Parabolic reciprocity gap for heat source identification

Nicolas Auffray, Marc Bonnet, Stéphane Pagano

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N. Auffray[1], M. Bonnet[1], S. Pagano[2]

[1] *LMS, CNRS UMR 7649, Ecole Polytechnique, France, {auffray,bonnet}@lms.polytechnique.fr*
[2] *LMGC, CNRS UMR 5508, Université Montpellier II, France, pagano@lmgc.univ-montp2.fr*

Motivation. The deformation of solid materials is nearly always accompanied with temperature variations. These variations, governed by the heat diffusion equation stemming from the first and second laws of thermodynamics, are induced by intrinsic dissipation of energy and thermomechanical coupling. Infrared thermography techniques provide an experimental means for measuring thermal fields on specimen boundaries. But even if thermal fields are related to the material behavior they are not intrinsic to it as they also depend on external factors such as boundary conditions. Inverting boundary thermal fields is thus needed to obtain valid insight into the specimen thermomechanical behavior. Such an operation belongs to the class of source inverse problem. Inverse source problems are known to be ill-posed in the sense of Hadamard: their solution does not depend continuously on the data, and is not unique for a general source distribution when using only boundary measurements [3]. Modeling hypotheses on the sought sources are thus needed to properly retrieve information [3, 4, 6].

Modeling assumptions.. Let Ω be a bounded regular domain of $\mathbb{R}^2$, with a sufficiently regular boundary Γ, and denote by n the unit outward normal to Γ. Considering thermal evolutions over a time interval $[0,T]$, let $Q_T = \Omega \times [O,T]$ denote the corresponding space-time cylinder and Σ_T its lateral boundary. This communication aims at introducing a non-iterative numerical source identification procedure by boundary measurement for the following parabolic evolution problem (made non-dimensional through suitable scaling of space and time coordinates):

$$\begin{cases} (\partial_t - \Delta)\,\theta(x,t) = f(x,t), \text{ in } Q_T \\ \theta + \partial_n = 0, \text{ in } \Sigma_T \\ \theta(x,0) = 0 \text{ in } \Omega \end{cases} \tag{1}$$

For physical reasons, coming from experimental observation, the source term $f(x,t)$ will be modeled as a linear combination of point sources with time-varying intensies described by rectangular functions:

$$f(x,t) = \sum_{j=1}^{N} p_j \delta(x - x_j)\Pi(\frac{t-t_j}{\ell_j}), \qquad \text{with} \quad \Pi(z) = \mathrm{H}(z+1/2) - \mathrm{H}(z-1/2) \tag{2}$$

where H stand for the Heaviside step function, p_j, x_j, t_j, ℓ_j are the unknown parameters for the j-th source: intensity, spatial location, temporal location and holding time. Finally N is the number of sources. Under these hypothesis the source term is uniquely determined by the observation.

Purpose of this communication. In this communication it will be shown that a robust estimation of spatial location and energy of the source term can be obtained using the parabolic reciprocity identity:

$$\int_{\Sigma_T} \left(\frac{\partial \psi}{\partial n}\overline{\theta} - \overline{\phi}\psi \right) \mathrm{d}s\mathrm{d}t + \int_{\Omega} [\theta_t(x)\psi_t(x)]_0^T \mathrm{d}x = \int_{Q_T} f\psi \mathrm{d}x\mathrm{d}t \tag{3}$$

wherein time-independent harmonic adjoint fields ar used as trial functions:

$$\psi \in H(\Omega) = \{\psi \in H^1(\Omega), \Delta\psi = 0\} \tag{4}$$

For given adjoint field ψ, the left-hand side of (3) is a known function of experimental data, and is usually referred to as the reciprocity gap $R(\psi)$. In the absence of any heat source, the right-hand side of (3) vanishes. Nonzero values of $R(\psi)$ hence indicate the presence of buried sources (or other departures from reference physical modelling assumptions). Such sources may then be identified by using $R(\psi)$ with appropriate adjoint fields. Moreover, it can be shown that combining (3) and (4) yields a reciprocity identity formulated at final time T linking time-integrated versions of the boundary data and unknown sources. Choosing T larger than the last source extinction time ensures that information on sources is properly embedded in the reciprocity identity. Combining such an observation with the algebraic algorithm introduced in [4] for source identification in elliptic problems allows to reconstruct the source locations and their time-integrated intensities. The algorithm requires a preset number of trial point sources, and retains as actually identified sources only those whose identified time-integrated intensity is above a predefined threshold. Preliminary results on simulated data show that the proposed reciprocity-based source identification method yields results that are very stable with respect to significant data noise. One such result (Fig. 1 and Table 1) demonstrates the correct identification of three point sources using 10 trial point sources, with simulated Gaussian noise of up to 40% of average measurement level.

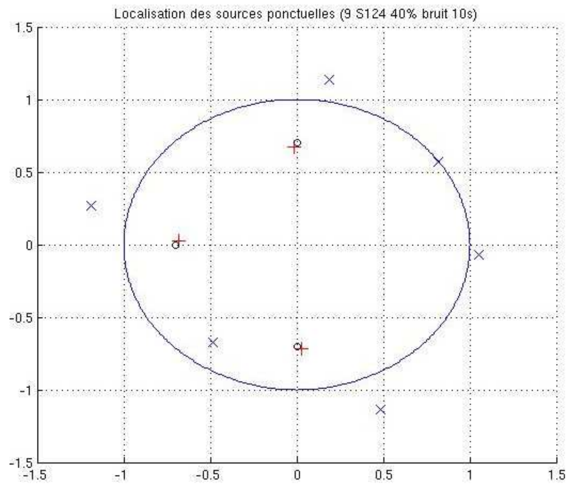

Figure 1: Identification of the location of three point sources, using 10 trial sources (instead of 3) and 40% of noise ratio (black circle: real source; red cross: identified source; blue cross: ghost source).

	0%	5%	10%	20%	40%
$\max(err_{rel}(\text{Location}))$	0.27%	1.20%	0.52%	1.70%	4.40%
$\max(err_{rel}(\text{Intensity}))$	0.84%	2.26%	1.03%	3.55%	10.37%

Table 1: Identification of the location of three point sources: maximum relative error on spatial location and time-integrated intensity for several levels of data noise, using 10 trial sources.

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