What can You Verify and Enforce at Runtime?

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Abstract

The underlying property, its definition and representation play a major role when monitoring a system. Having a suitable and convenient framework to express properties is thus a concern for runtime analysis. It is desirable to delineate in this framework the spaces of properties for which runtime analysis approaches can be applied to. This paper presents a unified view of runtime verification and enforcement of properties in the Safety-Progress classification. Firstly, we extend the Safety-Progress classification of properties in a runtime context. Secondly, we characterize the set of properties which can be verified (monitorable properties) and enforced (enforceable properties) at runtime. We propose in particular an alternative definition of "property monitoring" to the one classically used in this context. Finally, for the delineated spaces of properties, we obtain specialized verification and enforcement monitors.

Keywords: runtime verfication, runtime enforcement, monitor, property, r-property, safety-progress, finitary property, infinitary property, synthesis, Streett, enforcement monitor

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1 Introduction

In the past decades, we have seen the emergence of a world in which information systems are ubiquitous. System dissemination entails a growing need of confidence. System failures in history showed limits of existing engineering methodologies and enabled the emergence of *formal methods* [CW96]. Ideally, one would like to validate a program prior to its execution. However, static validation methods sucvh as model-checking [EC80] suffer from limits preventing their use in real large-scale applications. For instance, those techniques are often bound to the design stage of a system and hence they are not shaped to face off specification evolution. Even when those techniques (*e.g.*,static analysis [CC92]) do scale well, they are limited by the properties they can check, and may not be able to check interesting behavioral properties. Thus, the verification of some properties, and elimination of some faults, have to be done *complementary* using methods relying on dynamic analysis. In this paper, we are interested in runtime verification and runtime enforcement. These methods, said to be incomplete, operate on *one* execution of the system. Acknowledging the loss of completness enables to face-off the limitations of static validation methods.

Runtime-verification [Run09, PZ06, BLS09, BLS07, HG08] is an effective technique to ensure at execution time that a system meets a desirable behavior. It can be used in numerous application domains, and more particularly when integrating together untrusted software components. A possible approach for runtime verification consists in analyzing a run of the system under scrutiny in an incremental way using a decision procedure called a *monitor*. This monitor may be generated from a user-provided high level specification (*e.g.*, a temporal property, an automaton). The primary goal of this monitor is to detect violation or validation with respect to the given specification. It can be viewed as a state machine (with an output function) processing an execution sequence (step by step) of the monitored program, and producing a sequence of verdicts (truth values taken from a truth-domain) indicating specification fulfillment or violation. The major part of research endeavor was done on the monitoring of safety properties (stating that *something bad can never happen*), as seen for example in [RCB08, HR02]. However, the authors of [dR05] show that safety properties are not the only monitorable properties. Recently, a new definition of monitorability was given by Pnueli in [PZ06] and it has been proven in [BLS07] that safety and co-safety properties represent only a proper subset of the space of the monitorable properties.

Runtime enforcement is an extension of runtime verification aiming to circumvent property violations. It was initiated by the work of Schneider [Sch00] on what has been called *security automata*. In this work the enforcement monitor watches the current execution sequence and halts the underlying program whenever it deviates from the desired property. Such security automata are able to enforce the class of safety properties [HMS06]. Later, Viswanathan [Vis00] noticed that the class of enforceable properties is impacted by the computational power of the enforcement monitor. As the enforcement mechanism can implement no more than computable functions, the enforceable properties are included in the decidable ones.

More recently, Ligatti *et al.* [LBW09] showed that it is possible to enforce at runtime more than safety properties. Using more powerful enforcement mechanisms called *edit-automata*, it is possible to enforce the larger class of *infinite renewal properties*. Within the classical safety-liveness dichotomy, the renewal class is a super set of the safety class which contains some liveness properties (but not all). More than simply halting an underlying program, edit-automata can also suppress (*i.e.*, freeze) and insert (frozen) actions in the current execution sequence.

Several tools have been proposed in this context, and in practice there is not always a clear distinction between runtime-verification and runtime-enforcement. For instance a verification monitor may execute an exception handler when detecting an error, hence modifying the initial program execution.

Motivations and contributions. According to the amount of works published and existing tools now available within the runtime-validation community, it appears that this technique progressed a lot in the last ten years and seems now mature enough to address concrete industrial challenges. However, some interesting questions remain about its expressiveness. More precisely, the main questions we consider in this work are the following: what are the classes of properties that can be handled at runtime, and is there a distinct answer for runtime verification and runtime enforcement? These questions are not original in themselves, but we propose here to address them within a unified framework: the Safety-Progress (SP) classification of properties [MP90, CMP92b]. The paper contributions are then the following:

1. to propose a suitable framework for specifying and reasoning about properties in a runtime context;
2. to integrate within this framework some existing expressiveness results related to runtime monitoring [PZ06, BLS09, BLS07], and to propose an alternative definition of the notion of *monitorability*, leveraging the semantics of finite execution sequences;
3. and to improve some recent results related to property enforcement [FFM08, FFM09a], giving a more accurate classification of enforceable properties.

Let us illustrate a bit more the second motivation. Consider a system on which it is possible to evaluate two atomic propositions called p and q. At system runtime, system events are fed to a monitor. Each event is a pair containing the evaluations of p and q (either true or false). Now let us consider the following requirement: "Either p is always true or q is eventually true". This means that, for the observed sequence of events, either p is evaluated to true on every event, or there exists an event on which q is evaluated to true. Now consider the two following possible executions of the system, represented by their sequences of events of length 2:

- $\{p, \overline{q}\} \cdot \{p, \overline{q}\}$: on both events p is true, q is false;
- $\{p, \overline{q}\} \cdot \{\overline{p}, \overline{q}\}$: on the first event p is true and q is false, on the second event p and q are false.

After observation of the first sequence of events, one can reasonably state that the property is "currently" true. Thus, if the program execution stops after this observation, the requirement is satisfied. Indeed, p has been always true during the program execution. Conversely, after observing the second sequence of events, one can reasonably state that the property is "currently" false. Indeed, the last observed event does not fulfill the requirement (neither p nor q evaluate to true).

We will see in Section 5 that this kind of property is monitorable according to the classical definition of monitorability. Moreover, a monitor built following this definition of monitorability would produce the *same verdict* for those two sequences, namely a *don't know* verdict. This situation is undesirable from our point of view. Thus, we will propose an alternative definition of monitorability able to better cope with these kinds of properties, and to give more precise verdicts.

This paper is a revised and extended version of [FFM09b] which appeared in the 9^{th} international workshop on Runtime Verification. This new version brings the following additional contributions. First, it contains a more comprehensive theoretical basis by revisiting and extending results about the Safety-Progress classification of properties. Moreover, we provide additional results on monitorability. Furthermore, the synthesis of verification and enforcement monitors is given with full details (it was previously sketched). Finally, the presentation has been improved by means of additional examples, corrected results, and complete proofs.

Paper Organization. The remainder of this article is organized as follows. First, Section 2 introduces some preliminary notations used throughout this paper and Section 3 overviews related works on the issues we address. In Section 4, we propose an extension of the Safety-Progress classification of properties in a runtime verification context. Section 5 is dedicated to runtime monitoring, whereas Section 6 is dedicated to runtime enforcement. In both sections we provide some characterizations of the classes of properties that can be handled by these techniques, with respect to the Safety-Progress framework. Then, in Section 7, we show how to obtain runtime verification and enforcement monitors for the delineated sets of properties. Finally, we give some concluding remarks and future works in Section 8.

In order to facilitate article's reading, some of the proofs have been sketched. Complete proofs can be found in Appendix.

2 Preliminaries and notations

This section introduces some background, namely the notions of *program execution sequences* and *program properties*.

2.1 Sequences, and execution sequences

Sequences and execution sequences. Considering a finite set of elements E, we define notations about sequences of elements belonging to E. A sequence σ containing elements of E is formally defined by a total function $\sigma : I \to E$ where I is either the integer interval $[0, n]$ for some $n \in \mathbb{N}$, or $\mathbb{N}$ itself (the set of natural numbers). We denote by E^* the set of finite sequences over E (partial function from $\mathbb{N}$), by E^+ the set of non-empty finite sequences over E, and by E^ω the set of infinite sequences over E. The set $E^\infty = E^* \cup E^\omega$ is the set of all sequences over E. The empty sequence of E is denoted by ϵ_E or ϵ when clear from context. The length (number of elements) of a finite sequence σ is noted $|\sigma|$ and the $(i+1)$-th element of σ is denoted by σ_i. For two sequences $\sigma \in E^*, \sigma' \in E^\infty$, we denote by $\sigma \cdot \sigma'$ the concatenation of σ and σ', and by $\sigma \prec \sigma'$ the fact that σ is a strict prefix of σ' (resp. σ' is a strict suffix of σ). The sequence σ is said to be a strict prefix of $\sigma' \in E^\infty$ when $\forall i \in \{0, \ldots, |\sigma| - 1\}, \sigma_i = \sigma'_i$ and $|\sigma| < |\sigma'|$. When $\sigma' \in E^*$, we note $\sigma \preceq \sigma' \stackrel{\text{def}}{=} \sigma \prec \sigma' \vee \sigma = \sigma'$. For $\sigma \in E^\infty$ and $n \in \mathbb{N}$, $\sigma_{\ldots n}$ is the sub-sequence containing the $n+1$ first elements of σ. Also, when $|\sigma| > n$, the subsequence $\sigma_{n\ldots}$ is the sequence containing all elements of σ but the n first ones.The set of prefixes $pref(\sigma)$ of a sequence $\sigma \in E^\infty$ is defined as follows. If $\sigma \in E^*$, then $pref(\sigma) = \{\sigma' \in E^* \mid \sigma' \preceq \sigma\}$. If $\sigma \in E^\omega$, then $pref(\sigma) = \{\sigma' \in E^* \mid \sigma' \prec \sigma\}$. The set $Pref(X)$ of prefixes of a set of sequences X is the union of the sets of prefixes of X-sequences: $Pref(X) = \bigcup_{\sigma \in X} pref(\sigma)$. The σ-continuations, *i.e.*, the continuations of a sequence σ, are the finite and infinite sequences belonging to the set $\{\sigma' \in \Sigma^\infty \mid \sigma \prec \sigma'\}$.

A program $\mathcal{P}$ is considered as a generator of execution sequences. We are interested in a restricted set of operations the program can perform. These operations influence the truth value of properties the program is supposed to fulfill. Such execution sequences can be made of access events on a secure system to its resources, or kernel operations on an operating system. In a software context, these events may be abstractions of relevant instructions such as variable modifications or procedure calls. These events may also be fed from the underlying program and contain the evaluation of some propositions of the system under scrutiny. We abstract these operations by a finite set of *events*, namely a vocabulary Σ. We denote by $\mathcal{P}_\Sigma$ a program for which the vocabulary is Σ. The set of execution sequences of $\mathcal{P}_\Sigma$ is denoted by $Exec(\mathcal{P}_\Sigma) \subseteq \Sigma^\infty$. This set is *prefix-closed*, *i.e.*, $\forall \sigma \in Exec(\mathcal{P}_\Sigma), \forall \sigma' \in \Sigma^*, \sigma' \preceq \sigma \Rightarrow \sigma' \in Exec(\mathcal{P}_\Sigma)$. In the remainder of this article, we use a vocabulary Σ.

2.2 Properties

Properties as sets of execution sequences. A *finitary property* (resp. an *infinitary property*, a *property*) is a subset of execution sequences of Σ^* (resp. Σ^ω, Σ^∞). Considering a given finite (resp. infinite, finite or infinite) execution sequence σ and a property ϕ (resp. φ, θ), when $\sigma \in \phi$, noted $\phi(\sigma)$ (resp. $\sigma \in \varphi$, noted $\varphi(\sigma)$, $\sigma \in \theta$, noted $\theta(\sigma)$), we say that σ *satisfies* ϕ (resp. φ, θ). A consequence of this definition is that properties we will consider are restricted to *single* execution sequences[1], excluding specific properties defined on power-sets of execution sequences (like fairness, for instance).

Runtime properties. In this paper we will focus on properties to be evaluated at *runtime*. As stated in the introduction, this means that we would have to consider finite and infinite execution sequences (that a program may produce). A runtime verification technique should address both kinds of sequences in a uniform way. As so, we introduce a notion of "runtime property" (*r*-property) as a pair of finite/infinite execution sequence sets[2]:

[1] This is the distinction, made by Schneider [Sch00], between properties and (general) policies. The set of properties (defined over single execution sequences) is a subset of the set of policies (defined over sets of execution sequences).

[2] Using a pair of sets makes the distinction between the finitary and infinitary parts of the property more explicit.

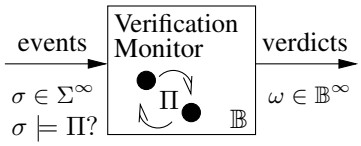


Figure 1: Principle of runtime verification

DEFINITION 2.1 (RUNTIME PROPERTIES) *A r-property is a pair $(\phi, \varphi) \subseteq \Sigma^* \times \Sigma^\omega$. The property ϕ is called the finitary part of the r-property, whereas φ is called the infinitary part of the r-property.*

Intuitively, the finitary property ϕ represents the desirable property that finite execution sequences should fulfill, whereas the infinitary property φ is the expected property for infinite execution sequences. Notations for *r*-properties follow from the notations for finitary and infinitary properties. For a *r*-property (ϕ, φ), its negation, noted $\overline{(\phi, \varphi)}$, is defined as $(\Sigma^* \setminus \phi, \Sigma^\omega \setminus \varphi)$. Boolean combinations of *r*-properties are defined in a natural way: $(\phi_1, \varphi_1) \vee (\phi_2, \varphi_2) = (\phi_1 \cup \phi_2, \varphi_1 \cup \varphi_2)$, and $(\phi_1, \varphi_1) \wedge (\phi_2, \varphi_2) = (\phi_1 \cap \phi_2, \varphi_1 \cap \varphi_2)$. Considering an execution sequence $\sigma \in \mathit{Exec}(\mathcal{P}_\Sigma)$, we say that σ satisfies (ϕ, φ) when $\sigma \in \Sigma^* \wedge \phi(\sigma) \vee \sigma \in \Sigma^\omega \wedge \varphi(\sigma)$. For a *r*-property $\Pi = (\phi, \varphi)$, we note $\Pi(\sigma)$ (resp. $\neg\Pi(\sigma)$) when σ satisfies (resp. does not satisfy) (ϕ, φ). The prefixes of a *r*-property $\Pi = (\phi, \varphi)$ is defined as: $\mathit{Pref}(\Pi) = \mathit{Pref}(\phi) \cup \mathit{Pref}(\varphi)$. Intersection between finitary, infinitary properties and *r*-properties is straightforward and denoted using operator $\sqcap$, *e.g.*, $\Sigma^* \sqcap (\phi, \varphi) = \phi$.

Evaluation of *r*-properties. Monitorability, enforceability, and monitor synthesis are based on the evaluation of *r*-properties by a monitor. Evaluating an execution sequence σ wrt. a *r*-property consists in producing a verdict regarding the current property-satisfaction of σ or future satisfactions of the possible σ-continuations. As a matter of facts, the verdicts produced by monitors are not necessarily usual boolean values: they are truth-values taken from a *truth-domain*. A truth-domain is a lattice, *i.e.*, a partially ordered set with an upper-bound and a lower-bound. Examples of truth-domains are the classical boolean domain $\{\mathit{true}, \mathit{false}\}$ or the real-number interval $[0, 1]$, or any relevant set of values used for evaluating properties. Considering a truth-domain $\mathbb{B}$, a *r*-property Π and a finite execution sequence σ, the evaluation of $\sigma \in \Sigma^*$ wrt. Π in $\mathbb{B}$, noted $[\![\Pi]\!]_\mathbb{B}(\sigma)$, is an element of $\mathbb{B}$ depending on $\Pi(\sigma)$ and satisfaction of σ-continuations wrt. Π.

The sets of monitorable and enforceable properties (Sections 5 and 6) rely both upon the truth-domain and evaluation function we consider.

3 Related Works

This section overviews some related works on the topics we will discuss in this paper. In particular we summarize the basic concepts used for runtime verification and runtime enforcement, and we recall the existing results in terms of set of properties that can be addressed by each of these techniques.

3.1 Runtime verification

Basic concepts. As stated in the introduction, the notion of runtime verification can be formalized by a *verification monitor* (see Fig. 1) whose behavior consists in translating a sequence of events $\sigma \in \Sigma^\infty$ into a sequence of verdicts $\omega \in \mathbb{B}^\infty$, where $\mathbb{B}$ is a given truth domain. This monitor is defined with respect to a *r*-property Π, and the sequence of verdicts ω is expected to give some information on the evaluation of Π on σ with respect to $\mathbb{B}$. Thus, one of the problems to be addressed is that each evaluation $[\![\Pi]\!]_\mathbb{B}(\sigma_{\cdots n}) = \omega_{\cdots n}$ of a *finite* sequence should not only give some relevant information on $\Pi(\sigma_{\cdots n})$, but also possibly on $\Pi(\sigma)$. In this context several notions of *monitorability* were proposed in the past, and we review below the most important results.

Monitorability in the sense of [VK04]. The first characterization of monitorable properties was given by Viswanathan and Kim in [VK04]. Monitorable properties were characterized as a strict subset of safety properties. The authors show that, due to the indecidability of some problems, a verification monitor is limited by some computability constraints. Monitorable properties are precisely defined as the safety decidable properties[3]. The authors establish the equality between this space of properties and the class Π_1^0 of the arithmetical hierarchy which is the class of co-recursively enumerable properties.

Monitorability in the sense of [PZ06]: Pnueli *et al.* give a more general notion of monitorable properties relying on the notion of verdict determinacy for an *infinite* sequence. More precisely, considering a finite sequence $\sigma \in \Sigma^*$, a property $\theta \subseteq \Sigma^\infty$ is negatively determined (resp. positively determined) by an execution sequence σ if σ and *each of its possible extension* does not satisfy (resp. does satisfy) θ. Then, θ is σ-monitorable if σ has an extension *s.t.* θ is negatively or positively determined by this extension. Finally, θ is monitorable, if it is σ-monitorable for every $\sigma \in \Sigma^*$. The idea is that it becomes unnecessary to continue the execution of a θ-monitor after reading σ if θ is not σ-monitorable.

In Section 5, we give the corresponding formal definition in the context of *r*-properties.

Monitorability in the sense of [BLS07]: Bauer *et al.*, inspired from Pnueli's definition of monitorable properties, proposed a slightly different one based on the notion of good and bad prefix introduced in model-checking [KYV01] by Kupferman and Vardi. The intuitive idea is that with monitorable properties it is possible to "detect" a violation or validation of infinitary properties with finite sequences. More precisely, the definition of monitorable properties comes in the following way. Considering an infinitary property $\varphi \subseteq \Sigma^\omega$, a prefix σ is said to be a bad prefix, noted $bad\_prefix(\sigma, \varphi)$ (resp. good prefix, noted $good\_prefix(\sigma, \varphi)$) of φ if $\forall w \in \Sigma^\omega, \neg\varphi(\sigma \cdot w)$ (resp. $\forall w \in \Sigma^\omega, \varphi(\sigma \cdot w)$). Then, a prefix σ is said to be ugly if it does not have good nor bad continuation, *i.e.*, $\nexists v \in \Sigma^\omega, bad\_prefix(\sigma \cdot v, \varphi) \vee good\_prefix(\sigma \cdot v, \varphi)$. Finally, a property is said to be monitorable if it has no ugly prefix, formally: $\forall \sigma \in \Sigma^*, \exists v \in \Sigma^*, bad\_prefix(\sigma \cdot v, \varphi) \vee good\_prefix(\sigma \cdot v, \varphi)$.

About previous characterizations of monitorable properties: The first characterization of monitorable properties given in [VK04] may seem arbitrary. It characterizes the space of monitorable properties directly as a class of properties. This characterization is not related to the practical constraints that a monitor has to face off. However, let us remark that, the idea behind this definition is that a monitor is dedicated to the detection of "bad behaviors" from a finite observation. It seems reasonable that a verification monitor is used to detect "good" behaviors as well, *e.g.*, the satisfaction of a desired property. This is actually the idea behind the definition given in [PZ06]. The last definition, given in [BLS07], is equivalent to the previous one on Σ^ω. We will refer to the definition given in [PZ06] as the *classical definition* as it has been enunciated firstly. Furthermore, Bauer *et al.* have shown that, according to this definition, the set of monitorable properties is a strict super set of safety and co-safety properties. These classes of properties are taken from the classical safety-liveness classification of properties [Lam77, AS84]. They also gave an example of request/acknowledge property which is not monitorable. Such a property can be framed in the set of response properties (see Section 4) wrt. the SP classification (see Example 5.2 in Section 5).

3.2 Runtime enforcement

Basic concepts. Regarding *runtime enforcement*, the purpose of an *enforcement monitor* (see Fig. 2) used at runtime is to transform an input sequence $\sigma \in \Sigma^\infty$ into an output sequence $o \in \Sigma^\infty$ with respect to a *r*-property Π. The expected constraints on o are (usually) the following:

soundness: o should be a *correct* execution sequence, i.e., Π should evaluate to true on o ;

transparency: the enforcement operation should preserve as much as possible the initial program behavior by modifying the input sequence *in a minimal way*. A possible interpretation is that when σ does not satisfy Π then o should be the *longest correct prefix* of σ.

[3] Roughly speaking, a non-decidable safety property is a safety property for which the test used to decide whether a given sequence belongs to the property is not computable. See [VK04] for more details.

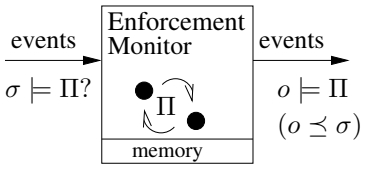


Figure 2: Principle of runtime enforcement

According to this definition, the set of properties that can be enforced at runtime clearly depends on the capabilities of the enforcement mechanism. To this purpose, the authors of [HMS06] proposed a very general classification of enforceable properties: a program is viewed as a Turing machine and the enforcement mechanisms they considered were based respectively on static analysis, program rewriting and runtime enforcement monitors. Other works [Sch00, Vis00, LBW09, LBW05, FFM09a] focused more specifically on runtime enforcement monitors and proposed a characterization of enforceable properties in this context. We summarize these results below.

Security automata and decidable safety properties: Schneider introduced security automata (a variant of Büchi automata) as the first runtime mechanism for enforcing properties in [Sch00]. The set of enforceable properties with this kind of security automata is the set of safety properties. Then [HMS06] Schneider, Hamlen, and Morrisett refined the set of enforceable properties and showed that these security automata were in fact restrained by some computational limits. Indeed, Viswanathan [Vis00] noticed that the class of enforceable properties is impacted by the computational power of the enforcement monitor. As the enforcement mechanism can implement no more than computable functions, the enforceable properties are included in the decidable ones. Hence, they showed in [HMS06] that the set of safety properties is a strict upper limit of the power of (execution) enforcement monitors defined as security automata.

Edit-automata and infinite renewal properties: Ligatti *et al.* [LBW09, LBW05] introduced *edit-automata* as runtime monitors. Depending on the current input and its control state, an edit-automaton can either insert a new action by replacing the current input, or suppress it. The properties enforced by edit-automata are called *infinite renewal* properties: it is a superset of safety properties and contains some liveness properties (but not all). More precisely, a property θ is said to be an infinite renewal property iff each valid infinite sequence σ has an infinite number of valid prefixes:
$\forall \sigma \in \Sigma^{\infty}, \theta(\sigma) \Rightarrow$
$\quad \forall \sigma' \in \Sigma^{*}, \sigma' \prec \sigma \Rightarrow \exists \sigma'' \in \Sigma^{*}, \sigma' \preceq \sigma'' \prec \sigma \wedge \theta(\sigma'')$.

Shallow History Automata and an information-based lattice of enforceable policies. Fong [Fon04] studied the effect of restraining the capacity of the runtime execution monitor and the effect on the enforcement power. Shallow History Automata (SHA) keep as history a set of access events the underlying program made. Fong showed that these automata can enforce a set of properties strictly contained in the set of properties enforceable by Schneider's automata. The result has been generalized by using abstraction mechanisms on an equivalent variant of Schneider's automata. It raised up an information-based lattice of enforceable policies. At the top of this lattice is the set of properties enforceable by security automata (SHA keeps history of all events). At the bottom of this lattice is the set of policies prohibiting a set of events (SHA do not distinguish between prefixes of execution sequences made of the same events).

Fong's classification has a practical interest in the sense that it studies the effect of practical programming constraint (limited memory). It also shows that some classical security policies remain enforceable using such shallow automata.

Generic runtime enforcers and response properties: In [FFM09a, FMFR10] we introduced a generic notion of enforcement monitor encompassing previous mechanisms and gave a lower-bound on the space of properties they can enforce in the Safety-Progress classification (see Section 4). In this paper, we will

show that this bound is tight. Furthermore, in [FMFR10], we have studied the question of enforcement monitor composition.

3.3 Synthesis of monitors

For runtime verification: Generally, runtime verification monitors are generated from LTL-based specifications, as seen recently in [BLS07, CR07]. Alternatively, ω-regular expressions have been used as a basis for generating monitors, as for example in [dR05]. An exhaustive list of works on monitor synthesis is far beyond the scope of this paper. We refer to [Run09, LS08, HG08] for more information on this topic.

For runtime enforcement: In [MM07] Martinelli and Matteucci tackle the synthesis of enforcement mechanisms as defined by Ligatti. More generally the authors consider security automata and edit-automata. The monitor is modeled by an algebraic operator expressed in CCS. The program under scrutiny is then a term $Y \rhd_K X$ where X is the target program, Y the controller program and $\rhd_K$ the operator modeling the monitor where K is the kind of monitor (truncation, insertion, suppression or edit). The desired property for the underlying system is formalized using μ-calculus. In [Mat07] Matteucci extends the approach in the context of real-time systems. In [FFM08, FMFR10] we defined transformations for some classes of the safety-progress classification of properties. Those class-specific transformations take as input a Streett automaton recognizing a property and produce an enforcement monitor for this property. In this paper, we will provide a unified class-independant transformation.

4 The SP classification in a runtime context

This section recalls and extends some results about the Safety-Progress [CMP92a, CMP92b, MP90] classification of properties. In the original papers this classification introduced a hierarchy between *regular* properties[4] defined as sets of *infinite* execution sequences. We extend the classification to deal with finite-length execution sequences. As so we revisit this classification for *r*-properties.

4.1 Informal description

The Safety-Progress classification is made of four basic classes over execution sequences. Informally, the classes were defined as follows:

- *safety* properties are the properties for which whenever a sequence satisfies a property, *all its prefixes* satisfy this property.
- *guarantee* properties are the properties for which whenever a sequence satisfies a property, *there are some prefixes* (at least one) satisfying this property.
- *response* properties are the properties for which whenever a sequence satisfies a property, *an infinite number of its prefixes* satisfy this property.
- *persistence* properties are the properties for which whenever a sequence satisfies a property, *all but finitely many* of its prefixes satisfy this property, *i.e.*, a finite number of its prefixes does not satisfy the property.

Furthermore, two extra classes can be defined as finite boolean combinations (union and intersection) of basic classes.

- The *obligation class* can be defined as the class obtained by boolean combinations of safety and guarantee properties.
- The *reactivity class* can be defined as the class obtained by boolean combinations of response and persistence properties. This is the more general class containing all linear temporal properties [CMP92a].

[4] In the following, the term property will stand for regular property.

A *r*-property of a given class is said to be *pure* when it is a property of none of the others sub-classes.

The Safety-Progress classification is an alternative to the more classical safety-liveness [Lam77, AS84] dichotomy. Unlike this later, the Safety-Progress classification is a hierarchy and not a partition. It provides a finer-grain classification, and the properties of each class are characterized according to four *views* [CMP92a]: a language-theoretic view, a topological view, a temporal logic view, and an automata-based view. The language-theoretic view describes the hierarchy according to the way each class can be constructed from sets of finite sequences. The topological view characterizes the classes as sets with topological properties. The third vision links the classes to their expression in temporal logic. At last, the automata-base view gives syntactic characterization on automata recognizing properties of a given class. We will consider here only the language-theoretic and the automata views dedicated to *r*-properties.

4.2 The language-theoretic view of *r*-properties

4.2.1 Construction of *r*-properties

The language-theoretic view of the Safety-Progress classification is based on the construction of infinitary properties and finitary properties from finitary ones. It relies on the use of four operators A, E, R, P (building infinitary properties) and four operators A_f, E_f, R_f, P_f (building finitary properties) applying to finitary properties. In the original classification of Mana and Pnueli, operators A, E, R, P, A_f, E_f were introduced. In this paper, we add operators R_f and P_f and give a formal definition of all operators. In these definitions ψ is a finitary property over Σ.

DEFINITION 4.1 (OPERATORS A, E, R, P) *The operators are defined as follows:*

- $A(\psi) = \{\sigma \in \Sigma^\omega \mid \forall \sigma' \in \Sigma^*, \sigma' \prec \sigma \Rightarrow \psi(\sigma')\}$.
- $E(\psi) = \{\sigma \in \Sigma^\omega \mid \exists \sigma' \in \Sigma^*, \sigma' \prec \sigma \wedge \psi(\sigma')\}$.
- $R(\psi) = \{\sigma \in \Sigma^\omega \mid \forall \sigma' \in \Sigma^*, \exists \sigma'' \in \Sigma^*, \sigma' \prec \sigma'' \prec \sigma \wedge \psi(\sigma'')\}$.
- $P(\psi) = \{\sigma \in \Sigma^\omega \mid \exists \sigma' \in \Sigma^*, \forall \sigma'' \in \Sigma^*, \sigma' \prec \sigma'' \prec \sigma \Rightarrow \psi(\sigma'')\}$.

$A(\psi)$ consists of all infinite words σ s.t. *all* prefixes of σ belong to ψ. $E(\psi)$ consists of all infinite words σ s.t. *some* prefixes of σ belong to ψ. $R(\psi)$ consists of all infinite words σ s.t. *infinitely many* prefixes of σ belong to ψ. $P(\psi)$ consists of all infinite words σ s.t. *all but finitely many* prefixes of σ belong to ψ.

Operators A_f, E_f, R_f, P_f build finitary properties from finitary ones.

DEFINITION 4.2 (OPERATORS A_f, E_f, R_f, P_f) *The operators are defined as follows:*

- $A_f(\psi) = \{\sigma \in \Sigma^* \mid \forall \sigma' \in \Sigma^*, \sigma' \preceq \sigma \Rightarrow \psi(\sigma')\}$.
- $E_f(\psi) = \{\sigma \in \Sigma^* \mid \exists \sigma' \in \Sigma^*, \sigma' \preceq \sigma \wedge \psi(\sigma')\}$.
- $R_f(\psi) = \{\sigma \in \Sigma^* \mid \psi(\sigma) \wedge \forall n \in \mathbb{N}, \exists \sigma' \in \Sigma^*, n \leq |\sigma| \Rightarrow \sigma \prec \sigma' \wedge |\sigma'| \geq n \wedge \psi(\sigma')\}$.
- $P_f(\psi) = \{\sigma \in \Sigma^* \mid \psi(\sigma) \wedge \exists \sigma' \in \Sigma^*, \sigma \preceq \sigma' \wedge \forall n \in \mathbb{N}, \exists \sigma'' \in \Sigma^*, |\sigma''| = n \wedge \psi(\sigma' \cdot \sigma'')\}$.

$A_f(\psi)$ consists of all finite words σ s.t. *all* prefixes of σ belong to ψ. One can observe that $A_f(\psi)$ is the largest prefix-closed subset of ψ. $E_f(\psi)$ consists of all finite words σ s.t. *some* prefixes of σ belong to ψ. One can observe that $E_f(\psi) = \psi \cdot \Sigma^*$. $R_f(\psi)$ consists of all finite words σ s.t. $\psi(\sigma)$ and there exists an infinite number of continuations σ' of σ also belonging to ψ. $P_f(\psi)$ consists of all finite words σ belonging to ψ s.t. there exists a continuation σ' of σ s.t. σ' persistently has extensions σ'' staying in ψ (*i.e.*, $\sigma' \cdot \sigma''$ belongs to ψ).

Based on these operators, each class can be seen from the language-theoretic view.

DEFINITION 4.3 *A r-property* $\Pi = (\phi, \varphi)$ *is defined to be*

- *A* safety *r-property if* $\Pi = (A_f(\psi), A(\psi))$ *for some finitary property* ψ*. That is, all prefixes of a finite word* $\sigma \in \phi$ *or of an infinite word* $\sigma \in \varphi$ *belong to* ψ*.*
- *A* guarantee *r-property if* $\Pi = (E_f(\psi), E(\psi))$ *for some finitary property* ψ*. That is, each finite word* $\sigma \in \phi$ *or infinite word* $\sigma \in \varphi$ *is guaranteed to have some prefixes (at least one) belonging to* ψ*.*
- *A* response *r-property if* $\Pi = (R_f(\psi), R(\psi))$ *for some finitary property* ψ*. That is, each infinite word* $\sigma \in \varphi$ *recurrently has (infinitely many) prefixes belonging to* ψ*.*
- *A* persistence *r-property if* $\Pi = (P_f(\psi), P(\psi))$ *for some finitary property* ψ*. That is, each infinite word* $\sigma \in \varphi$ *persistently has (continuously from a certain point on) prefixes belonging to* ψ*.*

In all cases, we say that Π *is built over* ψ*. Furthermore, obligation (resp. reactivity) r-properties are obtained by boolean combinations of safety and guarantee (resp. response and persistence) r-properties.*

Given a set of events Σ, we note $\mathrm{Safety}(\Sigma)$ (resp. $\mathrm{Guarantee}(\Sigma)$, $\mathrm{Obligation}(\Sigma)$, $\mathrm{Response}(\Sigma)$, $\mathrm{Persistence}(\Sigma)$) the set of safety (resp. guarantee, obligation, response, persistence) *r*-properties defined over Σ.

We illustrate in the following example the construction of infinitary properties from finitary ones (described as regular expressions) for each of the four operators.

EXAMPLE 4.1 (BUILDING INFINITARY AND FINITARY PROPERTIES FROM FINITARY ONES; *r*-PROPERTIES) *We consider a client-server application, with a set of observable events* $\Sigma \subseteq \{r, g, d\}$ *where* r *denotes a client request of a given resource and* g *(resp.* d*) denotes a corresponding grant (resp. deny) of this resource provided by the server.*

- *For the finitary property* $\psi = \epsilon + r^+ \cdot g^*$*,* $A_f(\psi) = \epsilon + r^+ \cdot g^*$*,* $A(\psi) = r^\omega + r^+ \cdot g^\omega$*,* $\Pi_1 = (A_f(\psi), A(\psi))$ *is a safety r-property. This language contains all the words that have either only occurrences of* r *or a finite number of occurrences of* r *(at least one) followed only by occurrences of* g*. According to this property a resource should be requested at least once to be granted, and, when granted once, it should not be requested anymore.*
- *For the finitary property* $\psi = r^+ \cdot g$*,* $E_f(\psi) = r^+ \cdot g \cdot \Sigma^*$*,* $E(\psi) = r^+ \cdot g \cdot \Sigma^\omega$*,* $\Pi_2 = (E_f(\psi), E(\psi))$ *is a guarantee r-property. This property tells that the client will issue some requests and will receive a positive answer later on.*
- *For the finitary property* $\psi = g + (r \cdot g)^*$*,* $R_f(\psi) = (r \cdot g)^*$*,* $R(\psi) = (r \cdot g)^\omega$*,* $\Pi_3 = (R_f(\psi), R(\psi))$ *is a response r-property. This language contains all the words that have infinitely many occurrences of* $r \cdot g$*. This property tells that clients will repeatedly send requests and receive back a positive answer (the pattern* $r \cdot g$ *can be seen here as a* transaction*).*
- *For the finitary property* $\psi = g + r \cdot g \cdot (r + r \cdot g)^*$*,* $P_f(\psi) = r \cdot g \cdot (r + r \cdot g)^*$*,* $P(\psi) = r \cdot g \cdot (r + r \cdot g)^\omega$*,* $\Pi_4 = (P_f(\psi), P(\psi))$ *is a persistence r-property. This language contains all the words starting with* $r \cdot g \cdot r$ *and ending with occurrences of* $r + r \cdot g$*. According to this property, after a first granted resource, at some point this resource should be granted forever.*

4.2.2 Some useful facts in the language view

Now, we give some useful facts about *r*-properties in the language view. Those facts will be used in the remainder when characterizing the set of monitorable properties.

Basic classes were defined in a constructive fashion. It is sometimes interesting to have a direct characterization for the properties of those classes. The following property gives a characterization for safety and guarantee *r*-properties. The proof is a direct adaptation of the proof given in [CMP92a].

PROPERTY 4.1 (CHARACTERIZATION OF SAFETY AND GUARANTEE *r*-PROPERTIES) *A r-property* $\Pi = (\phi, \varphi)$ *is a*

- *safety iff* $\Pi = (A_f(\mathit{Pref}(\phi)), A(\mathit{Pref}(\varphi)))$*,*

- *guarantee iff* $\Pi = \big(E_f(\overline{Pref(\overline{\phi})}), E(\overline{Pref(\overline{\varphi})})\big)$.

We expose the closure of safety and guarantee *r*-properties as a straightforward consequence of definitions of safety and guarantee *r*-properties.

PROPERTY 4.2 (CLOSURE OF *r*-PROPERTIES) *Considering a r-property* $\Pi = (\phi, \varphi)$ *defined over an alphabet* Σ *built from a finitary property* ψ*, the following facts hold:*

1 *If* Π *is a safety r-property, all prefixes of a sequence belonging to* Π *also belong to* Π*. That is,* $\forall \sigma \in \Sigma^{\infty}, \Pi(\sigma) \Rightarrow \forall \sigma' \prec \sigma, \Pi(\sigma')$.

2 *If* Π *is a guarantee r-property, all continuations of a finite sequence belonging to* Π *also belong to* Π*. That is,* $\forall \sigma \in \Sigma^{*}, \Pi(\sigma) \Rightarrow \forall \sigma' \in \Sigma^{\infty}, \Pi(\sigma \cdot \sigma')$.

Proof We prove the two facts successively:

1 We have either $\phi(\sigma)$ or $\varphi(\sigma)$, *i.e.*, all prefixes σ' of σ belong to ψ. Necessarily, all prefixes σ'' of σ' also belong to ψ, that is $\psi(\sigma'')$. By definition, that means $\sigma' \in A_f(\psi)$, *i.e.*, $\phi(\sigma')$ and $\Pi(\sigma')$.

2 $\Pi(\sigma)$ implies that σ has at least one prefix $\sigma_0 \preceq \sigma$ belonging to ψ: $\sigma \in E_f(\psi)$. Then, any continuation of σ built using any finite or infinite sequence σ' has at least the same prefix belonging to ψ. If $\sigma' \in \Sigma^{*}$, we have $\sigma_0 \preceq \sigma \preceq \sigma \cdot \sigma'$ and $\sigma \cdot \sigma' \in E_f(\psi)$. If $\sigma' \in \Sigma^{\omega}$, we have $\sigma_0 \preceq \sigma \prec \sigma \cdot \sigma'$ and $\sigma \cdot \sigma' \in E(\psi)$.

The following lemma (inspired from [CMP92a]) provides a decomposition of each obligation properties in a normal form.

LEMMA 4.1 *Any obligation r-property can be represented as the intersection*

$$\bigcap_{i=1}^{k} (\mathrm{Safety}_i \cup \mathrm{Guarantee}_i)$$

for some $k > 0$*, where* Safety_i *and* $\mathrm{Guarantee}_i$ *are respectively safety and guarantee r-properties. We refer to this presentation as the conjunctive normal form of obligation r-properties.*

When a *r*-property Π is expressed as $\cap_{i=1}^{k}(\mathrm{Safety}_i \cup \mathrm{Guarantee}_i)$, Π is said to be a k-obligation *r*-property. The set of k-obligation *r*-properties ($k \geq 1$) is denoted $Obligation_k$. Similar definitions and properties hold for reactivity *r*-properties which are expressed by combination of response and persistence *r*-properties.

4.3 The automata view of *r*-properties

For the automata view of the Safety-Progress classification, we follow [CMP92a] and define *r*-properties using Streett automata. Furthermore, for each class of the Safety-Progress classification it is possible to syntactically characterize a recognizing finite-state automaton. Moreover, we introduce transformations able to take a deterministic finite state automaton and modification pattern so as to obtain a Streett automaton. These transformations are the representatives in the automata view of the operators defined in the language view.

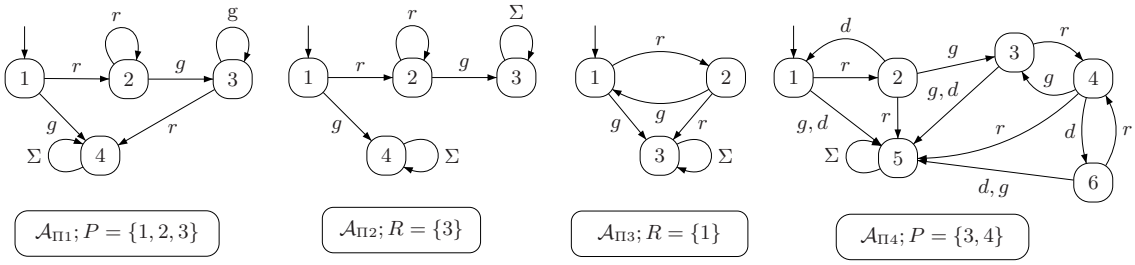


Figure 3: Examples of Streett automata

4.3.1 Streett automata

We define[5] a variant of deterministic and complete Streett automata (introduced in [Str81] and used in [CMP92a]) for property recognition. These automata process events and decide properties of interest. We add to original Streett automata a finite-sequence recognizing criterion in such a way that these automata uniformly recognize *r*-properties.

DEFINITION 4.4 (STREETT AUTOMATON) *A deterministic finite-state Streett automaton is a tuple* $(Q, q_{\text{init}}, \Sigma, \longrightarrow, \{(R_1, P_1), \ldots, (R_m, P_m)\})$ *defined relatively to a set of events* Σ*. The set* Q *is the set of automaton states,* $q_{\text{init}} \in Q$ *is the initial state. The function* $\longrightarrow$*:* $Q \times \Sigma \to Q$ *is the (complete) transition function. In the following, for* $q, q' \in Q, e \in \Sigma$ *we abbreviate* $\longrightarrow (q, e) = q'$ *by* $q \xrightarrow{e} q'$*. The set* $\{(R_1, P_1), \ldots, (R_m, P_m)\}$ *is the set of accepting pairs, for all* $i \leq n$*,* $R_i \subseteq Q$ *are the sets of recurrent states, and* $P_i \subseteq Q$ *are the sets of persistent states.*

We refer to an automaton with m accepting pairs as a m-automaton. When $m = 1$, a 1-automaton is also called a *plain*-automaton, and we refer to R_1 and P_1 as R and P. Moreover, for $\sigma = \sigma_0 \cdots \sigma_{n-1}$ a word of Σ of length n and $q, q' \in Q^{\mathcal{A}}$ two states, we note $q \xrightarrow{\sigma} q'$ when $\exists q_1, \ldots, q_{n-2} \in Q^{\mathcal{A}}, q \xrightarrow{\sigma_0} q_1 \wedge \ldots \wedge q_{n-2} \xrightarrow{\sigma_{n-1}} q'$. In the following $\mathcal{A} = (Q^{\mathcal{A}}, {q_{\text{init}}}^{\mathcal{A}}, \Sigma, \longrightarrow_{\mathcal{A}}, \{(R_1, P_1), \ldots, (R_m, P_m)\})$ designates a deterministic finite state Streett m-automaton.

For $q \in Q^{\mathcal{A}}$, $Reach_{\mathcal{A}}(q)$ is the set of reachable states from q with at least one transition in $\mathcal{A}$, that is $Reach_{\mathcal{A}}(q) = \{q' \in Q^{\mathcal{A}} \mid \exists \sigma \in \Sigma^+, q \xrightarrow{\sigma}_{\mathcal{A}} q'\}$. For $\sigma \in \Sigma^\infty$, the *run* of σ on $\mathcal{A}$ is the sequence of states involved by the execution of σ on $\mathcal{A}$. It is formally defined as $run(\sigma, \mathcal{A}) = q_0 \cdot q_1 \cdots$ where $\forall i, (q_i \in Q^{\mathcal{A}} \wedge q_i \xrightarrow{\sigma_i}_{\mathcal{A}} q_{i+1}) \wedge q_0 = {q_{\text{init}}}^{\mathcal{A}}$. The *trace* resulting in the execution of σ on $\mathcal{A}$ is the unique sequence (finite or not) of tuples $(q_0, \sigma_0, q_1) \cdot (q_1, \sigma_1, q_2) \cdots$ where $run(\sigma, \mathcal{A}) = q_0 \cdot q_1 \cdots$.

For an execution sequence $\sigma \in \Sigma^\omega$ on a Streett automaton $\mathcal{A}$, we define $vinf(\sigma, \mathcal{A})$, as the set of states appearing infinitely often in $run(\sigma, \mathcal{A})$. It is formally defined as follows: $vinf(\sigma, \mathcal{A}) = \{q \in Q^{\mathcal{A}} \mid \forall n \in \mathbb{N}, \exists m \in \mathbb{N}, m > n \wedge q = q_m$ with $run(\sigma, \mathcal{A}) = q_0 \cdot q_1 \cdots\}$.

For a Streett automaton, the notion of acceptance condition is defined using the accepting pairs.

DEFINITION 4.5 (ACCEPTANCE CONDITION FOR INFINITE SEQUENCES) *For* $\sigma \in \Sigma^\omega$*, we say that* $\mathcal{A}$ *accepts* σ *if* $\forall i \in [1, m], vinf(\sigma, \mathcal{A}) \cap R_i \neq \emptyset \vee vinf(\sigma, \mathcal{A}) \subseteq P_i$.

To deal with *r*-properties we need to define also an acceptance criterion for *finite* sequences: a finite sequence is accepted by a a Streett automaton if and only if it terminates on a distinguished state R_i or P_i for each accepting pair i.

DEFINITION 4.6 (ACCEPTANCE CONDITION FOR FINITE SEQUENCES) *For a finite-length execution sequence* $\sigma \in \Sigma^*$ *s.t.* $|\sigma| = n$*, we say that the* m*-automaton* $\mathcal{A}$ *accepts* σ *if* $(\exists q_0, \ldots, q_{n-1} \in Q^{\mathcal{A}}, run(\sigma, \mathcal{A}) = q_0 \cdots q_{n-1} \wedge q_0 = {q_{\text{init}}}^{\mathcal{A}}$ *and* $\forall i \in [1, m], q_{n-1} \in P_i \cup R_i)$.

[5] There exist several equivalent definitions of Streett automata dedicated to infinite sequences recognition. We choose here to follow the definition used in [CMP92a].

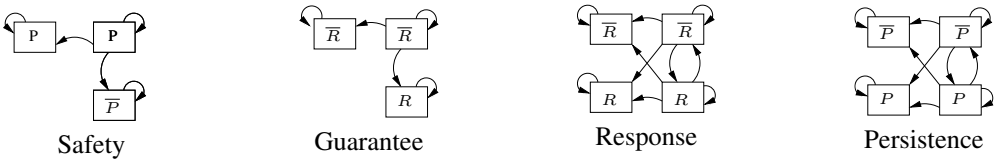


Figure 4: Schematic illustrations of the shapes of Streett automata for basic classes

4.3.2 The hierarchy of automata.

An interesting feature of Streett automata is that the class of property they recognize can be easily characterized by some syntactic considerations.

- A *safety automaton* is a plain automaton s.t. $R = \emptyset$ and there is no transition from a state $q \in \overline{P}$ to a state $q' \in P$.
- A *guarantee automaton* is a plain automaton s.t. $P = \emptyset$ and there is no transition from a state $q \in R$ to a state $q' \in \overline{R}$.
- An m*-obligation automaton* is an m-automaton s.t. for each i in $[1, m]$:
  - there is no transition from $q \in \overline{P_i}$ to $q' \in P_i$,
  - there is no transition from $q \in R_i$ to $q' \in \overline{R_i}$.
- A *response automaton* is a plain automaton s.t. $P = \emptyset$.
- A *persistence automaton* is a plain automaton s.t. $R = \emptyset$.
- A *reactivity automaton* is any unrestricted automaton.

The syntactic restrictions are illustrated in Fig. 4: shapes of Streett automata for basic classes are depicted. One may remark that these syntactic restrictions hold for the automata represented in Fig. 3.

Automata and properties. We now link Streett automata to *r*-properties.

DEFINITION 4.7 (AUTOMATA AND *r*-PROPERTIES) *We say that a Streett automaton $\mathcal{A}$* defines *a r-property $(\phi, \varphi) \in 2^{\Sigma^* \times \Sigma^\omega}$ if and only if the set of finite (resp. infinite) execution sequences accepted by $\mathcal{A}$ is equal to ϕ (resp. φ), which is noted $\mathcal{L}(\mathcal{A}) = (\phi, \varphi)$. Conversely, a r-property $(\phi, \varphi) \in 2^{\Sigma^* \times \Sigma^\omega}$ is said to be* specifiable *by an automaton $\mathcal{A}$ if the set of finite (resp. infinite) execution sequences accepted by the automaton $\mathcal{A}$ is ϕ (resp. φ).*

EXAMPLE 4.2 (STREETT AUTOMATA) *In Fig. 3 are represented Streett plain-automata for the properties presented in Example 4.1.*

- *$\mathcal{A}_{\Pi 1}$ is a safety automaton, its set of recurrent states is empty, its set of persistent states is $P = \{1, 2, 3\}$. A finite sequence is accepted if its runs ends in either states $1, 2$ or 3, meaning that, if a grant happened there was at least one request previously. An infinite sequence is accepted if the only states visited infinitely often are states $1, 2$ or 3, meaning that requests have been made and they were followed by only grants.*
- *$\mathcal{A}_{\Pi 2}$ is a guarantee automaton, its set of persistent states is empty, its set of recurrent states is $R = \{3\}$. A finite sequence is accepted if its run ends in state 3. An infinite sequence is accepted if the state 3 is visited infinitely often. In both cases, it means that requests have been issued and then have been granted.*

- *$\mathcal{A}_{\Pi 3}$ is a response automaton, its set of persistent states is empty, its set of recurrent states is $R = \{1\}$. A finite sequence is accepted if its run ends in state 3, meaning that every request has been followed by a grant (in this order). An infinite sequence is accepted if it visits the state 1 infinitely often, meaning that this infinite sequence contains a succession of action sequences "one request followed by one grant".*
- *$\mathcal{A}_{\Pi 4}$ is a persistence automaton, its set of recurrent states is empty, its set of persistent states is $P = \{3, 4\}$. A finite sequence is accepted if its run ends in state 3, meaning that a first successful request has been made and, after that, the user performs only successful requests (if he makes a requests, this request is granted). An infinite sequence is accepted if it visits infinetely often only state 3, meaning that after a first sucessful request all user's requests have been granted.*

In Section 4.5 we link the syntactic characterizations on the automata to the semantic characterization of the properties they specify.

4.3.3 From a DFA to a Streett automaton

We now introduce four transformations allowing to obtain a Streett automaton, given a deterministic finite-state automaton and a "pattern" for this underlying property. These patterns are inspired from the different classes of the Safety-Progress hierarchy. These simple transformations correspond, in the automata view, to the operators in the language view and the temporal modalities in the logical view[6]. We start by first defining those transformations and then prove their soundness.

A deterministic finite-state automaton (DFA) [HU79], is given relatively to an alphabet Σ, and is here formally defined as a tupe $(Q, q_{\text{init}}, \longrightarrow, F)$ where Q is a finite set of states, $q_{\text{init}} \in Q$ is the initial state, $\longrightarrow: Q \times \Sigma \to Q$ is the transition function, and $F \subseteq Q$ is the set of accepting states.

Definitions of the transformations. In the following, $\mathcal{A}_\psi = (Q^{\mathcal{A}_\psi}, {q_{\text{init}}}^{\mathcal{A}_\psi}, \longrightarrow_{\mathcal{A}_\psi}, F^{\mathcal{A}_\psi})$ designates a complete DFA recognizing a finitary regular property ψ. We define a transformation for each basic class of the hierarchy.

Synthesis of safety automata. For this class of *r*-properties, the transformation is defined as follows:

DEFINITION 4.8 (DFA TO STREETT SAFETY AUTOMATON) *The transformation of $\mathcal{A}_\psi$ into a Strett safety automaton is* $\text{DFA2S\_Saf}(\mathcal{A}_\psi) = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \to_{\mathcal{A}_\Pi}, \{(\emptyset, P)\})$ *and defined by:*

- $Q^{\mathcal{A}_\Pi} = F \cup \{sink\}$*, where* $sink \notin Q^{\mathcal{A}_\psi}$*,*
- ${q_{\text{init}}}^{\mathcal{A}_\Pi} = {q_{\text{init}}}^{\mathcal{A}_\psi}$ *if* ${q_{\text{init}}}^{\mathcal{A}_\psi} \in F^{\mathcal{A}_\psi}$*, and sink otherwise,*
- $\to_{\mathcal{A}_\Pi}$ *is defined as the smallest relation verifying:*
  - $q \xrightarrow{a}_{\mathcal{A}_\Pi} q'$ *if* $q \in F \wedge q' \in F \wedge q \xrightarrow{a}_{\mathcal{A}_\psi} q'$ (TSAFE1)
  - $q \xrightarrow{a}_{\mathcal{A}_\Pi} sink$ *if* $\exists q' \in Q^{\mathcal{A}_\psi}, q' \notin F \wedge q \xrightarrow{a}_{\mathcal{A}_\psi} q'$ (TSAFE2)
  - $\forall a \in \Sigma, sink \xrightarrow{a}_{\mathcal{A}_\Pi} sink$ (TSAFE3)
- $P = Reach_{\mathcal{A}_\Pi}({q_{\text{init}}}^{\mathcal{A}_\Pi}) \setminus \{sink\}$, $(m = 1)$

One can remark that the resulting automaton is indeed a Strett safety automaton since $R = \emptyset$ and there is no transition from the states in $\overline{P}$ to the states in P. This transformation adds a sink state and modifies the transition function in order to redispatch the transitions outgoing from accepting states to the sink state. Furthermore, the transitions outgoing from a state not belonging to a F-state have been removed. The set of persistent states is the set of accepting states of the DFA.

One may remark that the resulting set of states, belonging to the Streett automaton is the smallest solution X of the equation $X = \{q_{\text{init}}\} \cup X \cup post(X) \cap F$ if $q_{\text{init}} \in F$ or the equation $X = \{sink\}$ if $q_{\text{init}} \notin F$. Moreover, according to the syntactic restrictions of the obtained Streett safety automata, the following property holds: for a sequence $\sigma \in \Sigma^\omega$ and an automaton resulting of the transformation $\mathcal{A}_\Pi$, if $sink \in vinf(\sigma, \mathcal{A}_\Pi)$, then $vinf(\sigma, \mathcal{A}_\Pi) = \{sink\}$; else $vinf(\sigma, \mathcal{A}_\Pi) \subseteq P$.

[6] *i.e.*, operators A, E, R, P (and their finitary versions) of the language view and the temporal modalities $\Box$, $\Diamond$, $\Box\Diamond$, $\Diamond\Box$ of the logical view.

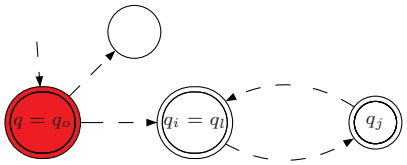


Figure 5: Principle for tagging recurrent states in DFA2S_Res

Synthesis of Streett guarantee automata. For this class of *r*-properties, the transformation is defined as follows:

DEFINITION 4.9 (DFA TO STREETT GUARANTEE AUTOMATON) *The transformation of $\mathcal{A}_\psi$ into a Strett guarantee automaton is* $\text{DFA2S\_Guar}(\mathcal{A}_\psi) = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \rightarrow_{\mathcal{A}_\Pi}, \{(R, \emptyset)\})$ *and defined by:*

- $Q^{\mathcal{A}_\Pi}$ *is the smallest subset of* $Q^{\mathcal{A}_\psi}$ *containing the reachable states from the initial state* ${q_{\text{init}}}^{\mathcal{A}_\Pi}$ *with* $\longrightarrow_{\mathcal{A}_\Pi}$ *(defined below),*
- ${q_{\text{init}}}^{\mathcal{A}_\Pi} = {q_{\text{init}}}^{\mathcal{A}_\psi}$,
- $\rightarrow_{\mathcal{A}_\Pi}$ *is defined as the smallest relation verifying:*
  - $q \xrightarrow{a}_{\mathcal{A}_\Pi} q$ *if* $\exists q' \in Q^{\mathcal{A}_\psi} \cdot q \xrightarrow{a}_{\mathcal{A}_\psi} q' \wedge q \in F$ (TGUAR1)
  - $q \xrightarrow{a}_{\mathcal{A}_\Pi} q'$ *if* $q \notin F \wedge q \xrightarrow{a}_{\mathcal{A}_\psi} q'$ (TGUAR2)
- $R = F$, $(m = 1)$

One may remark that the resulting automaton is indeed a Streett guarantee automaton since $P = \emptyset$ and there is no transition from the R-state to the $\overline{R}$-states. This automaton may not be minimal for the R-states. They can be merged into one unique state since they are all equivalent wrt. property recognition. This transformation modifies the transition function in the following manner: outgoing transitions from the accepting states (to an accepting state or not) are modified into a loop on the same state. Indeed, when a run reaches a state in F, this suffix suffices in order to satisfy the guarantee property. The initial state is not modified, and the set of states of the Streett automaton is defined as the smallest set of reachable set from the initial state with the new transition function.

Synthesis of Streett response automata. For this class of *r*-properties, the transformation is defined as follows:

DEFINITION 4.10 (DFA TO STREETT RESPONSE AUTOMATON) *The transformation of $\mathcal{A}_\psi$ into a Streett response automaton is* $\text{DFA2S\_Resp}(\mathcal{A}_\psi) = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \rightarrow_{\mathcal{A}_\Pi}, \{(R, \emptyset)\})$ *and defined by:*

- $Q^{\mathcal{A}_\Pi} = Q^{\mathcal{A}_\psi}$,
- ${q_{\text{init}}}^{\mathcal{A}_\Pi} = {q_{\text{init}}}^{\mathcal{A}_\psi}$,
- $\rightarrow_{\mathcal{A}_\Pi}$ *is defined as* $\rightarrow_{\mathcal{A}_\psi}$,
- $R = \{q \in F \mid \exists l \in \mathbb{N}^*, \exists q_0, \ldots, q_l \in Q^{\mathcal{A}_\psi},\ (1)\ \wedge\ (2)\ \} \cup \{q \in F \mid q \longrightarrow_{\mathcal{A}_\psi} q\}$

$$\forall j \in [0, l-1], q_j \longrightarrow_{\mathcal{A}_\psi} q_{j+1} \tag{1}$$

$$\exists i \in [0, l], \exists j \in [i, l-1], q_j \in F \wedge q_i = q_l \wedge q_0 = q \tag{2}$$

The resulting automaton is indeed a Streett response automaton since $P = \emptyset$. This transformation does not modify the set of states nor the transition function. It marks as recurrent states (cf. Fig. 5) every accepting state of the DFA s.t. it is possible from this state to reach a cycle containing at least one accepting state.

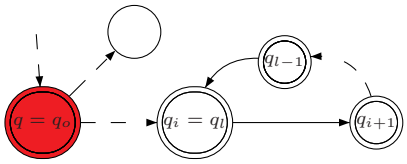


Figure 6: Principle for tagging persistent states in DFA2S_Per

Synthesis of Streett persistence automata. For this class of *r*-properties, the transformation is defined as follows:

DEFINITION 4.11 (DFA TO STREETT PERSISTENCE AUTOMATON) *The transformation of $\mathcal{A}_\psi$ into a Streett persistence automaton is* $\mathrm{DFA2S\_Per}(\mathcal{A}_\psi) = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \rightarrow_{\mathcal{A}_\Pi}, \{(\emptyset, P)\})$ *and defined by:*

- $Q^{\mathcal{A}_\Pi} = Q^{\mathcal{A}_\psi}$,
- ${q_{\text{init}}}^{\mathcal{A}_\Pi} = {q_{\text{init}}}^{\mathcal{A}_\psi}$,
- $\rightarrow_\Pi$ *is defined as* $\rightarrow_{\mathcal{A}_\psi}$,
- $P = \{q \in F \mid \exists l \in \mathbb{N}^*, \exists q_0, \ldots, q_l \in Q^{\mathcal{A}_\psi},\ (1)\ \wedge\ (3)\ \} \cup \{q \in F \mid q \longrightarrow_{\mathcal{A}_\psi} q\}$

$$\exists i \in [0, l], \forall j \in [i, l-1], q_j \in F \wedge q_i = q_l \wedge q_0 = q \tag{3}$$

The resulting automation is indeed a Streett persistence automaton since $R = \emptyset$. This transformation does not modify the set of states nor the transition function. It marks (cf. Fig. 6) as persistent state every accepting state of the DFA from which it is possible to reach a cycle of accepting states.

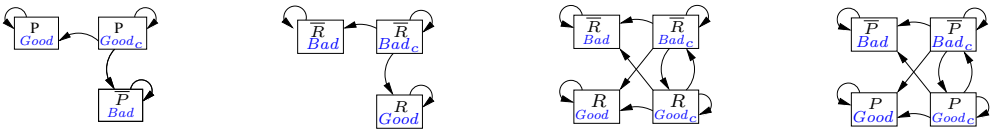
Soundness of the transformations. Given a finitary property ψ, defining a regular language over an alphabet Σ and specified by a DFA $\mathcal{A}_\psi$, the safety (resp. guarantee, response, persistence) *r*-property $(X_f(\psi), X(\psi))$ where $X \in \{A, E, R, P\}$ is specified by the Streett automaton obtained by the transformation *DFA2S* specific to safety (resp. guarantee, response, persistence) properties. This is stated formally by the following theorem:

THEOREM 4.1 (Soundness of the transformations of DFAs to Streett automata) *The transformation DFA2S_Saf (resp. DFA2S_Guar, DFA2S_Resp, DFA2S_Per) in the automata view "corresponds" to the operator A_f and A (resp. E_f and E, R_f and R, P_f and P) in the language view. More precisely, when $\mathcal{L}(\mathcal{A}_\psi) = \psi$,*

$$\begin{aligned}
\mathcal{A}_\Pi = \mathit{DFA2S_Saf}(\mathcal{A}_\psi) &\Rightarrow \mathcal{L}(\mathcal{A}_\Pi) = (A_f(\psi), A(\psi)) \\
\mathcal{A}_\Pi = \mathit{DFA2S_Guar}(\mathcal{A}_\psi) &\Rightarrow \mathcal{L}(\mathcal{A}_\Pi) = (E_f(\psi), E(\psi)) \\
\mathcal{A}_\Pi = \mathit{DFA2S_Resp}(\mathcal{A}_\psi) &\Rightarrow \mathcal{L}(\mathcal{A}_\Pi) = (R_f(\psi), R(\psi)) \\
\mathcal{A}_\Pi = \mathit{DFA2S_Per}(\mathcal{A}_\psi) &\Rightarrow \mathcal{L}(\mathcal{A}_\Pi) = (P_f(\psi), P(\psi))
\end{aligned}$$

Proof Proofs are conducted for each class of properties and the associated transformation by using the acceptance criteria and examining runs of accepted sequences. The complete proof can be found in Appendix A.1.1.

REMARK 4.1 *Let us remark that these transformation may entail a loss of information. It is in general not possible to find again the finitary language from which a Streett automaton has been built. Consider for exemple the Streett guarantee automaton represented on Fig. 3. There exists an infinite number of finitary languages from which this automata can be constructed. Indeed, to obtain them, it suffices to retransform this Streett automaton into a minimal DFA by forgetting accepting pairs and changing the R-state into an accepting state. Then, from this accepting state, we can add arbritary transitions. The automata produced by doing so will always be transformed by DFA2S_Guar into $\mathcal{A}_{\Pi 2}$.*



Safety Guarantee Response Persistence

Figure 7: Characterization of states for basic classes

	Language view	**Automata view**
"base brick"	$\psi \subseteq \Sigma^+$	$\mathcal{A}$ (DFA)
***r*-property**	$(X_f(\psi), X(\psi))$ $X \in \{A, E, R, P\}$	$DFA2S\_X(\mathcal{A})$ $X \in \{Saf., Guar., Resp., Persit.\}$

Table 1: Ways to specify properties according to the views

4.4 Characterizing states of Streett automata

To better identify particular execution sequences on a Streett automaton we characterize some subsets of its states in terms of reachability of distinguished states. More precisely, the set $\mathbb{P}^{\mathcal{A}} = \{Good^{\mathcal{A}}, Good_c^{\mathcal{A}}, Bad_c^{\mathcal{A}}, Bad^{\mathcal{A}}\}$ is a set of subsets of $Q^{\mathcal{A}}$, s.t. $Good^{\mathcal{A}}$, $Good_c^{\mathcal{A}}$, $Bad_c^{\mathcal{A}}$, $Bad^{\mathcal{A}}$ designate respectively the good (resp. currently good, currently bad, bad) states. The set $\mathbb{P}^{\mathcal{A}}$ is defined as follows:

- q is in $Good^{\mathcal{A}}$ iff it terminates an accepted sequence and every sequence starting from q is accepted: $Good^{\mathcal{A}} = \{q \in \bigcap_{i=1}^{m}(R_i \cup P_i) \mid Reach_{\mathcal{A}}(q) \subseteq \bigcap_{i=1}^{m}(R_i \cup P_i)\}$;
- q is in $Good_c^{\mathcal{A}}$ iff it terminates an accepted sequence and there exist non accepted sequences starting from q: $Good_c^{\mathcal{A}} = \{q \in \bigcap_{i=1}^{m}(R_i \cup P_i) \mid Reach_{\mathcal{A}}(q) \not\subseteq \bigcap_{i=1}^{m}(R_i \cup P_i)\}$;
- q is in $Bad_c^{\mathcal{A}}$ iff it terminates a non accepted sequence and there exist accepted sequences starting from q: $Bad_c^{\mathcal{A}} = \{q \in \bigcup_{i=1}^{m}(\overline{R_i} \cap \overline{P_i}) \mid Reach_{\mathcal{A}}(q) \not\subseteq \bigcup_{i=1}^{m}(\overline{R_i} \cap \overline{P_i})\}$;
- q is in $Bad^{\mathcal{A}}$ iff it terminates a non accepted sequence and every sequence starting from q is not accepted: $Bad^{\mathcal{A}} = \{q \in \bigcup_{i=1}^{m}(\overline{R_i} \cap \overline{P_i}) \mid Reach_{\mathcal{A}}(q) \subseteq \bigcup_{i=1}^{m}(\overline{R_i} \cap \overline{P_i})\}$.

The subsets are illustrated for basic classes in Fig. 7. Note that $Q^{\mathcal{A}} = Good^{\mathcal{A}} \cup Good_c^{\mathcal{A}} \cup Bad_c^{\mathcal{A}} \cup Bad^{\mathcal{A}}$.

EXAMPLE 4.3 (CHARACTERIZATION OF STREETT AUTOMATA STATES) *We illustrate the characterization on the states of the Streett automata presented in Example 4.2:*

- $Bad^{\mathcal{A}_{\Pi 1}} = \{4\}$, $Good_c^{\mathcal{A}_{\Pi 1}} = \{1, 2, 3\}$,
- $Bad^{\mathcal{A}_{\Pi 2}} = \{4\}$, $Bad_c^{\mathcal{A}_{\Pi 2}} = \{1, 2\}$, $Good^{\mathcal{A}_{\Pi 2}} = \{3\}$,
- $Bad^{\mathcal{A}_{\Pi 3}} = \{3\}$, $Bad_c^{\mathcal{A}_{\Pi 2}} = \{2\}$, $Good_c^{\mathcal{A}_{\Pi 3}} = \{1\}$,
- $Bad^{\mathcal{A}_{\Pi 4}} = \{5\}$, $Bad_c^{\mathcal{A}_{\Pi 4}} = \{1, 2, 6\}$, $Good_c^{\mathcal{A}_{\Pi 4}} = \{3, 4\}$.

REMARK 4.2 *For a Streett automaton $\mathcal{A}_\Pi$, all states in $Bad^{\mathcal{A}_\Pi}$ (resp. $Good^{\mathcal{A}_\Pi}$) are equivalent wrt. property recognition; and can thus be merged into one single state.*

This characterization of states will be usefull in the following sections when characterizing monitorable properties and when synthesizing monitors.

4.5 Summary

A graphical representation of the Safety-Progress hierarchy of properties is depicted in Fig. 8. A link between two classes means that the higher class contains strictly the lower one. Furthermore, for each class, we have recalled and uniformly extended the characterizations in the language-theoretic and automata views.

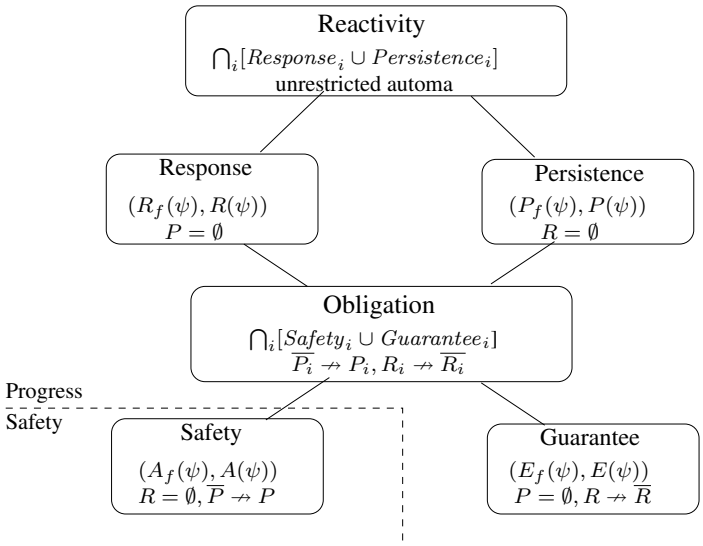


Figure 8: The Safety-Progress classification of *r*-properties

	Language view	**Automata view**
Finite seq	$\in X_f(\psi)$ (Def. 4.1)	Finite seq criterion (Def. 4.6)
Infinite seq	$\in X(\psi)$ (Def. 4.2)	Infinite seq criterion (Def. 4.5)

Table 2: Recognizing criteria according to the considered view

In table 1 is represented each "base brick", *i.e.*, the element used to build a *r*-property. In the language view, *r*-properties are built from a finitary language ψ, and using operators X_f, and X, with $X \in \{A, E, R, P\}$. In the automata view, a finite state automaton is transformed, by one of the transformations DFA2S specific to a class of properties, into a Streett automaton which recognizes $(X_f(\psi), X(\psi))$ according to the class of properties.

REMARK 4.3 *It is worth noticing that property interpretation of finite sequences extends to infinite sequences in a consistent way, depending on the class of properties under consideration:*

- *for a safety property* Π, $\forall i \in \mathbb{N}, \Pi(\sigma_{...i}) \Rightarrow \Pi(\sigma)$
- *for a guarantee property* Π, $\exists i \in \mathbb{N}, \Pi(\sigma_{...i}) \Rightarrow \neg\Pi(\sigma)$
- *for a response property* Π, $\overset{\infty}{\exists}\, i \in \mathbb{N}, \Pi(\sigma_{...i} \Rightarrow \Pi(\sigma)$
- *for a persistence property* Π, $\neg(\overset{\infty}{\exists}\, i \in \mathbb{N}, \neg\Pi(\sigma_{...i})) \Rightarrow \neg\Pi(\sigma)$

5 Monitorability wrt. the SP classification

As stated in the introduction, studying the question of monitorabily amounts to study the expressiveness of runtime verification, *i.e.*, characterize the classes of properties that are worth verifying at runtime. In this section we first recall and extend existing monitorability results in the Safety-Progress classification of properties. Second, we propose to parameterize the classical definition with a truth-domain. Third, we propose an alternative definition of monitorability.

In fact, characterizing the space of "monitorable" properties depends on several parameters: the property semantics for *finite* sequence, the set of monitor verdicts we consider, and the exact definition of monitoring.

5.1 According to the classical definition of monitoring

In this section, we express the classical definition of monitorability given by Pnueli and Zaks in the SP framework introduced in the previous section. Then, we characterize the set of monitorable properties according to this classical definition.

5.1.1 The classical definition of monitorability

The main objective of monitoring, in its classical definition, is to evaluate an (infinitary) property φ on a possibly infinite execution sequence from one of its *finite* prefix. Intuitively, the idea is to be able to detect verdicts, *i.e.*, find an evaluation, wrt. an infinitary property, from a finite observation of a system behavior. This is formalized as follows:

DEFINITION 5.1 (POSITIVE/NEGATIVE DETERMINACY [PZ06]) *A r-property* $\Pi \subseteq \Sigma^* \times \Sigma^\omega$ *is said to be:*

- *negatively determined by* $\sigma \in \Sigma^*$ *if* $\neg\Pi(\sigma) \wedge \forall \mu \in \Sigma^\infty, \neg\Pi(\sigma \cdot \mu)$*;*
- *positively determined by* $\sigma \in \Sigma^*$ *if* $\Pi(\sigma) \wedge \forall \mu \in \Sigma^\infty, \Pi(\sigma \cdot \mu)$.

A *r*-property is negatively (resp. positively) determined if the current sequence does not (resp. does) satisfy the property and if every future possible continuations do not (resp. do) satisfy the property. The practical meaning is the following: when a monitor observes a system in order to verify a property, if this property is negatively or positively determined, then the observation of the system can be stopped. A monitor, dedicated to a *r*-property Π, associates emits the verdict $\bot$ (resp. the verdict $\top$) after reading σ if the property is negatively (resp. positively determined) by σ. The obtained verdict is definitive. In others cases, a monitor issues the value "?", meaning that no definitive verdict can be produced.

DEFINITION 5.2 (MONITORABLE *r*-PROPERTIES, "CLASSICAL" DEFINITION [PZ06]) *A r-property* Π *is:*

- σ*-monitorable, if there exists a (finite)* $\mu \in \Sigma^*$ *s.t.* Π *is positively or negatively determined by* $\sigma \cdot \mu$*;*
- *monitorable, if it is* σ*-monitorable for every* $\sigma \in \Sigma^*$.

The set of monitorable properties, according to the classical definition is noted MP_c. A *r*-property is monitorable if, for any execution sequence that can be observed, a possible extension of this sequence determines negatively or positively the property.

Remark that this definition of monitorability is hard to use in practice. Given a *r*-property, the definition does not afford an easy way to determine if this property is monitorable or not.

5.1.2 Characterization of monitorable properties according to the classical definition

One of our first objective is to characterize the subset of *monitorable properties* within the Safety-Progress classification.

We start by first enunciating a lemma that we will use later on. This lemma states that the set of MP_c-monitorable properties is closed under boolean operations.

LEMMA 5.1 (CLOSURE OF MONITORABLE PROPERTIES UNDER BOOLEAN OPERATIONS) *Given two r-properties* Π_1, Π_2, *we have:*

$$\Pi_1, \Pi_2 \in MP_c \Rightarrow \Pi_1 \wedge \Pi_2 \in MP_c$$
$$\Pi_1, \Pi_2 \in MP_c \Rightarrow \Pi_1 \vee \Pi_2 \in MP_c$$
$$\Pi_1 \in MP_c \Rightarrow \neg\Pi_1 \in MP_c$$

Proof The complete proof is given in Appendix A.2. Let us consider two *r*-properties $\Pi_1, \Pi_2 \in MP_c$.

- Proof of $\Pi_1 \wedge \Pi_2 \in MP_c$. It consists in showing that $\Pi_1 \wedge \Pi_2$ is σ-monitorable for any sequence $\sigma \in \Sigma^*$. Let $\sigma \in \Sigma^*$, let us exhibit an extension $\mu \in \Sigma^*$ s.t. $\Pi_1 \wedge \Pi_2$ is negatively or positively determined by $\sigma \cdot \mu$.

  The proof is conducted by using the fact that Π_1 is monitorable which gives us a sequence μ_1 s.t. Π_1 is positively or negatively determined by $\sigma \cdot \mu_1$. Then, using the fact that Π_2 is monitorable gives us a sequence μ_2 s.t. $\Pi_1 \wedge \Pi_2$ is negatively or positively determined by $\sigma \cdot \mu_1 \cdot \mu_2$. Then, one has to analyze the different boolean combinations to obtain the expected result.
- The proof of $\Pi_1 \vee \Pi_2 \in MP_c$ is similar.
- The proof of $\neg\Pi_1 \in MP_c$ is straightforward by noticing that for any sequence $\sigma \in \Sigma^*$, if Π_1 is positively (resp. negatively) determined by σ, then $\neg\Pi$ is negatively (resp. positively) determined by σ.

We are now able to establish that the set of monitorable properties according to the classical definition strictly contains the set of obligation properties.

THEOREM 5.1 (*Obligation*$(\Sigma) \subset MP_c$) *The obligation properties are strictly contained in the set of monitorable properties.*

Proof Obligation *r*-properties are obtained by boolean combinations of safety and guarantee *r*-properties. For $k \in \mathbb{N}$, a k-obligation *r*-property is expressed:

$$\bigcap_{i=1}^{k} (\text{Safety}_i \cup \text{Guarantee}_i),$$

where Safety_i and Guarantee_i are safety and guarantee *r*-properties. The set of all k-obligation *r*-properties for $k \in \mathbb{N}$ is the set of obligation *r*-properties.

Let $\Pi \in \text{Obligation}(\Sigma)$, there exists $k \in \mathbb{N}$ s.t. $\Pi \in k\text{-Obligation}(\Sigma)$. The proof relies on an induction on k and uses the following facts:

- Safety and guarantee properties are monitorable. Here is the proof[7]:
  - Let $\Pi = (A_f(\psi), A(\psi))$ be a safety *r*-property, let us prove that Π is monitorable. Let $\sigma \in \Sigma^*$, let us prove that Π is σ-monitorable. The proof is done by distinguishing two cases : either there exists a continuation $\sigma' \in \Sigma^*$ of σ s.t. $\neg\Pi(\sigma')$, or there does not exist. In the first case, we have that $\neg A_f(\psi)(\sigma')$, *i.e.*, σ' does not have all of its prefixes in ψ. Then, the same hold for every continuation σ'' of σ': $\forall \sigma'' \in \Sigma^*, \sigma' \preceq \sigma'' \Rightarrow \neg A_f(\psi)(\sigma'')$. It follows that $\forall \sigma'' \in \Sigma^*, \sigma \preceq \sigma' \preceq \sigma'' \Rightarrow \neg\Pi(\sigma'')$. That is to say Π is negatively determined by σ'. In the second case, every continuation of σ satisfies Π. That is Π is positively determined by $\sigma \cdot \epsilon$.
  - Let $\Pi = (E_f(\psi), E(\psi))$ be a guarantee *r*-property, let us prove that Π is monitorable. The proof can be similarly conducted. It suffices to consider $\sigma \in \Sigma^*$ and show that there exists an extension which makes that Π is negatively or positively determined by this extension. Similarly, two cases can be distinguished whether there exists an extension of σ which satisfies the property.
- Union and intersection of two monitorable properties are monitorable (Lemma 5.1).
- Example 5.2 shows that the inclusion is strict.

Thus, we have extended the previous bound established by Bauer et al. in [BLS07] stating that $\text{Safety}(\Sigma) \cup \text{Guarantee}(\Sigma) \subset MP_c$[8]. Indeed, the set of obligation properties is a strict super set of

[7] The proof can also be done by examining the syntactic restriction applying to an automaton recognizing a safety or a guarantee property: for all $\sigma \in \Sigma^*$, there exists a continuation μ s.t. this property is negatively or positively determined by $\sigma \cdot \mu$. For instance, in a safety automaton, for each state there exists a path which leads either to a terminal strongly connected component of states in which the property is satisfied or in a terminal strongly connected component in which the property is not satisfied.

[8] In [BLS07], guarantee properties are named co-safety properties.

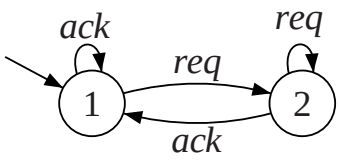


Figure 9: Non-monitorable response property - $R = \{1\}, P = \emptyset$

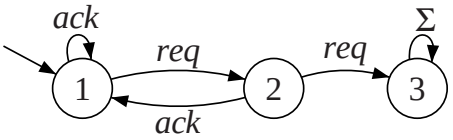


Figure 10: Monitorable response property - $R = \{1\}, P = \emptyset$

the union of safety and guarantee properties.

EXAMPLE 5.1 (CLASSICAL MONITORING OF AN OBLIGATION PROPERTY) *We go back on the example presented in the introduction, defined using two atomic propositions p or q, stating that p should always hold or q should eventually hold. This is a 1-obligation r-property*[9]*, defined by the disjunction of a safety r-property ("p should always hold") and a guarantee r-property ("q should eventually hold"). According to the classical definition of monitorability, this property is monitorable. Indeed, for any sequence σ, this property can be positively determined by $\sigma \cdot \{\overline{p}, q\}$ or by $\sigma \cdot \{p, q\}$, i.e., by completing σ with an event in which q is true.*

Beyond Obligation properties. Following the classical definition of monitorability, it is possible to show that there exist non-monitorable and monitorable properties for super classes of the Obligation class. The above two properties are pure response properties, one is not monitorable, the other one is.

EXAMPLE 5.2 (NON-MONITORABLE RESPONSE PROPERTY [BLS07]) *The (response) property "Every request should be acknowledged" is not monitorable. This property is represented by the Streett (response) automaton depicted in Fig. 9 with $R = \{1\}$. Using the acceptance criteria for finite and infinite sequences,one can reasonably be convinced that this automaton defines the considered property. Indeed, a finite sequence is accepted iff previous requests have been acknowledged. An infinite sequence is accepted iff state 1 is visited infinitely often which means for an infinite sequence that requests have been acknowledged.*

For this property, there are two limitations for monitoring with the considered truth-domain and definition of monitorability. First, it is impossible to distinguish correct (ending in state 1) and incorrect finite sequences (ending in state 2): both evaluate to "?". Second, for all finite sequences, it is never possible to decide $\top$ or $\bot$ since every finite sequence can be extended to correct or incorrect infinite continuations. In other words, it is never possible to satisfy or falsify this property with a finite observation.

EXAMPLE 5.3 (MONITORABLE RESPONSE PROPERTY) *The (response) property "Every request should be acknowledged, and it is forbidden to send two successive requests (without acknowledgment)" is monitorable. This property is represented by the Streett (response) automaton depicted in Fig. 10 with $R = \{1\}$. Intuitively, given an execution sequence, this r-property can always be negatively determined by one of its extension. Indeed, for any $\sigma \in \Sigma^*$, the property is negatively determined by $\sigma \cdot req \cdot req$, and thus σ-monitorable.*

Thus there exist monitorable (pure) response properties. Consequently, using Lemma 5.1, there exist also monitorable pure persistence and reactivity properties. Indeed, monitorable properties are closed under boolean operations.

[9] Seen in the logical view, this property can be defined by the temporal logic formula $\Box p \vee \Diamond q$.

5.2 Considering other truth domains ?

As we will see, the characterization of monitorable properties may also depend on the truth domain $\mathbb{B}$ we consider when evaluating an execution sequence. Thus we parametrize the classical definition of monitorable properties with a truth domain. For a truth-domain $\mathbb{B}$, we will note $MP(\mathbb{B})$ the space of monitorable properties, according to the classical definition of monitorability.

The first truth domain we have considered is a 3-valued truth domain $\mathbb{B}_3 = \{\top, ?, \bot\}$, *i.e.*, $MP_c = MP(\mathbb{B}_3)$. This truth-domain is inherent in the classical definition. Value "$\top$" is used to express property satisfaction when the property is positively determined. Value "$\bot$" is used to express property violation when the property is negatively determined. Whereas value "?" is used to express that no verdict can be produced. $\mathbb{B}_3$ can be viewed as a complete lattice, those minimal value is $\bot$ and maximal value is $\top$. Boolean operators $\vee$ and $\wedge$ are then defined respectively as upper and lower bounds.

We know tackle the question of how the underlying truth domain we consider may influence the class of monitorable properties.

Monitorability with $\mathbb{B}_2$. Restraining $\mathbb{B}_3$ to a truth-domain of cardinality 2 allows only either positive or negative determinacy, and hence reduces the set of monitorable properties. Indeed, the purpose of the monitor is then to only detect bad behaviors *or* good behaviors (but not both). In the sequel we consider two subsets of $\mathbb{B}_3$, namely $\mathbb{B}_2^{\bot} = \{\bot, ?\}$ and $\mathbb{B}_2^{\top} = \{?, \top\}$.

DEFINITION 5.3 (MONITORABLE *r*-PROPERTIES WITH TRUTH-DOMAINS OF CARDINALITY 2) *A r-property* Π *is:*

- *σ-monitorable with $\mathbb{B}_2^{\bot}$, if there exists a (finite) $\mu \in \Sigma^*$ s.t. Π is negatively determined by $\sigma \cdot \mu$;*
- *σ-monitorable with $\mathbb{B}_2^{\top}$, if there exists a (finite) $\mu \in \Sigma^*$ s.t. Π is positively determined by $\sigma \cdot \mu$;*
- *monitorable with $\mathbb{B}_2^{\bot}$ (resp. $\mathbb{B}_2^{\top}$), if it is σ-monitorable with $\mathbb{B}_2^{\bot}$ (resp. $\mathbb{B}_2^{\top}$) for every $\sigma \in \Sigma^*$.*

However, there is no simple characterization of these properties in the Safety-Progress hierarchy. Intuitively one may think that with $\mathbb{B}_2^{\bot} = \{?, \bot\}$, the set of monitorable properties would be the set of safety properties. But in fact, there are numerous safety properties which can never be negatively determined. For example, the *r*-property $true = (\Sigma^*, \Sigma^\omega) = (A_f(\Sigma^*), A(\Sigma^*))$ cannot be negatively determined nor falsified. Moreover all safety properties which are valid forever for execution sequences longer than a given k are not $\sigma - \mathbb{B}_2^{\bot}$-monitorable when $|\sigma| > k$. For these kinds of properties a monitor would produce only verdict sequences of "?" when evaluating an execution sequence. Similarly, there exist many guarantee properties that cannot be positively determined, and therefore are not monitorable with $\mathbb{B}_2^{\top} = \{?, \top\}$.

It appears that there is no simple characterization, in term of classes of the Safety-Progress classification, for monitorable properties. However, in Section 5.4, we will provide a syntactic criterion on Streett automata in order to decide whether the *r*-property recognized by this automaton is monitorable according to the mentionned truth-domains.

5.3 According to an alternative definition of monitorability

The interest of previous definitions of monitorability is due to two facts: the underlying truth-domain is 2-valued or 3-valued and the aim is the detection of verdict of infinitary properties. Although it is possible to give a semantics to all reactive properties with either a 2-valued or 3-valued truth-domain, the question is whether those values make sense for some properties in a monitoring context.

As noticed in [BLS07, LS08], it seems interesting to investigate further the space of monitorable properties, and to answer more precisely questions like "what verdict to issue if the program execution stops here". This means a better distinction between finite sequences which evaluate to "?" in a 2-valued or 3-valued truth-domain.

Hence, the authors of [BLS07, LS08] proposed to consider a 4-valued truth-domain $\mathbb{B}_4 = \{\top, \top_c, \bot_c, \bot\}$. The truth-value $\top_c$ (resp. $\bot_c$) denotes "currently true" (resp. "currently false") and it expresses "Π-satisfaction (resp. Π-violation) if the program execution stops here". Boolean operators $\vee$ and $\wedge$ are defined in [BLS07]. Using $\mathbb{B}_4$ leads to an alternative definition of monitoring. This new definition leverages the evaluation of finite sequences in the Safety-Progress classification framework.

5.3.1 Property evaluation in a truth-domain.

We first introduce how, given a *r*-property, we evaluate an execution sequence in the truth-domains we considered so far.

DEFINITION 5.4 (PROPERTY EVALUATION WRT. A TRUTH-DOMAIN) *For each of the possible truth-domain $\mathbb{B}$, we define the evaluation functions $[\![\cdot]\!]_{\mathbb{B}}(\cdot) : 2^{\Sigma^* \times \Sigma^\omega} \times \Sigma^* \to \mathbb{B}$ as follows:*

For $\mathbb{B}_2^\bot$:

$[\![\Pi]\!]_{\mathbb{B}_2^\bot}(\sigma) = \bot$ *if* $\neg\Pi(\sigma) \wedge \forall \mu \in \Sigma^\infty, \neg\Pi(\sigma \cdot \mu)$;

$[\![\Pi]\!]_{\mathbb{B}_2^\bot}(\sigma) = ?$ *otherwise.*

For $\mathbb{B}_2^\top$:

$[\![\Pi]\!]_{\mathbb{B}_2^\top}(\sigma) = \top$ *if* $\Pi(\sigma) \wedge \forall \mu \in \Sigma^\infty, \Pi(\sigma \cdot \mu)$;

$[\![\Pi]\!]_{\mathbb{B}_2^\top}(\sigma) = ?$ *otherwise.*

For $\mathbb{B}_3$:

$[\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = \bot$ *if* $\neg\Pi(\sigma) \wedge \forall \mu \in \Sigma^\infty, \neg\Pi(\sigma \cdot \mu)$;

$[\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = \top$ *if* $\Pi(\sigma) \wedge \forall \mu \in \Sigma^\infty, \Pi(\sigma \cdot \mu)$;

$[\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = ?$ *otherwise.*

For $\mathbb{B}_4$:

$[\![\Pi]\!]_{\mathbb{B}_4}(\sigma) = [\![\Pi]\!]_{\mathbb{B}_3}(\sigma)$ *if* $[\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = \bot$ *or* $[\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = \top$,

$[\![\Pi]\!]_{\mathbb{B}_4}(\sigma) = \top_c$ *if* $[\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = ?$ *and* $\Pi(\sigma)$,

$[\![\Pi]\!]_{\mathbb{B}_4}(\sigma) = \bot_c$ *if* $[\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = ?$ *and* $\neg\Pi(\sigma)$.

REMARK 5.1 *The defined property evaluation wrt. $\mathbb{B}_4$ is similar to the semantics of the LTL variant RV-LTL defined in [BLS09].*

5.3.2 An alternative definition of monitorability.

Intuitively, the monitorability notion we propose relies on the ability of a given monitor to distinguish between *good* and *bad* finite execution sequences with respect to a property Π.

DEFINITION 5.5 (ALTERNATIVE MONITORABILITY) *A r-property $\Pi = (\phi, \varphi)$ is said to be monitorable with the truth-domain $\mathbb{B}$, or $\mathbb{B}$-monitorable iff*

$$\forall \sigma_{good} \in \phi, \forall \sigma_{bad} \in \overline{\phi}, [\![\Pi]\!]_{\mathbb{B}}(\sigma_{good}) \neq [\![\Pi]\!]_{\mathbb{B}}(\sigma_{bad})$$

We note $MP^*(\mathbb{B})$, the set of monitorable properties with truth domain $\mathbb{B}$ according to this definition.

Thus, a *r*-property is monitorable with a given truth-domain $\mathbb{B}$ iff evaluations of good and bad finite execution sequences lead to *distinct* values. Remark that this definition does not rely directly on the infinitary part of the *r*-property (although this infinitary part is taken into account in the evaluation function).

5.3.3 Characterization of monitorable properties

LEMMA 5.2 ($MP^*(\mathbb{B}_3)$, SAFETY, AND GUARANTEE PROPERTIES) *The set of monitorable properties (according to Definition 5.5) with $\mathbb{B}_3$ is included in the union of safety and guarantee properties. Formally:*

$$MP^*(\mathbb{B}_3) \subseteq \text{Safety}(\Sigma) \cup \text{Guarantee}(\Sigma)$$

Proof The complete proof can be found in Appendix A.2. It is done by *reductio ad absurdum* and supposing the existence of a *r*-property $\Pi = (\phi, \varphi)$ defined on Σ which is neither a safety nor a guarantee *r*-property. The proofs shows the existence of two execution sequences, one good, the other bad, for Π s.t. these sequences are evaluated to "?".

THEOREM 5.2 (Multi-valued characterization of alternative monitorability) *The sets of monitorable properties according to the truth domains considered so far are the following:*

(i) $MP^*(\mathbb{B}_2^{\perp}) = \text{Safety}(\Sigma)$

(ii) $MP^*(\mathbb{B}_2^{\top}) = \text{Guarantee}(\Sigma)$

(iii) $MP^*(\mathbb{B}_3) = \text{Safety}(\Sigma) \cup \text{Guarantee}(\Sigma)$

(iv) $MP^*(\mathbb{B}_4) = \text{Reactivity}(\Sigma)$

Proof We prove each of this facts successively. Let $\Pi = (\phi, \varphi)$ be a *r*-property.

Proof of (i).

- Let $\Pi \in \text{Safety}(\Sigma)$, we show that $\Pi \in MP^*(\mathbb{B}_2^{\perp})$. As $\Pi \in \text{Safety}(\Sigma)$, there exists a finitary property $\psi \subseteq \Sigma^*$, s.t. $\Pi = (A_f(\psi), A(\psi))$. Let us consider $\sigma_{good} \in \phi$ and $\sigma_{bad} \in \overline{\phi}$, we want to prove that the evaluations in $\mathbb{B}_2^{\perp}$ of these two sequences differ. On one hand, we have that $\Pi(\sigma_{good})$ (since $\sigma_{good} \in \phi$) and thus $[\![\Pi]\!]_{\mathbb{B}_2^{\perp}}(\sigma_{good}) =?$. On the other hand, we have that $\neg\Pi(\sigma_{bad})$ and $\sigma_{bad} \notin A_f(\psi)$ (since $\sigma_{bad} \notin \phi$). Using Property 4.2, we have $\forall \mu \in \Sigma^{\infty}, \neg\Pi(\sigma_{bad} \cdot \mu)$, *i.e.*, $[\![\Pi]\!]_{\mathbb{B}_2^{\perp}}(\sigma_{bad}) = \perp$.
- Let $\Pi \in MP^*(\mathbb{B}_2^{\perp})$, we show that $\Pi \in \text{Safety}(\Sigma)$. According to the characterization of safety properties given in Property 4.1, showing that Π is a safety *r*-property amounts to show that it verifies $\Pi = (A_f(Pref(\phi)), A(Pref(\varphi)))$. This is what we do by showing the inclusion in both ways.
  - $\Pi \sqcap \Sigma^* \subseteq A_f(Pref(\phi))$ is immediate as for every sequence $\sigma \in \Pi \sqcap \Sigma^*$ (*i.e.*, $\sigma \in \phi$), σ has all of its prefixes in $Pref(\phi)$. The same holds for $\Pi \sqcap \Sigma^{\omega} \subseteq A(Pref(\varphi))$.
  - Let us show that $A_f(Pref(\phi)) \subseteq \Pi \sqcap \Sigma^*$. Let $\sigma \in A_f(Pref(\phi))$, we prove that $\sigma \in \Pi \sqcap \Sigma^*$. As $\sigma \in A_f(Pref(\phi))$, all prefixes of σ belong to $Pref(\phi)$. That is, all prefixes of σ are the prefixes of a sequence in ϕ. Let σ_{min} the smallest word of ϕ which is an extension of a prefix of σ. We distinguish two cases. If $\sigma_{min} = \sigma$, then $\sigma \in \Pi$. Else ($\sigma \prec \sigma_{min}$), as $\sigma_{min} \in \phi$, we have $[\![\Pi]\!]_{\mathbb{B}_2^{\perp}}(\sigma_{min}) =?$; and consequently $[\![\Pi]\!]_{\mathbb{B}_2^{\perp}}(\sigma) =?$. Using $\Pi \in MP^*(\mathbb{B}_2^{\perp})$, we obtain $\sigma \in \phi$ and consequently $\sigma \in \Pi$.
    The same reasoning can be conducted to show that $A(Pref(\varphi)) \subseteq \Pi \cap \Sigma^{\omega}$.

  Finally, according to the definition of *r*-properties (Definition 2.1), we know that $\Pi = (\phi, \varphi)$ can be written $\Pi = (A_f(Pref(\phi)), A(Pref(\varphi)))$, which gives the expected result.

Proof of (ii).

- The reasoning to prove that $\text{Guarantee}(\Sigma) \subseteq MP^*(\mathbb{B}_2^{\top})$ is similar to the reasoning used to prove $\text{Safety}(\Sigma) \subseteq MP^*(\mathbb{B}_2^{\perp})$. It suffices to show that all bad executions sequences are evaluated to "?". Furthermore, all good execution sequences are evaluated to $\top$. Indeed, once a sequence satisfies a guarantee *r*-property, all its continuations also satisfy it.
- Proving that $MP^*(\mathbb{B}_2^{\top}) \subseteq \text{Guarantee}(\Sigma)$ can be done following the reasoning used to prove $MP^*(\mathbb{B}_2^{\perp}) \subseteq \text{Safety}(\Sigma)$, by showing that if $\Pi \in MP^*(\mathbb{B}_2^{\top})$, then Π verifies $\Pi = \big(E_f(\overline{Pref(\overline{\phi})}), E(\overline{Pref(\overline{\varphi})})\big)$.

Proof of (iii)

- The proof of $\text{Safety}(\Sigma) \cup \text{Guarantee}(\Sigma) \subseteq MP^*(\mathbb{B}_3)$ is evident by noticing that $MP^*(\mathbb{B}_2^{\bot}) \subset MP^*(\mathbb{B}_3)$ and $MP^*(\mathbb{B}_2^{\top}) \subset MP^*(\mathbb{B}_3)$.
- The fact that $MP^*(\mathbb{B}_3) \subseteq \text{Safety}(\Sigma) \cup \text{Guarantee}(\Sigma)$ is given by Lemma 5.2.

Proof of (iv). The proof is straightforward by noticing that every *r*-property can be evaluated by effectively distinguishing good and bad sequences. In others words, every reactivity *r*-property can be evaluated consistently with $\mathbb{B}_4$. Indeed, a good sequence σ_{good} is evaluated to $\top_c$ or $\top$ according to its continuations. A bad sequence σ_{bad} is evaluated to $\bot_c$ or $\bot$ depending on its continuations. As we can see here, the truth values $\bot_c$ and $\top_c$ refine the verdict "?". ∎

EXAMPLE 5.4 (ALTERNATIVE MONITORING OF AN OBLIGATION PROPERTY) *Let us go back again to the property considered in the introduction, stating that "p should* always *hold or q should* eventually *hold". We examine again the execution sequences: $\sigma_{good} = \{p, \overline{q}\} \cdot \{p, \overline{q}\}$ and $\sigma_{bad} = \{p, \overline{q}\} \cdot \{\overline{p}, \overline{q}\}$. In $\mathbb{B}_3$, we have $[\![\Pi]\!]_{\mathbb{B}_3}(\sigma_{good}) = [\![\Pi]\!]_{\mathbb{B}_3}(\sigma_{bad}) = ?$. Thus, Π is not $\mathbb{B}_3$-monitorable. However, Π is $\mathbb{B}_4$-monitorable and $[\![\Pi]\!]_{\mathbb{B}_4}(\sigma_{good}) = \top_c$ and $[\![\Pi]\!]_{\mathbb{B}_4}(\sigma_{bad}) = \bot_c$.*

This example shows how the finite sequence semantics leverages the interest of monitoring. Furthermore, it shows that under our definition of monitoring, ambiguous situations, such as those encountered with the classical definition, are avoided.

Our definition of monitorability has the advantage of being able to identify the properties which should not be monitored with a truth-domain "not fine enough". Indeed the last property shows that if we build a monitor for such a property with the truth-domain $\mathbb{B}_3$, this monitor would produce an evaluation "?" for correct and incorrect execution sequences wrt. the property. This seems not desirable to us.

Furthermore, we have shown in Section 4.3 that, for a given finite sequence σ, $[\![\Pi]\!]_{\mathbb{B}_4}(\sigma)$ is easy to compute from the set of states of a Streett automaton recognizing Π.

5.4 Characterizations in the automata view

Although some spaces of monitorable properties we considered cannot be precisely expressed in terms of Safety-Progress classes, it is still possible to characterize them with some syntactic criteria on Streett automata. It relies on the characterization of the states of Streett automata introduced in Section 4.3.

PROPERTY 5.1 (CORRESPONDANCE BETWEEN STREETT AUTOMATA STATES AND $\mathbb{B}_4$) *Given a Streett m-automaton recognizing a r-property Π and a sequence $\sigma \in \Sigma^*$ of length n s.t. $run(\sigma, \mathcal{A}_\Pi) = q_0 \cdots q_{n-1}$, we have that:*

$$
\begin{array}{c}
q_{n-1} \in Good^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_4}(\sigma) = \top, \\
q_{n-1} \in Good_c^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_4}(\sigma) = \top_c, \\
q_{n-1} \in Bad_c^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_4}(\sigma) = \bot_c, \\
q_{n-1} \in Bad^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_4}(\sigma) = \bot.
\end{array}
$$

Proof The proof of this property is given in Appendix A.2. The proof uses the acceptance criteria of Streett automata to ground the correspondance.

REMARK 5.2 (Correspondance with $\mathbb{B}_3$, $\mathbb{B}_2^{\bot}$, and $\mathbb{B}_2^{\top}$) *From Property 5.1 and Definition 5.4, one can easily deduce a correspondance between the set of states and the evaluation in the truth-domain of a lower cardinality:*

- *For $\mathbb{B}_3$:*
  - $q_{n-1} \in Good^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = \top$,
  - $q_{n-1} \in Good_c^{\mathcal{A}_\Pi} \cup Bad_c^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = ?$,

  - $q_{n-1} \in Bad^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_3}(\sigma) = \bot$.

- *For* $\mathbb{B}_2^\top$ *:*
  - $q_{n-1} \in Good^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_2^\top}(\sigma) = \top$,
  - $q_{n-1} \in Good_c^{\mathcal{A}_\Pi} \cup Bad_c^{\mathcal{A}_\Pi} \cup Bad^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_2^\top}(\sigma) =?$.
- *For* $\mathbb{B}_2^\bot$ *:*
  - $q_{n-1} \in Bad^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_2^\bot}(\sigma) = \bot$,
  - $q_{n-1} \in Bad^{\mathcal{A}_\Pi} \cup Good_c^{\mathcal{A}_\Pi} \cup Bad_c^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!]_{\mathbb{B}_2^\bot}(\sigma) =?$.

Now we are able to give an exact characterization of monitorable properties in the automata view.

THEOREM 5.3 (Automata view of classical monitorability) *The r-property* Π *recognized by the Streett* m*-automaton* $\mathcal{A}_\Pi = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \rightarrow_{\mathcal{A}_\Pi}, \{(R_1, P_1), \ldots, (R_m, P_m)\})$ *is*

- $MP(\mathbb{B}_2^\bot)$*-monitorable iff*
  $\forall q \in Reach({q_{\text{init}}}^{\mathcal{A}_\Pi}), Reach(q) \cap Bad^{\mathcal{A}_\Pi} \neq \emptyset$
- $MP(\mathbb{B}_2^\top)$*-monitorable iff*
  $\forall q \in Reach({q_{\text{init}}}^{\mathcal{A}_\Pi}), Reach(q) \cap Good^{\mathcal{A}_\Pi} \neq \emptyset$
- $MP(\mathbb{B}_3)$*-monitorable iff*
  $\forall q \in Reach({q_{\text{init}}}^{\mathcal{A}_\Pi}), Reach(q) \cap (Bad^{\mathcal{A}_\Pi} \cup Good^{\mathcal{A}_\Pi}) \neq \emptyset$

Proof This property is established by noticing that it is a consequence of Property 5.1 and by noticing that we are considering deterministic and complete Streett automata. Thus the two following facts are equivalent:

- being able to reach from every accessible state, a bad (resp. good, bad or good);
- every finite sequence has a continuation that determines negatively (resp. positively, negatively or positively) the underlying property.

EXAMPLE 5.5 (CLASSICAL MONITORABILITY IN THE AUTOMATA VIEW) *We illustrate the use of the previous theorem to state whether the following properties, with their automata provided, are monitorable according to the classical definition:*

- *The property* Π_1 *specified by* $\mathcal{A}_{\Pi_1}$ *is* $\mathbb{B}_2^\bot$*-monitorable, and thus* $\mathbb{B}_3$*-monitorable; but not* $\mathbb{B}_2^\top$*-monitorable.*
- *The property* Π_2 *specified by* $\mathcal{A}_{\Pi_2}$ *is* $\mathbb{B}_2^\top$*-monitorable and* $\mathbb{B}_2^\bot$*-monitorable, and thus* $\mathbb{B}_3$*-monitorable.*
- *The property* Π_3 *specified by* $\mathcal{A}_{\Pi_3}$ *is* $\mathbb{B}_2^\bot$*-monitorable, and thus* $\mathbb{B}_3$*-monitorable; but not* $\mathbb{B}_2^\top$*-monitorable.*
- *The property* Π_4 *specified by* $\mathcal{A}_{\Pi_4}$ *is* $\mathbb{B}_2^\bot$*-monitorable, and thus* $\mathbb{B}_3$*-monitorable; but not* $\mathbb{B}_2^\top$*-monitorable.*

THEOREM 5.4 (Automata view of alternative monitorability) *The r-property* Π *recognized by the Streett* m*-automaton* $\mathcal{A}_\Pi = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \rightarrow_{\mathcal{A}_\Pi}, \{(R_1, P_1), \ldots, (R_m, P_m)\})$ *is*

- $MP^*(\mathbb{B}_2^\bot)$*-monitorable iff* $Bad^{\mathcal{A}_\Pi} = \bigcup_{i=1}^m \overline{R_i} \cap \overline{P_i}$*;*
- $MP^*(\mathbb{B}_2^\top)$*-monitorable iff* $Good^{\mathcal{A}_\Pi} = \bigcap_{i=1}^m R_i \cup P_i$*;*
- $MP^*(\mathbb{B}_3)$*-monitorable iff*
  $\nexists q \in Reach({q_{\text{init}}}^{\mathcal{A}_\Pi}) \cap \bigcap_{i=1}^m R_i \cup P_i, \nexists q' \in Reach({q_{\text{init}}}^{\mathcal{A}_\Pi}) \cap \bigcup_{i=1}^m \overline{R_i} \cap \overline{P_i}, q \in Good_c^{\mathcal{A}_\Pi} \wedge q' \in Bad_c^{\mathcal{A}_\Pi}$

Proof The proof is conducted in three stages:

- The expressed condition generalizes the syntactic restriction of Streett safety automata and is also a condition when a given not minimal Streett m-automaton is recognizing a safety property and can be minimized so as to be represented as a Streett safety automaton.

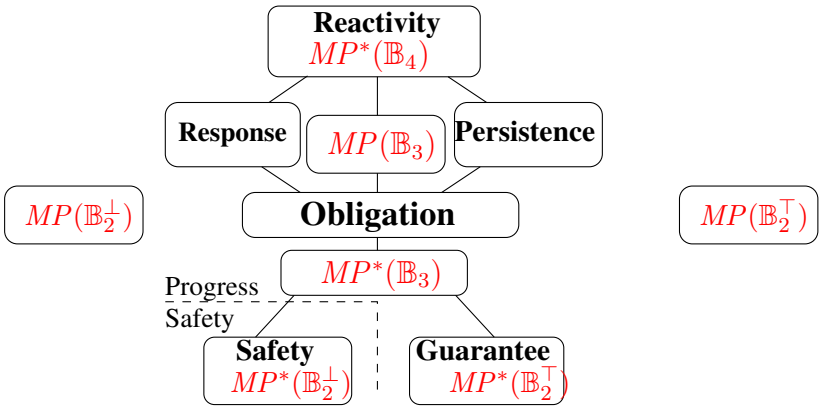


Figure 11: Monitorable *r*-properties in the Safety-Progress classification

- The expressed condition generalizes the syntactic restriction of Streett guarantee automata and is also a condition when a given not minimal Streett m-automaton is recognizing a guarantee property and can be minimized so as to be represented as a Streett safety automaton.
- The third condition can be established using the two following facts:
  - A *r*-property is not monitorable according to this theorem iff for two sequences, a good and a bad sequences evaluate to "?". Others evaluations are not simultaneously possible for a bad and good sequences.
  - We are considering deterministic and complete Streett automata.

5.5 Summary

We depict in Fig. 11 the main results obtained in this section, which can be summarized as follows:

- The classes of monitorable properties, according to the classical definition, are:
  - $MP(\mathbb{B}_2^\top)$, which can not be compared directly with any other class;
  - $MP(\mathbb{B}_2^\bot)$, which can not be compared directly with any other class;
  - and $MP(\mathbb{B}_3) = MP_c$ which contains strictly the class of obligation properties.
- The classes of monitorable properties, according to the new definition we introduced are:
  - $MP^*(\mathbb{B}_2^\bot)$ is the set of safety *r*-properties;
  - $MP^*(\mathbb{B}_2^\top)$ is the set of guarantee *r*-properties;
  - $MP^*(\mathbb{B}_3)$, is strictly contained in the class of obligation *r*-properties;
  - and $MP^*(\mathbb{B}_4)$ is the set of reactivity *r*-properties.

Remark that using the truth-domain $\mathbb{B}_4$ does not add any expressiveness to the classical definition of monitorability (*i.e.*, $MP(\mathbb{B}_4) = MP(\mathbb{B}_3)$). Indeed, this definition is bound to the notion of positive and negative determinacy. However using this domain would permit to better distinguish execution sequences and to avoid ambuiguity exposed in Example 5.1. Note also that some obligation properties (between $MP(\mathbb{B}_3)$ and $MP^*(\mathbb{B}_3)$) should not be monitored unless with a truth domain equipped with an interpretation of finite sequences allowing to distinguish good and bad finite sequences (*e.g.*, with truth-values $\bot_c$ and $\top_c$).

6 Enforceability wrt. the SP classification

Now we adress the question of the expressiveness of runtime enforcement, that is we characterize the class of enforceable properties. In Section 3, we have seen that the previous proposed classes were delineated according to the mechanism used to enforce the properties. Such mechanisms should obey the soundness and transparency constraints. We choose here to take an alternative approach. Indeed we believe that the set of enforceable properties can be characterized independently from any enforcement mechanism complying to these constraints; provided that this memory is unbounded but finite. This will give us an upper-bound of the set of enforceable properties.

6.1 Enforcement criteria

The enforcement constraints exposed in Section 3, namely **soundness** and **transparency**, express a relation between the input sequence (submitted to an enforcement monitor) and an output sequence (produced by this monitor). We interpret these constraints in the following way: if the input sequence already verifies the property, then it should remain unchanged (up to a given equivalence relation), otherwise its *longest prefix* satisfying the property should be issued[10].

A consequence is that a *r*-property (ϕ, φ) will be considered as *enforceable* only if each incorrect infinite sequence has a *longest* correct prefix. This means that any infinite incorrect sequence should have only a finite number of correct prefixes[11]. We give two enforcement criteria, in the language and automata views.

DEFINITION 6.1 (ENFORCEMENT CRITERION (LANGUAGE VIEW)) *A r-property (ϕ, φ) is said to be enforceable iff* $\forall \sigma \in \Sigma^\omega$,

$$\neg\varphi(\sigma) \Rightarrow (\exists \sigma' \in \Sigma^*, \sigma' \prec \sigma, \forall \sigma'' \in \Sigma^*, \sigma' \prec \sigma'' \Rightarrow \neg\phi(\sigma'')) \tag{4}$$

Alternatively, a *r*-property Π recognized by a Streett automaton $\mathcal{A}_\Pi$ is said to be enforceable iff every strongly-connected component (SCC) of $\overline{R}$-states contain (only) either P-states or $\overline{P}$-states.

DEFINITION 6.2 (ENFORCEMENT CRITERION (AUTOMATA VIEW)) *Denoting $\mathcal{S}(\mathcal{A}_\Pi)$ the set of SCC of $\mathcal{A}_\Pi$, an m-automaton, recognizing Π, Π is said to be enforceable iff*

$$\forall i \in [1, m], \forall s \in \mathcal{S}(\mathcal{A}_\Pi), s \subseteq \overline{R_i} \Rightarrow (s \subseteq \overline{P_i} \vee s \subseteq P_i) \tag{5}$$

Enforcement criteria of Definitions 6.1 and 6.2 are equivalent for basic classes of properties, as stated below.

PROPERTY 6.1 (EQUIVALENCE BETWEEN ENFORCEMENT CRITERIA (BASIC CLASSES)) *Considering a r-property $\Pi = (\phi, \varphi)$ of a basic class, recognized by a Streett automaton $(Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \Sigma, \rightarrow^{\mathcal{A}_\Pi}, \{(R, P)\})$, we have that:*

$$(4) \Leftrightarrow (5).$$

Proof For basic classes $\Leftrightarrow \forall s \in \mathcal{S}(\mathcal{A}_\Pi), s \subseteq \overline{R} \Rightarrow (s \subseteq \overline{P} \vee s \subseteq P)$. The complete proof is given in Sect. A.3.1. This proof relies on the computation of strongly connected components [Tar72] of a Streett automaton (SCC). The proof is in two stages by proving implications in both ways.

- $(4) \Rightarrow (5)$ is done by considering a SCC of $\mathcal{A}_\Pi$ containing only $\overline{R}$-states. The proofs shows that there cannot exist two states q, q' in this SCC s.t. $q \in P$ and $q' \notin P$.

[10] Another possible interpretation could consist in correcting an erroneous sequence by adding extra events.

[11] Note that those criteria differ from the existence of bad prefixes. Bad prefixes are sequences which cannot be extended to correct (finite or infinite) ones, *i.e.*, sequences that determine negatively the underlying property.

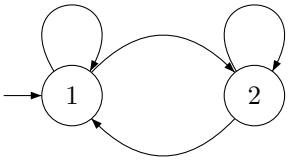


Figure 12: 2-reactivity automaton for which (4) $\not\Rightarrow$ (5)

- (5) $\Rightarrow$ (4). is done by by considering a sequence $\sigma \in \Sigma^\omega$ s.t. $\neg\varphi(\sigma)$ and analyzing the run of this infinite sequence on the last visited SCC.

However, these two enforcement criteria are not equivalent for general reactivity properties, as stated below.

PROPERTY 6.2 (COMPARING ENFORCEMENT CRITERIA FOR COMPOUND CLASSES) *Considering a Streett automaton* $(Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \Sigma, \rightarrow^{\mathcal{A}_\Pi}, \{(R,P)\}$ *recognizing a r-property* $\Pi = (\phi, \varphi)$*, we have that:*

$$(4) \Leftrightarrow (5), \textit{ for Obligation properties}$$
$$(4) \Leftarrow (5), \textit{ for Reactivity properties}$$

Proof We sketch the proof for both classes of properties.

For Obligation properties. Similarly to the proof of Property 6.1, the proof relies on the fact that in a m-obligation automaton, for $i \in [1,m]$, there is no transition from R_i-states to $\overline{R_i}$-states, and no transition from $\overline{P_i}$-states to P_i-states.

For Reactivity properties. Let us consider $\sigma \in \Sigma^\omega$ s.t. $\neg\varphi(\sigma)$. Similarly to the proof of Property 6.1 ($\Leftarrow$ direction), the run of σ on $\mathcal{A}_\Pi$ visits a SCC infinitely often and can be expressed:

$run(\sigma, \mathcal{A}_\Pi) = q_0 \cdots q_{k-1} \cdot (q_i + \cdots + q_{i+l})^n$ with $k \leq i \wedge l \leq |Q|$.

The states $q_i, \ldots, q_{i+l}$ are the last states visited by the run of σ on $\mathcal{A}_\Pi$, *i.e.*, they are the states visited infinitely often. Moreover, we know that $\forall i \leq j \leq i+l, q_j \in P \vee \forall i \leq j \leq i+l, q_j \in \overline{P}$. We have that $\forall \sigma' \in \Sigma^*, \sigma_{\ldots k} \preceq \sigma', \neg\Pi(\sigma')$. Indeed, otherwise it would mean that $\forall i \in [1,m], \forall j \geq k, q_j \in P_i$, which would lead to $\varphi(\sigma)$ using the infinite-sequence acceptance condition of Streett automata. ∎

REMARK 6.1 *We give an example of 2-reactivity Streett automaton for which* (4) $\not\Rightarrow$ (5)*. This automaton is depicted in Fig. 12. It has two states, its alphabet does not matter, and the pairs of accepting states* (R_1, P_1) *and* (R_2, P_2) *are defined as follows:*

$R_1 = \emptyset$, $P_1 = \emptyset$,

$R_2 = \emptyset$, $P_2 = \{1\}$.

The property recognized by this Streett automaton verifies condition (4)*, but not condition* (5)*.*

The set of enforceable *r*-properties is denoted EP. We will now characterize this set wrt. the SP classification. Though, we will prove that the class of enforceable properties is *exactly* the class of response properties. Note that the enforcement criterion in the automata view is still useful as it provides a syntactic procedure to determine whether a property is enforceable or not.

6.2 Enforceable properties

We start first by proving that response properties (defined in Section 4) are enforceable. Then, we give an example of a persistence property which is not enforceable. Thus, we prove that the set of response properties is exactly the set EP.

THEOREM 6.1 (Response are enforceable) $Response(\Sigma) \subseteq EP$

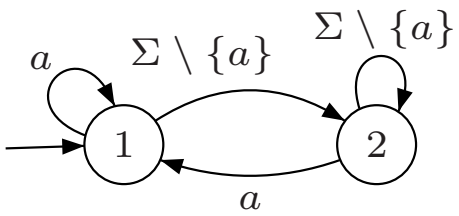


Figure 13: Non-enforceable persistence *r*-property

Proof Indeed consider a response *r*-property $\Pi = (\phi, \varphi)$ and an execution sequence $\sigma \in \Sigma^\omega$. Π can be expressed as $(R_f(\psi), R(\psi))$ for a given finitary language ψ. Let us suppose that $\neg\varphi(\sigma)$. It means that $\sigma \notin R(\psi)$, *i.e.*, σ has finitely many prefixes belonging to ψ. Consider the set $S = \{\sigma' \in \Sigma^* \mid \forall \sigma'' \in \Sigma^*, \sigma' \prec \sigma'' \prec \sigma \wedge \neg\psi(\sigma'')\}$ of finite sequences from which all finite continuations do not satisfy ψ. As $\neg R(\psi)(\sigma)$, this set is not empty. Let us note σ_0 the smallest element of S regarding $\prec$. We have $\forall \sigma' \in \Sigma^*, \sigma_0 \prec \sigma' \Rightarrow \neg\psi(\sigma')$. Since $\forall \psi \subseteq \Sigma^*, R_f(\psi) \subseteq \psi$ (cf. the definition of operators building finitary properties), it implies that $\forall \sigma' \in \Sigma^*, \sigma_0 \prec \sigma' \Rightarrow \neg\phi(\sigma')$. ∎

A straightforward consequence is that safety, guarantee and obligation *r*-properties are enforceable. We prove that, in fact, pure persistence properties are not enforceable.

For $\Sigma \supseteq \{a, b\}$, an example of pure persistence *r*-property is $\Pi = (\Sigma^* \cdot a^+, \Sigma^* \cdot a^\omega)$ stating that "it will be eventually true that a always occurs". One can notice that this property is neither a safety, guarantee nor obligation property. Π is recognized by the Streett automaton $\mathcal{A}_\Pi$ depicted on Figure 13 (with $P = \{1\}$). One can understand the enforcement limitation intuitively with the following argument: if this property was enforceable it would imply that an EM can decide from a certain point that the underlying program will always produce the event a. However such a decision can never be taken by a monitor without memorizing the entire execution sequence beforehand. This is unrealistic for an infinite sequence. More formally, as stated in the previous section, a *r*-property (ϕ, φ) is enforceable if for all infinite execution sequences σ when $\neg\varphi(\sigma)$, the longest prefix of σ satisfying ϕ always exists. For the above automaton, the execution sequence $\sigma'_{bad} = (a \cdot b)^\omega$ does not satisfy the property whereas an infinite number of its prefixes do (prefixes ending with a).
Applying enforcement criteria (Definitions 6.1 and 6.2) on persistence properties, it turns out that the enforceable persistence properties are in fact response properties.

THEOREM 6.2 (Enforceable persistence properties are response properties)

$$\text{Persistence}(\Sigma) \cap EP \subseteq \text{Response}(\Sigma)$$

Proof A *r*-property becomes non-enforceable as soon as there exists a SCC of $\overline{R}$-states containing a P-state and a $\overline{P}$-state on its recognizing automaton (see Definition 6.2). Indeed, on a Streett automaton it allows infinite invalid execution sequences with an infinite number of valid prefixes. When removing this possibility on a Streett automaton, the constrained automaton can be easily translated to a response automaton. Indeed, on this constrained automaton, the states visited infinitely often are either all in P or $\overline{P}$, that is: $\forall \sigma \in \Sigma^\omega, \mathit{vinf}(\sigma) \cap P \neq \emptyset \Leftrightarrow \mathit{vinf}(\sigma) \subseteq P$. On such automaton there is no difference between R-states and P-states. Consequently by re-tagging P-states to R, this automaton recognizes the same property. The re-tagged automaton is a response automaton. ∎

COROLLARY 6.1 *Pure persistence are not enforceable:*

$$(\text{Persistence}(\Sigma) \setminus \text{Response}(\Sigma)) \cap EP = \emptyset$$

Proof This is a direct consequence of Theorem 6.2. ∎

COROLLARY 6.2 *Pure reactivity are not enforceable:*

$$\text{Reactivity}(\Sigma) \not\subseteq EP$$
$$\text{Reactivity}(\Sigma) \setminus (\text{Persistence}(\Sigma) \cup \text{Response}(\Sigma)) \cap EP = \emptyset$$

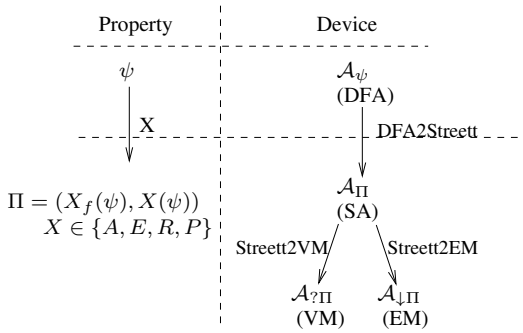


Figure 14: Automaton transformations

Proof This is a direct consequence of Corollary 6.1. A general reactivity property can be expressed as the composition of response and persistence properties. As a consequence, pure persistence properties are included in the set of reactivity properties. And consequently, the persistence part of a reactivity property is not enforceable. ∎

COROLLARY 6.3 *Enforceable properties are exactly response properties:*

$$EP = \text{Response}(\Sigma)$$

Proof It remains to prove that the set of enforceable properties is included in the set of response properties. Suppose that there exists an enforceable property which is not a response one. Then, according to the definition of the Safety-Progress hierarchy, this property would be a pure persistence or reactivity property. Consequently this property would not be enforceable. ∎

EXAMPLE 6.1 (ENFORCEABLE AND NOT ENFORCEABLE PROPERTIES) *We illustrate the enforcement criterion on the properties introduced in Example 4.1 and represented by their Streett automata in Example 4.2.*

- *Properties* Π_1, Π_2, Π_3 *are enforceable;*
- *Property* Π_4 *is not enforceable; e.g., the infinite sequence* $r \cdot g \cdot (r \cdot d \cdot r \cdot g)^\omega$ *is not accepted while this sequence has an infinite number of correct prefixes: e.g., all sequences belonging to* $r \cdot g \cdot (r \cdot d \cdot r \cdot g)^*$.

Being enforceable or not can be determined rather easily by observing the automata and using the accepting criteria for finite and infinite sequences.

7 Monitor synthesis

Now we show how it is possible to obtain easily a monitor either for verifying or enforcing a property thanks to the framework introduced in Section 4. Generally speaking, a monitor is a device processing an input sequence of events or states in an incremental fashion. It is purposed to yield a property-specific decision according to its goal. In (classic) runtime verification such a decision is a truth-value taken from a truth-domain. This truth-value states an appraisal of property satisfaction or violation by the input sequence. For runtime enforcement, the monitor produces a sequence of enforcement operations. The monitor uses an internal memory and applies enforcement operations to the input event and its current memory so as to modify input sequence and produce an output sequence. The relation between the input and output sequences should follow enforcement monitoring constraints: soundness and transparency (Sect. 3.2). In the following we consider a Streett m-automaton $\mathcal{A}_\Pi = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \rightarrow_{\mathcal{A}_\Pi}, \{(R_1, P_1), \ldots, (R_m, P_m)\}$ and Π the r-property recognized by $\mathcal{A}_\Pi$. Also we evaluate properties only in $\mathbb{B}_4$, and consequently we abbreviate $[\![\Pi]\!]_{\mathbb{B}_4}(\cdot)$ by $[\![\Pi]\!](\cdot)$.

The general monitor synthesis procedure is depicted in Fig. 14. From a "pattern" X corresponding to one of the basic classes of the hierarchy and a DFA $\mathcal{A}_\psi$ recognizing a finitary property $\psi \subseteq \Sigma^*$,

DFA2S_X yields a Streett automaton recognizing the *r*-property $(X_f(\psi), X(\psi))$. Then using *Streett2VM* (resp. *Streett2EM*) one is able to obtain a verfication (resp. enforcement) monitor for the *r*-property $(X_f(\psi), X(\psi))$.

7.1 Monitor: a general definition

A monitor is a procedure consuming events fed by an underlying program and producing an appraisal in the current state depending on the sequence read so far. Considered monitors are deterministic finite-state machines producing an output in a relevant domain. This domain will be refined for special-purpose monitors (verification and enforcement). For verification monitors, this output function gives a truth-value (a verdict) in $\mathbb{B}_4$ regarding the evaluation of the current sequence relatively to the desired property. For enforcement monitors (EMs), this output function gives an enforcement operation inducing a modification on the input sequence so as to enforce the desired property.

DEFINITION 7.1 (MONITOR) *A* monitor $\mathcal{A}$ *is a 5-tuple* $(Q^{\mathcal{A}}, {q_{\text{init}}}^{\mathcal{A}}, \longrightarrow_{\mathcal{A}}, X^{\mathcal{A}}, \Gamma^{\mathcal{A}})$ *defined relatively to a set of events* Σ*. The finite set* $Q^{\mathcal{A}}$ *denotes the control states and* ${q_{\text{init}}}^{\mathcal{A}} \in Q^{\mathcal{A}}$ *is the initial state. The complete function* $\longrightarrow_{\mathcal{A}}$*:* $Q^{\mathcal{A}} \times \Sigma \to Q^{\mathcal{A}}$ *is the transition function. In the following we abbreviate* $\longrightarrow_{\mathcal{A}} (q, a) = q'$ *by* $q \xrightarrow{a}_{\mathcal{A}} q'$*. The set of values* $X^{\mathcal{A}}$ *depends on the purpose of the monitor (verification or enforcement). The function* $\Gamma^{\mathcal{A}} : Q^{\mathcal{A}} \to X^{\mathcal{A}}$ *is an output function, producing values in* $X^{\mathcal{A}}$ *from states.*

7.2 Synthesizing monitors for runtime verification

In the following, we consider monitorable *r*-properties $\Pi, (\phi, \varphi)$.

DEFINITION 7.2 (VERIFICATION MONITOR) *A* verification monitor (VM) $\mathcal{A}_?$ *is a monitor, i.e., a 5-tuple* $(Q^{\mathcal{A}_?}, {q_{\text{init}}}^{\mathcal{A}_?}, \longrightarrow_{\mathcal{A}_?}, \mathbb{B}_4, \Gamma)$*.* $\Gamma : Q^{\mathcal{A}_?} \to \mathbb{B}_4$ *is a classical output function, producing truth-values of* $\mathbb{B}_4$ *from states.*

Such monitors are independent from any specification formalism, and can be easily adapted to the specification formalism from which they are generated. We define, the notion of *verification sequence* produced by a monitor and what it means to verify a property for a monitor.

DEFINITION 7.3 (SEQUENCE VERIFICATION) *We define the verification performed by a VM* $\mathcal{A}_?$ *while reading an input* $\sigma \in \Sigma^*$ *(produced by* $\mathcal{P}_\Sigma$*) and producing a sequence* $b \in {\mathbb{B}_4}^+$*. The verification function* $[\![\mathcal{A}_?]\!](\cdot) : \Sigma^* \to {\mathbb{B}_4}^+$*, defining the verification performed by* $\mathcal{A}_?$*, is the verification sequence produced by* $\mathcal{A}_?$ *while reading* σ*. This verification sequence is obtained by the output of the monitor, in the following way:*

$$\forall \sigma \in \Sigma^*, [\![\mathcal{A}_?]\!](\sigma) = \Gamma^{\mathcal{A}_?}(q) \tag{6}$$

with ${q_{\text{init}}}^{\mathcal{A}_?} \xrightarrow{\sigma}_{\mathcal{A}_?} q$

The sequence σ is verified by $\mathcal{A}_?$ if it is verified from the initial state (6).

DEFINITION 7.4 (MONITOR SOUNDNESS) *A monitor* $\mathcal{A}_?$ *is sound wrt.* $\Pi = (\phi, \varphi) \in MP^*(\mathbb{B}_4)$ *on* $\mathcal{P}_\Sigma$*, noted* $Ver(\mathcal{A}_?, \Pi, \mathcal{P}_\Sigma)$*, iff*

$$\forall \sigma \in Exec(\mathcal{P}_\Sigma) \cap \Sigma^*, [\![\mathcal{A}_?]\!](\sigma) = [\![\Pi]\!]_{\mathbb{B}_4}(\sigma).$$

where $[\![\cdot]\!]_{\mathbb{B}_4}$ *is defined in Definition 5.4.*

This definition states that the verification sequence produced by $\mathcal{A}_?$ matches the evaluation function of a sequences wrt. a *r*-property.

Using the set $\mathbb{P}^{\mathcal{A}_\Pi}$ of a Streett automaton $\mathcal{A}_\Pi$, we show how it is possible to obtain a verification monitor for the *r*-property Π.

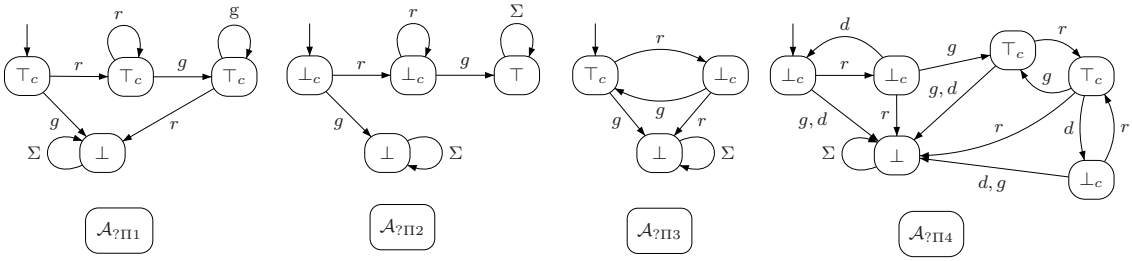


Figure 15: Examples of Verification Monitors

DEFINITION 7.5 (STREETT2VM TRANSFORMATION) *Given a Streett m-automaton $\mathcal{A}_\Pi = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \Sigma, \longrightarrow_{\mathcal{A}_\Pi}, \{(R_1, P_1), \ldots, (R_m, P_m)\})$ recognizing $\Pi \in MP^*(\mathbb{B}_4)$, we define the transformation* $\text{Streett2VM}(\mathcal{A}_\Pi) = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \rightarrow_{\mathcal{A}_\Pi}, \mathbb{B}_4, \Gamma)$ *s.t.* $\Gamma : Q^{\mathcal{A}_?} \rightarrow \mathbb{B}_4$ *produces truth-values from states depending on the set* $\mathbb{P}^{\mathcal{A}_\Pi}$*:* $\forall q \in Q^{\mathcal{A}_\Pi}$,

$$q \in Good^{\mathcal{A}_\Pi} \Rightarrow \Gamma(q) = \top \; ; \; q \in Good_p^{\mathcal{A}_\Pi} \Rightarrow \Gamma(q) = \top_p$$
$$q \in Bad_p^{\mathcal{A}_\Pi} \Rightarrow \Gamma(q) = \bot_p \; ; \; \forall q \in Bad^{\mathcal{A}_\Pi} \Rightarrow \Gamma(q) = \bot$$

A *r*-property Π is verifiable on $\mathcal{P}_\Sigma$ by a VM $\mathcal{A}_{?\Pi}$ obtained by the application of Streett2VM on the automaton recognizing Π.

EXAMPLE 7.1 (VERIFICATION MONITORS) *In Fig. 15 are represented VMs for the properties introduced in Example 4.1, specified by Streett automata of Fig. 3, and synthesized with the Streett2VM transformation.*

THEOREM 7.1 (Correctness of Streett2VM) *Given $\mathcal{A}_\Pi$ recognizing Π, we have:*

$$(\Pi \in MP^*(\mathbb{B}_4) \wedge \mathcal{A}_{?\Pi} = \text{Streett2VM}(\mathcal{A}_\Pi)) \Rightarrow Ver(\mathcal{A}_{?\Pi}, \Pi, \mathcal{P}_\Sigma)$$

Proof The proof of this theorem relies on the correctness of the computation performed while obtaining $\mathbb{P}^{\mathcal{A}_\Pi}$ for $\mathcal{A}_\Pi$ (Prop. 5.1).

7.3 Synthesizing monitors for runtime enforcement

In the remainder, we consider enforceable *r*-properties (ϕ, φ) and $\Pi \in EP$. An EM is producing enforcement operations depending on its current state.

DEFINITION 7.6 (ENFORCEMENT MONITOR) *An EM $\mathcal{A}_!$ is a 5-tuple $(Q^{\mathcal{A}_!}, {q_{\text{init}}}^{\mathcal{A}_!}, \longrightarrow_{\mathcal{A}_!}, Ops, \Gamma)$. Enforcement operations of Ops performed by the EM are aimed to operate a modification of the internal memory and potentially produce an output, i.e., each enforcement operation is a function: $\Sigma \times \Sigma^* \rightarrow \Sigma^* \times \Sigma^*$. Then $\Gamma : Q^{\mathcal{A}_\downarrow} \rightarrow Ops$ is the output function, producing enforcement operations from states.*

The considered enforcement operations allow enforcement monitors either to halt the target program (when the current input sequence irreparably violates the property), or to store the current event in a *memory device* (when a decision has to be postponed), or to dump the content of the memory device (when the target program went back to a correct behavior), or to switch off the monitor when all possible continuations of the current input sequence are correct wrt. the property under scrutiny[12].

DEFINITION 7.7 (ENFORCEMENT OPERATIONS) *We define a set of enforcement operations $Ops = \{halt, store, dump, off\}$ as follows:* $\forall a \in \Sigma \cup \{\epsilon_\Sigma\}, \forall m \in \Sigma^*$,

$$halt(a, m) = (\epsilon_\Sigma, m) \qquad store(a, m) = (\epsilon_\Sigma, m.a)$$
$$dump(a, m) = (m.a, \epsilon_\Sigma) \qquad off(a, m) = (m.a, \epsilon_\Sigma)$$

[12] Although, *dump* and *off* have the same definition, distinguishing them is useful in practice. Indeed, the *off* operation is intended to be produced when all continuations of the current execution sequence are correct wrt. the property. It allows to determine when the EM is not needed anymore. Consequently, it helps reducing the performance impact on the underlying program

(a designates the input event of the monitor and m the memory device content.)

Note that the *off* and *dump* operations have the same definitions. From a theoretical perspective, the *off* is indeed not necessary. However, it has a practical interest. In order to limit the monitor's impact on the original program (performance wise), it is of interest to know when the monitor is not needed anymore.

We define the transformation performed by an EM $\mathcal{A}_!$ while reading an input sequence $\sigma \in \Sigma^*$ and producing an output sequence $o \in \Sigma^*$.

DEFINITION 7.8 (SEQUENCE TRANSFORMATION) *The transformation function $[\![\mathcal{A}_!]\!](\cdot) : \Sigma^* \to \Sigma^*$ relies on the function $[\![\mathcal{A}_!]\!](\cdot,\cdot,\cdot) : \Sigma^* \times Q^{\mathcal{A}_!} \times \Sigma^* \to \Sigma^*$ defining the transformation performed on the current state and the internal memory content: $[\![\mathcal{A}_!]\!](\sigma, q, m)$ is the output sequence produced while reading σ from state q and (initial) memory content m.*

$$\forall q \in Q^{\mathcal{A}_!}, \forall m \in \Sigma^*, [\![\mathcal{A}_!]\!](\epsilon_\Sigma, q, m) = \epsilon_\Sigma \tag{7}$$

$$[\![\mathcal{A}_!]\!](a \cdot \sigma, q, m) = o \cdot [\![\mathcal{A}_!]\!](\sigma, q', m') \tag{8}$$

with $q \xrightarrow{a}_{\mathcal{A}_!} q' \wedge \Gamma(q') = \alpha \in Ops \wedge \alpha(a, m) = (o, m')$

The empty sequence ϵ_Σ is transformed into itself by $\mathcal{A}_!$, this is the case when the underlying program does not produce any event (7). An execution sequence $a \cdot \sigma$ is (incrementally) transformed according to the transition fired by the input a: the current memory content and the input a are applied to the enforcement operation of the arriving-state transition, it induces a new memory content and an output o (8).

We define now the notion of property-enforcement by an EM. The notion of enforcement relates the input sequence produced by the program and given to the EM and the output sequence allowed by the EM (correct wrt. the property under consideration)[13].

DEFINITION 7.9 (PROPERTY-ENFORCEMENT) *For $\Pi = (\phi, \varphi) \in EP$, we say that $\mathcal{A}_!$ enforces Π on $\mathcal{P}_\Sigma$, noted $Enf(\mathcal{A}_!, \Pi, \mathcal{P}_\Sigma)$, iff for all $\sigma \in Exec(\mathcal{P}_\Sigma) \cap \Sigma^*$, there exists $o \in \Sigma^\infty$, s.t. the following constraints hold:*

$$[\![\mathcal{A}_!]\!](\sigma, {q_{\text{init}}}^{\mathcal{A}_!}, \epsilon) = o \tag{9}$$

$$\Pi(\sigma) \Rightarrow \sigma = o \tag{10}$$

$$\neg\Pi(\sigma) \wedge Pref_\prec(\phi, \sigma) = \emptyset \Rightarrow o = \epsilon \tag{11}$$

$$\neg\Pi(\sigma) \wedge Pref_\prec(\phi, \sigma) \neq \emptyset \Rightarrow o = Max(Pref_\prec(\phi, \sigma)) \tag{12}$$

(9),(10),(11) and (12) ensure soundness and transparency of $\mathcal{A}_!$: (9) stipulates that the sequence σ is transformed by $\mathcal{A}_!$ into a sequence o; (10) ensures that if σ satisfied already the property then it is not transformed. When there is no correct prefix of σ satisfying the property, (11) ensures that the EM outputs nothing (the empty sequence ϵ_Σ). If there exists a prefix of σ satisfying the property (12) ensures that o is the longest prefix of σ satisfying the property.

Soundness is due to the fact that the produced sequence o, when different from ϵ_Σ, always satisfies the property ϕ. Transparency is ensured by the fact that correct execution sequence are not changed, and incorrect ones are restricted to their longest correct prefix.

One may remark that we could have set $Max(Pref_\prec(\phi, \sigma))$ to ϵ_Σ when $Pref_\prec(\phi, \sigma) = \emptyset$ and merge the two last constraints. However, we choose to distinguish explicitly the case in which $Pref_\prec(\phi, \sigma) = \emptyset$ as it highlights some differences when an EM produces ϵ_Σ. Sometimes it corresponds to the only correct prefix of the property. But it can be also an incorrect sequence wrt. the property. In practice, when implementing an EM for a system, this sequence can be "tagged" as incorrect[14].

Finally, since we have to deal with potentially infinite input sequences, the output sequence should be produced in an incremental way[15]: for each current prefix σ of the input sequence read by the EM,

[13] In the general case, the comparison between input and output sequences is performed up to some equivalence relation $\approx \subseteq \Sigma^\infty \times \Sigma^\infty$. Note that the considered equivalence relation should preserve the *r*-property under consideration.

[14] This latter case is avoided in [LBW09] by assuming that properties under consideration always contain ϵ_Σ.

[15] From a more general perspective, we can see this limitation from a runtime verification point of view. Verifying infinitary properties at runtime on a produced execution sequence, in essence, should be done by checking finite prefixes of the current sequence.

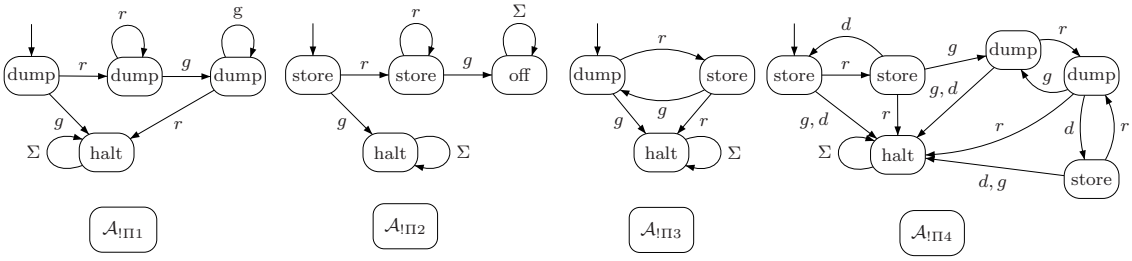


Figure 16: Examples of Enforcement Monitors

the *current* output o produced should be sound and transparent with respect to Π and σ. This means that deciding wether a finite sequence σ verifies Π or not should be computable in a finite amount of time (and reading only a finite continuation of σ).

We synthesize EMs from Streett automata in the framework of *r*-properties. This transformation was previously introduced in [FFM08] in the form of several transformations specific to each class of enforceable properties. Here we generalize those transformations into a unique one.

DEFINITION 7.10 (STREETT2EM TRANSFORMATION) *Given a Streett m-automaton $\mathcal{A}_\Pi = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \Sigma, \longrightarrow_{\mathcal{A}_\Pi}, \{(R_1, P_1), \ldots, (R_m, P_m)\})$ recognizing an enforceable r-property $\Pi \in EP$.*
We define the transformation $\text{Streett2EM}(\mathcal{A}_\Pi) = (Q^{\mathcal{A}_\Pi}, {q_{\text{init}}}^{\mathcal{A}_\Pi}, \rightarrow_{\mathcal{A}_\Pi}, Ops, \Gamma)$ *s.t.* $\Gamma : Q^{\mathcal{A}_{!\Pi}} \rightarrow Ops$ *produces enforcement operations:* $\forall q \in \mathcal{A}_\Pi$

$$q \in Good^{\mathcal{A}_\Pi} \Rightarrow \Gamma(q) = off \ ; \ q \in Good_p^{\mathcal{A}_\Pi} \Rightarrow \Gamma(q) = dump$$
$$q \in Bad_p^{\mathcal{A}_\Pi} \Rightarrow \Gamma(q) = store \ ; \ q \in Bad^{\mathcal{A}_\Pi} \Rightarrow \Gamma(q) = halt$$

EXAMPLE 7.2 (VERIFICATION MONITORS) *In Fig. 16 are represented EMs for the properties introduced in Example 4.1, specified by Streett automata of Fig. 3, and synthesized with the Streett2EM transformation.*

A *r*-property $\Pi \in EP$ is enforceable on $\mathcal{P}_\Sigma$ by an EM obtained by the application of Streett2EM on the automaton recognizing Π.

THEOREM 7.2 (Correctness of Streett2EM) *Given $\mathcal{A}_\Pi$ recognizing Π, we have:*
$(\Pi \in EP \wedge \mathcal{A}_{!\Pi} = \text{Streett2EM}(\mathcal{A}_\Pi)) \Rightarrow Enf(\mathcal{A}_{!\Pi}, \Pi, \mathcal{P}_\Sigma)$

Proof Correctness of the general transformation relies on the correctness (proved in [FFM08]) of the transformations specific to each class of properties. Indeed, this general transformation reduces to the specific transformation when applied to a specific class of properties.

Such a unified transformation is usefull in practice (from an implementation point of view) as it can be applied to any Streett automaton (regardless of the class of the recognized property).

7.4 Discussion

One of the important challenges in runtime verification is the practical interest in the specification formalisms. An ideal specification language should be easy to use by end-users. Moreover, a desirable feature, advocated by this paper, is the need for adressing both infinite and finite execution sequences.

One could reproach the two following facts to the proposed synthesis approach:

- First, several mechanisms are needed: DFAs, Streett automata, and finally verification and enforcement monitors.
- Second, one may question the usefulness of the DFA to Streett transformations. Arguably, it would be possible to generate monitors directly for properties specified by Streett automata (written by hand or generated from temporal logic formula).

In our point of view, using the DFA2Streett transformations in the synthesis process has the two following advantages:

- The direct translation from a supposed ideal specification formalism to a Streett automaton would be difficult. This formalism would have to adress both finite and infinite behaviors; requiring a design expertise. It is likely that such a translation would be error-prone or lead to ambiguous specifications.
- One of the underlying arguments for using our approach is that the end-user is only required to specify a finite behavior and indicate the wished pattern (used with the finitary property). Thus, the infinite behavior is comprehended by the user by only seeing patterns like "always", "at least once", "regularly", or "persistently". The user is thus kept from specifying the infinite behavior with Streett automata.

A complex translation is avoided, replaced by two simpler transformations, and the work required to the user seem to be simpler.

8 Conclusion and future works

Conclusion. We have extended the Safety-Progress classification of properties in a runtime verification context. This hierarchical organization of properties turned out to be a convenient framework for specifying properties purposed to be used at runtime. We addressed the problem of monitorability and enforceability of properties at runtime using this general framework. We characterized the sets of monitorable and enforceable properties in a unified way. We introduced a new definition of monitorability based on distinguishability of good and bad execution sequences. This definition is based on positive and negative determinacy as well. However, we believe that it better corresponds to practical needs and tool implementations and fits better in the hierarchy of properties. Moreover, this alternative definition is able to better distinguish equivocal situations that a monitor would have to face off without a finite sequence interpretation. Furthermore, we have delineated the space of enforceable properties wrt. the SP classification. This set of properties was characterized independently from any mechanism. It is thus an upper-bound for the set of properties that could be addressed by any enforcement mechanism.

Future works. The proposed approach raises new research perspectives and open questions.

First, it seems interesting to consider this approach in the *testing* perspective. A monitor (passively) observes the execution of the program. Notably it has no control on the produced events and their sequencing. In a testing context, the notion of controllable event is introduced. An interesting issue would be to characterize the set of testable properties in the SP framework. Note that the definitions of positive and negative determinacy could be rather appropriate in this context. Indeed, in a test campaign one is concerned with the set of all execution sequences that can be produced by the underlying program. Notions of positive and negative determinacy seem to be a first approximation of the set of possible execution sequences of a program.

An additional issue to take into consideration is to deal with a reduced observability on the system under scrutiny. In practical situations, the desired property may refer to events out of the observation scope of a monitor. Similarly, it seems interesting to see how it is possible to characterize the space of properties for which other runtime-verification derived techniques can be applied (*e.g.*, runtime reflection [LS08]).

Furthermore, an interesting question would be to investigate how the SP classication transposes to other spaces of properties such as context-free languages. The classification used in this paper focuses on regular properties. In the quest of expressiveness for specification languages, relying on a classification appears as a good way to delineate monitorable and enforceable properties.

Finally, the question of expressiveness is somehow related to *parametric properties*, *i.e.*, the properties depending on events with parameters whose values depend on the program execution. Recently [CR09], a framework has been proposed to reason about parametric properties and monitor them efficiently. The space of considered properties is the set of regular properties: monitors are expressed using a FSM-like formalism. It is rather clear that being able to express parametric properties is an asset in runtime verification and is surely desirable from a practical point of view. Now, as runtime verification is always concerned

with efficiency, the question is to balance between the gained expressiveness and the induced overhead. Moreover, another question is to investigate whether it is more efficient to enhance expressiveness by considering parametric properties or using a more expressive specification formalism.

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A Proofs

A.1 Proofs for Section 4

A.1.1 Proof of Theorem 4.1

In the following proofs, for a finite sequence σ of length n, we may use the notion of run of σ on a DFA $\mathcal{A}_\psi$ or on a Streett automaton $\mathcal{A}_\Pi$ obtained by the transformations. We note:

- $run(\sigma, \mathcal{A}_\psi) = q_0 \cdots q_{n-1}$
- $run(\sigma, \mathcal{A}_\Pi) = q'_0 \cdots q'_{n-1}$

For Safety properties. We show that the *r*-property accepted by $\mathcal{A}_\Pi$ (obtained using $DFA2S\_Saf$) is exactly $(A_f(\psi), A(\psi))$.

Let $\sigma \in \Sigma^\infty$ s.t. $(A_f(\psi), A(\psi))(\sigma)$, let us prove that the sequence σ is accepted by $\mathcal{A}_\Pi$. We have two cases: σ is a finite sequence or not.

- Let us consider $\sigma \in \Sigma^*$ s.t. $|\sigma| = n$, then by definition of *r*-properties: $\sigma \in A_f(\psi)$, *i.e.*, every prefix of σ belongs to ψ. Let us examine $run(\sigma, \mathcal{A}_\psi) = q_0 \cdots q_{n-1}$. As $\mathcal{L}(\mathcal{A}_\psi) = \psi$, we have $\forall i \in [0, n-1], q_i \in F$. By definition of the transformation DFA2S_Saf, we have $\forall i \in [0, n-1], q_i \in P$. According to (TSAFE1), we have $run(\sigma, \mathcal{A}_\Pi) = q_0 \cdots q_{n-1}$. Using the acceptance criterion of finite sequences, σ is accepted by $\mathcal{A}_\Pi$.

- Let $\sigma \in \Sigma^\omega$, then by definition of *r*-properties: $\sigma \in A(\psi)$, *i.e.*, every finite prefixe of σ belong to ψ. Let us suppose that σ is not accepted by $\mathcal{A}_\Pi$. According to the acceptance criterion for infinite sequences (Definition 4.5), we would have that $vinf(\sigma, \mathcal{A}_\Pi) \not\subseteq P$ (as $\mathcal{A}_\Pi$ is a safety automaton, $R = \emptyset$). By definition of the transformation $DFA2S\_Saf$ and the shape of the obtained automaton $\mathcal{A}_\Pi$, we have that $vinf(\sigma, \mathcal{A}_\Pi) = \{sink\}$. Using (TSAFE2), we know that there exists a smallest prefix σ' of σ, s.t. the run of σ' on $\mathcal{A}_\Pi$ reaches the state $sink$. With the definition of $DFA2S\_Saf$, we can deduce that the run of σ' on $\mathcal{A}_\psi$ ends in a state in $\overline{F}$. As $\mathcal{L}(\mathcal{A}_\psi) = \psi$, $\sigma' \notin \psi$. We obtain a contradiction with $\sigma \in A(\psi)$, and σ is actually accepted by $\mathcal{A}_\Pi$.

Let σ be a sequence accepted by $\mathcal{A}_\Pi$, let us prove that $\sigma \in (A_f(\psi), A(\psi))$. We distinguish again two cases: σ is a finite sequence or not.

- Let $\sigma \in \Sigma^*$ s.t. $|\sigma| = n$, then by definition of the acceptance criterion for finite sequences of Streett automata (Definition 4.6), we have that $q'_{n-1} \in P$. As $\mathcal{A}_\Pi$ is a safety automaton, we can deduce that $\forall i \in [0, n-1], q'_i \in P$. Following the definition of $DFA2S\_Saf$, we find that the states visited during the run of σ on $\mathcal{A}_\psi$ are all in F: $\forall i \in [0, n-1], q_i \in F$ (and $q_i = q'_i$). By definition of the acceptance criterion for DFAs, we can deduce that every prefixes of σ are accepted by $\mathcal{A}_\psi$. As $\mathcal{L}(\mathcal{A}_\psi) = \psi$, we can deduce that all prefixes of σ belong to ψ, *i.e.*, $\sigma \in A_f(\psi)$.

- If $\sigma \in \Sigma^\omega$, then by definition of the acceptance criterion for infinite sequences (Definition 4.5), we know that $vinf(\sigma, \mathcal{A}_\Pi) \subseteq P$. Let us suppose that $\sigma \notin A(\psi)$, by definition of the operator A (see Section 4.2), we would have the existence of a strict prefix σ' of σ not belonging to ψ. Let $n' = |\sigma'|$. As $\mathcal{L}(\mathcal{A}_\psi) = \psi$, then the run of σ' on $\mathcal{A}_\psi$, $run(\sigma', \mathcal{A}_\psi) = q_0 \cdots q_{n'-1}$, would verify $q_0 = {q_{\text{init}}}^{\mathcal{A}_\psi} \wedge q_{n'-1} \notin F$. According to the definition of the transformation $DFA2S\_Saf$ and the rule (TSAFE2), we would have that $q'_{n'-1} = sink \notin P$. Furthermore, using (TSAFE3), every continuation of σ' would have its run ending in $sink$. We would deduce that $vinf(\sigma, \mathcal{A}_\Pi) = \{sink\} \not\subseteq \{P\}$. Which is a contradiction with the initial hypothesis, and gives us $\sigma \in A(\psi)$.

For Guarantee properties. We show that the sets of sequences accepted by $\mathcal{A}_\Pi$ obtained by $DFA2S\_Guar$ is exactly $(E_f(\psi), E(\psi))$.

Let $\sigma \in \Sigma^\infty$ s.t. $(E_f(\psi), E(\psi))(\sigma)$, let us prove that the sequence σ is accepted by $\mathcal{A}_\Pi$. We have two subcases: σ is a finite sequence or not.

- Let us consider $\sigma \in \Sigma^*$ s.t. $|\sigma| = n$, then by definition of *r*-properties: $\sigma \in E_f(\psi)$, *i.e.*, σ has at least one prefix which belongs to ψ. Let us consider $S_{sat} = \{\sigma' \in \Sigma^* \mid \sigma' \preceq \sigma \wedge \sigma' \in \psi\}$, the set of prefixes of σ which belong to ψ. As $\sigma \in E_f(\psi)$, we can deduce that $S_{sat} \neq \emptyset$, S_{sat} has thus a smallest element σ_{min}. Let $n' = |\sigma_{min}|$. We have, by definition of σ_{min}, $\forall \sigma' \in \Sigma^*, \sigma' \prec \sigma_{min} \Rightarrow \sigma' \notin \psi$. Let us examine $run(\sigma_{min}, \mathcal{A}_\psi) = q_0 \cdots q_{n'-1}$. As $\mathcal{L}(\mathcal{A}_\psi) = \psi$, we have $\forall i \in [0, n'-2], q_i \notin F \wedge q_{n'-1} \in F$. According to (TGUAR2), we have $run(\sigma_{min}, \mathcal{A}_\Pi) = q_0 \cdots q_{n'-1}$ with $\forall i \in [0, n'-2], q_i \notin R \wedge q_{n'-1} \in R$. Following (TGUAR1), we have $\forall i \in [n'-1, n-1], q_i \in R$. According to the acceptance criterion for finite sequences, σ is accepted by $\mathcal{A}_\Pi$.

- Let $\sigma \in \Sigma^\omega$, then by definition of *r*-properties: $\sigma \in E(\psi)$, *i.e.*, (at least) one finite prefix of σ belongs to ψ. Let us suppose that σ is not accepted by $\mathcal{A}_\Pi$. According to the acceptance criterion for infinite sequences (Definition 4.5), we would have that $vinf(\sigma, \mathcal{A}_\Pi) \cap R = \emptyset$ (as $\mathcal{A}_\Pi$ is a guarantee automaton, $P = \emptyset$). In other words, we have $vinf(\sigma, \mathcal{A}_\Pi) \subseteq \overline{R}$. As $\mathcal{A}_\Pi$ is a guarantee automaton, every state visited by the run of σ on $\mathcal{A}_\Pi$ is in $\overline{R}$. Indeed, according to the shape of transition function of guarantee automata, if a state of R was visited, we would have $vinf(\sigma, \mathcal{A}_\Pi) \cap R \neq \emptyset$. Let us consider now the prefixes of σ. During the run of these prefixes on $\mathcal{A}_\Pi$, none of them visits a R-state. It follows that, according to (TGUAR2), none of the runs on $\mathcal{A}_\Pi$ of these prefixes visits a state in F. As $\mathcal{L}(\mathcal{A}_\psi) = \psi$, we would deduce that none of the prefixes of σ belongs to ψ. We obtain a contradiction with $\sigma \in E(\psi)$, and consequently σ is indeed accepted by $\mathcal{A}_\Pi$.

Let σ be a sequence accepted by $\mathcal{A}_\Pi$, let us prove that $\sigma \in (E_f(\psi), E(\psi))$. We distinguish two cases: σ is a finite sequence or not.

- Let $\sigma \in \Sigma^*$ s.t. $|\sigma| = n$, then by definition of the acceptance criterion for finite sequences (Definition 4.6), we have that $q_{n-1} \in R$. Let us suppose that $\sigma \notin E_f(\psi)$, *i.e.*, none of the prefixes of σ belongs to ψ. As $\mathcal{L}(\mathcal{A}_\psi) = \psi$, the run of σ on $\mathcal{A}_\psi$ would verify: $\forall i \in [0, n-1], q_i \notin F$. Starting from ${q_{\text{init}}}^{\mathcal{A}_\psi} = {q_{\text{init}}}^{\mathcal{A}_\Pi} \notin R$, and using (TGUAR2), we would find that $run(\sigma, \mathcal{A}_\Pi) = q_0 \cdots q_{n-1}$ with $\forall i \in [0, n-1], q_i \notin R$. This is a contradiction with $q_{n-1} \in R$, and thus $\sigma \in E_f(\psi)$.

- Let $\sigma \in \Sigma^\omega$, then by definition of the acceptance criterion for infinite sequences (Definition 4.5), we have that $vinf(\sigma, \mathcal{A}_\Pi) \cap R \neq \emptyset$. As $\mathcal{A}_\Pi$ is a guarantee automaton, it would mean that $vinf(\sigma, \mathcal{A}_\Pi) \subseteq R$. According to the shape of the transition function for guarantee automata, it means that there is a prefix σ' of σ on $\mathcal{A}_\Pi$ for which the run switches from states in $\overline{R}$ to states in R. More formally, $\exists \sigma' \in \Sigma^*, \sigma' \prec \sigma \wedge |\sigma'| = n' \wedge \forall i \in [0, n'-2], q_i \in \overline{R} \wedge \forall i > n', q_i \in R$. Let us suppose that $\sigma \notin E(\psi)$, *i.e.*, σ has no prefix belonging to ψ. As $\mathcal{L}(\mathcal{A}_\psi) = \psi$, the run of σ on $\mathcal{A}_\psi$ would verify: $\forall i \in \mathbb{N}, q_i \notin F$.

Similarly to the finitary case, and according to the transformation DFA2S_Guar (TGUAR2), it would question the existence of σ'. We deduce that $\sigma \in E(\psi)$.

For Response properties. We show that the *r*-property accepted by $\mathcal{A}_\Pi$, obtained with *DFA2S_Resp* is exactly $(R_f(\psi), R(\psi))$.

Let $\sigma \in \Sigma^\infty$ s.t. $(R_f(\psi), R(\psi))(\sigma)$, let us prove that the sequence σ is accepted by $\mathcal{A}_\Pi$. We have two subcases: σ is a finite sequence or not.

- Let $\sigma \in \Sigma^*$, thus $\sigma \in R_f(\psi)$. Proving that σ is accepted by $\mathcal{A}_\Pi$ amounts to show that the run of σ on $\mathcal{A}_\Pi$, *i.e.*, $run(\sigma, \mathcal{A}_\Pi)$, ends in a R-state ($q_{n-1} \in R$). First of all, let us remark that $\sigma \in R_f(\psi)$ gives us $\psi(\sigma)$. Furthermore, as $\mathcal{L}(\mathcal{A}_\psi) = \psi$, we can deduce that $q_{n-1} \in F$.

  As $\sigma \in R_f(\psi), \forall n \in \mathbb{N}, \exists \sigma' \in \Sigma^*, \sigma \prec \sigma' \wedge |\sigma'| \geq n \wedge \psi(\sigma')$ (cf. the definition in Sect. 4.2).

  Let $n' = |F|$ be the number of accepting states of $\mathcal{A}_\psi$. Now let us consider the set $S = \{\sigma" \in \Sigma^* \mid \sigma \prec \sigma'' \wedge |\{\sigma' \in \Sigma^* \mid \sigma \prec \sigma' \prec \sigma'' \wedge \sigma' \in \psi\}| > n'\}$. This set contains the sequences which are continuations of σ and have at least n' prefixes longer than σ and belonging to ψ. As $\sigma \in R_f(\psi)$, we know that $S \neq \emptyset$, thus S has a smallest element σ_{min}. Let us examine the run of σ_{min} on $\mathcal{A}_\psi$: $run(\sigma, \mathcal{A}_\psi) = run(\sigma, \mathcal{A}_\Pi) = q_0 \cdots q_{|\sigma_{min}|-1} = q_0 \cdots q_{n-1} \cdots q_{|\sigma_{min}|-1}$.

  Between q_{n-1} and $q_{|\sigma_{min}|-1}$, there are at least $n'+1$ accepting states. As $|F| = n'$, two states between q_{n-1} and $q_{|\sigma_{min}|-1}$ are identical. Moreover, we have $\forall i \in [n-1, |\sigma_{min}|-2], q_i \rightarrow_{\mathcal{A}_\psi} q_{i+1}$. Which allows us to deduce, using the definition of *DFA2S_Resp* that q_{n-1} is tagged as a R-state.

- Let $\sigma \in \Sigma^\omega$, thus $\sigma \in R(\psi)$, *i.e.*, $\forall \sigma' \in \Sigma^*, \exists \sigma'' \in \Sigma^*, \sigma' \prec \sigma'' \prec \sigma \wedge \psi(\sigma'')$ holds for σ. Let us examine the run of σ on $\mathcal{A}_\Pi$, we will show that this run visits at least one R-state infinitely often. Indeed, let us consider a prefix σ' of σ, we can find an unbounded number of extensions σ'' of σ', s.t. $\psi(\sigma'')$. Furthermore, for each of these extensions, it is possible to find an unbounded number of extensions σ''' s.t. $\psi(\sigma''')$. Using $\mathcal{L}(\mathcal{A}_\psi) = \mathcal{A}_\psi$, the runs of the automaton $\mathcal{A}_\psi$ on the sequences σ'' and the sequences σ''' ends on a F-state. Using the same reasonning as the one used for finite sequences, the state on which the run of σ'' on $\mathcal{A}_\Pi$ is a R-state. Thus we can build a series $(\sigma_i)_{i\in\mathbb{N}}$ of σ-prefixes (of strictly growing length) s.t. the run of each σ_i ends in a R-state. Thus an infinite number of prefixes of σ go through a R-state. As $|R| \in \mathbb{N}$, there exists a state in R visited infinitely often during the run of σ on $\mathcal{A}_\Pi$. According to the acceptance criterion for infinite sequences, σ is accepted by $\mathcal{A}_\Pi$.

Let σ be a sequence accepted by $\mathcal{A}_\Pi$, let us prove that $\sigma \in (R_f(\psi), R(\psi))$. We distinguish again two cases: σ is a finite sequence or not.

- Let $\sigma \in \Sigma^*$ s.t. $|\sigma| = n$, then by definition of the acceptance criterion for finite sequences (Definition 4.6), we have that $q_{n-1} \in R$. According to the definition of *DFA2S_Resp*, we deduce that $q_{n-1} \in F$ and $\exists q_0, \ldots, q_l \in Q^{\mathcal{A}_\psi}, \exists a_0, \ldots, a_l \in \Sigma$,

$$\forall j \in [0, l-1], q_j \xrightarrow{a_i}_{\mathcal{A}_\psi} q_{j+1} \tag{13}$$

$$\exists i \in [0, l], \exists j \in [i, l-1], q_j \in F \wedge q_i = q_l \wedge q_0 = q \tag{14}$$

  Thus we can build a series $(\sigma_j)_{j\in\mathbb{N}}$ of σ-extensions s.t. $\forall j \in \mathbb{N}, \psi(\sigma_j)$ and σ_j is defined as $\sigma_j = \sigma \cdot a_0 \cdots a_i \cdot (a_{i+1} \cdots a_{l-1} \cdot a_0 \cdots a_i)^j$. This series exhibits strictly growing continuations of σ belonging to ψ. According to the definition of the operator R_f, we can deduce that $\sigma \in R_f(\psi)$.

- Let $\sigma \in \Sigma^\omega$, then by definition of the acceptance criterion for infinite sequences (Definition 4.5), we have that $vinf(\sigma, \mathcal{A}_\Pi) \cap R \neq \emptyset$. Thus, σ has an infinite number of prefixes for which the run ends in a R-state. Using the definition of *DFA2S_Resp*, we know that all these prefixes are accepted by $\mathcal{A}_\psi$ (as by definition the ending state of their run is a R-state). Using $\mathcal{L}(\mathcal{A}_\psi) = \psi$, we know that all these prefixes belong and have an unrestricted number of extensions belonging to ψ. We can deduce that $\sigma \in R(\psi)$.

For Persistence properties. We show that the set of sequences accepted by $\mathcal{A}_\Pi$, obtained with $DFA2S\_Per$ is exactly $(P_f(\psi), P(\psi))$. Let us remark that, according to the definition of the transformation (the transition function is not changed), we have $\forall j \in [n-1, n'+n''-1], q_j \longrightarrow_{\mathcal{A}_\Pi} q_{j+1} \wedge q_j \longrightarrow_{\mathcal{A}_\psi} q_{j+1}$. Moreover, as $Q^{\mathcal{A}_\Pi} = Q^{\mathcal{A}_\psi}$, we can merge the states q_j and q'_j visited by the runs of σ on $\mathcal{A}_\psi$ and $\mathcal{A}_\Pi$.

Let $\sigma \in \Sigma^\infty$ s.t. $(P_f(\psi), P(\psi))(\sigma)$, let us prove that the sequence σ is accepted by $\mathcal{A}_\Pi$. We have two subcases: σ is a finite sequence or not.

- Proving that σ is accepted by $\mathcal{A}_\Pi$ amounts to show that the run of σ on $\mathcal{A}_\Pi$ ends in a P-state ($q_{n-1} \in P$). First of all, let us remark that $\sigma \in P_f(\psi)$ gives us $\psi(\sigma)$. Furthermore, as $\mathcal{L}(\mathcal{A}_\psi) = \psi$, we can deduce that $q_{n-1} \in F$.

  As $\sigma \in P_f(\psi)$, there exists $\sigma', \mu \in \Sigma^*$ s.t. (cf. the definition in Sect. 4.2):

  $$\sigma \preceq \sigma' \wedge (\sigma' \cdot \mu^* \cdot pref(\mu)) \subseteq \psi \tag{15}$$

  Let $n' = |\sigma'|$, and $n'' = |\mu|$. Then, the runs of σ' and $\sigma' \cdot \mu$ on $\mathcal{A}_\Pi$ can be expressed:

  $$run(\sigma', \mathcal{A}_\Pi) = q_0 \cdots q_{n-1} \cdots q_{n'-1}$$
  $$run(\sigma' \cdot \mu, \mathcal{A}_\Pi) = q_0 \cdots q_{n-1} \cdots q_{n'-1} \cdot q_{n'} \cdots q_{n'+n''-1}$$

  According to (15), we have $q_{n'-1} \in F$. We can show by induction that

  $run(\sigma \cdot \mu^*, \mathcal{A}_\Pi) = q_0 \cdots q_{n'-1} \cdot (q_{n'} \cdots q_{n'+n''-1})^*$.

  Moreover, we have $\forall j \in [n', n'+n''-1], q'_j \in F$ and $q_{n'+n''-1} \longrightarrow_{\mathcal{A}_\Pi} q_{n'}$.

  Thus, we can deduce, following the definition of $DFA2S\_Per$, that $q_{n-1} \in P$. Indeed, it is sufficient to take $l = n' + n'' - 1 - n$ and $i = n' - n$.

- In order to prove $vinf(\sigma, \mathcal{A}_\Pi) \subseteq P$, it is sufficient to see that σ can be expressed $\sigma' \cdot \mu^\omega$. From this, every prefix of σ longer than σ' satisfies ψ, and have their run which stops in a F-state on $\mathcal{A}_\psi$. Thus, we exhibit a strongly connected component of F-states which are tagged as P-states by DFA2S_Per. Thus, the states visited infinitely often during the run of σ on $\mathcal{A}_\Pi$ are the states of this strongly connected component. Which gives the expected result.

Let σ be a sequence accepted by $\mathcal{A}_\Pi$, let us prove that $\sigma \in (P_f(\psi), P(\psi))$. We distinguish two subcases: σ is a finite sequence or not.

- Let $\sigma \in \Sigma^*$ s.t. $|\sigma| = n$, then by definition of the acceptance criterion for finite sequences of Streett automata (Definition 4.6), we have that $q_{n-1} \in P$. Then, there exists two possibilities.

  - In the first one, we have on one hand that $q_{n-1} \in F$, and on the other hand $\exists n \in \mathbb{N}^*, \exists q_0, \cdots, q_n \in Q^{\mathcal{A}_\psi}$ s.t.:
    - $\forall j \in [0, n-1], q_j \longrightarrow_{\mathcal{A}_\psi} q_{j+1}$, and
    - $\exists i \in [0, n-1], \forall j \in [i, n-1], q_j \in F \wedge q_i = q_n \wedge q_0 = q_{n-1}$

    We have $\psi(\sigma)$ since $\mathcal{L}(\mathcal{A}_\psi) = \psi$. Moreover, there exist $a_0, \ldots, a_{n-1} \in \Sigma$ s.t. $\forall j \in [0, n-1], q_j \xrightarrow{a_j}_{\mathcal{A}_\psi} q_{j+1}$. We can deduce that $\psi(\sigma \cdot a_0 \cdots a_i), \psi(\sigma \cdot a_0 \cdots a_i \cdot a_{i+1}), \ldots, \psi(\sigma \cdot a_0 \cdots a_{n-1})$. Let us note $L_p = \sigma' \cdot \Big((a_0 \cdots a_n)^* \cdot a_0 + (a_0 \cdots a_n)^* \cdot a_0 \cdot a_1 + \ldots + (a_0 \cdots a_n)^* \cdot a_0 \cdots a_{n-1}\Big)$. As $q_i = q_n$ ($\{q_i, \ldots, q_n\}$ is a strongly connected component, we can prove by induction that $L_p \subseteq \psi$. Furthermore, $\forall \sigma' \in \Sigma^* \cap L_p, \sigma \cdot a_0 \cdots a_i \preceq \sigma' \Rightarrow \psi(\sigma')$. Which proves that $\sigma \in P_f(\psi)$. Indeed, it is sufficient to take $\sigma' = \sigma \cdot a_0 \cdots a_i$, and $\mu = a_{i+1} \cdots a_{n-1}$.

  - In the second one, we have that $q_{n-1} \in F$ and $q_{n-1} \longrightarrow_{\mathcal{A}_\psi} q_{n-1}$. Thus, $\exists e \in \Sigma, q_{n-1} \xrightarrow{e}_{\mathcal{A}_\psi} q_{n-1}$. We deduce that $\psi(\sigma)$ and $\sigma \cdot e^* \subseteq \psi$, as $\mathcal{L}(\mathcal{A}_\psi) = \psi$. Which allows us to deduce easily that $\sigma \in P_f(\psi)$.

- Let $\sigma \in \Sigma^\omega$, then by definition of the acceptance criterion for infinite sequences of Streett automata (Definition 4.5), we have that $vinf(\sigma, \mathcal{A}_\Pi) \subseteq P$. That is to say, all prefixes of σ from a certain point on have their run which ends in a P-state. As the automaton $\mathcal{A}_\Pi$ has a finite number of states, it means that there exists a strongly connected component C, s.t. the run of σ on $\mathcal{A}_\Pi$ "stays in". More formally, $\exists n, m \in \mathbb{N}, C = \{q'_0, \ldots, q'_n\} \subseteq Q^{\mathcal{A}_\Pi} \wedge run(\sigma, \mathcal{A}_\Pi) = q_0 \cdots q_m \cdots \wedge \forall i > m, q_i \in C$. Moreover, as $\{q'_0, \ldots q'_n\}$ is a SCC, from every state of C it is possible to reach any state of C. Let us suppose, without loss of generality, that $q'_0 \xrightarrow{a_0}_{\mathcal{A}_\Pi} q'_1 \xrightarrow{a_1}_{\mathcal{A}_\Pi} \cdots \xrightarrow{a_{n-1}}_{\mathcal{A}_\Pi} q'_n \xrightarrow{a_n}_{\mathcal{A}_\Pi} q'_0$, with $a_0, \ldots, a_n \in \Sigma$. According to the definition of $DFA2S\_Per$, we have the same transitions on $\mathcal{A}_\psi$, *i.e.*, $q'_0 \xrightarrow{a_0}_{\mathcal{A}_\psi} q'_1 \xrightarrow{a_1}_{\mathcal{A}_\psi} \cdots \xrightarrow{a_{n-1}}_{\mathcal{A}_\psi} q'_n \xrightarrow{a_n}_{\mathcal{A}_\psi} q'_0$.

  Let us note $L_p = \sigma' \cdot (a_0 \cdots a_n)^* \cdot (a_0 + a_0 \cdot a_1 + \ldots + a_0 \cdots a_{n-1}) = \sigma' \cdot (a_0 \cdots a_n) \cdot pref(a_0 \cdots a_{n-1})$. The sequence σ can be expressed $\sigma' \cdot (a_0 \cdots a_n)^\omega$ with the fact that for every sequence $\sigma'' \in L_p$ which is a continuation of σ', the run of σ'' ends in a P-state. Which implies that the runs of these same sequences σ'' on $\mathcal{A}_\psi$ end in a F-state. We deduce, as $\mathcal{L}(\mathcal{A}_\psi) = \psi$, that $\forall \sigma'' \in L_p, \sigma' \preceq \sigma'' \prec \sigma \Rightarrow \psi(\sigma'')$.

  Which allows to deduce, using the definition of the operator P (see Section 4.2), that $\sigma \in P(\psi)$.

A.2 Proofs for Section 5

A.2.1 Proof of Lemma 5.1

Proof Let us consider two *r*-properties $\Pi_1, \Pi_2 \in MP_c$.

- Proof of $\Pi_1 \wedge \Pi_2 \in MP_c$. It consists in showing that $\Pi_1 \wedge \Pi_2$ is σ-monitorable for any sequence $\sigma \in \Sigma^*$. Let $\sigma \in \Sigma^*$, let us exhibit an extension $\mu \in \Sigma^*$ s.t. $\Pi_1 \wedge \Pi_2$ is negatively or positively determined by $\sigma \cdot \mu$.

  As Π_1 is monitorable, there exists μ_1 s.t. Π_1 is negatively or positively determined by $\sigma \cdot \mu_1$, that is we have the two following possibilities:

  - $\exists \mu_1 \in \Sigma^*, \forall \mu'_1 \in \Sigma^\infty, \neg\Pi_1(\sigma \cdot \mu_1 \cdot \mu'_1)$, Π_1 is negatively determined by $\sigma \cdot \mu_1$
  - $\exists \mu_1 \in \Sigma^*, \forall \mu'_1 \in \Sigma^\infty, \Pi_1(\sigma \cdot \mu_1 \cdot \mu'_1)$, Π_1 is positively determined by $\sigma \cdot \mu_1$

  As Π_2 is also monitorable, it is $\sigma \cdot \mu_1$-monitorable, there exists μ_2 s.t. Π_2 is negatively or positively determined by $\sigma \cdot \mu_1 \cdot \mu_2$. That is we have the two following possibilities :

  - $\exists \mu_2 \in \Sigma^*, \forall \mu'_2 \in \Sigma^\infty, \neg\Pi_2(\sigma \cdot \mu_1 \cdot \mu_2 \cdot \mu'_2)$, Π_2 is negatively determined by $\sigma \cdot \mu_1 \cdot \mu_2$
  - $\exists \mu_2 \in \Sigma^*, \forall \mu'_2 \in \Sigma^\infty, \Pi_2(\sigma \cdot \mu_1 \cdot \mu_2 \cdot \mu'_2)$, Π_2 is positively determined by $\sigma \cdot \mu_1 \cdot \mu_2$

  By combination, there exist four possibilities depending on the facts that Π_1 is positively or negatively determined by $\sigma \cdot \mu_1$ and Π_2 is negatively or positively determined by $\sigma \cdot \mu_1 \cdot \mu_2$. We group those possibilities into two cases :

  - Let us distinguish the case where Π_1 is positively determined by $\sigma \cdot \mu_1$ and Π_2 is positively determined by $\sigma \cdot \mu_1 \cdot \mu_2$. Then, by taking $\mu = \sigma \cdot \mu_1 \cdot \mu_2$, we have that $\Pi_1 \wedge \Pi_2$ is positively determined by μ. This gives us the expected result.
  - In the others cases, it suffices to take $\mu = \sigma \cdot \mu_1 \cdot \mu_2$ to show that $\Pi_1 \wedge \Pi_2$ is negatively determined by μ.

- The proof of $\Pi_1 \vee \Pi_2 \in MP_c$ is similar.

- The proof of $\neg\Pi_1 \in MP_c$ is straightforward by noticing that for any sequence $\sigma \in \Sigma^*$, if Π_1 is positively (resp. negatively) determined by σ, then $\neg\Pi$ is negatively (resp. positively) determined by σ.

A.2.2 Proof of Lemma 5.2

We prove this property by *reductio ad absurdum*. Let suppose the existence of a reactivity *r*-property $\Pi = (\phi, \varphi)$ defined on Σ which is neither a safety nor a guarantee: $\Pi \in \text{Reactivity}(\Sigma) \setminus (\text{Safety}(\Sigma) \cup \text{Guarantee}(\Sigma))$ and which is monitorable according to Definition 5.5 with $\mathbb{B}_3$.

As $\Pi \in MP^*(\mathbb{B}_3)$, by definition we have that:

$$\forall \sigma_{good} \in \phi, \forall \sigma_{bad} \in \overline{\phi}, [\![\Pi]\!]_{\mathbb{B}_3}(\sigma_{good}) \neq [\![\Pi]\!]_{\mathbb{B}_3}(\sigma_{bad})$$

Let us remark that $\phi \neq \emptyset$ and $\overline{\phi} \neq \emptyset$ as Π is neither a safety nor a guarantee. Indeed, if $\phi = \emptyset$, then Π would be necessarily the *r*-property always false, which is a safety. Likewise, if $\overline{\phi} = \emptyset$, *i.e.*, $\phi = \Sigma^*$, Π would be the *r*-property always true which is a safety as well.

Then, we consider two sequences σ_{good} and σ_{bad} in Σ^∞ :

- Let $\sigma_{good} \in \phi$ s.t. there exists $\sigma'_g \in \Sigma^\infty$ with $\neg\Pi(\sigma_{good} \cdot \sigma'_g)$. We know that such a sequence exists since $\Pi \notin \text{Guarantee}(\Sigma)$. This is a consequence of Property 4.2.
- Let $\sigma_{bad} \in \overline{\phi}$ s.t. there exists $\sigma'_b \in \Sigma^\infty$ with $\Pi(\sigma_{bad} \cdot \sigma'_b)$. We know that such a sequence exists since $\Pi \notin \text{Safety}(\Sigma)$. This is a consequence of Property 4.2.

According to the definition of the evaluation function for *r*-properties in a truth-domain (Definition 5.4), we have that :

$$[\![\Pi]\!]_{\mathbb{B}_3}(\sigma_{good}) = [\![\Pi]\!]_{\mathbb{B}_3}(\sigma_{bad}) = ?$$

This is a contradiction with $\Pi \in MP^*(\mathbb{B}_3)$. ∎

A.2.3 Proof of Property 5.1

In this proof, $[\![\Pi]\!]$ stands for $[\![\Pi]\!]_{\mathbb{B}_4}$. Let us consider an execution sequence $\sigma \in \Sigma^*$ of length n.

Proof of $q_{n-1} \in Good^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!](\sigma) = \top$

- Let us suppose that $q_{n-1} \in Good^{\mathcal{A}_\Pi}$. Using the acceptance criterion on finite sequences, we have that σ is accepted by $\mathcal{A}_\Pi$. Furthermore, as $\mathcal{A}_\Pi$ specifies Π, we have that $\Pi(\sigma)$. Now, let us consider $\mu \in \Sigma^+$ s.t. $|\sigma| + |\mu| = n' > n$ and $run(\sigma \cdot \mu, \mathcal{A}_\Pi) = q_0 \cdots q_{n'-1}$. As $q_{n-1} \in Good^{\mathcal{A}_\Pi}$, we have that $\forall k \in \mathbb{N}, n \leq k \leq n' - 1 \Rightarrow q_k \in \bigcap_{i=1}^m R_i \cup P_i$ and consequently $\Pi(\sigma \cdot \mu)$. Let us consider $\mu \in \Sigma^\omega$, one may remark that $\forall i \in [1, m], inf(\sigma \cdot \mu, \mathcal{A}_\Pi) \cap R_i \neq \emptyset \vee inf(\sigma \cdot \mu, \mathcal{A}_\Pi) \subseteq P_i$, which implies $\Pi(\sigma \cdot \mu)$. We have $\Pi(\sigma) \wedge \forall \mu \in \Sigma^\infty, \Pi(\sigma \cdot \mu)$, *i.e.*, $[\![\Pi]\!](\sigma) = \top$.
- Conversly, let us suppose that $[\![\Pi]\!](\sigma) = \top$. By definition, it means $\forall \mu \in \Sigma^\infty, \Pi(\sigma \cdot \mu)$. According to the acceptance criterion of Streett automata, we deduce that $\forall k \geq n, \forall \mu \in \Sigma^*, run(\sigma \cdot \mu, \mathcal{A}_\Pi) = q_0 \cdots q_{n-1} \cdots q_k \Rightarrow q_k \in \bigcap_{i=0}^m R_i \cup P_i$. That is to say $Reach_{\mathcal{A}_\Pi}(q_{n-1}) \subseteq \bigcap_{i=1}^m (R_i \cup P_i)$, *i.e.*, $q_{n-1} \in Good^{\mathcal{A}_\Pi}$.

Proof of $q_{n-1} \in Good_p^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!](\sigma) = \top_p$**.** The proof of $q_{n-1} \in Good_p^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!](\sigma) = \top_p$ is straightforward by examining the acceptance criterion for finite sequences.

- Let us suppose that $q_{n-1} \in Good_p^{\mathcal{A}_\Pi}$. Using the acceptance criterion for finite sequences, we have that σ is accepted by $\mathcal{A}_\Pi$. Moreover, as $\mathcal{A}_\Pi$ specifies Π, we have that $\Pi(\sigma)$. Now, as $Reach_{\mathcal{A}}(q) \not\subseteq \bigcap_{i=1}^m (R_i \cup P_i)$, there exists a state q' of $\mathcal{A}_\Pi$ reachable from q and belonging to $\bigcup_{i=1}^m (\overline{R_i} \cap \overline{P_i})$. Consequently, there exists $\mu \in \Sigma^*$ s.t. $run(\sigma \cdot \mu) = q_0 \cdots q_{n-1} \cdots q'$. Still following the acceptance criterion we deduce that $\neg\Pi(\sigma \cdot \mu)$, *i.e.*, $[\![\Pi]\!](\sigma) = \top_p$.
- Conversely, the same reasonning can be used to proved the seeked result.

Proof of $q_{n-1} \in Bad_p^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!](\sigma) = \bot_p$**.** Similarly, proving that $q_{n-1} \in Bad_p^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!](\sigma) = \bot_p$ is straightforward by examining the acceptance criterion for finite sequences of Streett automata.

- Let us suppose that $q_{n-1} \in Bad_p^{\mathcal{A}_\Pi}$. using the acceptance criterion of finite sequences, we have that σ is not accepted by $\mathcal{A}_\Pi$. Furthermore, as $\mathcal{A}_\Pi$ specifies Π, we have that $\neg\Pi(\sigma)$. Now, as $Reach_{\mathcal{A}}(q) \not\subseteq \bigcup_{i=1}^{m}(\overline{R_i} \cup \overline{P_i})$, there exists a state q' of $\mathcal{A}_\Pi$ reachable from q and belonging to $\bigcap_{i=1}^{m}(R_i \cup P_i)$. Consequently, there exists $\mu \in \Sigma^*$ s.t. $run(\sigma \cdot \mu) = q_0 \cdots q_{n-1} \cdots q'$. Still following the acceptance criterion, we deduce that $\Pi(\sigma \cdot \mu)$, *i.e.*, $[\![\Pi]\!](\sigma) = \bot_p$.
- Conversely, the same reasonning can be conducted.

Proof of $q_{n-1} \in Bad^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!](\sigma) = \bot$**.** Proving that $q_{n-1} \in Bad^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!](\sigma) = \bot$ can be done following the same proof principle that the one used to prove $q_{n-1} \in Good^{\mathcal{A}_\Pi} \Leftrightarrow [\![\Pi]\!](\sigma) = \top$.

- Let us suppose that $q_{n-1} \in Bad^{\mathcal{A}_\Pi}$. Using the acceptance criterion on finite sequences, we have that σ is not accepted by $\mathcal{A}_\Pi$. Furthermore, as $\mathcal{A}_\Pi$ specifies Π, we have that $\neg\Pi(\sigma)$. Now, let us consider $\mu \in \Sigma^+$ s.t. $|\sigma| + |\mu| = n' > n$ and $run(\sigma \cdot \mu, \mathcal{A}_\Pi) = q_0 \cdots q_{n'-1}$. As $q_{n-1} \in Bad^{\mathcal{A}_\Pi}$, we have that $\forall k \in \mathbb{N}, n \leq k \leq n'-1 \Rightarrow q_k \in \bigcup_{i=1}^{m} \overline{R_i} \cap \overline{P_i}$ and consequently $\neg\Pi(\sigma \cdot \mu)$. Let us consider $\mu \in \Sigma^\omega$, one may remark that $\forall i \in [1, m]$, $vinf(\sigma \cdot \mu, \mathcal{A}_\Pi) \cap R_i = \emptyset \wedge vinf(\sigma \cdot \mu, \mathcal{A}_\Pi) \not\subseteq P_i$, which implies that $\neg\Pi(\sigma \cdot \mu)$. We have $\neg\Pi(\sigma) \wedge \forall \mu \in \Sigma^\infty, \neg\Pi(\sigma \cdot \mu)$, *i.e.*, $[\![\Pi]\!](\sigma) = \bot$.
- Conversely, let us suppose that $[\![\Pi]\!](\sigma) = \bot$. By definition, it means $\forall \mu \in \Sigma^\infty, \neg\Pi(\sigma \cdot \mu)$. According to the acceptance criterion of Streett automata, we deduce that $\forall k \geq n, \forall \mu \in \Sigma^*, run(\sigma \cdot \mu, \mathcal{A}_\Pi) = q_0 \cdots q_{n-1} \cdots q_k \Rightarrow q_k \in \bigcup_{i=0}^{m} \overline{R_i} \cap \overline{P_i}$. That is to say $Reach_{\mathcal{A}_\Pi}(q_{n-1}) \subseteq \bigcup_{i=1}^{m}(\overline{R_i} \cap \overline{P_i})$, *i.e.*, $q_{n-1} \in Bad^{\mathcal{A}_\Pi}$.

A.3 Proofs for Section 6

A.3.1 Proof of Property 6.1

Proof This proof relies on the computation of strongly connected components [Tar72] of a Streett automaton (SCC). The proof is in two stages by proving implications in both ways.

- (4) $\Rightarrow$ (5). Let us consider a SCC of $\mathcal{A}_\Pi$ containing only $\overline{R}$-states. Suppose that there exists two states q, q' in this SCC s.t. $q \in P$ and $q' \notin P$. As q and q' are in a SCC, there exists a path from q to q' and from q' to q in $\mathcal{A}_\Pi$. Then there would exist an infinite execution sequence σ s.t. the run of σ on $\mathcal{A}_\Pi$ contains infinite occurrences of q and q'. As this SCC is made of $\overline{R}$-states, σ is not accepted by $\mathcal{A}_\Pi$ (since $vinf(\sigma, \mathcal{A}_\Pi) \not\subseteq P$), *i.e.*, $\neg\varphi(\sigma)$. However σ has an infinite number of "good" prefixes: all prefixes s.t. the run ends in a P-state. This is contradictory with our initial assumption.
- (5) $\Rightarrow$ (4). Let us consider $\sigma \in \Sigma^\omega$ s.t. $\neg\varphi(\sigma)$. As $\mathcal{A}_\Pi$ recognizes Π, σ is not accepted by $\mathcal{A}_\Pi$. As $\mathcal{A}_\Pi$ is a finite state automaton, the run of σ on $\mathcal{A}_\Pi$ visits a last SCC infinitely often and can be expressed:

  $run(\sigma, \mathcal{A}_\Pi) = q_0 \cdots q_i \cdots = q_0 \cdots q_{k-1} \cdot (q_k \cdots q_{k+l})^n$

  where $q_k, \ldots, q_{k+l}$ are the states visited infinitely often, *i.e.*, $vinf(\sigma, \mathcal{A}_\Pi) = \{q_k, \ldots, q_{k+l}\}$.

  As $\mathcal{A}_\Pi$ recognizes Π, σ is not accepted by $\mathcal{A}_\Pi$, thus we have:

  $$vinf(\sigma, \mathcal{A}_\Pi) \cap R = \emptyset \wedge vinf(\sigma, \mathcal{A}_\Pi) \not\subseteq P$$

  From $vinf(\sigma, \mathcal{A}_\Pi) \cap R = \emptyset$, we deduce $vinf(\sigma, \mathcal{A}_\Pi) \subseteq \overline{R}$. As $vinf(\sigma, \mathcal{A}_\Pi)$ is a non-maximal SCC of $\mathcal{A}_\Pi$, using (5), we deduce that either $vinf(\sigma, \mathcal{A}_\Pi) \subseteq P$ or $vinf(\sigma, \mathcal{A}_\Pi) \subseteq \overline{P}$. However, $vinf(\sigma, \mathcal{A}_\Pi) \subseteq P$ is impossible as σ is not accepted by $\mathcal{A}_\Pi$ (cf. above). Thus, it remains that $vinf(\sigma, \mathcal{A}_\Pi) \subseteq \overline{P}$. We deduce that for the states q_i appearing in the run of σ on $\mathcal{A}_\Pi$ that $\forall i \geq k, q_i \notin P$. It follows that, using the finite sequence acceptance criterion of a Streett automaton: $\forall i \geq k, \neg\Pi(\sigma_{\ldots i})$. Which shows that Π is enforceable.