Though for the cases we are interested in, e.g. piecewise smooth coefficients, the more general case behave the same way. However, since for this case the

solution cannot be written explicitly the analysis would be more involved and beyond the scope of this paper. A rough estimate of the error in the calculations of the surface temperature can be obtained by studying a simplified case where an explicit solution can be found. Therefore, we consider the case when the differential equation is valid in the quarter plane. That is,

$$\begin{cases} \alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}, & 0 < x < \infty, t > 0 \\ T(1, t) = f_m(t), & t > 0, \\ T(x, 0) = 0, & 0 < x < \infty, \\ T(x, t) \text{ is bounded as } x \rightarrow \infty. \end{cases} \quad (6)$$

For this simplified calculation we use constant values for the material properties, i.e., for the thermal diffusivity  $\alpha = \lambda / (\rho \cdot c)$ . The thermal conductivity is assumed to be  $\lambda = 40 \text{ W/(m} \cdot \text{K)}$ , the density  $\rho = 7500 \text{ kg/m}^3$  and the specific heat capacity  $c = 680 \text{ J/(kg} \cdot \text{K)}$ . For a non-dimensional model problem with the measurement device located at  $x=1$ , the solution operator can be written as [3],

$$T_c(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\xi_c}^{\xi_c} e^{i\xi t} e^{(1-x)\sqrt{i\xi/\alpha}} \hat{f}_m(\xi) d\xi. \quad (7)$$

For our calculation we re-define the thermal diffusivity to incorporate the existing time- and length scale,

$$\alpha' = \frac{\lambda \cdot T0}{(\rho \cdot c \cdot L_1^2)}, \quad (8)$$

where  $L_1 = 5 \text{ mm}$  is the distance between the thermocouple and the surface and  $T0 = 1432 \text{ s}$ , is the duration of the experiment. Using the explicit solution we see that individual frequency components are magnified as,

$$|E| = |A| \exp(\sqrt{(i \cdot \xi_c / \alpha')}) = |A| \exp(\sqrt{(\xi_c / \alpha')}) \approx 1.6 \cdot |A|, \quad (9)$$

where  $A$  is a constant depending on the accuracy of the measurement device. In the heating experiments, a cut-off frequency of  $\xi_c = 100$  was used and the uncertainty of the thermocouples is estimated to be around  $5^\circ \text{C}$ .

This calculation shows that our problem is not very unstable numerically. Any random measurement error in a frequency component is at most magnified by a factor 1.6. Hence we do not expect the ill-posedness of the problem to manifest itself in our computations.

Note that although the measurement errors are estimated to be around  $5^\circ \text{C}$  it would be too pessimistic to assume that all the measurement errors are localized to the high frequency components of the data vectors. Also the initial data filtering would effectively remove any such errors.

Hence the above calculation is to be understood as stating that numerical errors during the computations, or magnification of random errors in the measured data, do not account for the main part of the error in our results. Thus, most of the errors are assumed to be because of systematic errors introduced into the mathematical model, i.e. inaccuracy in the positioning of the thermocouples, and errors in the values used for the material properties.

## 7. Discussion and concluding remarks

In this paper we have investigated the possibility of calculating the transient surface temperature of a slab, by using interior measurements.

The assumption of one-dimensional heat conduction is a fairly good assumption apart from near the edges of the slab where end effects may occur. The problem of end effects is also of importance to the Steel industry to study homogeneity of the heating on the slabs. However, this problem is not addressed in this paper. There exists models in 3D based on calculations using FEM, FDM, FVM etc., but those models are often quite simplified since they rely on many parameters that are usually unknown or measured with low accuracy. Also, the problem of oxide scale is often omitted in those models as described in the introductory of this paper. Such calculations are performed by using direct methods, not inverse calculations. Using a 3D inverse model would in most cases be unpractical. Too many temperature sensors would be needed and could damage the

material properties of the slab and would be quite expensive. The accuracy gained would probably be better by repeating the 1D approach at selected places.

The measurements inside the slab are by nature noisy and may introduce errors together with the diffusive nature of heat conduction. A rough estimate indicates that random measurement errors can be dealt with effectively by the regularization inherent in the inverse solution method.

The data vectors were sampled at only  $0.25\text{Hz}$  resulting in data vectors of length 358, while the numerical calculations were carried out on a much finer grid. The initial data-smoothing, as well as the solution of the inverse problem works better with more data available. With a higher sample rate an averaging filter would more effectively remove the random noise from the data, and also the need for the initial resampling. For future experiments a much higher sampling rate will be used.

The thermocouple TC1 is located only  $L_1 = 5\text{ mm}$  below the surface. In a 1-D Steady state calculation, with constant coefficients, an error in the distance  $L_1$  translates linearly into an error in the calculated temperature at the surface, and has no effect on the computed heat flux, since in the steady state model the heat flux would be a constant. Although the time-dependent case is more complicated, the same general behavior can be expected. Calculated surface temperatures should be more sensitive with respect to errors

in the location of the thermocouples, while the calculated heat fluxes should be less sensitive.

Note that in the 1-D model mentioned above, the heat flux at the thermocouple TC1 would be calculated using the measured temperatures at the thermocouples TC1 and TC2 respectively, and using the distance  $L_2 = 6\text{ mm}$  between these two thermocouples. An error in the assumed distance  $L_2$  would lead to the wrong heat flux being calculated. There would be a linear dependence between an error in  $L_2$  and the calculated heat flux. Again the 1-D steady state model is not an accurate description of our experiment. However, the low frequency components of the full transient model behave essentially as a 1-D steady state solution. Hence this reasoning gives an indication about how the errors in our calculations depend on errors in the placement of the thermocouples.

The results obtained for the calculated temperature versus the measured are very good and this shows that the method can be successfully applied to thermal applications for wide range of temperatures.



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## References

- [1]. J.V.Beck, B.Blackwell, *Handbook of Numerical Heat Transfer*, Wiley, New York, 1988, pp. 787-834.
- [2]. C.E. Meija, D. A. Murio, *Numerical Solution of Generalized IHCP by Discrete Mollification*, Computers Math. Appl., (1996), 32:33-50.
- [3]. F. Berntsson. *A spectral method for solving the sideways heat equation*, Inverse Problems, (1999), 15:891-906.
- [4]. P.Lamm, *Approximation of ill-posed Volterra problems via predictor-corrector regularization methods*. SIAM J. Appl. Math., (1996), 56(2):524-54.
- [5]. P. Lamm and L. Elden, *Numerical Solution of First-Kind Volterra Equations by Sequential Tikhonov Regularization*, SIAM J. Numer. Anal., (1997), 34:1432-1450.

- [6]. Lars Elden *et.al.* *Wavelet and Fourier methods for solving the sideways heat equation*, SIAM J. Sci. Comput., (2000), 21(6):2187-2205.
- [7]. F.Berntsson, L.Elden, *An Inverse Heat Conduction Problem and an Application to Heat Treatment of Aluminium*, in: M.Tanaka, G.S. Dulikravich (Ed.), *Inverse Problems in Engineering Mechanics II. International Symposium on Inverse Problem Engineering Mechanics 2000 (ISIP 2000)*, Elsevier, Nagano, Japan, (2000), pp. 99-106.
- [8]. M. Honner *et.al.* *Temperature and heat transfer measurement in continuous reheating furnaces*, Scandinavian Journal of Metallurgy (2003), 32:225-232.
- [9]. H.Engl, M.Hanke, A.Neubauer, *Regularization of Inverse Problems*, , Kluwer Academic Publishers, Netherlands, 1996.